

Operator ∇ (nabla)

$$(01) \nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right]$$

$$(02) \nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right] \text{ (gradient)}$$

$$(03) \nabla \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \text{ (dywergencja)}$$

$$(04) \nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right] \text{ (rotacja)}$$

$$(05) \Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ (laplasjan)}$$

$$(06) \Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$(07) \Delta \mathbf{A} = [\Delta A_x, \Delta A_y, \Delta A_z]$$

Podstawowe własności ∇

$$(08) \nabla(fg) = g\nabla f + f\nabla g$$

$$(09) \nabla(f\mathbf{A}) = \mathbf{A}\nabla f + f\nabla \mathbf{A}$$

$$(10) \nabla(\mathbf{A}\mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A}\nabla)\mathbf{B} + (\mathbf{B}\nabla)\mathbf{A}$$

$$(11) \nabla(\mathbf{A} \times \mathbf{B}) = \mathbf{B}(\nabla \times \mathbf{A}) - \mathbf{A}(\nabla \times \mathbf{B})$$

$$(12) \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(13) \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B}\nabla)\mathbf{A} - (\mathbf{A}\nabla)\mathbf{B} + \mathbf{A}(\nabla\mathbf{B}) - \mathbf{B}(\nabla\mathbf{A})$$

$$(14) \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla\mathbf{A}) - \Delta\mathbf{A}$$

$$(15) \nabla(\nabla \times \mathbf{A}) = 0$$

$$(16) \nabla \times (\nabla f) = 0$$

Inne

$$(17) \oint \mathbf{A} \, d\mathbf{a} = \int \nabla \mathbf{A} \, d\tau \text{ (twierdzenie Gaussa)}$$

$$(18) \oint \mathbf{A} \, d\mathbf{l} = \int \nabla \times \mathbf{A} \, d\mathbf{a} \text{ (twierdzenie Stokesa)}$$