Analyzing atomic noise with a consumer sound card

Carsten H. H. Schulte, Georg M. Müller, Hauke Horn, Jens Hübner, and Michael Oestreich

Institut für Festkörperphysik, Leibniz Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany

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We discuss an advanced undergraduate project on spin noise spectroscopy of atomic rubidium vapor. The spin noise is digitized using a consumer sound card and analyzed by a fast Fourier transform. Students gain competence in digital data acquisition and processing, and the idea of analyzing a noise signal is emphasized. © 2012 American Association of Physics Teachers. [DOI: 10.1119/1.3663275]

I. INTRODUCTION

In 1905 Einstein proposed a fundamental relation between noise and dynamics in terms of the stochastic Brownian motion of a particle suspended in a liquid.1 The universality of this relation is described by the fluctuation-dissipation theorem.2 This theorem links, for example, the magnetization fluctuations in an equilibrium system to its magnetic susceptibility. Analogously, the time correlations of the fluctuations of the magnetization reveals the underlying spin dynamics under an infinitesimal external perturbation.3 Hence, the full spin dynamics can be determined by investigating the fluctuations and time correlations of the system if the experimental probe creates only an infinitesimal perturbation.

Spin noise spectroscopy records the spin fluctuations via the optical Faraday effect, which enables such a perturbation-free mapping of the spin noise onto the rotation of the linear light polarization of a probe laser whose energy is slightly detuned from an appropriate optical transition.4,5 Spin noise spectroscopy is used in atomic optics as a non-destructive measurement of the macroscopic magnetization,6 and enables light-matter or matter-matter entanglement.7–9 In semiconductor physics the technique yields a perturbation-free probe of electron spin dynamics.10–12

An undergraduate project on spin noise spectroscopy introduces students to atomic physics (energy levels, Lorentz oscillator model, and the relation between dispersion and dissipation, that is, the Kramers-Kronig relation), statistical physics (fluctuations from the mean, correlation functions and their relation via Fourier transform to the spectral power density function), different types of physical and measurement noise (classical 1/f noise, photon shot noise, projection noise, and digitizer noise), and the basics of optical experiments (laser and polarization optics). Students also become familiar with concepts such as probing with off-resonant light13 and non-destructive measurement techniques.

In this paper, we discuss an implementation of spin noise spectroscopy that utilizes a consumer sound card for analyzing the noise. The usually most expensive part of the experiment is replaced by an inexpensive consumer product. Our experience is that the required programming expertise corresponds to the skills of most undergraduate physics students. The realization of this project can be a valuable resource for students in subsequent laboratory work because digital data acquisition and processing is of much importance in many experiments.

II. BASICS OF SPIN NOISE SPECTROSCOPY

Figure 1(a) depicts the basic setup. Spin noise spectroscopy measures the fluctuating macroscopic spin component of the system of electrons or atoms along the direction of light propagation $s$, by off-resonant Faraday rotation. To this end, linearly polarized laser light with its energy slightly detuned from the optical transition of interest is sent through the sample. The rotation of the light polarization is measured by a Wollaston prism oriented 45° with respect to the original polarization. The prism projects the linearly polarized laser light onto the +45° and −45° linear polarization states, which enables the sensitive measurement of small rotations of the linear polarization with a pair of balanced photodiodes. The difference of the electrical signal of the balanced photodiodes is digitized and spectrally analyzed.

Typical noise spectra are depicted in Fig. 1(b). In addition to the actual spin noise, the measured noise signal also contains random photon shot noise and electrical background noise. The subtraction of a reference curve without the spin noise yields the pure spin noise signal.

A. Spin noise

The magnetization of a system in thermal equilibrium is intrinsically subject to fluctuations, and the random deviations from the mean dephase after a finite time.14 Also the projection to basis states by the measurement introduces projection noise,15 even for the case of infinitely long spin coherence. (Projection noise results from the inherent uncertainty in the measurement of a superposition state.) It becomes important if a spin polarization transverse to the direction of light propagation is induced by optical pumping by a second laser.7 In the experiments discussed below, the dephasing of the incomplete random cancellation of up and down spins at thermal equilibrium yields the measured spin noise. The mean deviation of the magnetization at thermal equilibrium is given for noninteracting spins by the binomial distribution and is proportional to $1/\sqrt{N}$, where $N$ is the number of probed spins.16 Accordingly, spin noise spectroscopy is an especially interesting tool for small spin systems such as semiconductor nanostructures.11,17

B. Below band-gap Faraday rotation

The spin noise is mapped in the experiment onto the orientation of the linear polarization of the probe light via the Faraday effect. For rubidium vapor the spin polarization of the 5S valence electrons is probed. To this end, the D1 transition to $^5$P$_{1/2}$ (794.98 nm) or the D2 transition to $^5$P$_{3/2}$ (780.24 nm) is virtually excited by detuning the probe laser from the corresponding resonance (see Fig. 2 for the energy level diagram of rubidium). A random spin polarization in the 5S level introduces a different absorption of left ($\sigma^-$) and right ($\sigma^+$) circularly polarized probe light. This difference in absorbance yields a circular birefringence ($\Delta n = n^+ - n^-$) via
Fig. 1. (Color online) (a) Off-resonant linearly polarized laser light is transmitted through a rubidium gas cell. The direction of the linear light polarization acquires a rotation due to the random spin polarization in the sample based on the magneto-optical Faraday effect. The resulting polarization noise is converted to electrical noise which is digitized and spectrally analyzed on a computer using a fast Fourier transform. (b) The black curve shows a noise spectrum including contributions from spin noise, shot noise, electrical noise, and classical 1/f noise. Subtracting a reference spectrum containing only background noise yields the pure spin noise signal. The spikes in the spectrum slightly below 80 kHz result from electromagnetic interference with unknown sources. The noise power above 90 kHz drops due to the limited bandwidth of the sound card.

C. Spectral analysis

The resulting polarization is converted into an electrical signal by means of a polarizing beam splitter and a pair of balanced photodiodes. Besides the actual spin noise, the signal from the balanced receiver also contains photon shot noise of the probe laser, some electrical noise introduced by the electronic components, and classical laser noise that scales with the inverse frequency and is therefore referred to as 1/f noise. Although 1/f noise contributes only at very low frequencies, photon shot noise becomes evident as frequency independent noise. In contrast, the spectral shape of the spin noise is determined by the underlying spin dynamics. The power spectral density of the spin noise \( S(2\pi f) \propto \left| F\{s_z(t)\}\right|^2 \) (\( F \) denotes the Fourier transform) is proportional to the real part of the Fourier transform of the spin autocorrelation function. This link between the autocorrelation function and the power spectral density is universal and is known as the Wiener-Chintchin theorem.20,21 The spin noise spectrum, which is the power spectral density, is given by a Lorentzian curve centered at zero frequency with the full width at half maximum given by \( 1/(\pi T_2) \) for an exponential spin decay \( \langle s_z(0),s_z(t) \rangle \propto \exp(-t/T_2) \), where \( T_2 \) is the spin dephasing time.22 Applying a magnetic field \( B \) transverse to the direction of light propagation induces a Larmor precession of the spin polarization with the Larmor frequency \( \omega_L \). The Larmor precession is equivalent to a modulation of the detected spin noise. In other words, the spin correlation function in the presence of a magnetic field in the \( x \)-direction is given by \( \langle s_z(0),s_z(t) \rangle \propto \cos(\omega_L t)\exp(-t/T_2) \). Accordingly, the Lorentzian spin noise curve is centered at the angular frequency \( \omega_L \). This modulation allows for separation of the spin noise from classical 1/f noise which is both centered at zero frequency in the absence of a magnetic field. The Larmor frequency is given by \( \omega_L = g^*\mu_B B h^{-1} \), where \( \mu_B \) is the Bohr magneton, \( g^* \) is the effective electron Landé factor, and \( h \) is the reduced Planck constant. The effective Landé factor \( g^* \) can significantly differ from the free electron value \( g_0 \approx 2 \). This deviation originates from spin-orbit and/or hyperfine coupling. For atomic rubidium vapor, the coupling with the nuclear spin yields \( g = g_F = 1/2 \) for the isotope \(^{85}\text{Rb} \) and \( g = g_F = 1/3 \) for \(^{87}\text{Rb} \) (see Fig. 2).

D. Spin dephasing

The spin information in the probe laser volume decreases due to the spatial motion of the atoms within the cell. The resulting finite interaction time of light and atomic spins yields a broadening of the spin noise curve in accordance with the Fourier uncertainty principle.23 The helium buffer gas inside the cell reduces the mean free path of the rubidium atoms and lowers the efficiency of this time-of-flight broadening which still dominates spin dephasing in the experiment. Due to inter-atomic collisions, the atomic motion is diffusive and accordingly a \( 1/w^2 \) dependence of the measured spin dephasing rate on the laser beam waist \( w \) is observed.24 Inter-atomic collisions do not contribute to spin dephasing. Collisions among the rubidium atoms conserve the total spin of the ensemble, and no spin can be transferred to the helium atoms due to the noble gas configuration. In contrast, collisions with the internal walls of the cell are an important source of depolarization of the atomic spin.

In semiconductors the electron spin couples via hyperfine or spin-orbit interaction much more strongly to its
environment. Accordingly, spin dephasing times are significantly reduced and the collision-free time evolution of a conduction-band electron is usually not spin-conserving. A brief discussion on this topic is given in Ref. 12.

III. EXPERIMENTAL DETAILS

A. Probe laser

We use a laser diode with an external resonator in the Litrow configuration for probing the optical D2 transition in rubidium. The back facet of the laser diode and a diffraction grating form the laser cavity. The first-order diffracted light is sent back into the laser diode while the zeroth-order light is the specular reflection out of the cavity. Rotating the diffraction grating enables the tuning of the laser wavelength close to the narrow optical transition which lies within the gain spectrum of the laser diode. The reduction of the free spectral range of the laser due to the external cavity enables stable operation in a single laser mode without mode jumps. The optical frequency drifts in the kilohertz to megahertz range because it is not additionally stabilized, for example, by locking it to an external reference. The drift is not relevant to the experiment due to the large detuning from the atomic resonance. The Littrow configuration allows a sweeping of the laser wavelength over the rubidium absorption spectrum and is described in more detail in Ref. 25. Using a free running laser with a relatively broad laser line around a wavelength of 780 nm that is tuned by changing the diode temperature is sufficient to achieve a proof of principle realization of this experiment. A variety of AlGaAs-based laser diodes with the corresponding wavelength are commercially available. A low-cost battery-based laser diode driver kit is a sufficiently stable current source.

B. The rubidium cell

In this work, we use a quartz cell with purified $^{87}$Rb and helium buffer gas at a pressure of 1 mbar. The buffer gas reduces the mean free path of the rubidium and increases the measured effective spin dephasing times by decreasing the efficiency of the time-of-flight broadening. However, any rubidium reference cell can be used for this experiment. The cell is heated to around 450 K, and the corresponding rubidium vapor density can be calculated by the Clausius-Clapeyron relation and is approximately $2 \times 10^{14}$ cm$^{-3}$ (see Ref. 26).

C. Terrestrial and applied magnetic fields

A magnetic field is applied for modulation of the spin noise with the Larmor frequency. Because the Earth’s magnetic field yields a Larmor frequency of several hundred kHz (the Earth’s magnetic field is $\approx 49.2$ µT in Hannover), the magnetic field of the Earth needs to be compensated in all three directions by three pairs of Helmholtz coils. Magnetic fields of this strength can be measured with high accuracy by a low-cost fluxgate magnetometer ( Förster probe). Magnetic fields that correspond to Larmor frequencies greater than the detection bandwidth of the setup enable the recording of reference noise spectra which do not contain any spin noise contribution. These reference spectra can be subtracted from the noise spectra to remove the photon shot noise background [see Fig. 1(b)].

D. Photodetection

A commercial balanced photoreceiver based on silicon photodiodes with a relatively high bandwidth of 80 MHz

Fig. 2. Energy states of $^{87}$Rb and $^{85}$Rb. The different hyperfine splitting for the two isotopes results in different effective $g$-factors $g^* = g_F = 1/2$ ($^{87}$Rb) and $g^* = g_F = 1/3$ ($^{85}$Rb). The hyperfine splitting is negligible for the experiment.
was used. This high bandwidth is not needed for the experiment, and even a hand-made balanced photodetector using commercial silicon photodiodes can be utilized. A second balanced receiver can be used to measure the absorption of the rubidium cell. This extension of the setup is especially helpful if the laser energy is continuously swept and enables recording of the rubidium absorption spectrum and calibration of the laser energy via the sharp absorption lines.

E. Spectral analysis

We employed a hand-made fast Fourier transform (FFT) spectrum analyzer for this experiment. An FFT spectrum analyzer consists of an analog-to-digital (A/D) converter and an implementation of the FFT algorithm to convert the time series into the frequency domain. We use a consumer PCI sound card for A/D conversion (see the Appendix for details). The idea of analyzing the spin noise with a sound card is appealing to students and results in a significant cost reduction of the experiment. The actual FFT analysis can be easily realized by modules in the LABVIEW programming environment or the free GNU scientific library.\textsuperscript{27} We use a sound card with a sampling rate of 192 kHz and a resolution of 24 bit. According to the Nyquist-Shannon theorem,\textsuperscript{28,29} the sampling rate corresponds to a detection bandwidth of 96 kHz. The high dynamic resolution (bit depth) of the utilized digitizer enables FFT spectrum analysis without any influence of noise due to binning.\textsuperscript{30} In our case a FFT containing 4096 points gives sufficient frequency resolution. We average 100 of these spectra before the applied magnetic field is switched off to acquire a reference noise spectrum. Switching off the applied magnetic field shifts the spin noise out of our detection window due to the uncompensated terrestrial magnetic field. Measurement times of around 20 min are used including some idle time for switching the magnetic field and computation. A sweeping spectrum analyzer can also be used for spectral analysis and shows decent detection sensitivity for the experiment. However, at high bandwidths, the FFT spectrum analysis combines the advantages of more efficient data averaging and lower cost.\textsuperscript{12,31} The higher sensitivity results from the fact that the FFT spectrum analysis uses the complete data stream for averaging, while a usual sweeping spectrum analyzer acquires at a particular time only the frequency component of the signal at the corresponding frequency of the internal oscillator.

IV. EXPERIMENTAL RESULTS

Figure 1(b) depicts typical noise curves. The pure spin noise spectrum is obtained by subtracting a reference noise spectrum which contains no spin noise. The ratio of peak spin noise density and the white photon shot noise background is much larger than one, indicating a high signal strength, which facilitates the implementation of the experiment. The laser wavelength is adjusted by first tuning it to the D2 absorption line. For coarse tuning a low-resolution spectrometer can be used. The fine tuning is best achieved with an infrared viewer or a CCD camera because the resonant excitation of the D2 transition results in a strong fluorescence of the cell. In the second step the laser is slightly detuned from resonance because the spin dephasing rates under resonant excitation are increased due to the finite lifetime of the excited states. Figure 3 demonstrates the modulation of the spin noise with the Larmor frequency by applying a transverse magnetic field. A detailed evaluation of the spin noise spectra by fitting the spectra with two Lorentzian curves reveals residual $^{85}\text{Rb}$ in the reference cell as shown in Fig. 4. The ratio of the two frequencies of 2/3 corresponds to the ratio of the two g-factors. The extracted peak width in Fig. 4 yields a spin dephasing time of 36 $\mu$s. With a calculated laser beam waist of around 20 $\mu$m, this dephasing time is consistent with time-of-flight broadening due to transient effects. Increasing the beam waist increases the spin dephasing time and reduces the width of the spin noise curves. The height of the spin noise peak and, consequently, the signal-to-noise ratio is roughly independent of the probe laser beam waist, because the integrated spin noise power also scales with the reciprocal laser beam area.\textsuperscript{5}

V. DISCUSSION

We have discussed the use of spin noise spectroscopy on rubidium vapor as a project at the advanced undergraduate
level in which atomic noise is demonstrated with comparatively simple equipment. The project has been successfully realized in Hannover as an undergraduate practical project. The students gain understanding of a variety of different physical aspects. They learn, for example, how to tune semiconductor laser diodes by varying the temperature, the current, or the external cavity. They also gain experience with atomic energy states and optical transitions. In addition, they understand the optical Faraday effect, which is important for many experiments and applications. The students also learn about important concepts such as correlation functions and their relation to noise spectra. Moreover, the students implement efficient digital data acquisition and processing.

APPENDIX: METHODS

We used a AlGaAs-quantum-well laser diode with AR coating and a center wavelength of 770 nm manufactured by eagleyard Photonics GmbH. Alternatively, a non-AR coated laser diode may be used in the cavity, in which case the laser diode current and the grating angle must be used to tune the laser to the required wavelength. In our experiment, a Trace Alpha PCI sound card produced by Marian was used. A webcam is suitable for this purpose. The balanced photo receiver with silicon photo diodes was purchased from New Focus. The balanced receiver’s signal noise power are discussed in Ref. 5.

The balanced receiver’s signal was amplified with a voltage amplifier made by Femto. To exploit the complete input range of the sound card. As mentioned, a hand-made balanced photo receiver is easily built and shows satisfying performance. An amplifier could also be built into the balanced receiver, but is not essential due to the fact that the sound card worked with a resolution of 24 bits. We used a Trace Alpha PCI sound card produced by Marian.

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filter-stabilized diode laser systems for the manipulation of neutral atoms,”
3Wavelength reference cells from Triad Technology, 640 South Sunset St.,
Longmont, CO 80501, <www.triadtechno.com/>,
3Spectroscopy cells from Toptica Photonics AG, Lochhamer Schlag 19,
D-82166 Graefelfing (Munich), <www.toptica.com/>.
80 MHz balanced photoreceiver 1807 from New Focus, 2584 Junction
DHPVA-200 variable gain 200 MHz wideband voltage amplifier from Femto
Trace Alpha from Marian GmbH, Berggartenstrasse 12, D-04155 Leipzig,

Bird-Cage Electrometer. William Thomson (1824-1907; Lord Kelvin) developed the basic Quadrant Electrometer in
1867. This allowed absolute measurements to be made of electrostatic potentials. Inside the four-part brass box a light
aluminum paddle hangs from a torsion fiber. This paddle has been charged by the Leiden jar in the bottom of the
apparatus. This jar is filled with sulphuric acid. Each brass quadrant in the box is electrically connected to the one
diagonally opposite it. The wires from the potential difference to be measured are connected to the pairs of quadrants,
and the angle through which the paddle swings is proportional to the potential difference. The Faraday cage over the
mechanism gives the apparatus its name of bird-cage electrometer. The electrometer, in the Greenslade Collection, was
made by Elliott of London. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)