Diffusion and entropy production for multi-networks with fitness factors

ECCS'13 WarmUp FlashTalk Session

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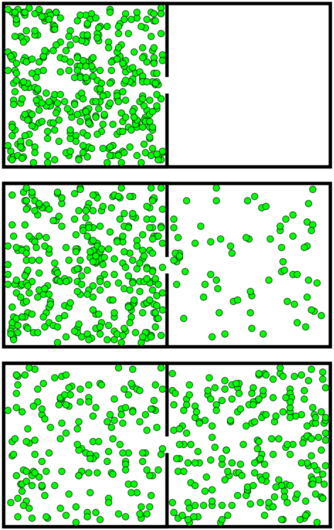
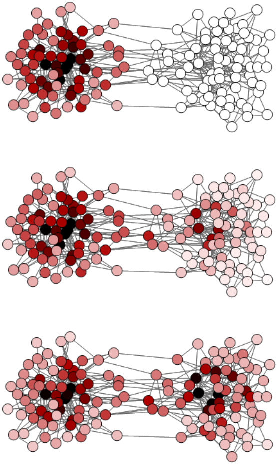
Diffusion

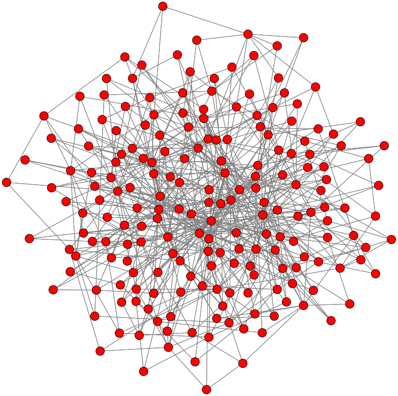


Diffusion on networks

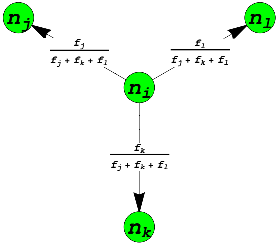


Networks - definitions

- A_{ij} - adjacency matrix of network,
- $n_i(t)$ - number of particles,
- $\sum_{i=1}^{M} n_i(t) = N$,
- f_i - fitness factor,



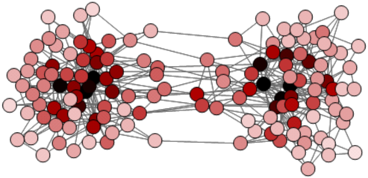
Diffusion on networks - Markov chain approach



$$n_i(t+1) = f_i \sum_{k=1}^{M} \frac{A_{ik}}{g_k} n_k(t), \qquad g_k = \sum_l A_{lk} f_l \qquad \mu_i = \frac{f_i g_i}{\sum_{k=1}^{M} f_k g_k},$$

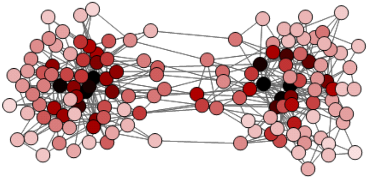
Equilibrium (ergodic) distribution

$$\mu_i = \frac{f_i g_i}{\sum_{k=1}^{M} f_k g_k}$$

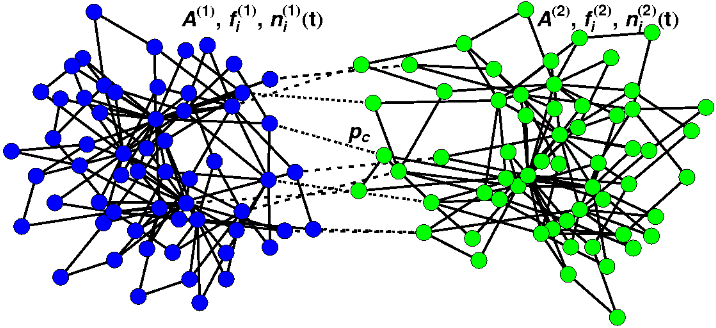


Entropy of equilibrium distribution

$$S_\infty = -\frac{(f_i g_i \log(f_i g_i))_{\mathrm{av}}}{(f_i g_i)_{\mathrm{av}}} + \log(f_i g_i)_{\mathrm{av}} + \log M.$$



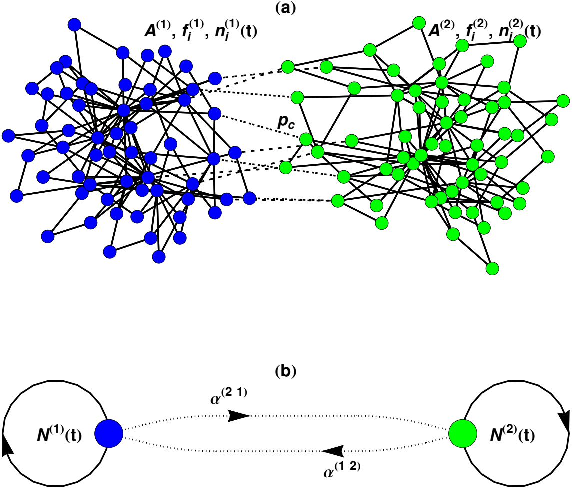
Coupled networks



Why diffusion on complex networks?

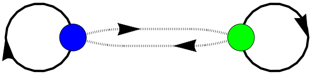
- Social and technical networks have this structure.
- ,,Physical" diffusion between two crystals.
- joke?

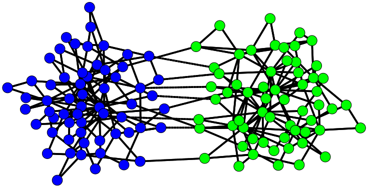
Weak connection



Weak connection - assumptions

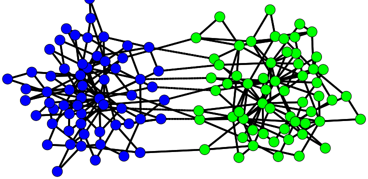
$$g_i^{(a)} \gg p_c M^{(b)} f_{\mathrm{av}}^{(b)},\ i = 1, 2, \ldots, M^{(a)},$$

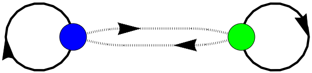




Separation of time scales

$$n_i^{(a)}(t) = N^{(a)}(t)\mu_i^{(a)}.$$



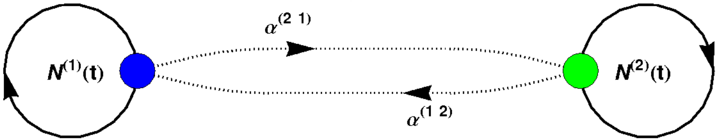


Result of our approach

$$N^{(1)}(t+1) = (1-\alpha^{(2\,1)})N^{(1)}(t) + \alpha^{(1\,2)}N^{(2)}(t),$$

where $\alpha^{(a\,b)}$ are constants dependent on $A^{(1)}$, $A^{(2)}$, p_c, $\mathbf{f}^{(1)}$, $\mathbf{f}^{(2)}$ as follows

$$\alpha^{(b\,a)} = p_c M^{(b)} \frac{f_{\rm av}^{(a)} f_{\rm av}^{(b)}}{(f_i^{(a)} g_i^{(a)})_{\rm av}}$$



First Fick's Law

For the diffusion on networks

$$\dot{N}^{(1)}(t) = -\left(N^{(1)}(t) - N^{(2)}(t)\right) D + FN,$$

where

$$D = \frac{\alpha^{(1\,2)} + \alpha^{(2\,1)}}{2}, \qquad F = \frac{\alpha^{(2\,1)} - \alpha^{(1\,2)}}{2}.$$

For classical diffusion

$$J = -D\frac{\partial \phi}{\partial x}$$

Entropy and entropy production

$$S_{\text{total}}(t) = \left[N^{(1)}(t)S_\infty^{(1)} + N^{(2)}(t)S_\infty^{(2)}\right] +$$
$$\left[-N^{(1)}(t)\log N^{(1)}(t) - N^{(2)}(t)\log N^{(2)}(t) + N\log N\right],$$

$$\sigma(t) = \dot{N}^{(1)}(t)\left(\log N^{(2)}(t) - \log N^{(1)}(t) + S_\infty^{(1)} - S_\infty^{(2)}\right),$$

where $S_\infty^{(a)}$, $a = 1, 2$, are equilibrium entropies per particle.

General case

Networks of Networks - NON

For m weakly[a] coupled networks

$$N^{(a)}(t+1) = \sum_{l=1}^{m} \alpha^{(a\,l)} N^{(l)}(t),$$

$$\alpha^{(a\,b)} = p_c^{(a\,b)} M^{(a)} \frac{f_{\mathrm{av}}^{(a)} f_{\mathrm{av}}^{(b)}}{(f_i^{(b)} g_i^{(b)})_{\mathrm{av}}}, \ \alpha^{(a\,a)} = 1 - \sum_{r \neq a} \alpha^{(r\,a)}.$$

[a]For m networks weak coupling means something different than for two.

Analogues of Fick's First Law for NONs

$$\dot{N}^{(a)}(t) = \sum_{l \neq a} [-D^{(l\,a)}(N^{(a)}(t) - N^{(l)}(t)) + F^{(l\,a)}(N^{(a)}(t) + N^{(l)}(t))],$$

where

$$D^{(a\,b)} = \frac{\alpha^{(a\,b)} + \alpha^{(b\,a)}}{2}, \qquad F^{(a\,b)} = \frac{\alpha^{(b\,a)} - \alpha^{(a\,b)}}{2}.$$

Hierarchical networks

$$\hat{f}_a = M^{(a)} f_{\text{av}}^{(a)}, \ \hat{A}_{ab} = p_c^{(a\,b)}, \text{ for } \ a \neq b,$$

$$\hat{A}_{aa} = \frac{(f_i^{(a)} g_i^{(a)})_{\text{av}}}{M^{(a)} \left(f_{\text{av}}^{(a)}\right)^2} \left(1 - \sum_{r \neq a} \alpha^{(r\,a)}\right).$$

I heard you like networks...

So we put a network into a network, so you can diffuse while you diffuse!

Conclusions

With our approach...

- we found Fick's Law analogous for the diffusion on weakly coupled networks,
- we calculate diffusion constant and external force,
- also for the NONs,
- propose tool for investigation diffusion on the hierarchical network, with different diffusion constant on every level of hierarchy.

For those who prefer read rather than listen ☺

G. Siudem, J. A. Hołyst, *Diffusion and entropy production for multi-networks with fitness factors*, arXiv:1303.2650 [nlin.CD], (2013).

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