APPARATUS AND DEMONSTRATION NOTES

Jeffrey S. Dunham, Editor
Department of Physics, Middlebury College, Middlebury, Vermont 05753

This department welcomes brief communications reporting new demonstrations, laboratory equipment, techniques, or materials of interest to teachers of physics. Notes on new applications of older apparatus, measurements supplementing data supplied by manufacturers, information which, while not new, is not generally known, procurement information, and news about apparatus under development may be suitable for publication in this section. Neither the American Journal of Physics nor the Editors assume responsibility for the correctness of the information presented. Submit materials to Jeffrey S. Dunham, Editor.

Studying collisions in the general physics laboratory with quadrature light emitting diode sensors

P. A. DeYoung and B. Mulder
Department of Physics and Engineering, Hope College, Holland, Michigan 49422-9000

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We have developed the means to measure position rapidly and precisely as a function of time in the general physics laboratory. These measurements are of sufficient quality that velocities and accelerations can be calculated from the position data using numerical derivatives. The precision of the measurements is such that any disagreement between theoretical expectations and experimental measurements is less than a few percent. Measurements of the system under study can be made as rapidly as every 200 μs, which is faster than the typical time scales over which the system changes. Measuring rapidly also allows one to investigate additional phenomena not previously accessible and to see features of the physics previously unobserved. The measurement system is based on commercially available sensors, computer hardware, and computer software (LABVIEW™). Many general physics laboratories based on this system have been developed but only an investigation of Newton’s second law will be described here. © 2002 American Association of Physics Teachers. [DOI: 10.1119/1.1482066]

I. INTRODUCTION

Computers and modern sensors can be used to improve introductory physics laboratory classes. In particular, the speed with which measurements can be made allows one to measure the behavior of colliding or other rapidly varying systems. Simultaneously, the precision with which measurements can be made allows one to investigate, not only the positions of objects, but also their velocities and accelerations. Making quality measurements also prevents students from erroneously appealing to friction or air resistance in order to explain away poor agreement between measurement and expectation.

There are many reports in the literature of measurements made with various sensors and data acquisition computers. Most of these reports involve systems that evolve continuously over a relatively long time scale so that measuring at a rate of 10–100 Hz is sufficient. There are descriptions of measuring systems with sonic ranger sensors,1–4 video camera technology,5–8 and a variety of other techniques.9–16 Others have developed systems which, while using computers, measure only crude properties of the system under study.17 Finally, there are systems that perform high speed measurements but are limited to electrical systems rather than mechanical ones.18 Of particular note are Refs. 19–21, which inspired the laboratory exercises described here.

The precision with which an object’s position as a function of time must be measured is important. It is relatively easy to obtain an adequate measure of the position of an object with time if position versus time is the only behavior being studied; however, most studies of moving systems in the general physics lab also need the velocity (for momentum and energy) and acceleration (when investigating forces and collisions). Velocities and accelerations can be found from the position data by computing numerical derivatives; but to achieve an accurate second derivative, a high quality original measurement is required. (In general, we have chosen to make precise position and time measurements and then calculate derivatives, rather than work with more complicated and expensive velocity and acceleration sensors.) The system detailed in the following, based on a quadrature light emitting diode (LED) sensor22 and finely ruled mylar strip,23 yields position data of sufficient precision for the calculation of the necessary derivatives.

In addition to sensors, a means of recording the data is important. The system detailed here controls the experiment and records the individual measurements with LABVIEW.24 This computer data acquisition system is expandable to a wide variety of complementary sensors and so forms a solid base upon which to build.

The need to calculate the velocity, acceleration, and other quantities is an important reason for making high-quality position measurements, but not the only one. With precise mea-
measurements, subtle features of the system under study are often revealed. The ease with which high quality data can be acquired by computer tempts one to design exercises that involve a great deal of investigation. We have used this approach with great success with our senior lab students. These students look for nuances in the results that are not ordinarily appreciated by the introductory student. The experiment described below is one performed in the first semester of the labs associated with both our calculus-based course and our algebra-based course.

II. EQUIPMENT

Momentum conservation is a concept often studied with two carts and an air track. Here, LED sensors are mounted above the air track and a mylar strip is affixed to each cart. This arrangement is shown in Fig. 1. Note that while the mylar strip passes through a slot in the sensor, there is no contact between the mylar and the sensor body. The quadrature sensors incorporate two infrared diodes (so that they are insensitive to ambient light) and two receivers which change their TTL output state when an opaque material blocks the infrared light. The spacing of the two diodes in the sensor is matched to the spacing of the dark lines on the mylar strip in such a way that as the strip moves past the sensor the two diodes do not register transitions at the same time; they are offset by a quarter period. This feature allows one to determine direction, as well as position, and provides finer position resolution. The strips are ruled with 200 black stripes per inch. The output of the quadrature sensor is fed directly to a LABVIEW counter board, which has been configured to interpret the two TTL diode signals and provide maximum resolution. Because the sensor contains two diodes which are out of phase and can sense the edges of the black stripes, the ultimate position resolution becomes approximately 0.03 mm. The same computer board that handles the quadrature sensor also has a precise clock. The computer’s internal clock (which counts at rates up to 80 MHz with 0.005% accuracy) along with the precise position measurements permit one to compute accurate numerical derivatives. The other major approach that is often used to measure position is one based on sonic ranger techniques. Many systems can be measured conveniently only with ranger techniques. For example, if the motion of the object under consideration is not constrained to linear motion our approach is not applicable. For linearly constrained systems the mylar tape approach is better because it allows one to measure at a higher rate and is more precise than the sonic ranger. The mylar tape system can be measured at rates in excess of 100 kHz while the sonic ranger systems are limited to approximately 50 Hz. We have not found that being restricted to linear systems is a hindrance to exploring physical systems in creative ways.

III. DATA PROCESSING

The quality of the data provided by the ruled strip and quadrature sensor is of primary importance; however, an understanding of the data processing done in the general physics laboratory is also important.

The high rate with which individual measurements can be recorded generates a substantial amount of data. Some processing must be done to reduce the amount of data presented to the students. The amount of data that can be measured in a small amount of time is significant. It is a simple matter to obtain 10 000 measurements of the positions of two colliding carts, including 400 measurements while the carts are in contact. The students are a bit overwhelmed by this amount of data (e.g., individual points cannot be resolved on a computer display). Obviously, any processing done to reduce the amount of data must preserve or improve the quality of the numerical derivatives we intend to calculate.

When taking numerical derivatives one must always remember that measurements yield discrete values rather than values with infinite precision. For the position measurement, these values are from the linear encoders. Since the step size is small, the discrete nature of the data is of no consequence until the data are used for finding derivatives. An accurate calculation of a derivative such as \( \frac{dx}{dt} \) requires that the interval between successive measurements be long enough that the measured variable changes significantly compared to the discrete measurement step size, yet be shorter than the characteristic time scale for changes in the system.

If the first condition is not met then between successive measurements it is possible that the physical system will have moved only a minute amount from just inside a black stripe to just outside a black stripe. The sensor will register this as a full step of 0.03 mm and compute a large derivative even though there was really very little change. There will be similar situations where the calculated derivative will be much less than the true value. Thus, one cannot base derivatives on measurements spaced too closely together in time.

If the second condition is not met, rather than seeing the behavior of the system, one gets only the time-averaged behavior and much of the relevant physics may be missed. Hence, one cannot base the study on measurements spaced too widely in time. The numerical second derivative is even more sensitive to both of these considerations.

Making measurements with infinite precision would overcome these problems, but of course that is not an option. We have chosen two numerical techniques to overcome these problems. Before the data are presented to the student the data are smoothed and repeated values are removed. Both of these steps are important if the student calculations are to be of high quality. The 10 000 original measurements are subjected to a cubic spline smoothing. In effect, this replaces the...
discrete data points with a continuous function that is a better representation of the true behavior of the system. This function smoothes out much of the noise associated with discrete digital measurements and is valid as long as the system is varying more slowly than the rate at which the original points are measured. This function is then evaluated at equally spaced time values and the functional values are presented to the student. The time step chosen can be matched to the system under study so that the calculated derivatives are accurate. Thus, the smoothing really becomes a way of adjusting the sampling rate with software, after the measurements are made, which is easier than adapting the original data acquisition rate for each experiment. It would be possible to differentiate the spline functions determined from the data and present the students with positions, velocities, and accelerations directly. This approach was not adopted. If the students calculate the derivatives, they are reminded about relationships between position, velocity, and acceleration and this type of calculation prevents the data from being too good.

When the rate of change in the system is high, the previous manipulations are sufficient. However, when a system is changing slowly, repeated points of the same position value must be removed before the cubic spline smoothing is done. Removing successive measurements that have the same value results in a data set for which the remaining points more accurately reflect the behavior of the system. The first occurrence of a value and its associated time is sensitive to when the physical system has changed enough to register in the sensors. When the carts are moving slowly, a change in the sensor output implies that the cart must be positioned very close to a clear-dark transition on the mylar strip. The subsequent repeated values are ambiguous. The cart could be almost to the next transition or not yet moved at all from the previous position. If the repeated points are not removed when the system is changing slowly, the smoothing process is not effective.

After the limited processing described above is complete, students perform their analysis with the aid of personal computers. The data are read directly into ORIGIN, a commercial analysis package, and then formulas are entered to manipulate the data to calculate the relevant quantities (velocities, accelerations, energy, etc.). Many common functions are built into the package, including numerical differentiation and fast Fourier transforms. It is important that students focus on the relevant physics rather than the complexities of the measurement process. The ease with which students can move from the measurement phase into interpretation (creating plots which reveal some relevant physics) allows them to focus on learning from the results.

IV. DESCRIPTION OF THE EXPERIMENT

Obviously the system described above is adaptable to a wide range of experiments. We describe one here that would be very difficult to perform without the sensors and the computer.

An example of a laboratory exercise which clearly benefits from rapid data acquisition is the investigation of the collision of two carts. The collision of two air track carts allows the study of conservation of linear momentum and the investigation of the conservation (or nonconservation) of kinetic energy during elastic and inelastic collisions. The more traditional approach to these studies is to measure the velocity, and from it calculate the energy and linear momentum, before the collision and then again after the collision. Much more insight into the concepts can be gained by measuring the momentum and energy while the two carts are in contact. To perform measurements during the time the carts are in contact, two air track carts are equipped with the mylar strips and two position sensors are mounted over the track. For studies of elastic collisions, one cart is positioned with its tape in the sensor and is initially stationary. The second sensor is arranged so that it will be located near the middle of the second cart at the time of the collision. For perfectly inelastic collisions, we have carts with velcro attached to the standard air track cart leaf springs. The combination of velcro and springs yields very interesting results which will be discussed more below.

The results from a typical measurement of linear momentum and kinetic energy during an elastic collision are shown in Fig. 2. The carts first make contact at approximately 0.72 s and remain in contact for about 20 ms. The top panel of Fig. 2 shows the momentum of each individual cart and the sum of the two. (The momentum is calculated by taking a numerical derivative of the position information and multiplying by the cart mass.) An examination of this plot shows that one cart was initially motionless. Following the collision, the cart which was initially motionless now has momentum in the same direction as the original moving cart and the cart which was originally moving has recoiled and is moving backward slowly. This is as expected given that the stationary cart had a mass of 0.485 kg and the initially mov-
before, the position of each cart is recorded as a function of time throughout the inelastic collision; the energies and momenta of the carts are computed after the velocity is calculated numerically. The individual and total momenta for a typical inelastic sticking collision (which occurs near 0.6 s) are shown in Fig. 3. The first thing to observe is that again the total momentum is conserved even though the individual momenta of the two carts exhibit complicated behavior. The complicated momentum plots of the individual carts is an accurate representation of the actual behavior of the carts. The carts come together and stick, but because the velcro is mounted on the standard springs, the carts oscillate as the combined system moves down the track. It is truly remarkable to the students that the sum of the two complicated momenta is a constant.

Since this collision is inelastic there is an expectation that the kinetic energy will not be conserved. When one examines the kinetic energy of the carts and the total kinetic energy, one sees that this is indeed the case. For the collision shown in Fig. 3, one finds that 59.6% of the initial kinetic energy is lost during the collision. Given that this collision is perfectly inelastic (the carts stick together), the expected energy loss, which depends only on the masses of the carts, is 59.5%.

During the development of this exercise there was one unanticipated finding. In order to slow down the collision time, we tried various springs that were longer and softer than the standard air cart bumpers. Some of these springs had significant mass. During elastic collisions, a small but not insignificant amount of the initial momentum of the system could be transferred into motion of the coils of the spring. As a result, it would appear as though momentum was not conserved during the collision. This was because we only measure the momenta of the carts themselves. If the coils of the spring are in motion, then it is not possible to measure all of the momentum, making it appear as if the total momentum is not constant. This is not easy for students to grasp so we have limited their selection of springs to those that do not exhibit this property.

V. OTHER APPLICATIONS

The quadrature sensor has found a number of other applications in the general physics laboratory and elsewhere. Within the laboratory, this approach has been combined with force transducers to verify Newton’s second law during a collision. A similar setup has provided an interesting study of the properties of anharmonic and coupled oscillators. A rotational version of the encoder combined with a ruled mylar disk has been used to measure the angular position, velocity, and acceleration of a physical pendulum. Also for the general physics laboratory, we have developed an apparatus which can be used to investigate the adiabatic compression of a gas. The compression stroke takes place in approximately 20 ms during which time the pressure, temperature, and piston position are recorded 200 times. Lastly, the encoder and tapes (or wheels) have been generally useful when combined with a commercially available counter and display, the SubCub. This device allows one to use the sensor in a variety of situations where position or rotation must be measured precisely but where the rapid (and more expensive) recording by a computer is not needed.

VI. CONCLUSION

In introductory laboratories it is important to measure physical systems as they change and evolve. A great deal
more insight into the true behavior of a system can be gained in this way, especially when compared to measurements of only “before” and “after” properties. In the case of collisions or other fast processes, this requires the ability to measure rapidly. It is also important to measure precisely. If the results are only approximately in agreement with expectations, the explanations and theories can seem to be only approximate descriptions of nature and it becomes difficult to look for interesting effects. An additional feature of our approach to measurement is that the students are able to make measurements quickly. This allows them more time to focus on the topics under study and allows time for the better students to explore subsidiary phenomena.

This approach to measurement also makes it easy to develop new laboratory experiments. Once familiar with the capabilities of the sensors, one finds a large number of applications in new and old settings where the precision and ease of use improves the laboratory exercise.

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Electronic mail: deyoung@hope.edu


L. Victoria, C. Molina, A. Arenas, and J. A. Ibañez, “Use of pressure
Tabletop thermoacoustic refrigerator for demonstrations

Daniel A. Russell and Pontus Weibull

Science and Mathematics Department, Kettering University, Flint, Michigan 48504

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An inexpensive (less than $25) tabletop thermoacoustic refrigerator for demonstration purposes was built from a boxed loudspeaker, acrylic tubing and sheet, a roll of 35 mm film, fishing line, an aluminum plug, and two homemade thermocouples. Temperature differences of more than 15 °C were achieved after running the cooler for several minutes. While nowhere near the efficiency of devices described in the literature, this demonstration model effectively illustrates the behavior of a thermoacoustic refrigerator. © 2002 American Association of Physics Teachers. [DOI: 10.1119/1.1485720]

I. INTRODUCTION

The basic workings of heat engines and refrigerators are commonly described in the undergraduate physics curriculum. Thermoacoustic heat engines and refrigerators, however, are topics usually reserved for graduate level courses and research. Recent articles in popular scientific journals have made the concepts behind such devices understandable to a much wider audience. A demonstration apparatus, as described in this note, can effectively introduce students to the physics behind thermoacoustic refrigerators. The basic operating principles are simple enough to be understood by beginning students, and this apparatus may be used to attract students to physics, or to upper level courses. The first author of this paper uses this apparatus regularly as a demonstration for prospective students and their parents during campus tours. This apparatus is not designed to be efficient; it is intended more as a proof of concept demonstration. However, it could readily serve as a starting point for a senior level research project complementing a thermodynamics or acoustics course.

II. CONSTRUCTION OF THE APPARATUS

The thermoacoustic refrigerator demonstration described in this note is of the standing wave variety, and consists of a quarter-wavelength resonator (an open-closed tube) driven by a loudspeaker. While this is the easiest resonator shape to build, it is the least efficient of the standing-wave type refrigerators. Since the primary purpose of this apparatus is to demonstrate the action of an acoustic refrigerator, efficiency was not a primary concern. A schematic drawing of the refrigerator is shown in Fig. 1(a).

The resonator for this refrigerator was a 23 cm length of acrylic tubing with an inner diameter of 2.2 cm. The length defines the resonance frequency of the system, which was 385 Hz for our apparatus. A hole was cut in the center of an acrylic cover sheet and the tube was glued to the cover sheet, which was then placed over the speaker. The speaker was a 4-inch boxed speaker capable of handling 40 W, and a 4-inch diameter o-ring was used to provide a seal around the edge of the speaker. An aluminum plug was milled to fit snugly into the end of the tube, forming the closed end.

The most important part of an acoustic refrigerator is the stack, which consists of a large number of closely spaced surfaces aligned parallel to the length of the resonator tube. The stack for this apparatus was constructed, as suggested Hofler, by winding a roll of 35-mm photographic film around a central spindle so that adjacent layers of the spirally wound film provide the stack surfaces. Lengths of 15-lb nylon fishing line separated adjacent layers of the spirally wound film stack so that air could move between the layers along the length of the stack parallel to the length of the resonator tube. Figure 1(b) shows a cross section of the rolled-film stack, with layers separated by fishing line.

The primary constraint in designing the stack is the fact that stack layers need to be a few thermal penetration depths apart, with four thermal penetration depths being the optimum layer separation. The thermal penetration depth, \( \delta_k \), is defined as the distance that heat can diffuse through a gas during the time \( t = 1/\pi f \), where \( f \) is the frequency of the standing wave. It depends on the thermal conductivity, \( k \), and density, \( \rho \), of the gas and the isobaric specific heat per unit mass, \( c_p \), according to

\[
\delta_k = \sqrt{\frac{k}{\pi f \rho c_p}}.
\]

If stack layers are too far apart the gas cannot effectively transfer heat to and from the stack walls. If the layers are too close together viscous effects hamper the motion of the gas particles. For a frequency of 385 Hz in air one thermal pen-
etration depth is $1.33 \times 10^{-4}$ m. The 15-lb nylon fishing line has a diameter of $3.40 \times 10^{-4}$ m; the stack layers in this apparatus were therefore separated by about 2.5 thermal penetration depths.

To construct the stack, a roll of 35-mm film was unrolled. Lengths of fishing line were glued across the width of the film at equal intervals using a spray adhesive. To keep lines straight the line was first wound onto a "loom," a cardboard frame with slits cut every 5 mm. After spraying the glue onto the lines, the frame was placed over the film and a teflon weight was placed on top, to press the lines against the film. Once the glue was set, the fishing line was cut flush with the edges of the film. This process was repeated for approximately 1 meter of film. The film was then rolled around a small diameter acrylic rod and layers were gradually peeled off until the film roll fit snugly into the tube. The stack was positioned in the tube approximately 4 cm from the closed end so as to be close to the pressure maximum, but away from the particle displacement minimum.

Two thermocouples were made by soldering copper and constantan wires together. One thermocouple was inserted through the outermost winding of the stack to detect the temperature below the stack, while the other was allowed to dangle just above the stack. Leads for both thermocouples passed through a small hole drilled in the aluminum plug at the end of the tube. The loudspeaker was driven by a sine wave generator through a 100 W audio amplifier. The pressure amplitude inside the resonator tube was not measured, but the power to the speaker was increased until a second harmonic became barely audible, indicating that the system was becoming nonlinear.

### III. TYPICAL RESULTS

Figure 2 shows a photograph of the apparatus after it had been running for about 10 minutes at a high sound level. The multimeter to the right of the apparatus shows the temperature below the stack; it started at 66 °F and dropped to 29 °F. The multimeter to the left of the apparatus shows the temperature above the stack; it started at 66 °F and increased to 75 °F. A temperature difference of 46 °F (25.6 °C) was obtained across the stack after just 10 minutes with air as the "coolant," and with the loudspeaker cone being the only moving mechanical part.

Figure 3 shows typical results for the temperatures above the stack ($T_{\text{hot}}$) and below the stack ($T_{\text{cold}}$) as a function of time. The starting temperatures were normalized to zero, so the plot shows the changes in temperature as measured by each thermocouple. To produce this plot the thermocouple leads were connected to a two-channel digital oscilloscope with an 8 minute capture time. The plot shows that the temperature below the stack ($T_{\text{cold}}$) begins decreasing immediately after the sound is turned on, dropping 4 °C in the first 15 seconds, with the rate of temperature change decreasing with time. After 4 minutes of operation the temperature below the stack has dropped by 10.5 °C and is still decreasing. The temperature above the stack ($T_{\text{hot}}$) increases, also more rapidly at first, as the heat is being pumped through the stack. After approximately 2 minutes the temperature above the stack has increased by 5 °C. After that it stops increasing as the rate at which heat is moved through the stack equals the rate at which heat is conducted through the aluminum cap into the surrounding room. After 4 minutes of operation, the temperature difference between the top and bottom of the stack is about 15.5 °C, a difference large enough to be detected by touching a finger along the outside of the acrylic tube. The trends in Fig. 3 are similar to those found in the literature.8

### IV. HOW IT WORKS

Figure 4 shows the basic operation of a heat engine and heat pump, or refrigerator. In a heat engine, heat is transferred from a high temperature reservoir to a lower temperature reservoir doing work in the process. In a heat pump, or refrigerator, externally applied work transfers heat from the lower temperature reservoir to the higher temperature reservoir. In the case of a thermoacoustic refrigerator the external work is supplied by the standing sound wave in the resonator.
The longitudinal standing sound wave causes the gas particles to oscillate back and forth parallel to the walls of the stack. The alternating compression and rarefaction of the gas causes the local temperature of the gas to oscillate due to the adiabatic nature of sound waves. If the local temperature of the gas becomes higher than that of the nearby stack wall, heat is transferred from the gas to the stack wall. If the local temperature of the gas drops below that of the stack wall, heat is transferred from the wall to the gas.

The second most important factor in the performance of a thermoacoustic refrigerator is the critical longitudinal temperature gradient

\[ \nabla T_{\text{crit}} = \frac{p}{\xi \rho c_p}, \]

where \( p \) and \( \xi \) are the acoustic pressure and displacement amplitudes, respectively. No heat is transferred when the peak-to-peak temperature variation caused by adiabatic compression of the gas, \( 2p/\rho c_p \), exactly matches the variation in the local wall temperature, \( 2\xi \nabla T_{\text{crit}} \), between the extremes of the gas particle motion. Only when the sound wave induced temperature variation in the gas is greater than the temperature gradient between the cold and hot ends of the stack will heat be moved from lower temperature to higher temperature causing refrigeration. This requires a rather intense sound wave inside the resonator. A boxed loudspeaker with as tight a seal as possible between the speaker and resonator helps to reduce the sound level in the room to tolerable levels.

The thermoacoustic refrigeration cycle is illustrated in Fig. 5. As the motion of the sound wave causes a gas parcel in the stack to move left (towards the closed end of the tube) the pressure increases and the gas is compressed. The compressed gas parcel is now hotter than the nearby stack wall so it damps heat to the cooler stack, thus shrinking in volume. As the standing wave continues through its cycle the parcel is pulled back to the right where the pressure is lower. The rarefied parcel is now cooler than the nearby stack wall so it absorbs heat from the warmer stack wall and expands. The cycle repeats with the net effect of a small amount of heat being moved a short distance along the stack from the colder towards the hotter end. A “bucket brigade” of particles can move a significant amount of heat from one end of the stack to the other.

\section*{V. INCREASING THE EFFICIENCY}

This simple and inexpensive thermoacoustic refrigerator effectively demonstrates the basic physical principles behind its operation. As shown, however, it is rather inefficient as a heat transfer device. If both ends of the stack were connected to heat exchangers, thus coupling the stack to a heat source or heat sink, the transfer of heat would be more efficient. Other improvements could be made by modifying the shape of the resonator\(^8\) or increasing the stack layer separation to an optimal four thermal penetration depths.\(^3\) One could also study the performance as a function of sound level inside the resonator. Such studies might make for an interesting senior research project.

\section*{ACKNOWLEDGMENTS}

This demonstration apparatus was built by the second author as part of a laboratory project for a senior level acoustics course. A similar thermoacoustic cooler, that served as the inspiration for this apparatus, was demonstrated by Dr. George Mozurkewich, of Ford Motor Co., at the Fall 1999 meeting of the Ohio Section of the American Physical Society. The authors would like to thank him for sharing his “blueprints” and for additional suggestions and discussion.

\(^{8}\)Current address: TXU Energy Trading, Dallas, TX 75201.

\(^{8}\)Formerly GMI Engineering & Management Institute.


\(^{5}\)Optimus XTS 40 loudspeakers (4-inch, 150–18 000 Hz) are available from Radio Shack (Catalog No. #40-991). They are quite often on sale for under $10.00.


A simple instrument for measuring time intervals with subnanosecond resolution

F. Fontanelli
Dipartimento di Fisica dell’Università di Genova and I.N.F.N. Sezione di Genova, Genova, Italy

L. Repetto and R. Chittofrati
Dipartimento di Fisica dell’Università di Genova, 16146 Genova, Italy

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We have built a simple and inexpensive instrument capable of measuring time intervals from 0 up to 100 ns with a 0.1-ns resolution. We use easy-to-find components, and the construction can be performed by an undergraduate student. © 2002 American Association of Physics Teachers. [DOI: 10.1119/1.1488639]

I. INTRODUCTION

The measurement of time intervals with a resolution of the order of 1 ns can be complicated and expensive, requiring specialized timing instruments or high-speed oscilloscopes. In this note we show that similar results can be achieved using a simple RC circuit and a few logic gates. The basic principle underlying our method can be understood from Fig. 1. Let’s suppose we want to measure the elapsed time between a start and a stop signal. Initially, the capacitor $C$ is discharged and switch $S$ is open. If we let the start signal close the switch $S$, the current will start charging $C$ according to the well-known law,\footnote{1}{
$$V(t) = V_0 (1 - e^{-t/\tau}),$$
where $\tau =RC$. In the same way, if the stop signal then opens switch $S$, the current flow, and consequently the charging, will cease.
}

At this point we have a certain amount of charge on capacitor $C$ which is strictly related, through Eq. (1), to the time elapsed between the start and stop signals. This charge will stay on the plates of the capacitor more or less indefinitely, depending on switch leakage and the insulating properties of the dielectric inside the capacitor.

Moreover, if the time elapsed $t$ is much smaller than $\tau$, Eq. (1) can be approximated by the first-order expansion term, resulting in a voltage buildup that is proportional to time:

$$V(t) = \alpha t,$$
where $\alpha = V_0/\tau$. The story sounds very simple, and indeed it is, provided a few (inexpensive) tricks are used: the choice of a suitable switch (fast, with low loss), the insertion of a buffer amplifier in front of our voltage reading instrument, and the choice of a well-behaved (from the point of view of losses and dielectric hysteresis) capacitor.

With the addition of a few logic functions and a general purpose data acquisition board, a system is produced that can substitute for an expensive, high-speed oscilloscope in nanosecond timing applications.

II. THE APPARATUS

Our apparatus is shown in Fig. 2. It is composed of a logic section to memorize the start and stop signals and an analog part to control the current charging the capacitor.

The logic section is built around two flip-flops; the upper one memorizes the start pulse, and the lower one does the same with the stop. The connection between the $\bar{Q}$ of the upper flip-flop and the $D$-input of the lower one protects against stop signals not preceded by start. This should never happen, but we take this precaution just in case it does. The AND gate (74LS08) controls the switch according to the state of the flip-flops.

The analog section is more delicate; it is built around a “transmission gate”, that is a switch made of metal–oxide–semiconductor field effect transistors (MOSFET). The device we used (HEF 4066) is apparently slow, with a turn-on time of $\approx 40$ ns, but it is very reliable and reproducible in its operation. This device controls the current flowing in the capacitor, which, as already stated, must have very good characteristics, i.e., very low leakage and dielectrical hysteresis. A few words are probably in order to clarify this point. Most dielectrics used by the electronics industry to make capacitors have a tendency to “remember” their state of polarization. This feature is not necessarily dangerous or undesirable in an electronic device, but in our case, it certainly is, because, after closing the switch, the voltage across the capacitor would change in a way depending on previous measurement cycles.\footnote{2}{
For this reason, ceramic capacitors must be avoided; in practice, we used a mica capacitor with good results.
}

The operational amplifier, which is connected as a unity gain buffer, separates the sensitive point of the capacitor from the voltmeter outside. A buffer is required to avoid any leakage from the capacitor and to keep the capacity of the node stable and well-known. In practice, any modern FET-input op-amp will do the job.

Before starting a new measurement cycle, the “reset” input is used to clear both flip-flops and to discharge the capacitor; the small resistor in series with the lower switch limits the peak discharge current so as to avoid damage to the MOSFET inside.

III. ACQUISITION SYSTEM AND CALIBRATION

The time-resolution of the proposed instrument is linearly related to the voltage-resolution of the acquisition system. In our case, with a time constant $\tau = 2.7 \mu s$ and a voltage supply of $+5$ V, we expect charging rate at the output of

$$\alpha_{\text{in}} = 1.85 \text{ mV/ns},$$
therefore we need a voltage-resolution of at least $100 \mu$V to achieve a time-resolution of 100 ps. The voltage reading must be completed quickly, otherwise leakage currents could
have a considerable affect on the measurement. To implement a voltmeter with these characteristics, we used a digitizing board that sampled at 100 kHz with 14-bit resolution (~30-μV resolution over 500 mV). By automating the readings of the digitizing board through a personal computer, we were able to repeat and average the measurements, so as to obtain an effective noise filtering.

At this point we have to clarify a question which has probably occurred to the reader: How can we obtain a 100-ps resolution using a switch that has a turn-on time of 40 ns? The answer is another simple, but essential, trick; we can add a fixed, but arbitrarily long, delay between the start and the stop signal simply by increasing the length of the coaxial cable that provides the stop signal to the apparatus. The zero-delay point will be moved up to a certain voltage, but, if closing and opening delays are reproducible, a differential reading will cancel out the constant delay.

The entire system is now ready to work: but before we can correctly associate a delay time to each (differential) voltage reading, we must perform a calibration, i.e., measure the value of \( \alpha \) appearing in Eq. (3).

The calibration curve shown in Fig. 3 has been obtained by sending precisely delayed pulse pairs to the apparatus. Pulses were generated by a commercial pulser (Lecroy model 9210) with a 10-ps timing resolution. The slope calculated by the least squares method on 20 data points is

\[
\alpha_{\text{exp}} = (1.760 \pm 0.002) \text{ mV/ns. (4)}
\]

The linearity is good, as can be inferred by a \( \chi^2 \) value of 25.7 for the 20 data points.

Looking at the uncertainty associated with \( \alpha \) we can deduce, as claimed, a time-resolution of 100 ps for a 100 ns delay. The \( y \) intercept of the calibration curve shown in Fig. 3 results from the need to delay the stop signal, by a few tens of nanoseconds, with respect to the start signal. In fact, as anticipated in Sec. II, the switch takes 40 ns to open or close. Moreover a few nanoseconds are required by the logic circuit to switch. To satisfy these requirements we put about 30 m of cable between the pulser and the stop input. This length, which corresponds to a delay of \( \approx 150 \text{ ns} \), is not critical at all.

IV. EXAMPLE OF MEASUREMENT

As a first application of our timing circuit we measured the time delay for a signal propagating along a coaxial cable of the RG174 type. We measured the time delay using lengths ranging from 10 cm to 3.00 m (Fig. 4), from which

\[
Y = 1.76 X + 261.66
\]
we can extract a propagation speed, \( v \), of \((200 \pm 2) \times 10^3 \) km/s in good agreement with the manufacturer’s data.\(^3\) Moreover, we used this measurement to extract the dielectric constant of the insulator employed in the cable; recalling that
\[
v = c / \sqrt{\varepsilon_r}
\]
we obtain a value of the dielectric constant, \( \varepsilon_r \), of \( \varepsilon_r = 2.25 \pm 0.04 \), which can be compared with the polyethylene dielectric constant \( \varepsilon_r = 2.26 \), which is normally used as the insulator in this type of cable.\(^4\)

\section*{A simple method for measuring atmospheric pressure}

S. Velasco, A. González, F. L. Román, and J. A. White
Departamento de Física Aplicada, Facultad de Ciencias, Universidad de Salamanca, 37008 Salamanca, Spain

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A simple and low-cost experimental method for measuring atmospheric pressure is proposed. A 100 ml glass syringe is used. The only theoretical requirement is Boyle’s law for ideal gases. © 2002 American Association of Physics Teachers.

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Modern glass syringes with ground glass plungers are common in undergraduate physics labs. When clean and dry, the plunger slides with little friction, but it does not completely seal the syringe against the escape or entry of air. Hence, when the plunger is placed in the zero volume position and the needle end is closed, if one pulls the plunger with a strong enough force the plunger moves slowly due to the entry of air. This \textit{deficiency} of glass syringes is used here to measure atmospheric pressure.

Figure 1 shows the experimental setup. A 100-ml glass syringe\(^1\) is mounted vertically in a metallic support with the help of two clamps. A plastic attachment is placed around the top of the syringe plunger and an inextensible thread is tied at the center of this attachment. The thread passes through a pulley (4-cm diam) and a metallic hook is tied at the other end, from which small metallic, circular masses (2.5, 5, and 10 g) are hung to counterbalance the weight of the plunger and the plastic attachment. With the syringe needle end open and the plunger placed in the middle (50-ml) position, the number of circular masses is adjusted until the plunger goes down only very slowly. In this way one also compensates partially for the small static frictional force between the plunger and the syringe wall.

Next, the plunger is placed at the zero volume position and the needle end is closed with the help of a silicone tube and a small pincer. A mass \( M \) is hung on the metallic hook. The plunger then begins to lift as a consequence of the slow entrance of air into the syringe. This motion is slow enough to assume that equilibrium prevails and that the pressure of the air within the syringe is given at any instant by
\[
P_f = \frac{Mg}{A} - P_a
\]
where \( P_a \) is atmospheric pressure, \( g \) is the acceleration due to gravity, and \( A \) is the cross-sectional area of the plunger. We assume the temperature remains constant and only the amount of air within the syringe changes during this process. The mass \( M \) is removed when the plunger reaches a given mark corresponding to a volume \( V_f \); the plunger then falls until it reaches an equilibrium position with practically no leak of air. To ensure that the air within the syringe is in thermal and mechanical equilibrium with the environment, one must wait for two or three minutes. The volume \( V_e \) corresponding to the final position of the plunger is recorded.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Experimental setup for measuring atmospheric pressure.}
\end{figure}
The pressure of the air within the syringe is now equal to the atmospheric pressure, \( P_a = P_2 \). Assuming that air behaves as an ideal gas, and the initial and final temperatures are the same, Boyle’s law, \( P_1 V_1 = P_2 V_2 \), allows one to obtain

\[
P_a = \frac{M g V_1}{A (V_1 - V_2)}.
\]

The volume \( V_2 \) is measured by an auxiliary scale (plotted on paper and glued to the syringe) with 0.5-ml scale divisions (the syringe has 1-ml scale divisions) that can be read with an uncertainty of \( \pm 0.25 \) ml. The cross-sectional area \( A \) was calculated from \( A = \pi d^2/4 \), where the diameter \( d \) of the plunger is measured by a vernier caliper. As an example, in a typical experiment we obtained \( V_2 = 60.25 \pm 0.25 \) ml, with \( M = 1.000 \) kg, \( V_1 = 70.00 \pm 0.25 \) ml, \( g = 9.80 \) m/s\(^2\), and \( d = 3.10 \pm 0.01 \) cm. Substituting these data into Eq. (1), one obtains \( P_a = 93.2 \pm 5.7 \) kPa. The record of atmospheric pressure in a standard mercury barometer (corrections included) was \( P_a = 696 \) mm Hg (92.8 kPa), and in a digital barometer (pressure transducer) was \( P_a = 92.7 \) kPa. Therefore, the value from this simple syringe apparatus differs by less than 1% from the values of standard lab barometers.

Other data of interest in the experiment (although not necessary for calculating \( P_a \)) are the lab temperature \( T_1 = T_2 = 293 \) K (20°C), the mass of the plunger and the plastic attachment, \( m_{p+a} = 153.2 \) g, the mass of the hook and the circular masses, \( m_{h+m} = 129.3 \) g, the distance moved by the plunger from the zero volume position to the \( V_1 \)-volume position, \( h_1 = 9.3 \) cm, and the time spent during this movement, \( t_1 = 821 \) s. These data allow one to obtain a number of illustrative additional results such as the static frictional force on the plunger, \( f_r = (m_{p+a} - m_{h+m}) g = 0.058 \) N, the mean speed of the plunger, \( v = d_1/t_1 = 0.11 \) mm/s, and the mean flow of air entering into the syringe \( n = n/t_1 = P_1 V_1/(RT_1 t_1) = 2.8 \times 10^{-6} \) mol/s (8.1 \( \times 10^{-5} \) g/s), where we have used the equation of state for an ideal gas with \( R = 8.314 \) J mol\(^{-1}\) K\(^{-1}\).

The experiment was completed by performing measurements for a fixed mass \( M = 1.000 \) kg and different values of \( V_1 \) (Table I), and for a fixed value of \( V_1 = 70 \) ml and different values of \( M \) (Table II). In the first case we obtained a mean atmospheric pressure \( P_a = 92.5 \) kPa with a standard deviation of \( \Delta P_a = 1.4 \) kPa. In the second case we obtained a mean atmospheric pressure \( P_a = 92.8 \) kPa with a standard deviation of \( \Delta P_a = 0.4 \) kPa.

In summary, we have described a method to measure atmospheric pressure without a barometer. The simplicity and low cost of the experiment makes it suitable for use in introductory physics laboratory courses.

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1The glass syringe used in this experiment is available from PHYWE SYSTEME GMBH, D-37070 Göttingen, Germany, model 02614.00.