Balanced polarimeter: A cost-effective approach for measuring the polarization of light

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(Received 21 July 2013; accepted 18 September 2014)

We have designed and built a fast and precise balanced polarimeter from components commonly found in undergraduate physics laboratories. Balanced polarimetry measures the orientation of linearly polarized light by splitting the beam into orthogonal polarization components and detecting each separately. Our polarimeter is capable of measuring the orientation of linearly polarized light with a precision of approximately $0.001^\circ/\sqrt{\text{Hz}}$. The apparatus cost less than $1000. Measurements of the specific rotation of sucrose and of the Faraday effect were performed, both of which produced results that were comparable to previously reported values. © 2015 American Association of Physics Teachers.

I. INTRODUCTION

Ever since Huygens first used polarization to explain the phenomena of the double image produced by the Iceland spar in 1678, the polarization of light has been an important topic in physics. In addition to elucidating an important aspect of the fundamental nature of light, polarimetry (the measurement of the polarization of light) is also used to explore the physical properties of matter from macroscopic to atomic samples. Polarimetry has been used to validate the Fresnel equations in the instructional laboratory and to investigate the Faraday effect. The rotation of polarized light by atoms has even been used to probe atomic parity violation. Despite the importance of polarization, we are unaware of a readily available, inexpensive, high-precision ($\leq 0.1^\circ$) instrument. Hopgood has reported a simple design which uses sheet polarizers and a protractor, a design which is inexpensive but lacks speed and precision. Over 70 years ago, Underwood presented a device in this journal which was more sophisticated, but only accurate to a tenth of a degree. Recently, Lisboa et al. designed a solution polarimeter that uses the near-extinction method. The precision rotation mount used in this approach may yield polarization angles as precise as $0.08^\circ$, but it will also be limited in its time resolution to the speed at which the operator can find and record the null angle. Each of these designs has its particular advantages, but none of them is inexpensive, simple, fast, and precise. Therefore, we have designed and built a polarimeter that meets all of these criteria.

Unlike null or near-extinction polarimetry, balanced polarimetry does not diminish the intensity of the incident light but rather analyzes its components. Polarized light is separated by a polarizing beamsplitter into two orthogonal polarization components. If the polarization of the incident light is oriented at $45^\circ$ to the principal axis of the beamsplitter, the intensities of the transmitted and reflected components are equal. If an additional optical element is introduced that rotates the polarization, the intensities of the two beams become unequal. The degree of rotation can be determined by measuring the intensities of the two individual components. The advantages of balanced polarimetry are discussed by Bouchiat et al.

Balanced polarimetry has been applied in fundamental and applied physics. For example, the nuclear weak force is known to cause optical activity in atoms, which has been directly measured with balanced polarimetry by Bouchiat and coworkers. Other applications of balanced polarimetry include laser frequency stabilization, detection of radio-frequency magnetic fields, and optical NMR detection. Therefore, in addition to providing a concrete demonstration of the nature of light and providing an apparatus that can be used to demonstrate other physical phenomena, such as the Faraday effect, this particular approach to polarimetry can provide students a window into the advanced physics research laboratory.
In this paper, we present the polarimeter design and the theory of its operation. The performance of the polarimeter is evaluated with two experiments, the optical rotation of sucrose solutions and the Faraday effect in fused silica, that demonstrate the precision, speed, and versatility of our apparatus.

II. EXPERIMENTAL

A. Apparatus

A schematic diagram of our balanced polarimeter is shown in Fig. 1. Light from a diode laser L (OS-8525A, PASCO, nominally 650 nm ± 10 nm) passes through a polarizing filter P (OS-8473, PASCO). An optional multi-order half-wave plate HWP (WPMH05M-633, Thorlabs) placed in a precision rotation mount between the polarizer and the sample area facilitates small adjustments in the polarization of the incident beam without affecting the intensity of transmitted light. After traversing the sample area, the beam is split into its polarization components by a polarizing beamsplitter cube PBS (PBS101, Thorlabs). The $s$ component is transmitted by the cube to photodiode detector D2 and the $p$ component is reflected by 90° to detector D1 (both detectors CI-6504A, PASCO). The photodiode signals are digitized and transferred to a personal computer for further analysis (750 Interface and DataStudio Software, PASCO). All experiments were carried out in ambient air with ambient room lighting. The total cost of the system is approximately $750, excluding the half-wave plate, the mounting surface, and the computer interface and software.

B. Theory

Assuming an ideal polarizer and ideal proportional detectors with equal sensitivity, the signals at detectors D1 and D2 will be $S_s = I_{tot} \cos^2 \phi$ and $S_p = I_{tot} \sin^2 \phi$, where $I_{tot}$ is the total intensity of the beam and $\phi$ is the angle of the polarization of the light incident on the beamsplitter measured relative to an axis that is perpendicular to the plane of incidence. The angle $\phi$ can be determined from the ratio of the two signals according to

$$\frac{S_s}{S_p} = \frac{\cos^2 \phi}{\sin^2 \phi}$$

$$\phi = \arctan \sqrt{\frac{S_p}{S_s}}. \quad \text{(1)}$$

The amount by which a material or perturbation (such as a magnetic field) in the sample area rotates the angle of polarization is easily determined by measuring $\phi_{\text{blank}}$ in the absence of the material or perturbation and $\phi_{\text{sample}}$ in the presence of the material or perturbation. The rotation owing to the material or perturbation is

$$\theta = \phi_{\text{blank}} - \phi_{\text{sample}}. \quad \text{(2)}$$

III. RESULTS

A. Performance

Figure 2 displays the smoothed signal from the two detectors with an acquisition rate of 1000 Hz over a period of approximately 45 s with only ambient air in the sample compartment. The data in this and all subsequent figures have been smoothed with a 50-point moving average algorithm. All numerical results reported are calculated using unsmoothed data. The raw data from Fig. 2 have been converted to polarization angles with Eq. (1) and are shown in Fig. 3. The slight difference in voltage between the two channels, apparent in Fig. 2, leads to an angle of polarization measurably different from 45° in Fig. 3. The standard deviation of the data shown in Fig. 3, before smoothing, is 0.019°.

To determine the effect of the sampling rate on noise, data were acquired with a 100-Hz sampling frequency. The calculated polarization angles at this frequency are shown in Fig. 4. The standard deviation of the polarization angle is approximately 0.005° and the angle is increasing at a rate of $4.13 \times 10^{-2}/s$. On occasion, the polarization angle fluctuated measurably (Fig. 4 inset). We attribute these large

![Fig. 1. Schematic of the experimental setup: L is a laser, P is a polarizer, HWP is a half-wave plate (optional), PBS is a polarizing beamsplitter cube, and D1, D2 are light sensors.](image)

![Fig. 2. The smoothed signals from detectors D1 and D2 measured at 1000 Hz during an interval of approximately 45 s with ambient air in the sample compartment.](image)

![Fig. 3. The smoothed polarization angle, calculated from the data in Fig. 2.](image)
fluctuations to the temperature dependence of the half-wave plate or other optics elements under the influence of the room air conditioning. Comparing the standard deviation of the measurements with 100-Hz and 1000-Hz acquisition rates indicates that the precision of our apparatus, defined as the standard deviation of many measurements divided by the square root of the sampling rate, is approximately $0.001/\sqrt{\text{Hz}}$. Therefore, our polarimeter can measure rotations with a precision of $0.01^\circ$ in approximately 10 ms.

B. Sample experiments

In order to demonstrate the capabilities of the polarimeter, we performed two experiments that can be adopted in the undergraduate laboratory.

1. Optical rotation of sucrose

Chiral molecules, including many biological molecules, display optical rotation. For small molecules in solution, optical rotation is quantified by the specific rotation $\alpha$, defined as the optical rotation induced in light traversing through one decimeter of a one-gram-per-milliliter solution.

We determined the specific rotation of sucrose at a wavelength of approximately 654 nm, measured by diffraction off a grating. Several solutions of granulated sugar from a local grocery store in deionized water were made by serial dilutions and placed in a 0.1000-m cell (Rudolph Research Analytical) in the sample compartment of our polarimeter. The optical rotation of deionized water and each solution was measured at ambient temperature. The measured values, five at each concentration, are shown in Fig. 5.

The data are well fit by the equation $\alpha = \frac{54.7(3)^\circ}{(g/\text{mL})} \times c + 0.081(4)^\circ$, where $c$ is the concentration of sucrose in g/ml and the uncertainties are standard deviations. This expression gives a value of $54.7 \pm 0.3^\circ$ for the specific rotation of sucrose at room temperature. This value is within 3% of the expected value of $53.3^\circ$ obtained from the Drude expression:

$$[\alpha]_T = \frac{A}{\lambda^2 - \lambda_0^2}$$

using $A$ and $\lambda_0$ from Lowry et al.\textsuperscript{19} The small discrepancy between the expected and measured values may be due to temperature variation of the optics or the specific rotation, impurities in the sugar sample, uncertainty in the laser wavelength, or unequal gains in the two detectors. The latter could be eliminated with a double-subtraction procedure.\textsuperscript{12}

2. The Faraday effect

Many materials display the Faraday effect: optical rotation induced by a magnetic field.\textsuperscript{20} The magnitude of the rotation $\theta$ depends on the Verdet constant $V$ of the material, the magnetic flux density $B$ in the direction of propagation, and the distance $d$ light travels through the material according to the following equation:

$$\theta = V B d.$$  \hfill (4)

There are several reports of undergraduate activities related to the Faraday effect.\textsuperscript{3–7} Experimental measurements typically employ lock-in amplification.\textsuperscript{3,6} With our balanced polarimeter, however, it was possible to measure the Faraday effect using both DC and AC magnetic fields of only hundreds of gauss without lock-in amplification.

We measured the optical rotation of light passing through a 6.35-mm thick fused silica optical window (CVI, PW1-1025-UV) in a magnetic field. The window was housed in a 1 in. diameter lens tube (Thorlabs, SM1L20) which formed the scaffold for a solenoid which was approximately 200 mm long with 840 turns of 24 AWG copper wire. We applied currents ranging from 0.5–1.75 amperes. The magnetic field strength was estimated using the long-solenoid
approximation $B = \mu_0 N I / L$, where $N$ is the number of turns, $I$ is the current, and $L$ is the length of the solenoid.

Figure 6 shows the optical rotation by fused silica as a function of the estimated magnetic field strength. These data are fit by the line $\theta = [1.01(6) \times 10^{-4} \text{ o}/\text{gauss}] \times B - 4(2) \times 10^{-3} \text{ o}$ (uncertainties are standard deviations). The slope of this line yields a Verdet constant of $197\text{ o}/\text{T} \cdot \text{m}$. Fujioka et al. provide an expression for $V(\lambda)$ which predicts a value of approximately $197\text{ o}/\text{T} \cdot \text{m}$ at 654 nm, approximately 25% larger than our measured value.21 This discrepancy is possibly due to variations from one sample of fused silica to the next or to our estimates of the magnetic field strength. There may also be significant error in the values estimated from the literature; two methods of estimating silica to the next or to our estimates of the magnetic field strength. There may also be significant error in the values estimated from the literature; two methods of estimating $\chi$ in Ref. 21 differ by approximately 8%.

IV. CONCLUSION

We have constructed a fast and accurate polarimeter for less than $1000 using readily available parts. The design permits direct observations of the polarization components of light and can be used to demonstrate physically important phenomena such as the optical rotation by a biological molecule in solution and the Faraday effect.