Social Networks & Recommendation Systems

II. Historical overview of the complex network science. Examples of the real-life networks.

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MSc program in Data Science has been developed
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co-funded by European Union from European Social Fund.

Before classes

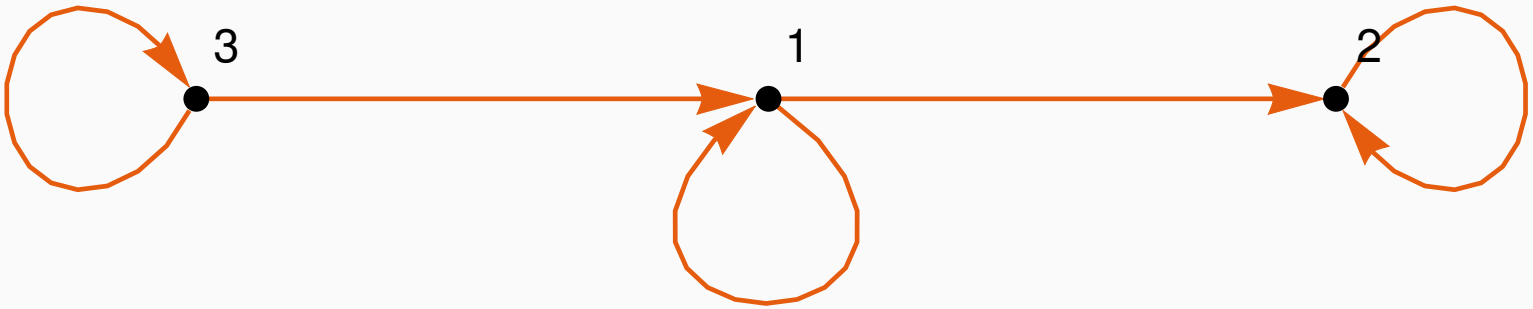
Remember: graph representation methods

Adjacency matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Adjacency list

$$L = \{\{1,2\},\{2\},\{1,3\}\}.$$



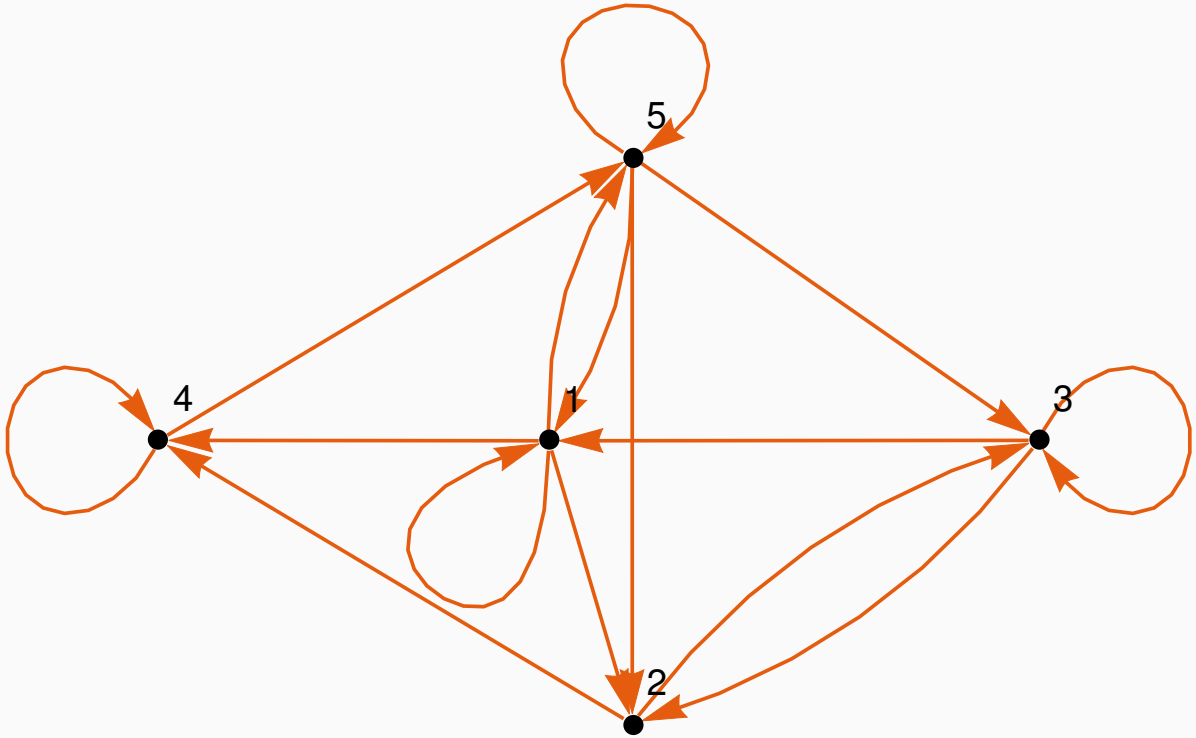
Remember: vertex degree

Vertex degree

Number of incoming or outcoming edges.

$$k_{out} = \{4, 2, 3, 2, 4\},$$

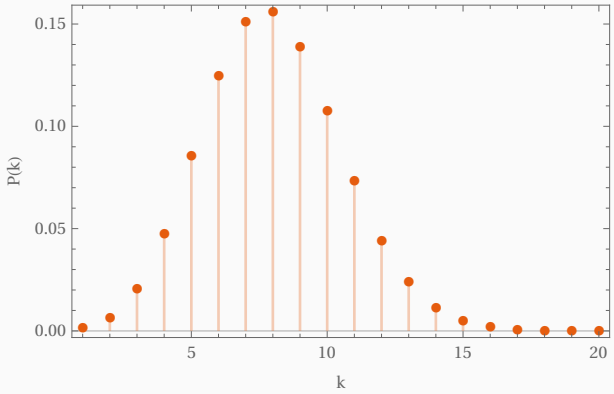
$$k_{in} = \{3, 3, 3, 3, 3\}.$$

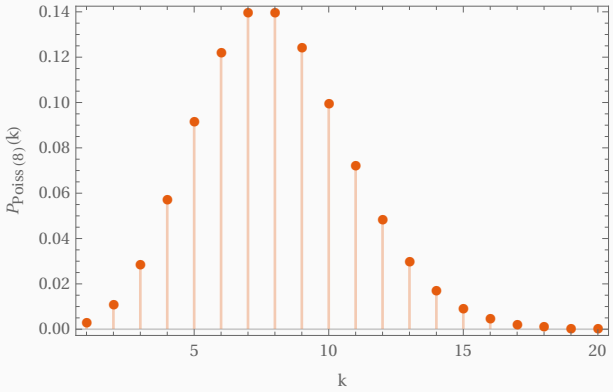


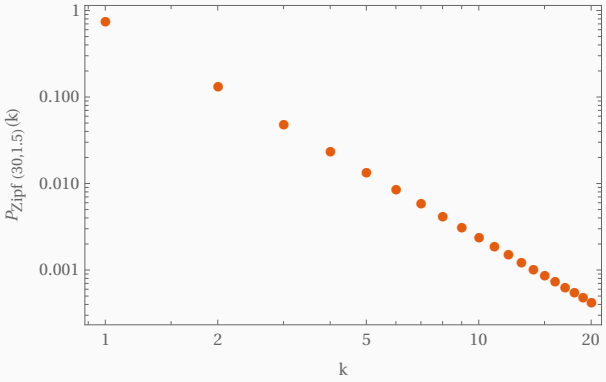
Remember: discrete probability distributions

Discrete probability distributions – examples

- binomial distribution,
- Poissona distribution,
- Zipf distribution.







$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(k) = \frac{1/k^s}{H_{N,s}}$$

Reminder

$$\text{Binom}(N, p) \to \text{Poiss}(Np)$$

Exercise to think about - continuous distributions

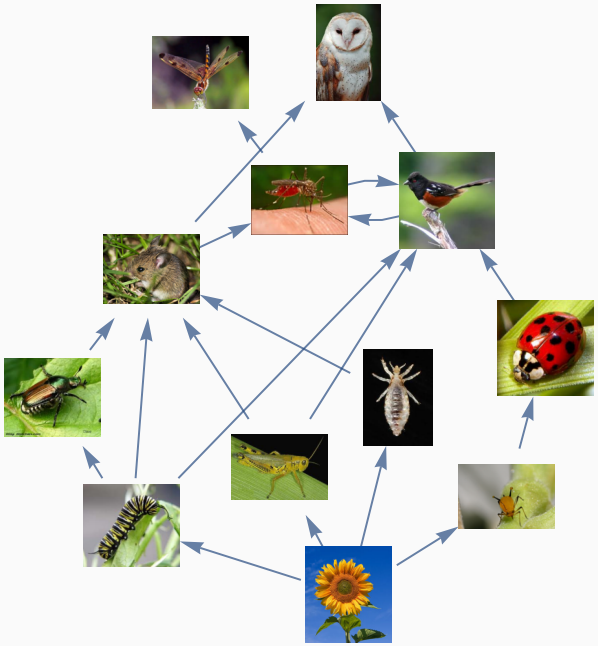
In complex networks, you often replace discrete distributions with continuous ones (this is the way we'll think during class).
Find continuous analogs of distributions from the previous slide.

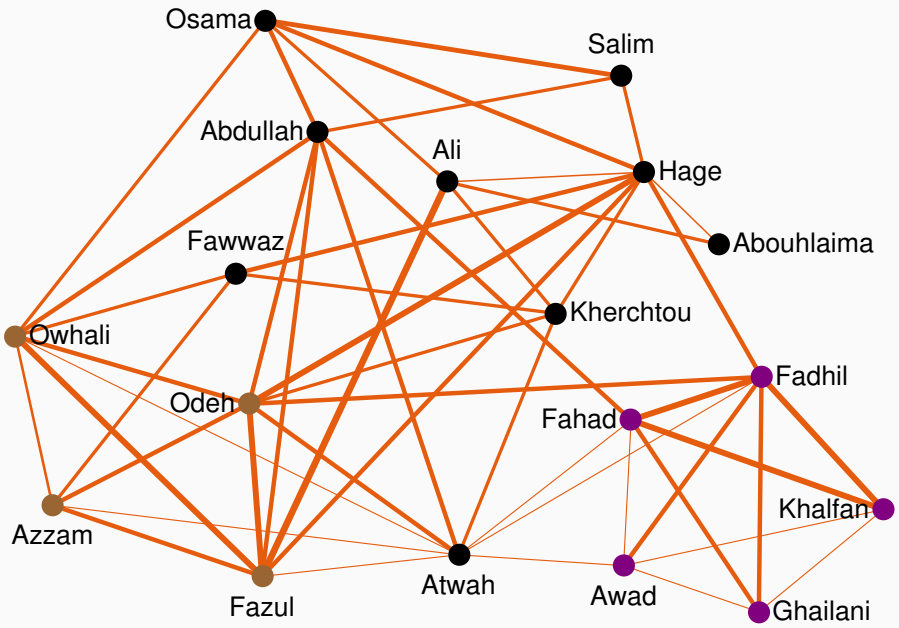
Lecture

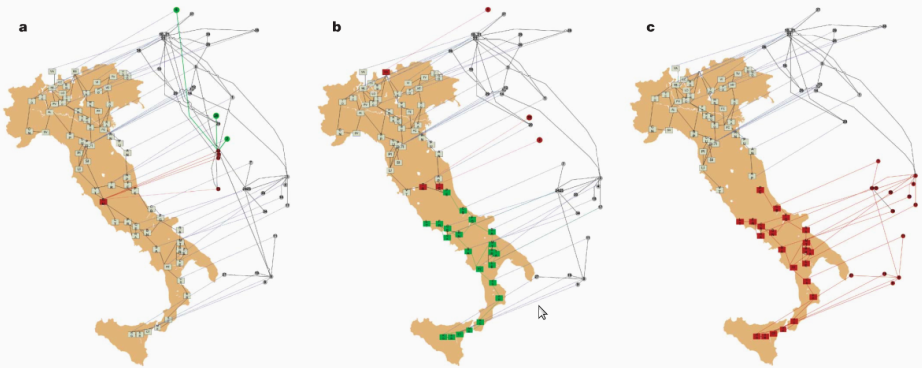
Why are networks/graphs so useful?

Graphs are an illustration of a relationship

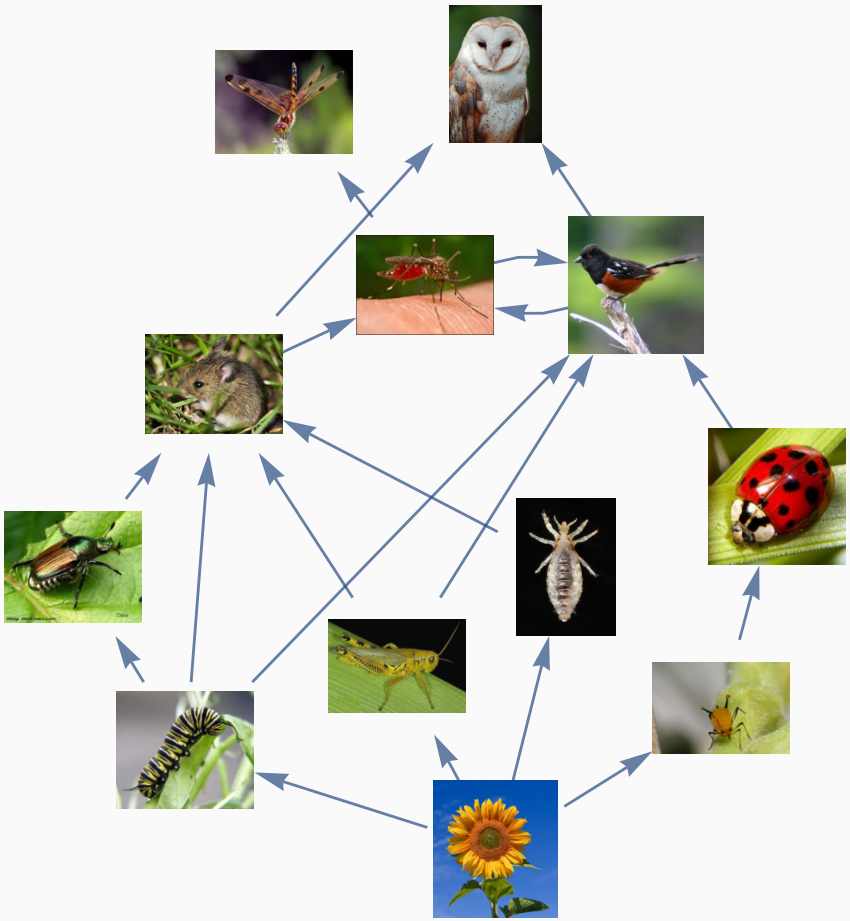
What relations are illustrated in the following graphs?

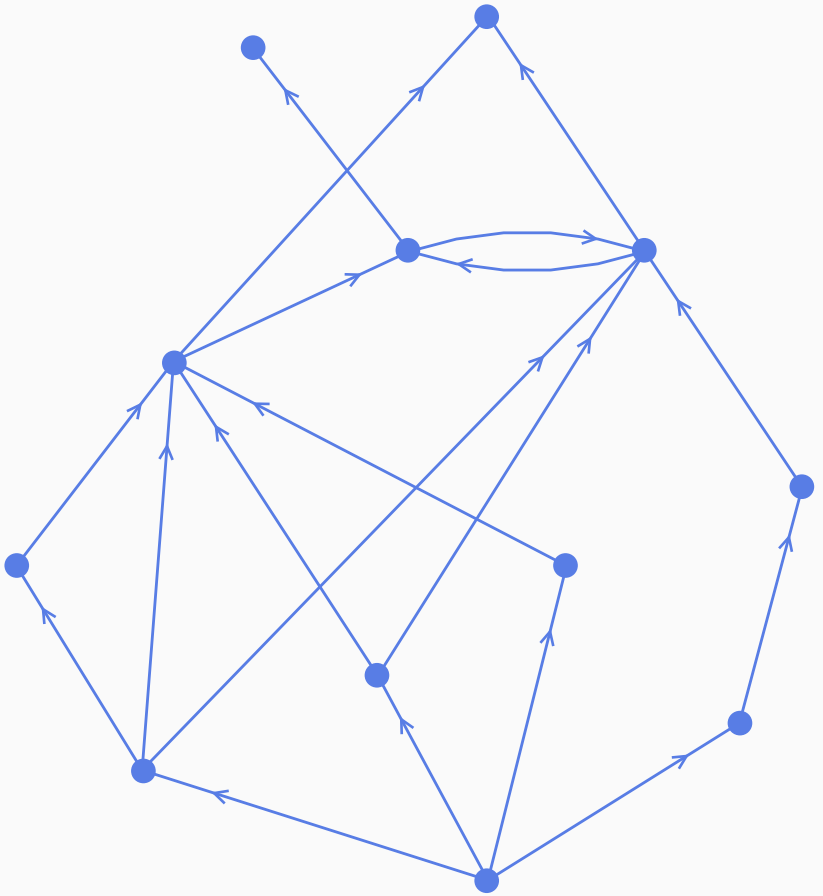






What is the difference between graphs and networks?





Why networks are complex?

Properties of the real networks

- distributions with fat tails.

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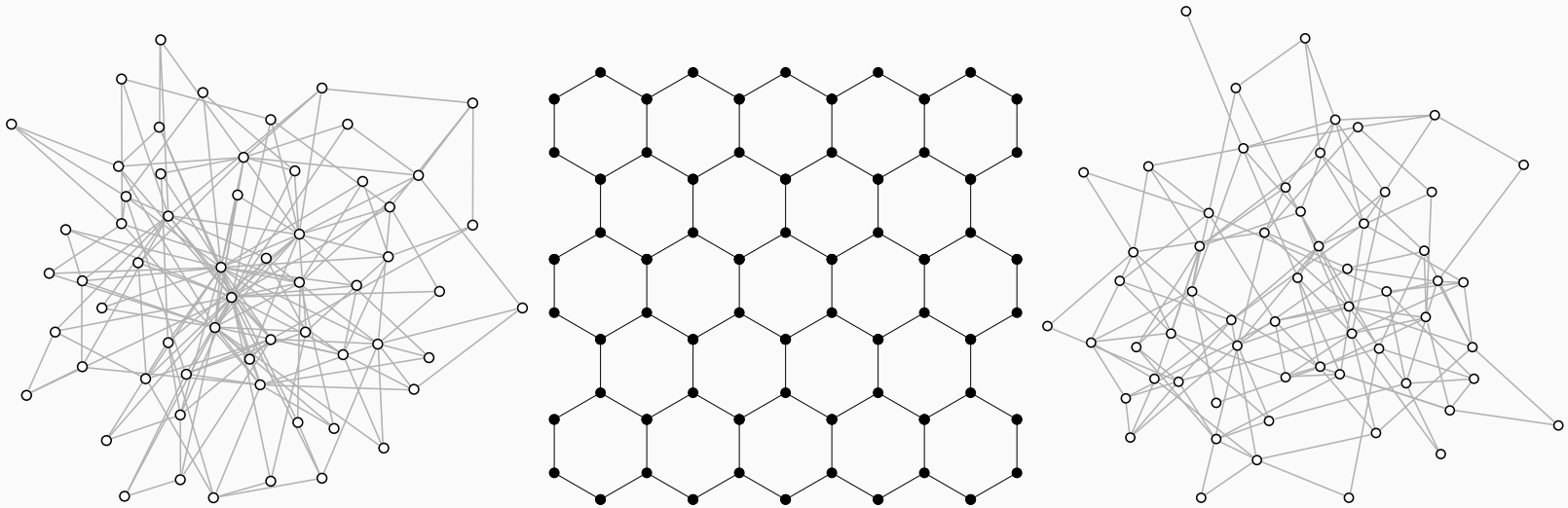

Which of these graphs represents the real network?

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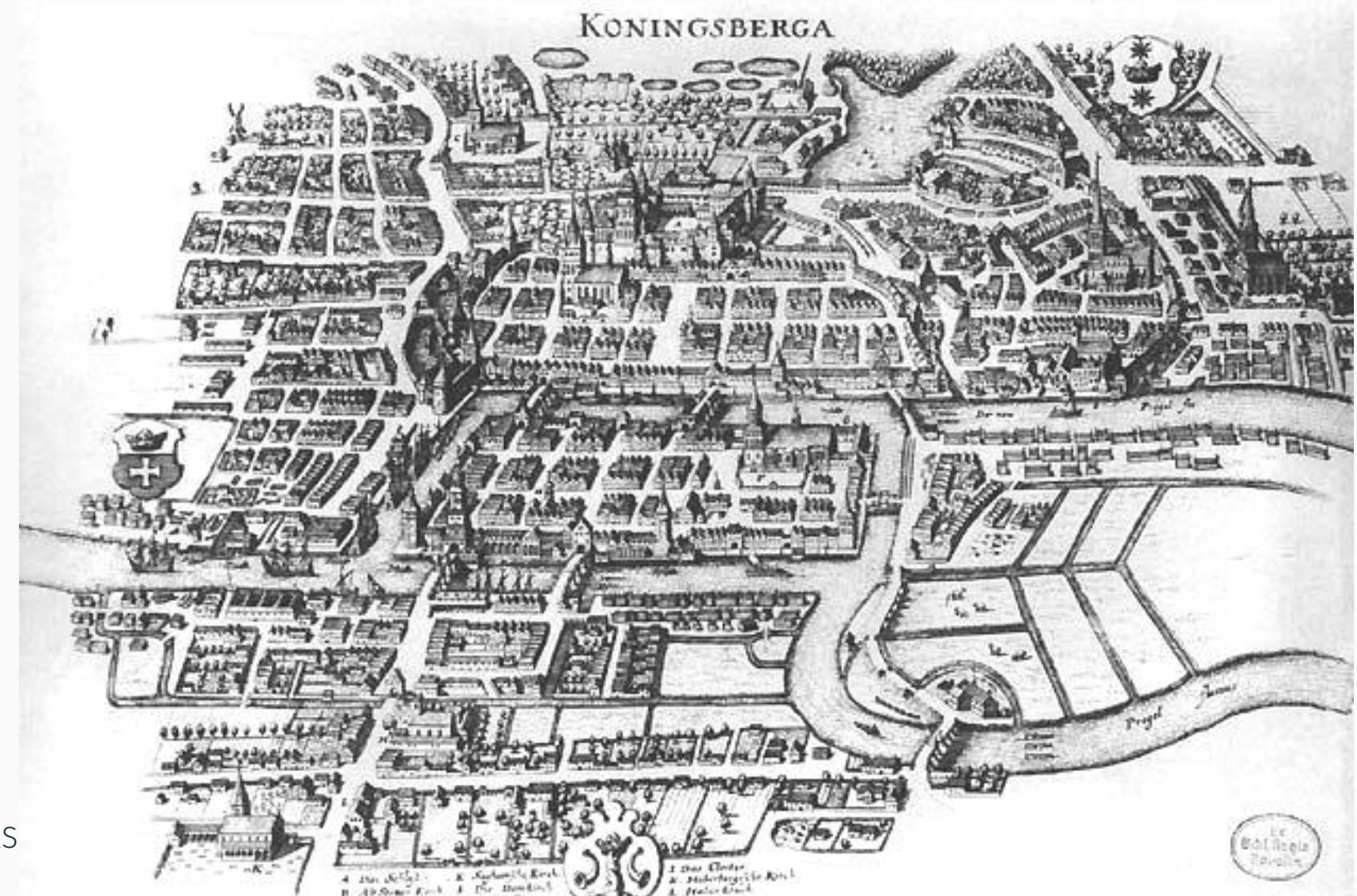
with expert knowledge from

- sociology,
- economics,
- biology,
- medicine,
- engineering,
- and many other...

The story of network science – Seven Bridges of Königsberg (1736)



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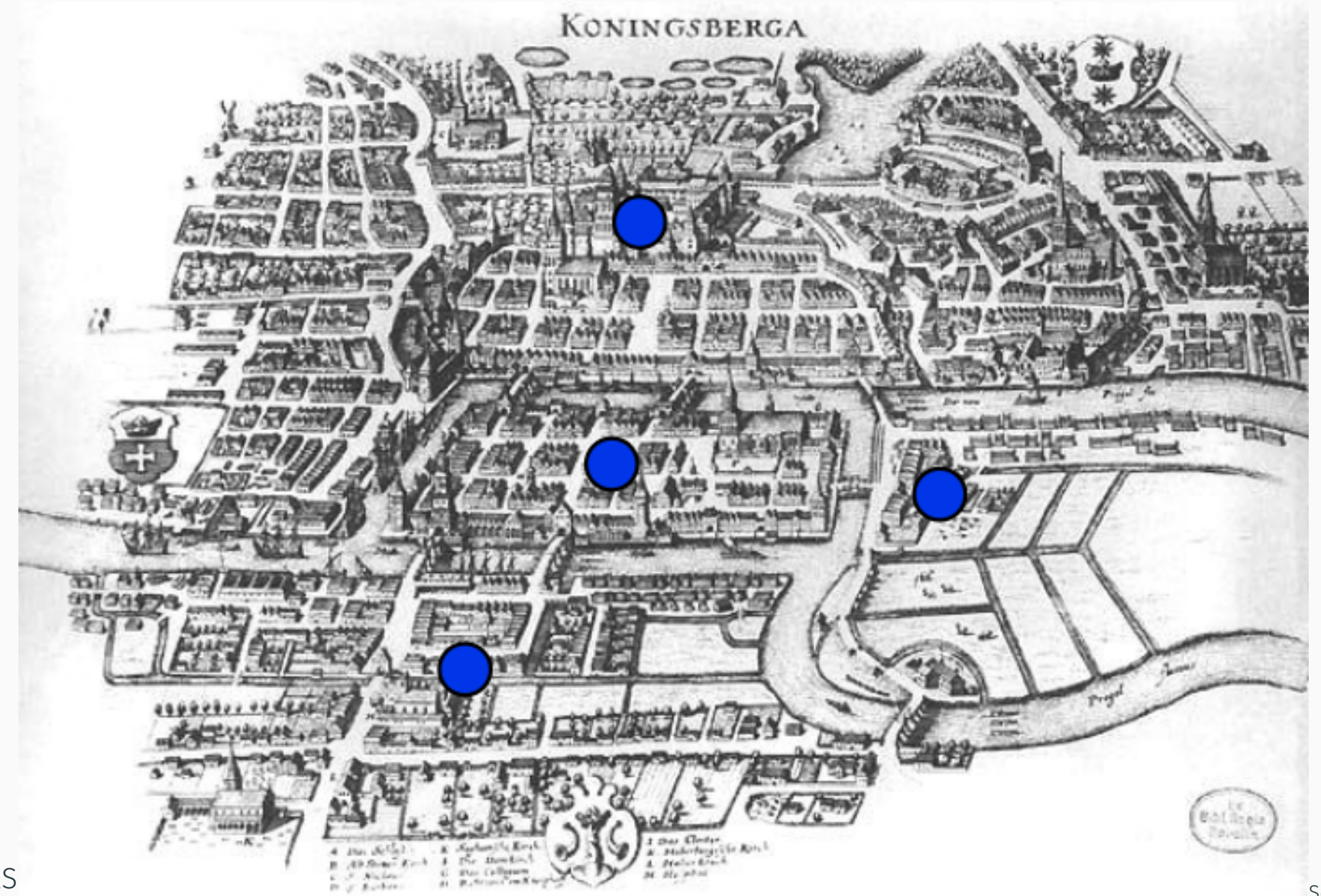

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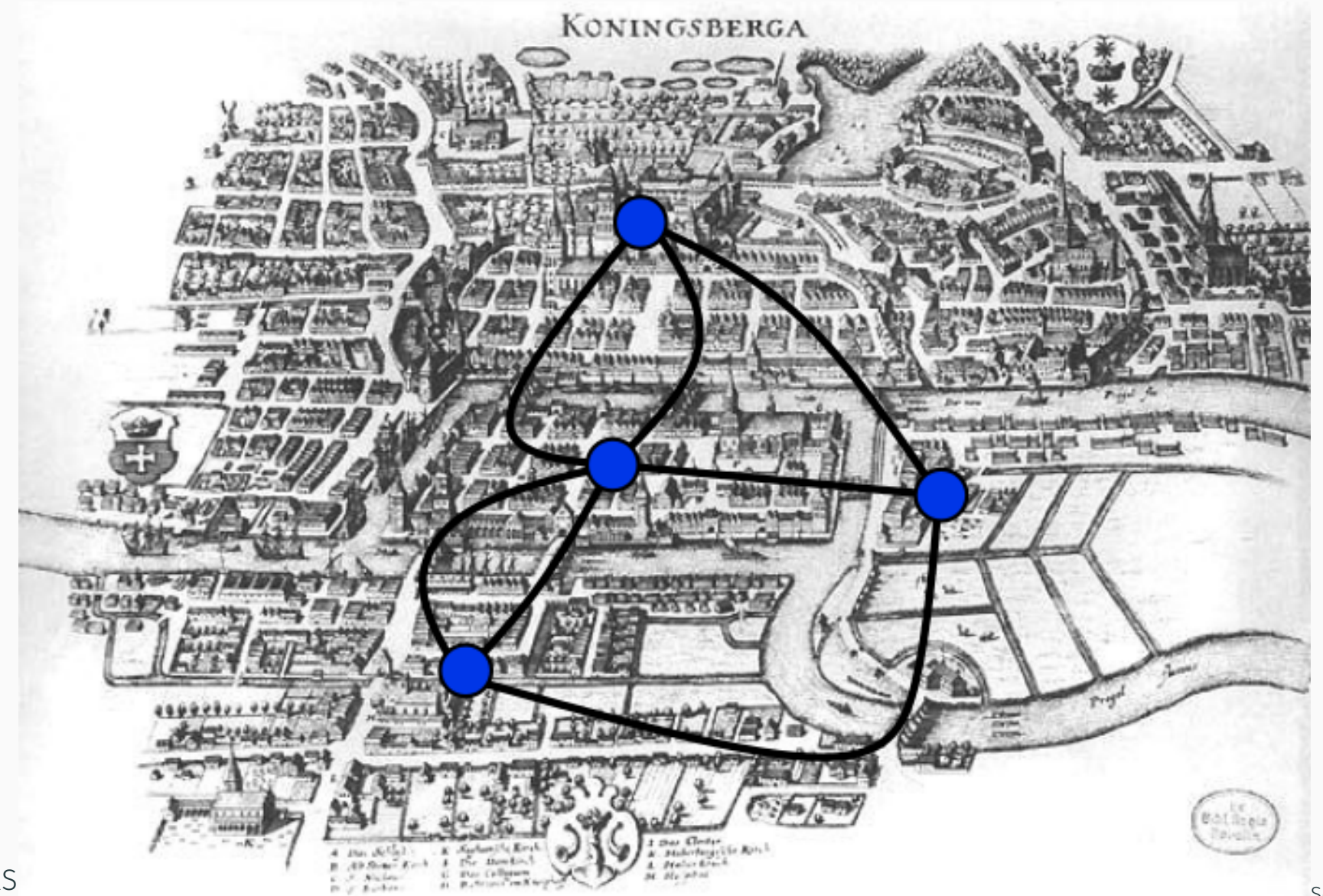

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- we do not focus on this line of research (with one exception!).

The story of network science – Erdös i Rényi (1960)

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More details on Lecture 5.

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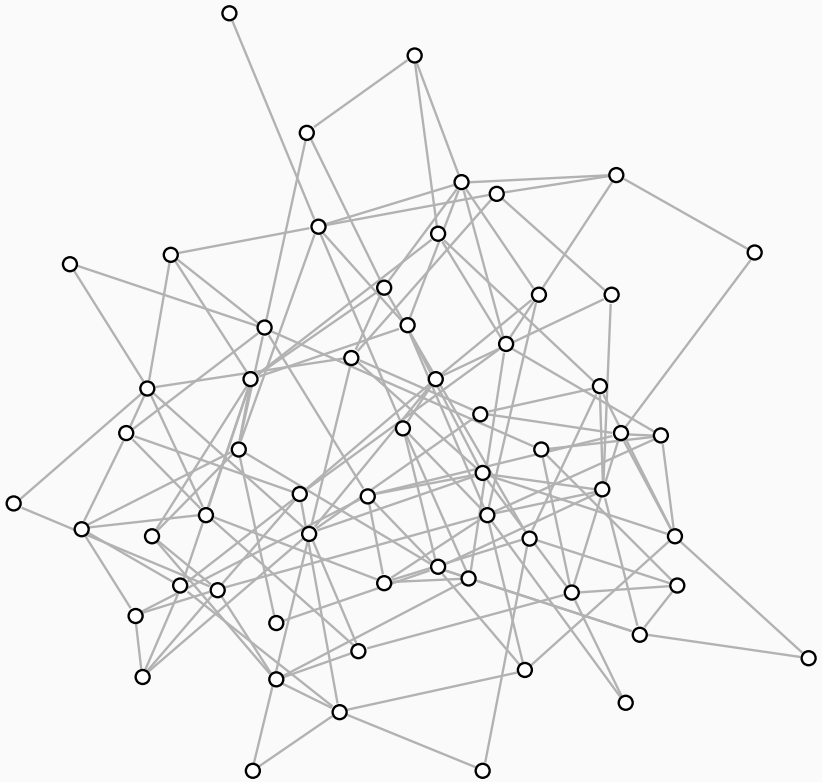
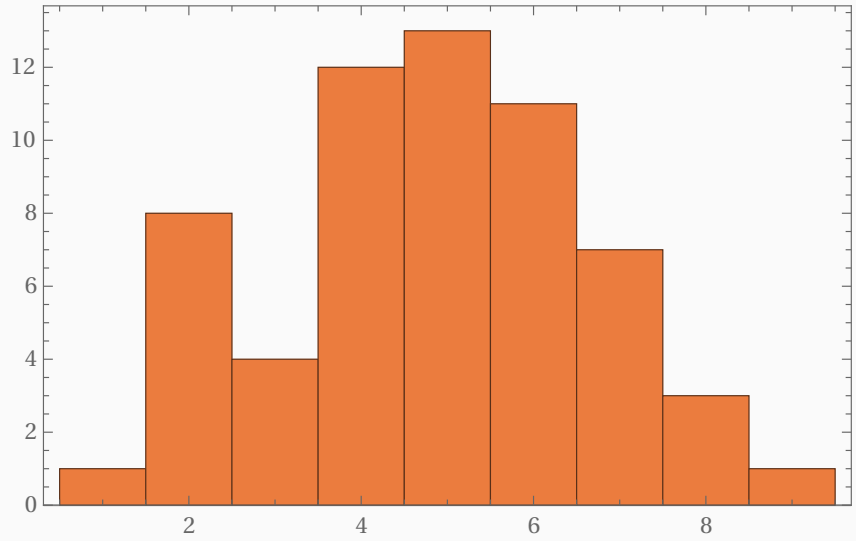
Degree distribution

$$P(k) = \text{fraction of vertices with degree } k$$





Is this a real network?

The story of network science – Milgram experiment (1967)

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Do not confuse the *correspondence* experiment with the famous experiment of obedience to authorities!

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- If the recipient of the letter knew the addressee personally, he/she should deliver it to him/her, otherwise he/she should send it to another person who she/he suspected might know the addressee.

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More details during Lecture 4.

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Problem:

Poissonian graphs do not have small world property.

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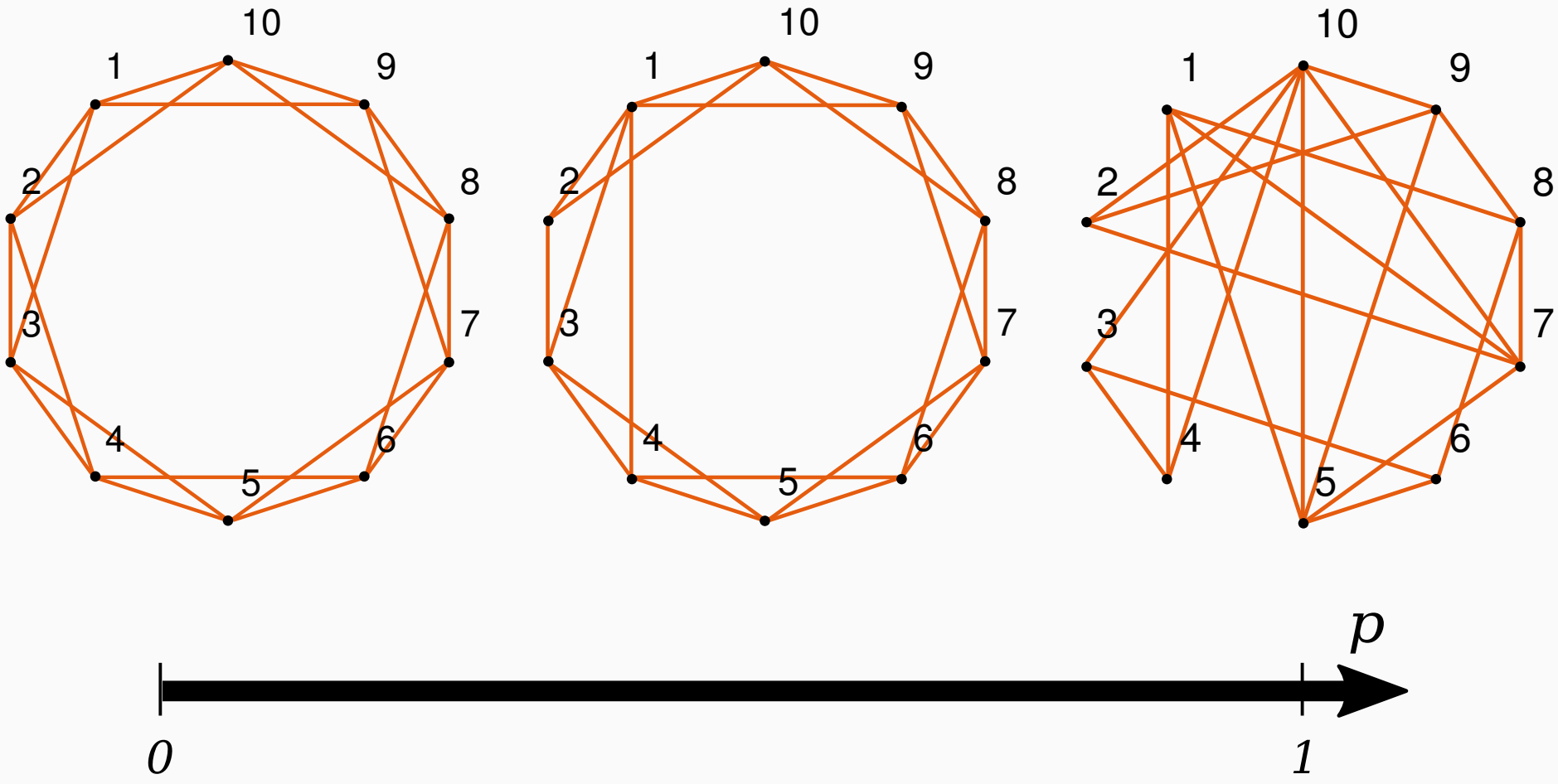
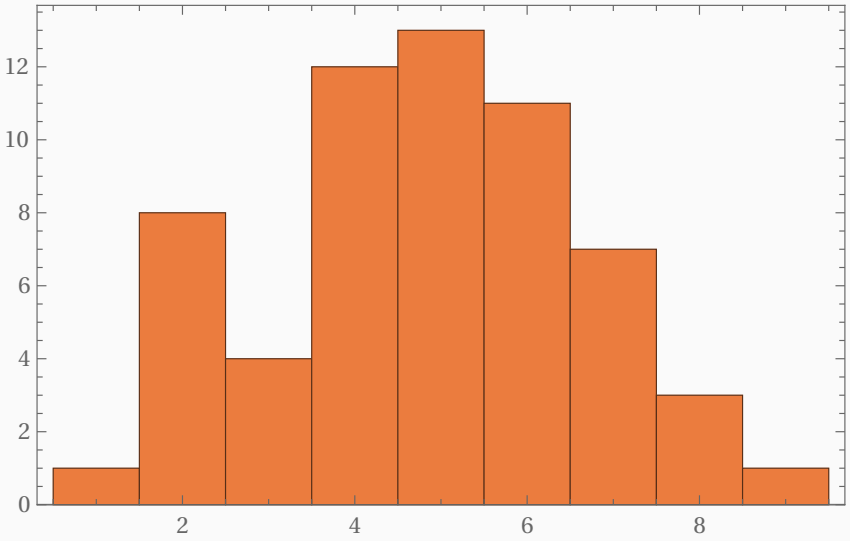
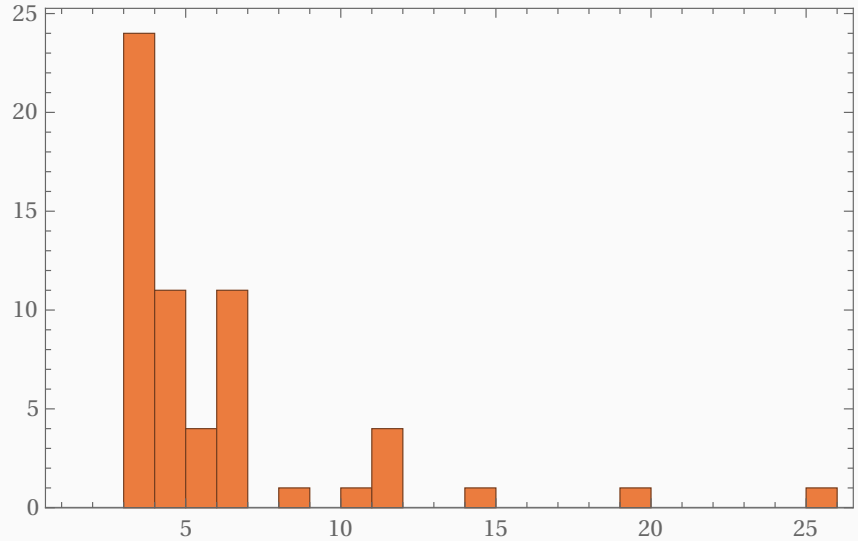


The story of network science – Barabási-Albert model (1999)

Problem:

Real-world networks usually have power law like distributions.





What does it means?

- No typical scale.
- Fat tails.
- Fast spreading epidemics...

The story of network science – Barabási-Albert model (1999)

BA model

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Both assumptions are necessary!

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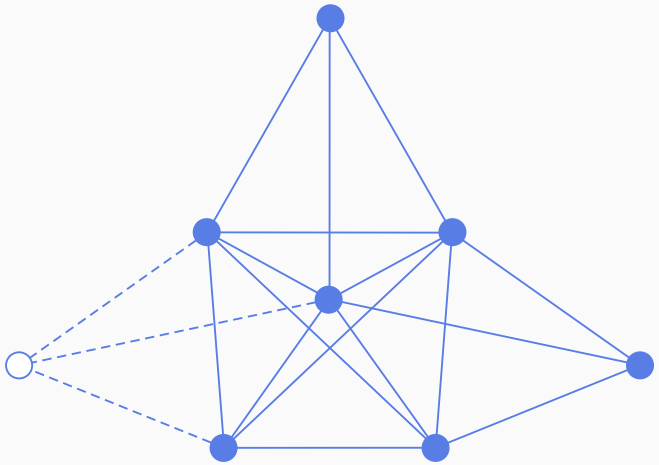

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More about BA model on Lecture 5.

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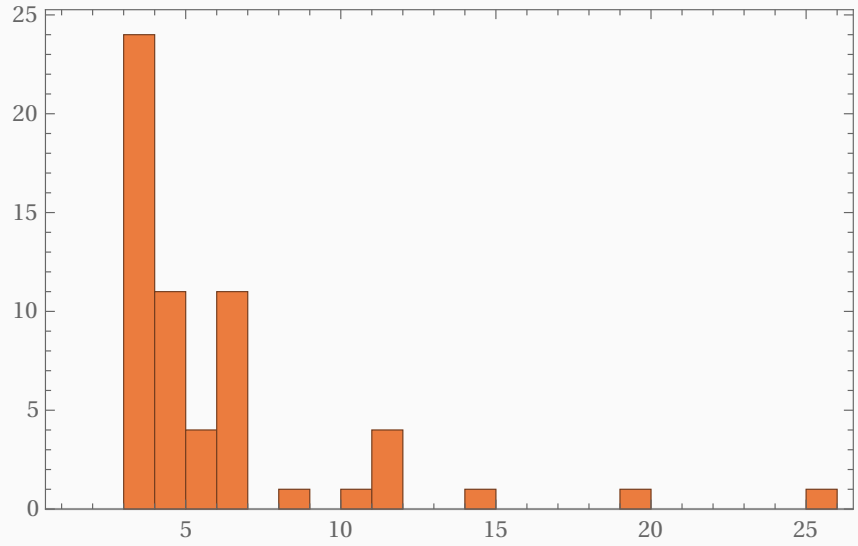
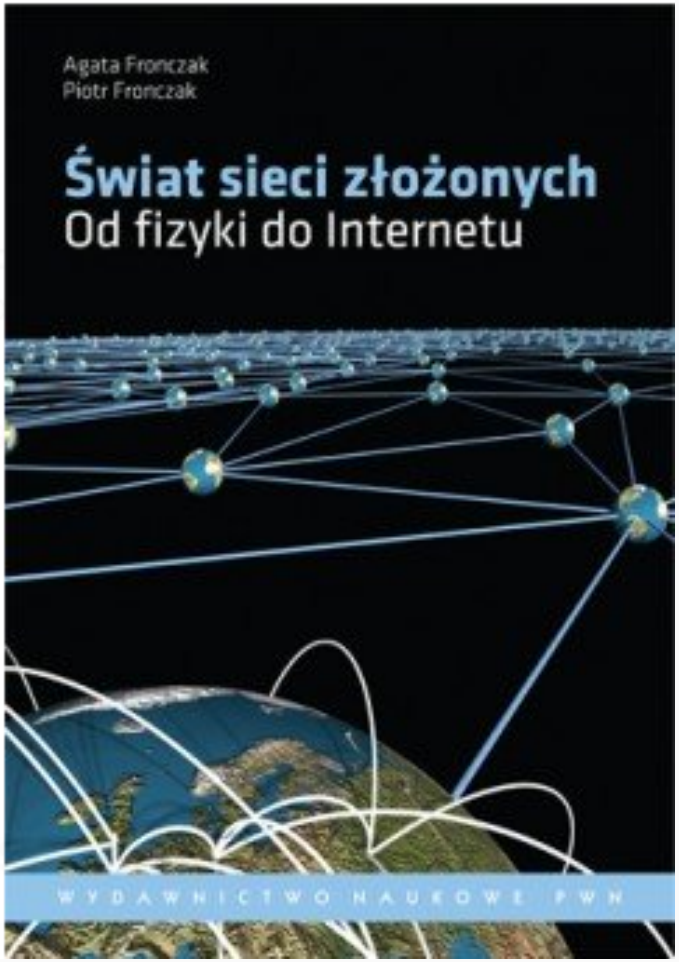
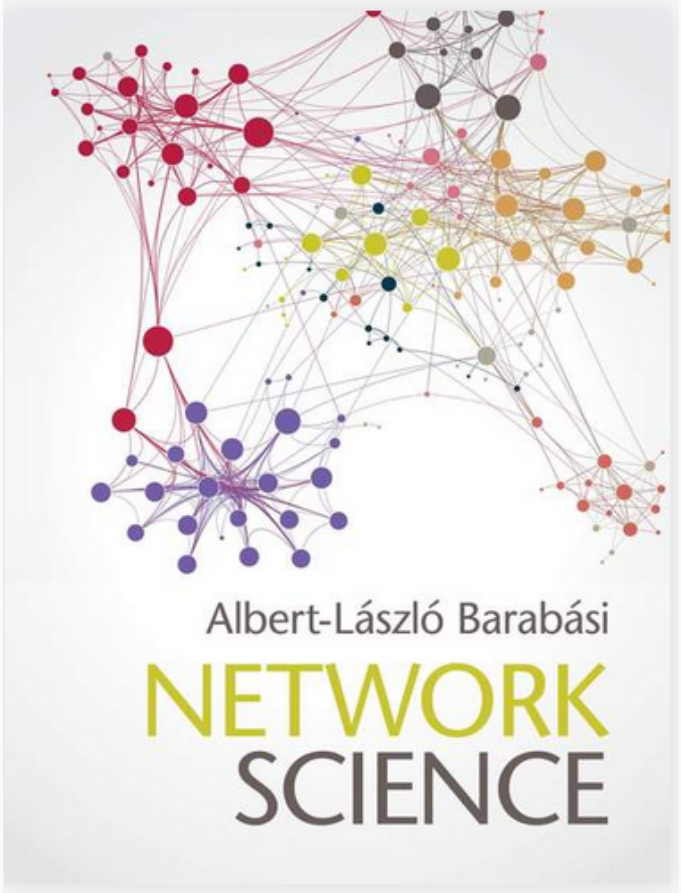


More on Lecture 5.

If you are interested in the history of complex networks read





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- Theoretical complex networks models (Lectures 5,6,9,10).
- Theoretical models of the dynamics in networks (Lectures 11,12).

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The main task of network science is to model real phenomena and processes using complex networks, which are appropriate graphs.

Typical tools:

- Quantitative description of the network, including
  - proposing new network metrics (Lecture 4).
  - discussion of their properties (Lectures 4,8).
  - measuring with them real networks (Lectures 4,8, 13-15).
  - Description of the dynamical processes within real networks (e.g. epidemics, news spreading, etc.) (Lecture 8,11-15).
- Theoretical complex networks models (Lectures 5,6,9,10).
- Theoretical models of the dynamics in networks (Lectures 11,12).
- Visualization of networks (both theoretical and real) (Lecture 3).

Thank you for your attention!

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