Fermionic turbulence within the framework of time dependent density functional theory

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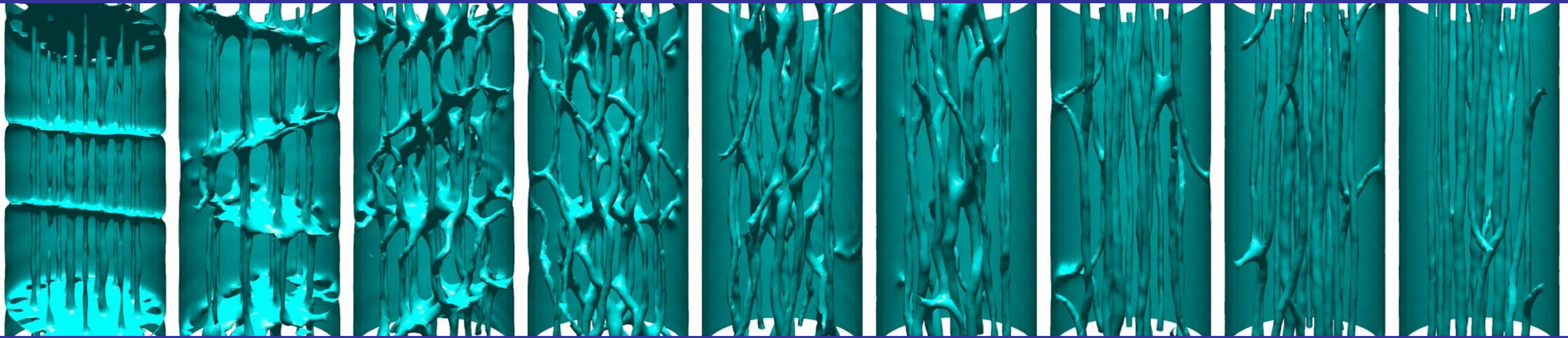

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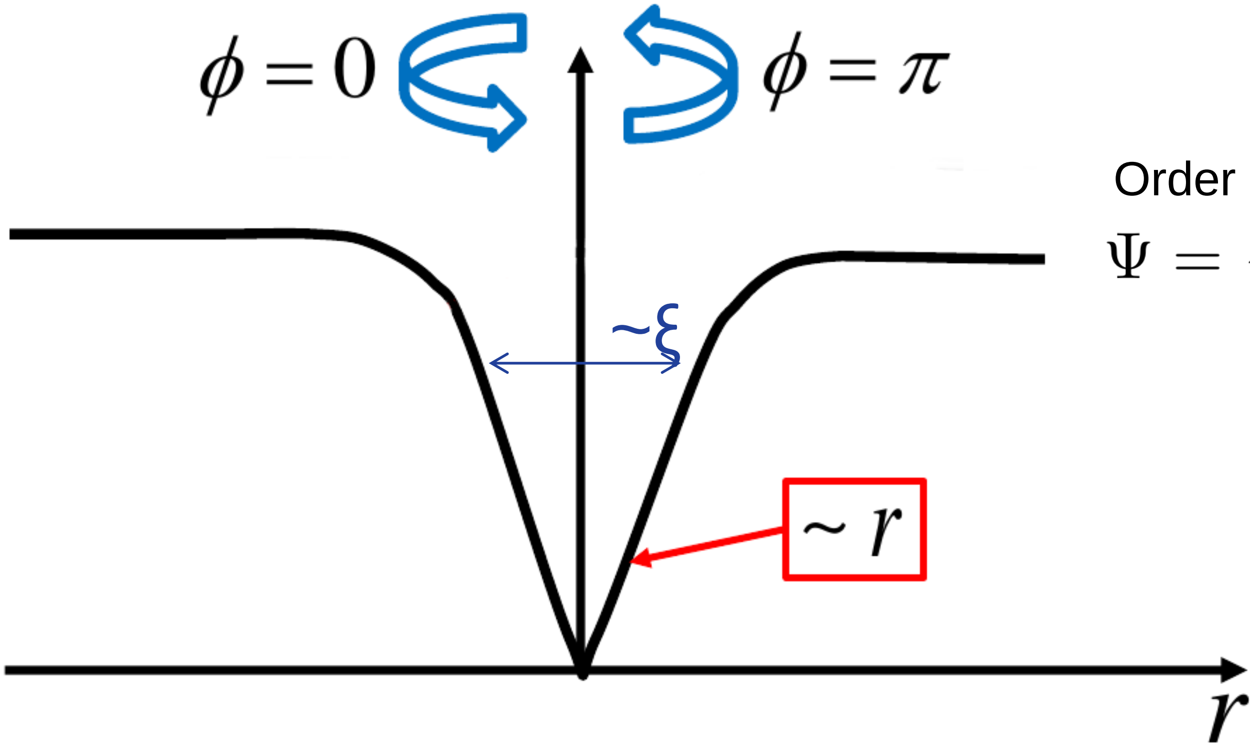
OUTLINE

- Vortex structure in spin imbalanced systems
- Time dependent density functional theory for superfluid Fermi gas
- Solitonic cascades: theory vs experiment
- Preliminary studies of vortex tangles

Anatomy of the vortex core

BOSONS:

Vortex structure: Bose gas → Gross-Pitaevskii eq. (GPE)



Order parameter:

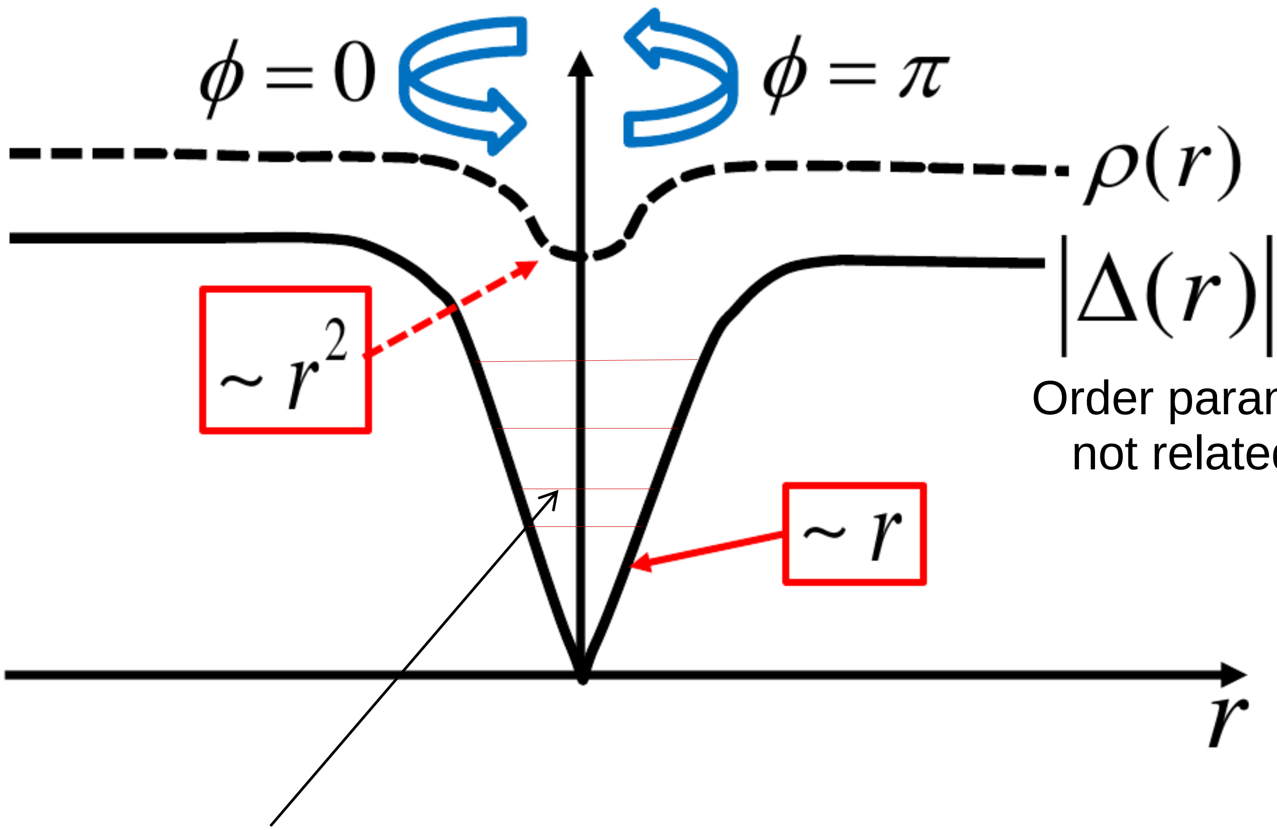
$$\Psi = \sqrt{\rho(r)}e^{i\phi}$$

$$\boldsymbol{v}_s = \frac{\hbar}{M}\boldsymbol{\nabla}\phi$$

$$\kappa = \oint d\boldsymbol{l}\cdot\boldsymbol{v}_s = \frac{h}{M}$$

FERMIONS:

Vortex structure: Fermi gas → BdG eq.



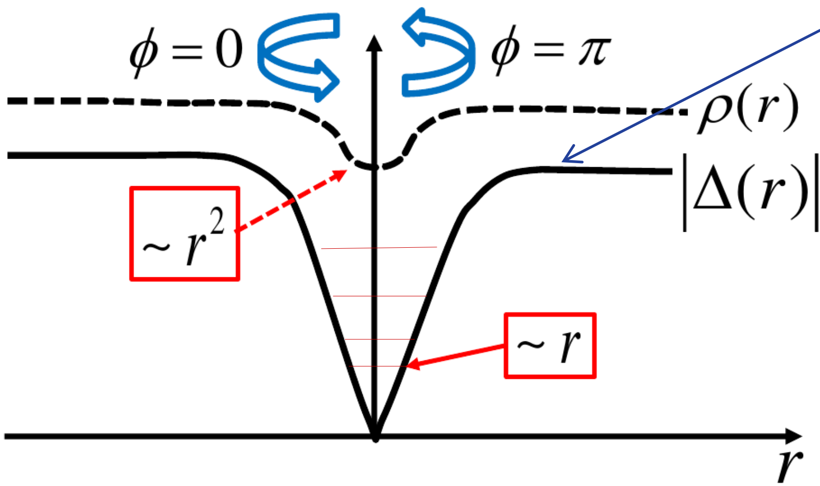
Order parameter:
not related directly to density

Andreev states affect the density distribution inside the core.

Vortex solution: Fermi gas

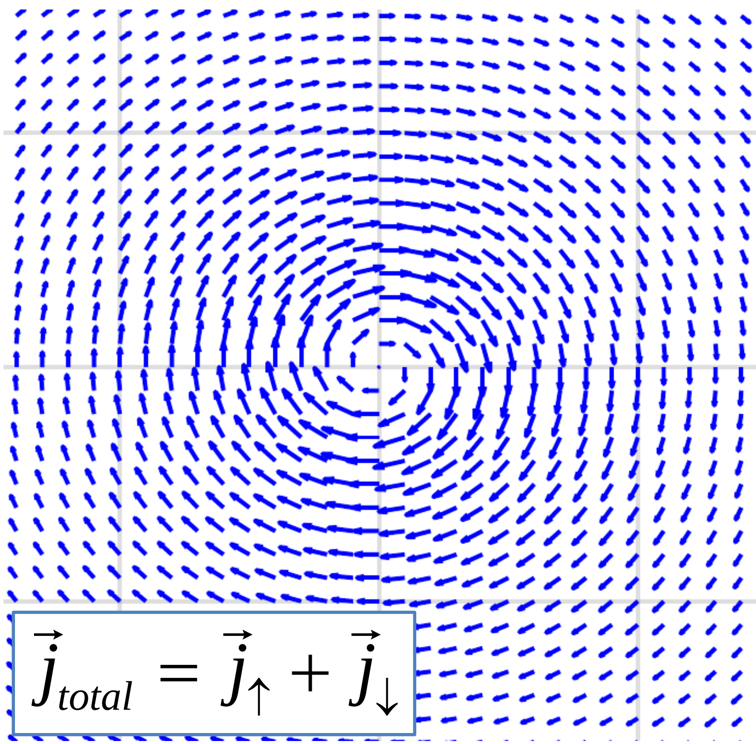
→ BdG eq. with ansatz for the pairing field

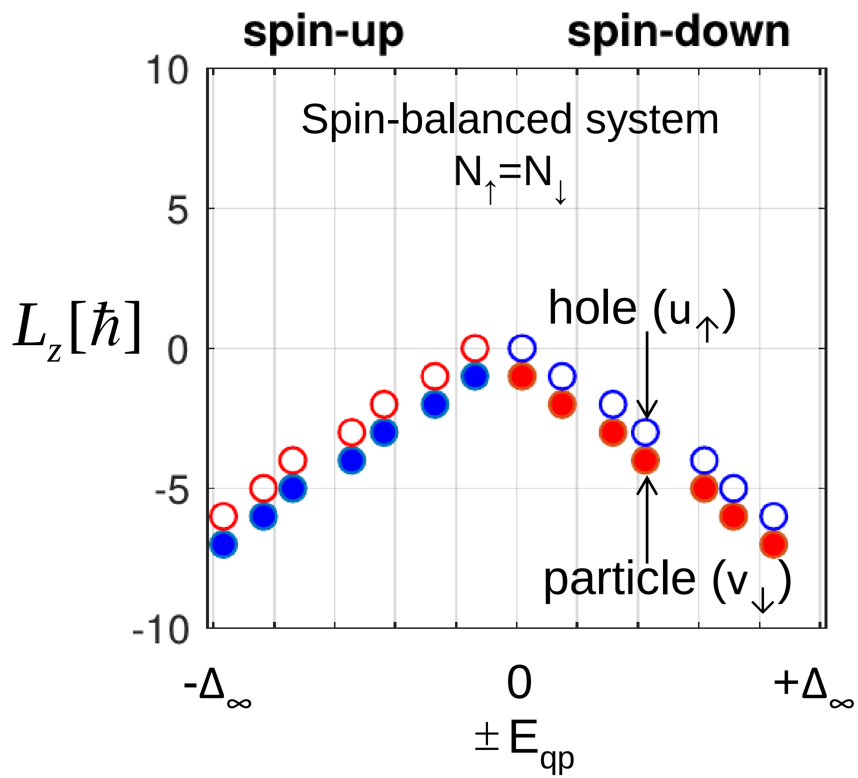
$$\begin{pmatrix} h_{\uparrow} & \Delta \\ \Delta^{*} & -h_{\downarrow}^{*} \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$

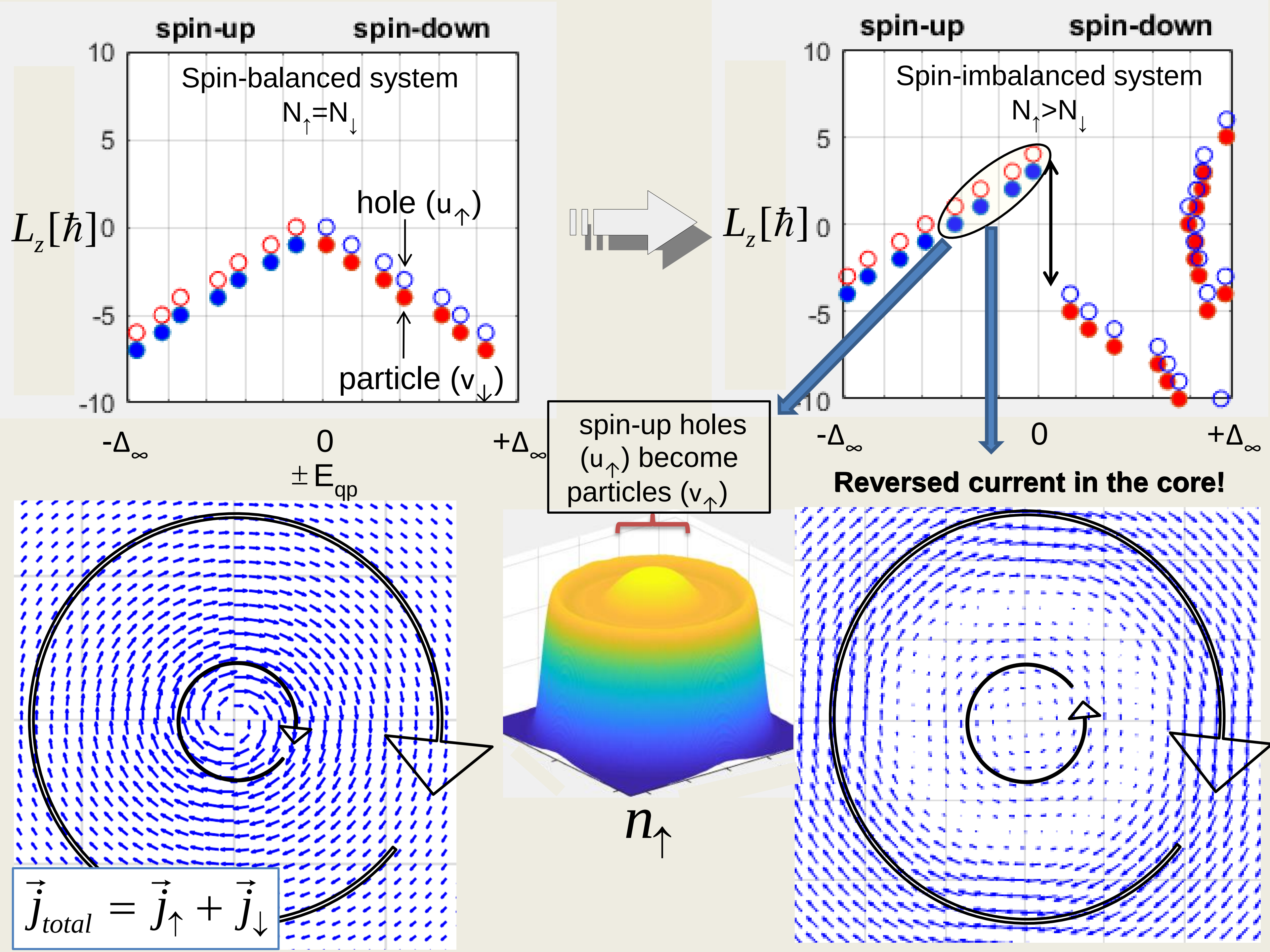


Form of the vortex-like solutions:

$$u_{\eta}(\mathbf{r}) = u_{nmk_z}(\rho)e^{im\varphi}e^{ik_z z}$$
$$v_{\eta}(\mathbf{r}) = v_{nmk_z}(\rho)e^{i(m+1)\varphi}e^{ik_z z}$$

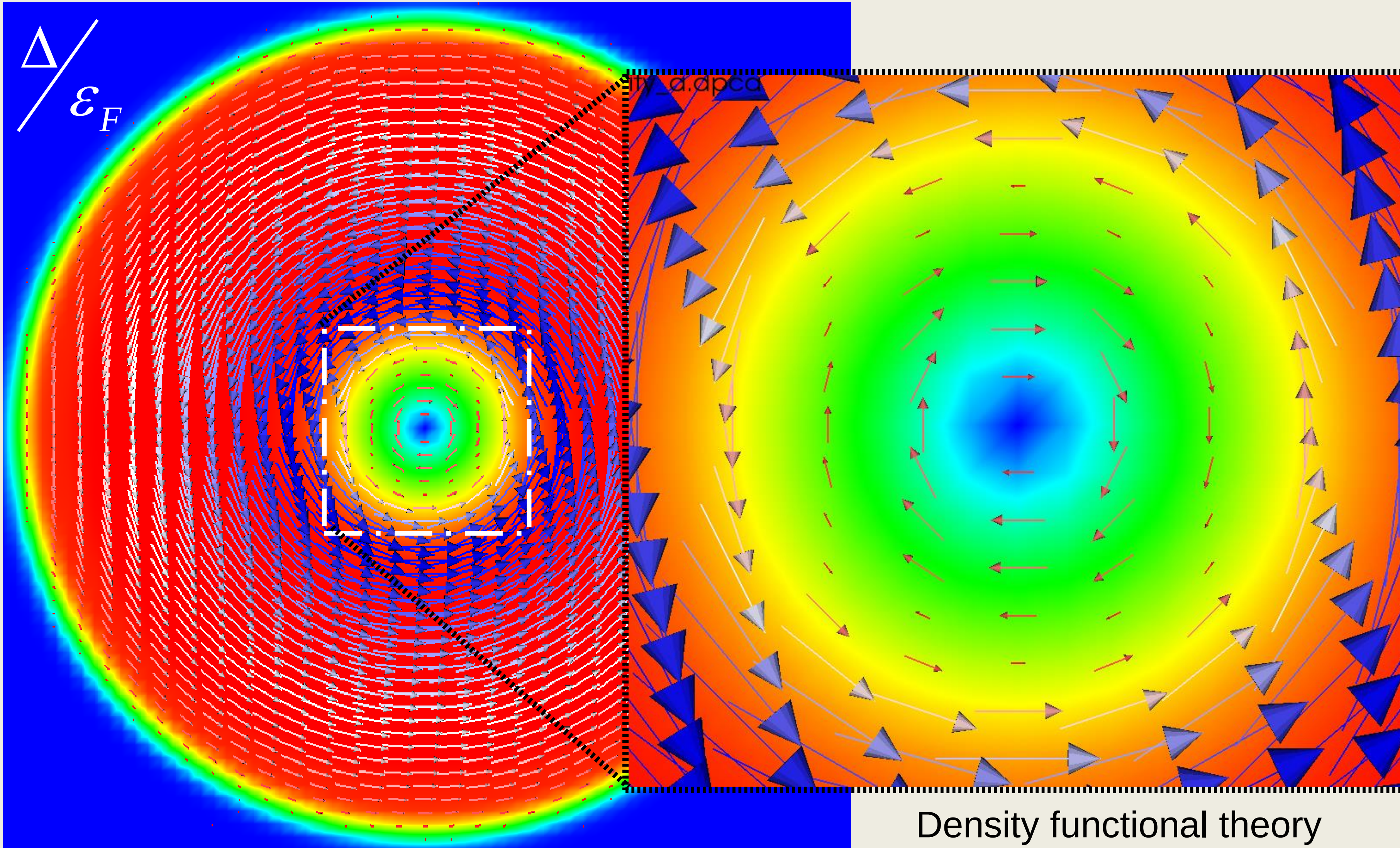






spin-up
spin-down
Spin-balanced system
N↑=N↓
hole (u↑)
particle (v↓)
Lz[ħ]
10
5
0
-5
-10
-Δ∞
0
+Δ∞
±Eqp
spin-up
spin-down
Spin-imbalanced system
N↑>N↓
Lz[ħ]
10
5
0
-5
-10
-Δ∞
0
+Δ∞
spin-up holes (u↑) become particles (v↑)
Reversed current in the core!
n↑
j_total = j↑ + j↓

Vortex in spin imbalanced Fermi gas: $N_\uparrow \neq N_\downarrow$



Due to increased spin-polarization the core expands

see eg. Hu,Liu,Drummond, PRL98,060406(2007)

Density functional theory
self-consistent solution.

Spin-polarization effects: does the structure of the core affect vortex dynamics?

Density Functional Theory (DFT):

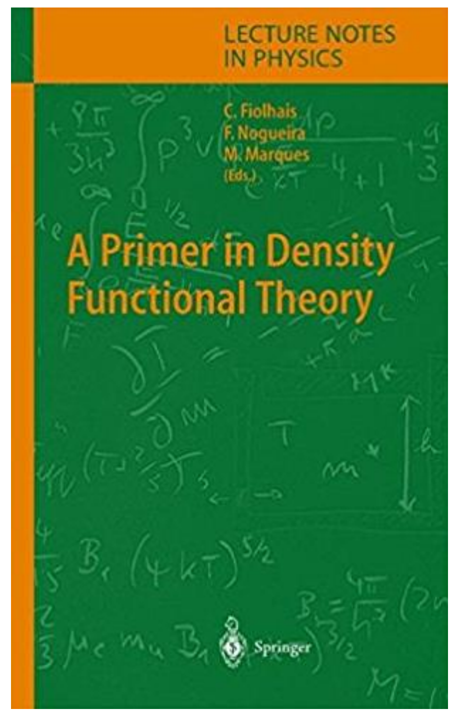
Quantitatively accurate

Unified description of static and dynamic properties of **large Fermi systems**

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi}$$

We know what Eq. should be solved...
The only problem:
How to do it in practice?

Input:
energy density functional



Methods:

- QMC (static)
- *DFT* (static and *dynamic*)
- ...

feed

DFT Basics - ground state

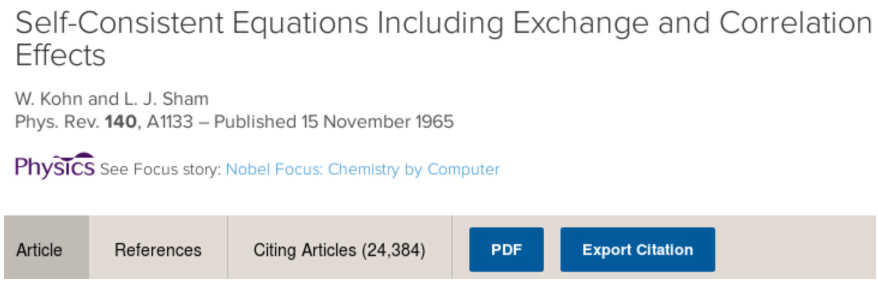
$$\begin{tikzcd} U_{ext}(\vec{r}) \arrow[r] \arrow[rr, bend left, "T"] & \Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \arrow[r] & n(\vec{r}) \end{tikzcd}$$

Hohenberg – Kohn theorem (1964):

$$T : U_{ext}(\vec{r}) \rightarrow n(\vec{r}) \text{ is invertible}$$

Consequences:

- Energy density functional $E[n]$ exists and observables can be determined from densities
- Kohn-Sham method (1965) provides practical way of extracting some of observables (energy and one-body observables)


Self-Consistent Equations Including Exchange and Correlation Effects

W. Kohn and L. J. Sham
Phys. Rev. **140**, A1133 – Published 15 November 1965

Physics See Focus story: Nobel Focus: Chemistry by Computer

Article	References	Citing Articles (24,384)	PDF	Export Citation

Most frequently cited paper in physics
(within Physical Review journals)

Time Dependent DFT Basics - excited states

$$\begin{tikzcd} U_{ext}(\vec{r},t) \arrow[r, Rightarrow] \arrow[rr, bend left, "T"] & \Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N,t) \arrow[r, Rightarrow] & n(\vec{r},t) \end{tikzcd}$$

Runge-Gross mapping(1984):

$$i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle, \quad \left|\psi_0\right\rangle = \left|\psi(t_0)\right\rangle$$

$$\frac{\partial n}{\partial t} + \nabla\cdot\vec{j} = 0$$

$$n(\vec{r}) \leftrightarrow e^{i\alpha(t)}\Psi[n](\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)$$

TDDFT variational principle also exists but it is more tricky:

$$F[\psi_0,n] = \int_{t_0}^{t_1}\left\langle\psi[n]\right|\left(i\hbar\frac{\partial}{\partial t} - \hat{H}\right)\left|\psi[n]\right\rangle dt$$

E. Runge, E.K.U Gross, PRL 52, 997 (1984)
B.-X. Xu, A.K. Rajagopal, PRA 31, 2682 (1985)
G. Vignale, PRA77, 062511 (2008)

Energy density functional (EDF) for Unitary Fermi Gas (UFG): Superfluid Local Density Approximation (SLDA)

EDF for UFG: Superfluid Local Density Approximation (SLDA)

Increasing quality and computing cost

Local Density Approximation (LDA)

$$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}))$$

Generalized Gradient Approximation (GGA)

$$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}), \boldsymbol{\nabla} n(\boldsymbol{r}))$$

Stationarity condition:

$$\frac{\delta E}{\delta n} = 0$$

EDF for UFG: Superfluid Local Density Approximation (SLDA)

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Meta - GGA

$$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}), \boldsymbol{\nabla} n(\boldsymbol{r}), \tau(r), \ldots)$$

...

where: $n(\boldsymbol{r}) = \sum_i |\phi_i(\boldsymbol{r})|^2$ $\quad \tau(\boldsymbol{r}) = \sum_i |\nabla \phi_i(\boldsymbol{r})|^2$

Stationarity condition:

$$\frac{\delta E}{\delta \phi_i} = 0$$

EDF for UFG: Superfluid Local Density Approximation (SLDA)

Increasing quality and computing cost

Local Density Approximation (LDA)	$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}))$
Generalized Gradient Approximation (GGA)	$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}), \boldsymbol{\nabla} n(\boldsymbol{r}))$
Meta - GGA	$E = \int d\boldsymbol{r}\, \mathcal{H}(n(\boldsymbol{r}), \boldsymbol{\nabla} n(\boldsymbol{r}), \tau(r), \ldots)$
...	

Stationarity condition:

$$\frac{\delta E}{\delta n} = 0$$

where: $n(\boldsymbol{r}) = \sum_i |\phi_i(\boldsymbol{r})|^2 \quad \tau(\boldsymbol{r}) = \sum_i |\nabla \phi_i(\boldsymbol{r})|^2$

Stationarity condition:

$$\frac{\delta E}{\delta \phi_i} = 0$$

$$E_{\mathrm{SLDA}} = \int d\boldsymbol{r}\, \mathcal{H}(\overbrace{n(\boldsymbol{r}), \boldsymbol{\nabla} n(\boldsymbol{r}), \tau(\boldsymbol{r}), \ldots}^{\text{Meta-GGA}}, \overbrace{\nu(\boldsymbol{r})}^{\text{LDA}})$$

order parameter $\Leftarrow$ anomalous density

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r}) \nabla v_{n,\sigma}(\boldsymbol{r})],$$

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EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$+ g(n_\uparrow, n_\downarrow)\nu$$

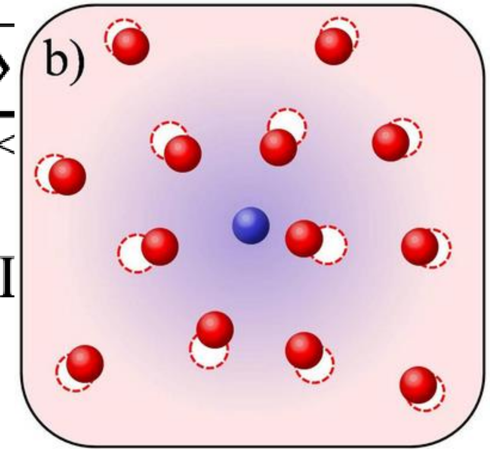
$$+ [1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<}$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{I}$$



EDF:

$$\mathcal{H} = \boxed{\alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$+ g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$+ [1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{\quad}{2n_\downarrow}$$

Kinetic term:
Effective mass α_σ of the particle depends on local polarization

$$p(\boldsymbol{r}) = \frac{n_\uparrow(\boldsymbol{r}) - n_\downarrow(\boldsymbol{r})}{n_\uparrow(\boldsymbol{r}) + n_\downarrow(\boldsymbol{r})}$$

and guarantees that correct limit is attained for $n_\uparrow >> n_\downarrow$,
where the problem reduces
to the *polaron* problem

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities: $$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

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EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$\boxed{+D(n_\uparrow, n_\downarrow)}$$

$$+g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$+[1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

Normal interaction energy:

$$D(n_\uparrow, n_\downarrow) \sim (n_\uparrow + n_\downarrow)^{5/3}\beta(p)$$

in order to get the proper scaling:

$$E = \xi E_{FFG}$$

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r})\nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$\boxed{+ g(n_\uparrow, n_\downarrow)\nu^\dagger\nu}$$

$$+ [1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

Pairing energy:

$$g(n_\uparrow, n_\downarrow) = \frac{\gamma(p)}{(n_\uparrow + n_\downarrow)^{1/3}}$$

in order to get proper scaling:

$$\Delta/\varepsilon_F = \mathrm{const} \approx 0.5$$

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r})\nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$+ g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$\boxed{+[1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}}$$

In order to restore Galilean invariance of the functional

More details:
A. Bulgac, M.M. Forbes, P. Magierski,
The Unitary Fermi Gas: From Monte Carlo to Density Functionals,
Lecture Notes in Physics 836
ed. W. Zwerger, Springer (2011).

Normal State				Superfluid State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)	(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
$(3,1)$	6.6 ± 0.01	6.687	1.3%	$(1,1)$	2.002 ± 0	2.302	15%
$(4,1)$	8.93 ± 0.01	8.962	0.36%	$(2,2)$	5.051 ± 0.009	5.405	7%
$(5,1)$	12.1 ± 0.1	12.22	0.97%	$(3,3)$	8.639 ± 0.03	8.939	3.5%
$(5,2)$	13.3 ± 0.1	13.54	1.8%	$(4,4)$	12.573 ± 0.03	12.63	0.48%
$(6,1)$	15.8 ± 0.1	15.65	0.93%	$(5,5)$	16.806 ± 0.04	16.19	3.7%
$(7,2)$	19.9 ± 0.1	20.11	1.1%	$(6,6)$	21.278 ± 0.05	21.13	0.69%
$(7,3)$	20.8 ± 0.1	21.23	2.1%	$(7,7)$	25.923 ± 0.05	25.31	2.4%
$(7,4)$	21.9 ± 0.1	22.42	2.4%	$(8,8)$	30.876 ± 0.06	30.49	1.2%
$(8,1)$	22.5 ± 0.1	22.53	0.14%	$(9,9)$	35.971 ± 0.07	34.87	3.1%
$(9,1)$	25.9 ± 0.1	25.97	0.27%	$(10,10)$	41.302 ± 0.08	40.54	1.8%
$(9,2)$	26.6 ± 0.1	26.73	0.5%	$(11,11)$	46.889 ± 0.09	45	4%
$(9,3)$	27.2 ± 0.1	27.55	1.3%	$(12,12)$	52.624 ± 0.2	51.23	2.7%
$(9,5)$	30 ± 0.1	30.77	2.6%	$(13,13)$	58.545 ± 0.18	56.25	3.9%
$(10,1)$	29.4 ± 0.1	29.41	0.034%	$(14,14)$	64.388 ± 0.31	62.52	2.9%
$(10,2)$	29.9 ± 0.1	30.05	0.52%	$(15,15)$	70.927 ± 0.3	68.72	3.1%
$(10,6)$	35 ± 0.1	35.93	2.7%	$(1,0)$	1.5 ± 0.0	1.5	0%
$(20,1)$	73.78 ± 0.01	73.83	0.061%	$(2,1)$	4.281 ± 0.004	4.417	3.2%
$(20,4)$	73.79 ± 0.01	74.01	0.3%	$(3,2)$	7.61 ± 0.01	7.602	0.1%
$(20,10)$	81.7 ± 0.1	82.57	1.1%	$(4,3)$	11.362 ± 0.02	11.31	0.49%
$(20,20)$	109.7 ± 0.1	113.8	3.7%	$(7,6)$	24.787 ± 0.09	24.04	3%
$(35,4)$	154 ± 0.1	154.1	0.078%	$(11,10)$	45.474 ± 0.15	43.98	3.3%
$(35,10)$	158.2 ± 0.1	158.6	0.27%	$(15,14)$	69.126 ± 0.31	62.55	9.5%
$(35,20)$	178.6 ± 0.1	180.4	1%				

Table 9.2 Comparison between the ASLDA density functional as described in this section and the FN-DMC calculations [136, 137] for a harmonically trapped unitary gas at zero temperature. The normal state energies are obtained by fixing $\Delta = 0$ in the functional: In the FN-DMC calculations, this is obtained by choosing a nodal ansatz without any pairing. In the case of small asymmetry, the resulting "normal states" may be a somewhat artificial construct as there is no clear way of preparing a physical system in this "normal state" when the ground state is superfluid.

Figure from: A. Bulgac, M.M. Forbes, P. Magierski,
Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\boldsymbol{\nabla}^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\boldsymbol{\nabla} + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where *h* and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\nu(\boldsymbol{r},t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

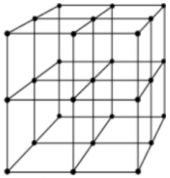
To execute superfluid TDDFT we need supercomputers...

https://www.topa500.org/

TOP 500 The List.

Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM DOE/SC/Oak Ridge National Laboratory United States	2,397,824	143,500.0	200,794.9	9,783
2	**Sierra** - IBM Power Syst… NVIDIA Volta GV100, Dual… NVIDIA / Mellanox DOE/NNSA/LLNL United States				
3	**Sunway TaihuLight** - Sun… Sunway , NRCPC National Supercomputin… China				
4	**Tianhe-2A** - TH-IVB-FEP… TH Express-2, Matrix-20… National Super Compute… China				
5	**Piz Daint** - Cray XC50, Xe… interconnect , NVIDIA Te… Swiss National Supercom… Switzerland				
6	**Trinity** - Cray XC40, Xeon… 68C 1.4GHz, Aries interc… DOE/NNSA/LANL/SNL United States				
7	**AI Bridging Cloud Infrastructure (ABCI)** - PRIMERGY CX2570 M4, Xeon Gold 6148 20C 2.4GHz, NVIDIA Tesla V100 SXM2, Infiniband EDR , Fujitsu National Institute of Advanced Industrial Science and Technology	391,680	19,880.0	32,576.6	1,649

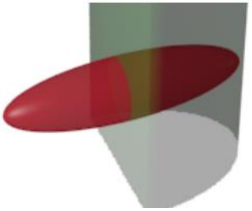
Present computing capabilities:



- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps
  (for cold atomic systems it gives
  a trajectory of length of a few ms)

All further results shown here were generated on Piz Daint (CSCS)

PRACE

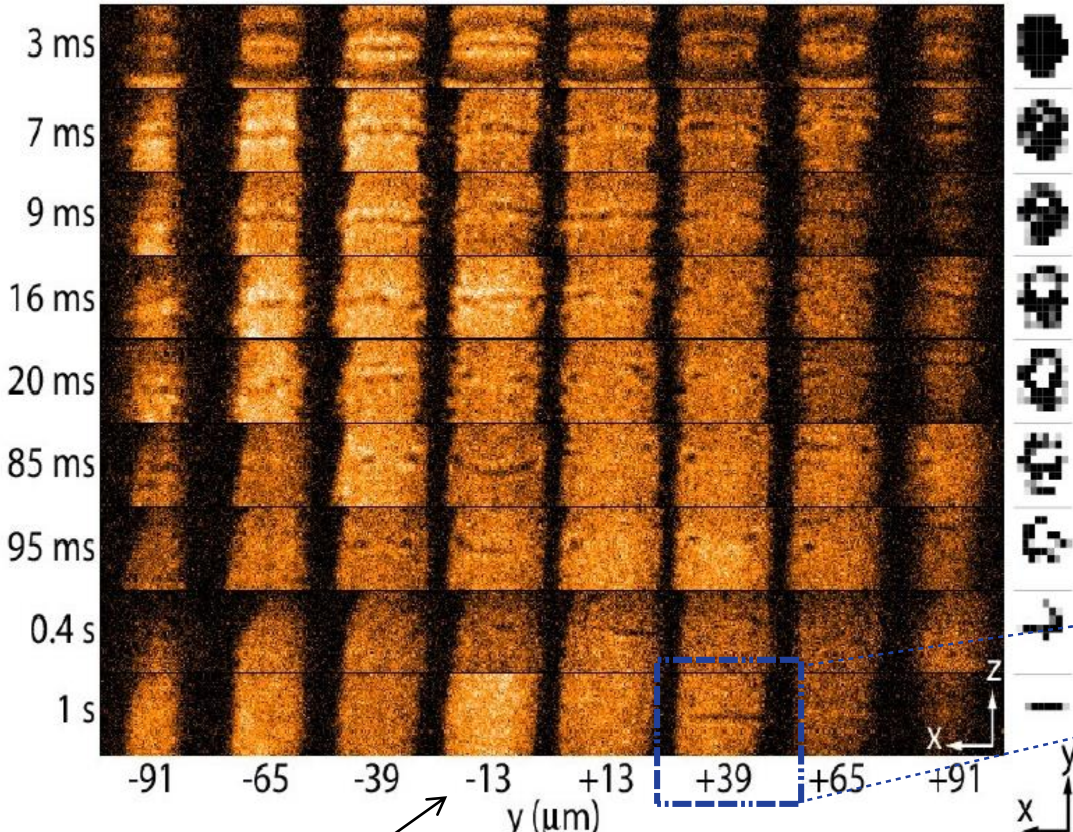


(a)

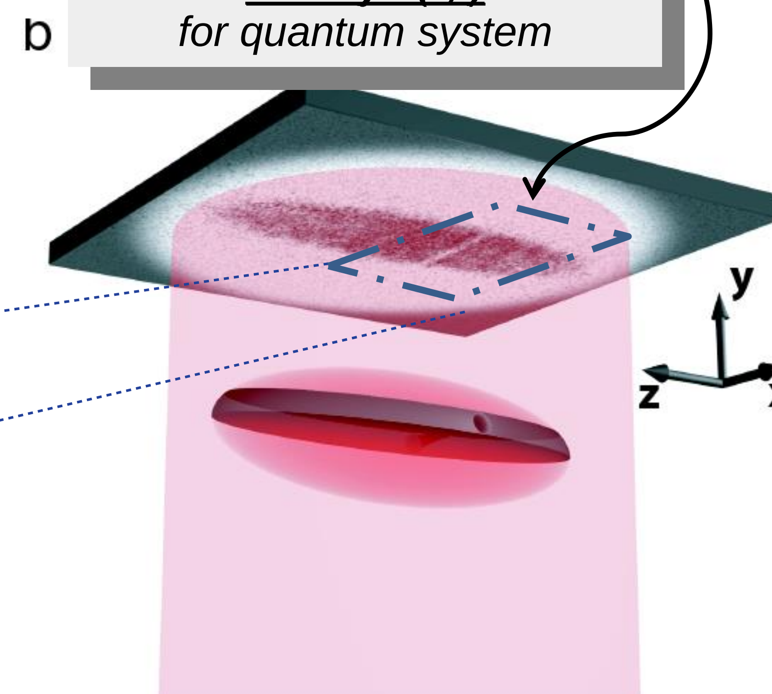
experiment

Phys. Rev. Lett. 116, 045304 (2016)

Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line

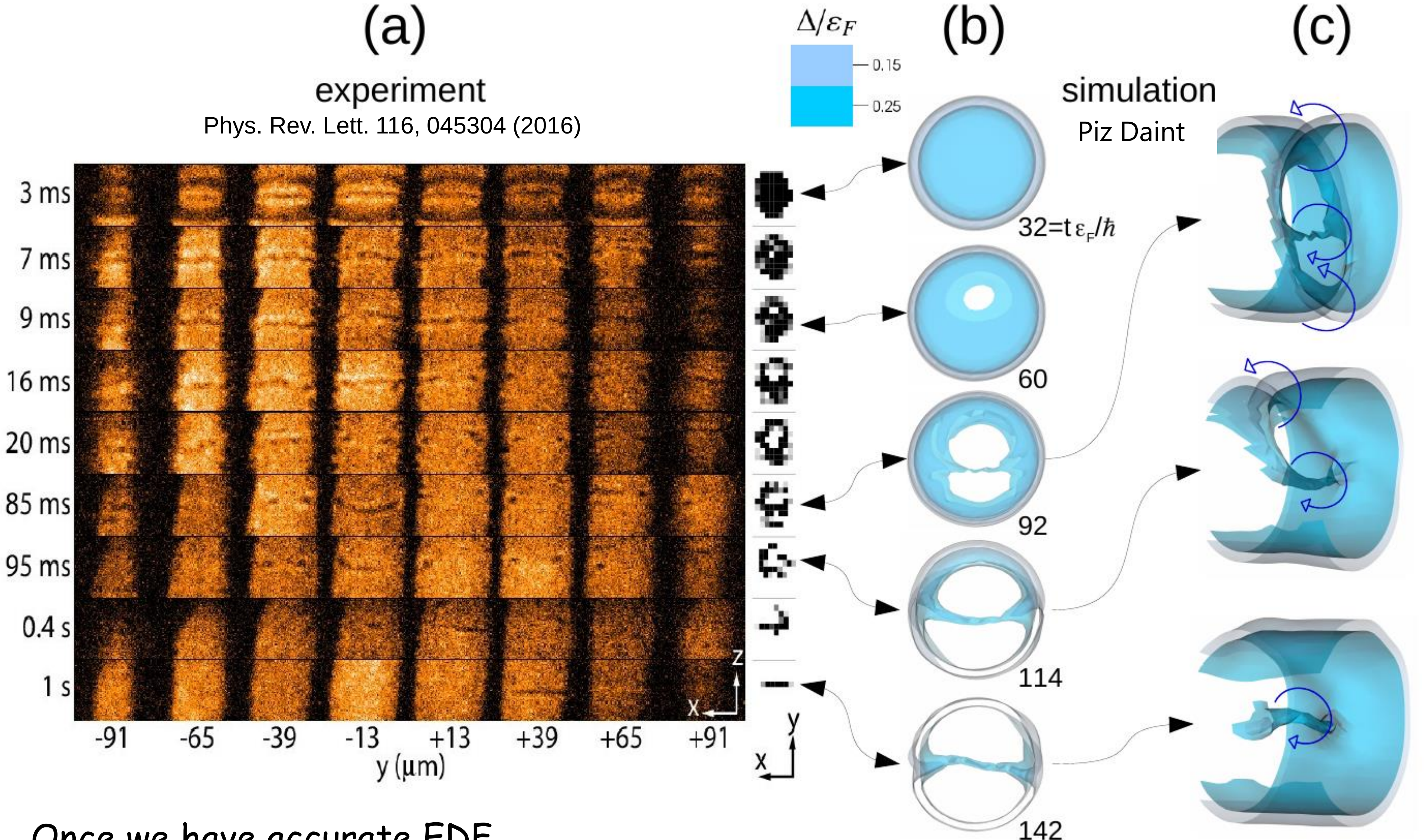


Note:
here we observe directly
time evolution of
density n(r,t)
for quantum system



unitary Fermi gas

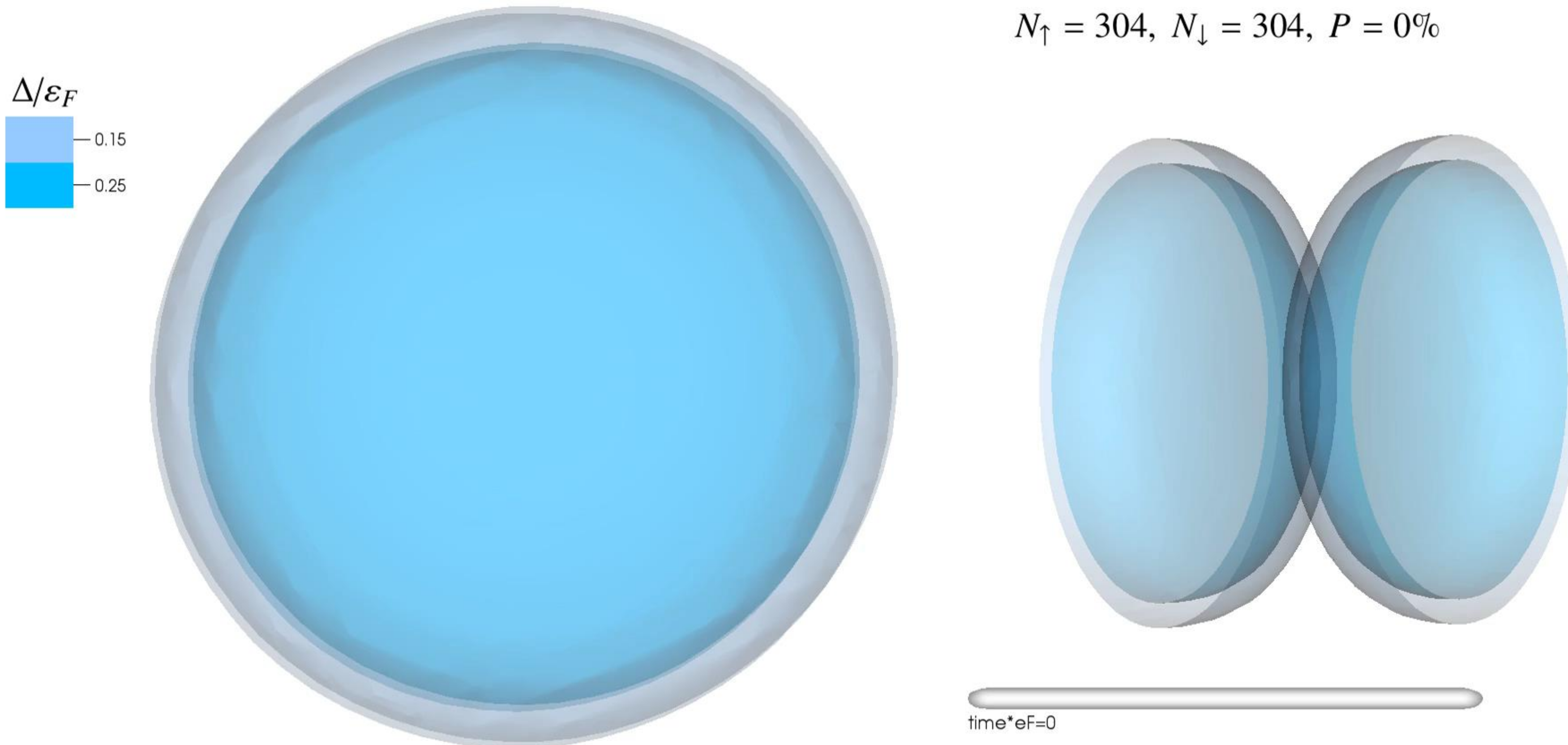
(superfluid properties manifest here in the form of topological defects)



Once we have accurate EDF
→ remarkable agreement between theory and data!

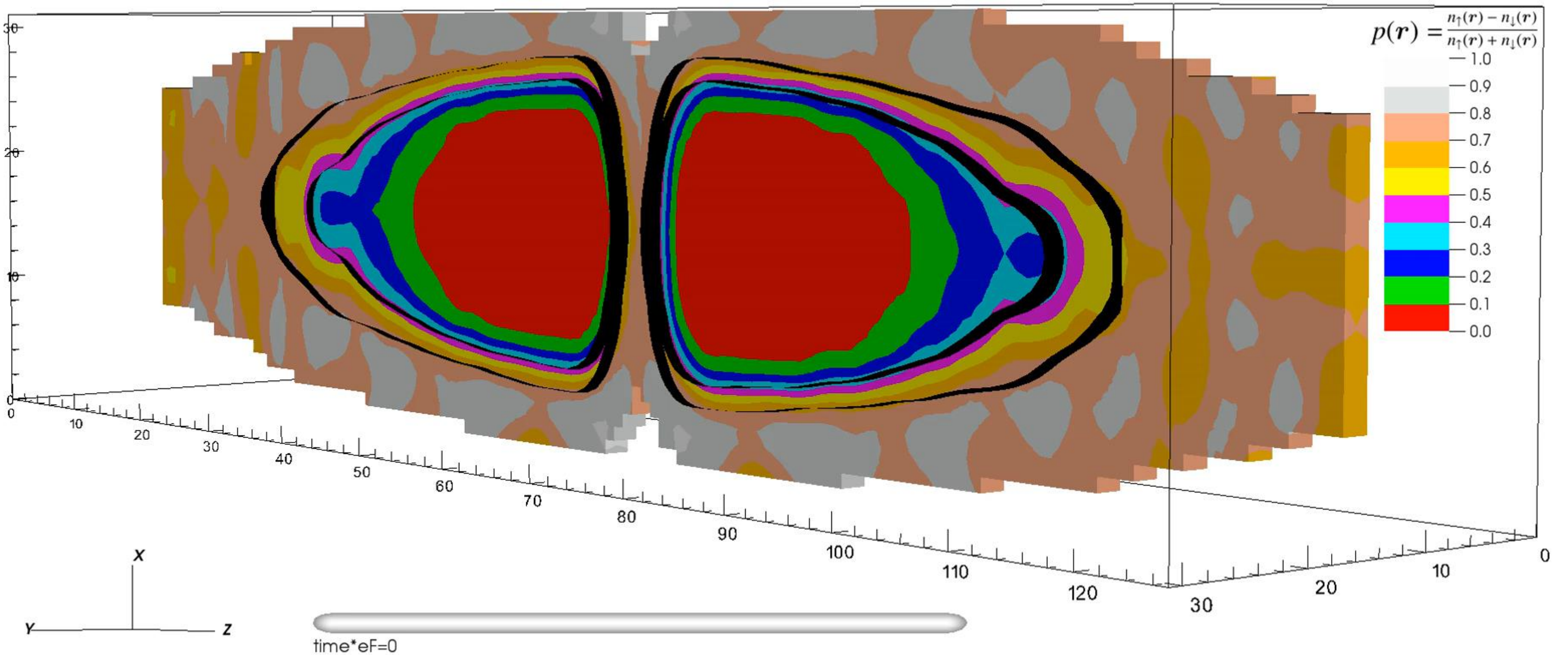
No adjusting parameters to the experiment!

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski,
Phys. Rev. Lett. **120**, 253002 (2018)


Δ/ε_F
0.15
0.25
$N_\uparrow = 304,\ N_\downarrow = 304,\ P = 0\%$
time*eF=0

Spin imbalanced UFG

$N_\uparrow = 304,\ N_\downarrow = 202,\ P = 20\%$



The vortex core becomes polarized!

This may be understood noting that the most energetically favorable place to store excess of unpaired spins is at the core of the vortex where, $\Delta = 0$ - no Cooper pairs need to be broken.

New effects predicted for spin-polarized systems:

Impact on the solitonic cascade:
final product of the cascade depends on the spin imbalance in the system

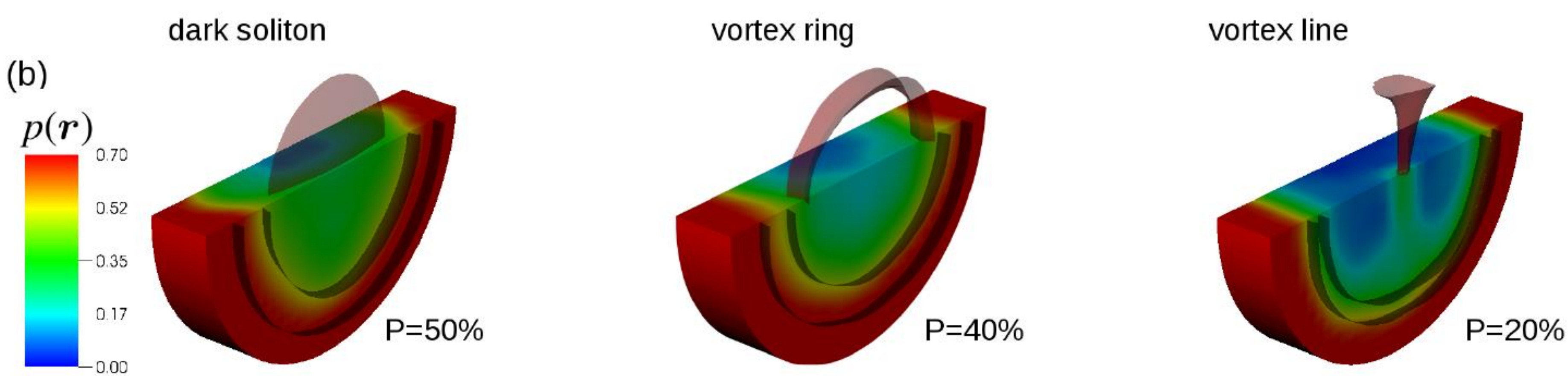
(can be verified experimentally with present setups)

P=20%: Dark soliton ⇨ Vortex ring ⇨ Vortex line

P=40%: Dark soliton ⇨ Vortex ring

P=50%: Dark soliton

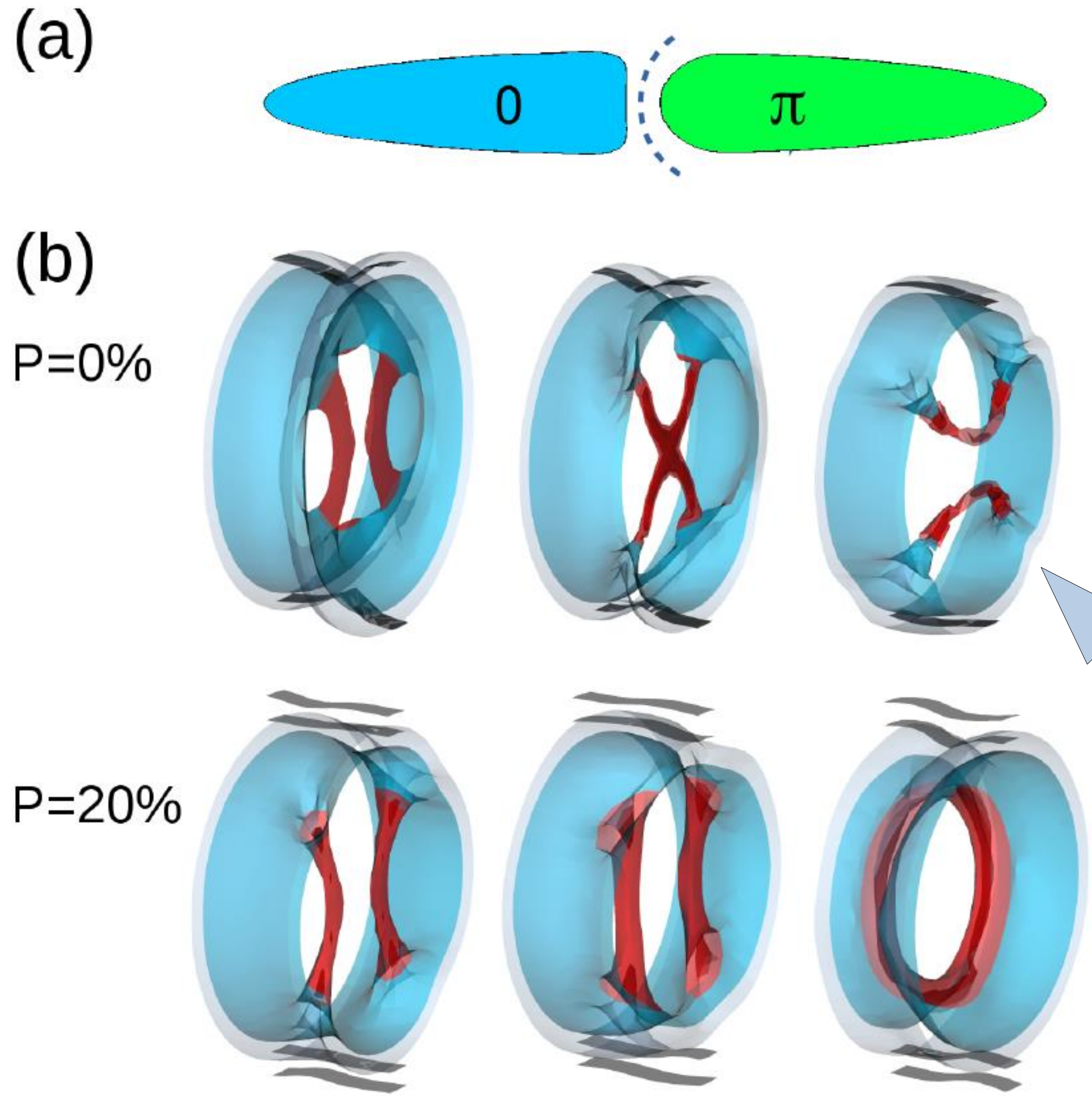
Cascade is suppressed by the polarization effects



Stability of topological defect depends on its internal structure…

→ For sufficiently large spin-imbalance dark solitons become stable
(no snake instability) (see also: Reichl & Mueller, PRA 95, 053637; Lombardi, et. al., PRA 96, 033609)

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski,
Phys. Rev. Lett. **120**, 253002 (2018)



Spin-imbalanced atomic gases may offer a new platform for studies of the *quantum turbulence* phenomenon in fermionic systems.

- **Both superfluid and normal component coexist even at zero temperature limit.**
- **Changes of rigidity of topological defects → Kelvin waves.**
- **Changes for vortex-vortex interaction → vortex reconnections**

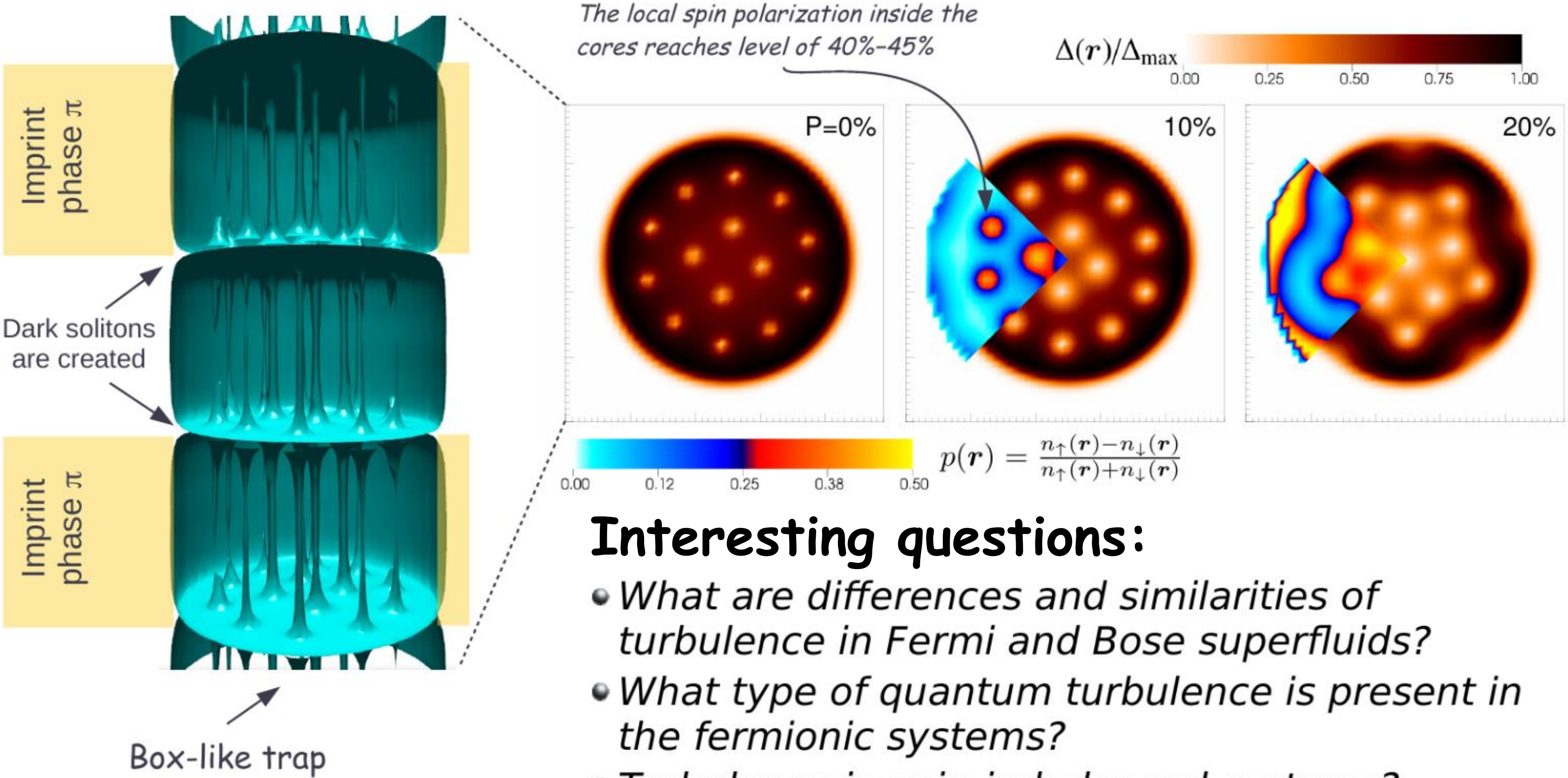
We observe that mutual dynamics of two vortices depend on its internal structure...

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski,
Phys. Rev. Lett. **120**, 253002 (2018)

Towards quantum turbulence in fermionic gas

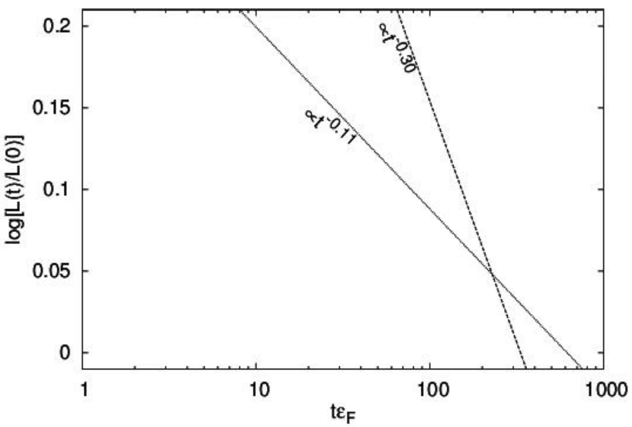
Problem 1: how to generate the turbulence?

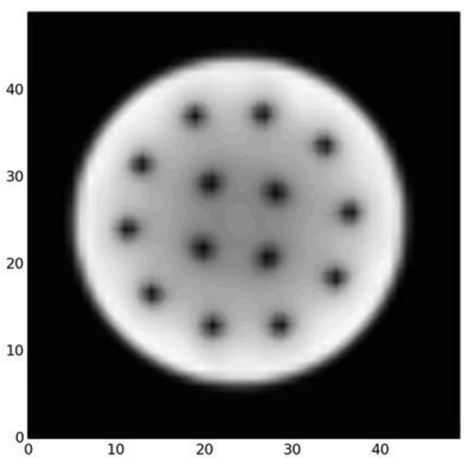
→ Our suggestion: *imprint few dark solitons on existing vortex lattice*
→ *rotating turbulence* *(nonzero total angular momentum)*

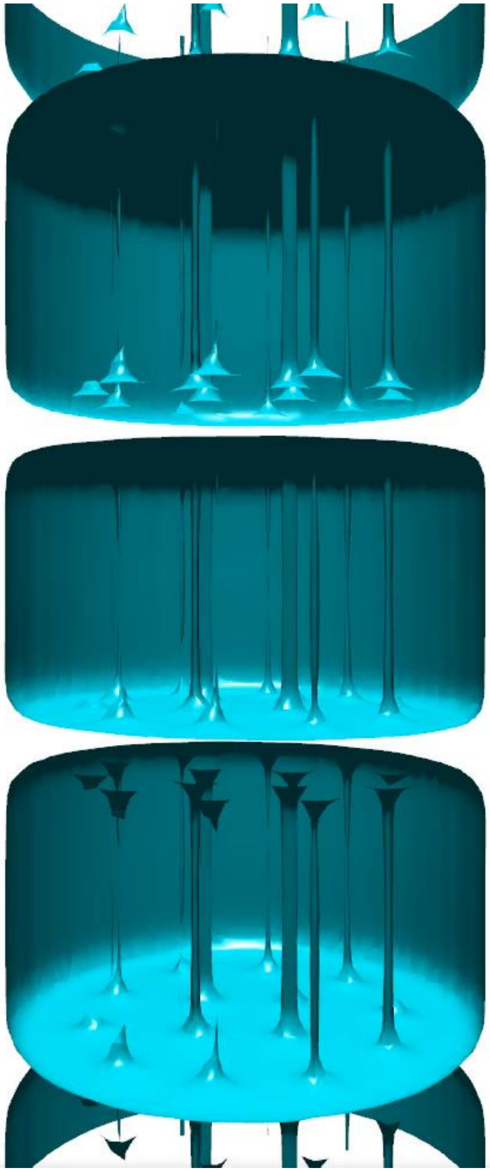


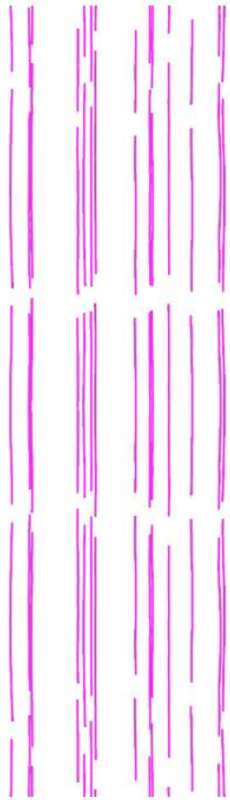
Interesting questions:

- *What are differences and similarities of turbulence in Fermi and Bose superfluids?*
- *What type of quantum turbulence is present in the fermionic systems?*
- *Turbulence in spin-imbalanced systems?*

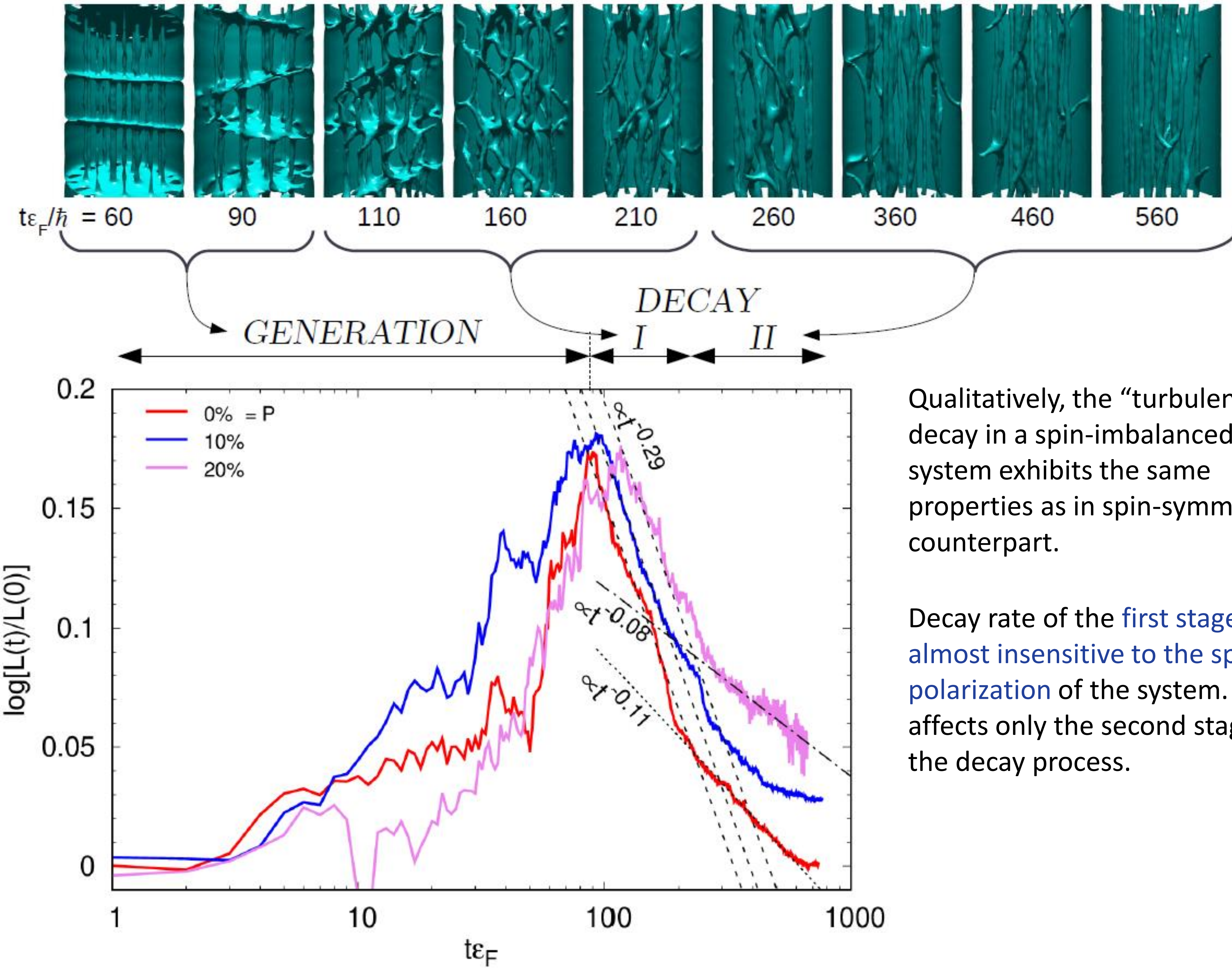








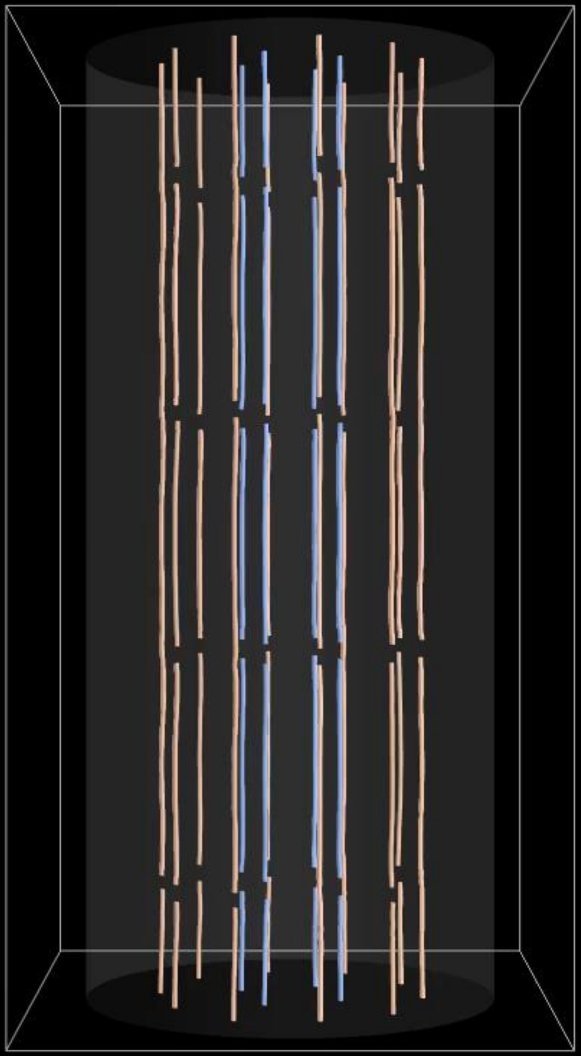
time = 0 $[\varepsilon_F^{-1}]$



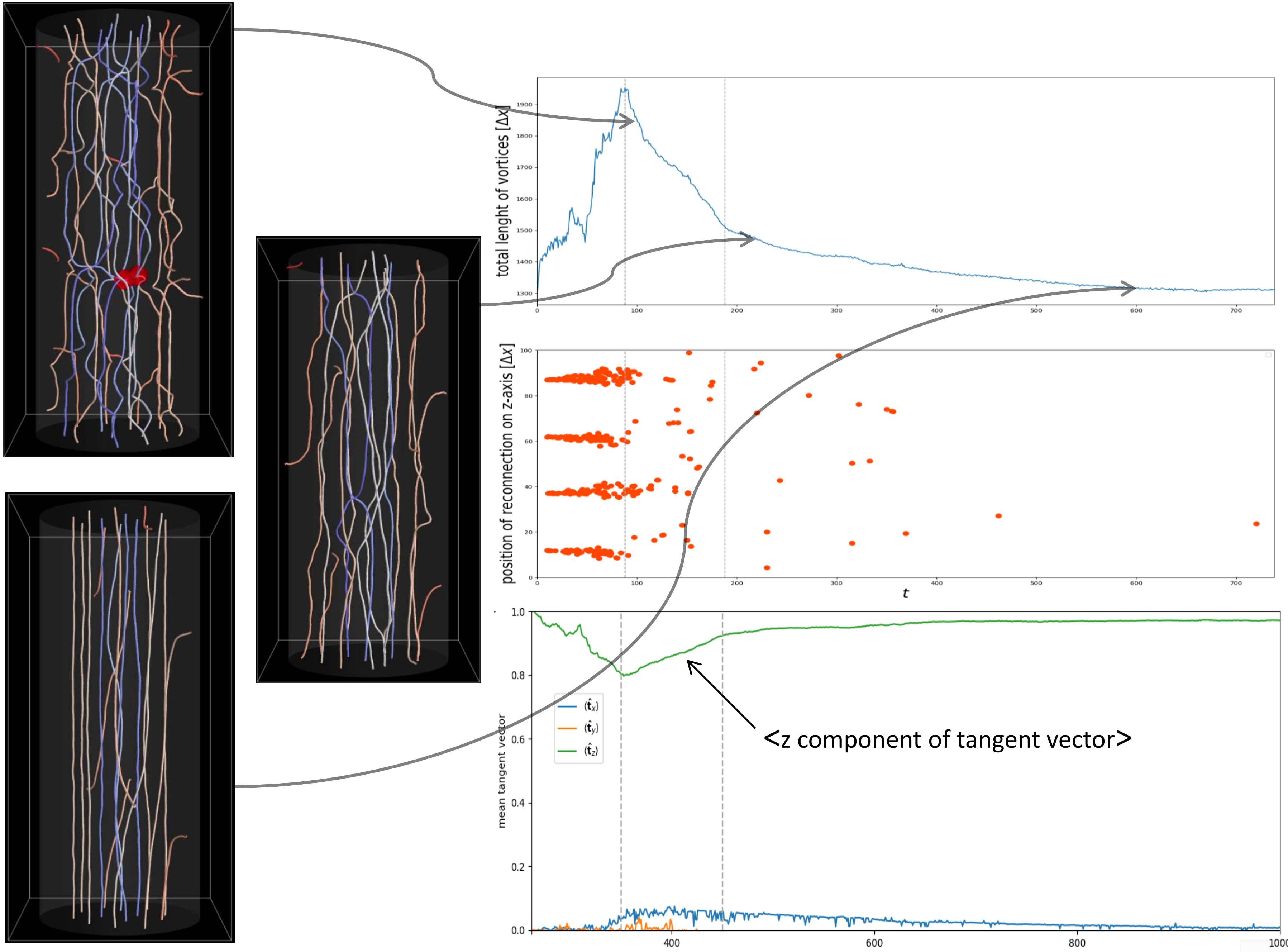
Qualitatively, the "turbulence" decay in a spin-imbalanced system exhibits the same properties as in spin-symmetric counterpart.

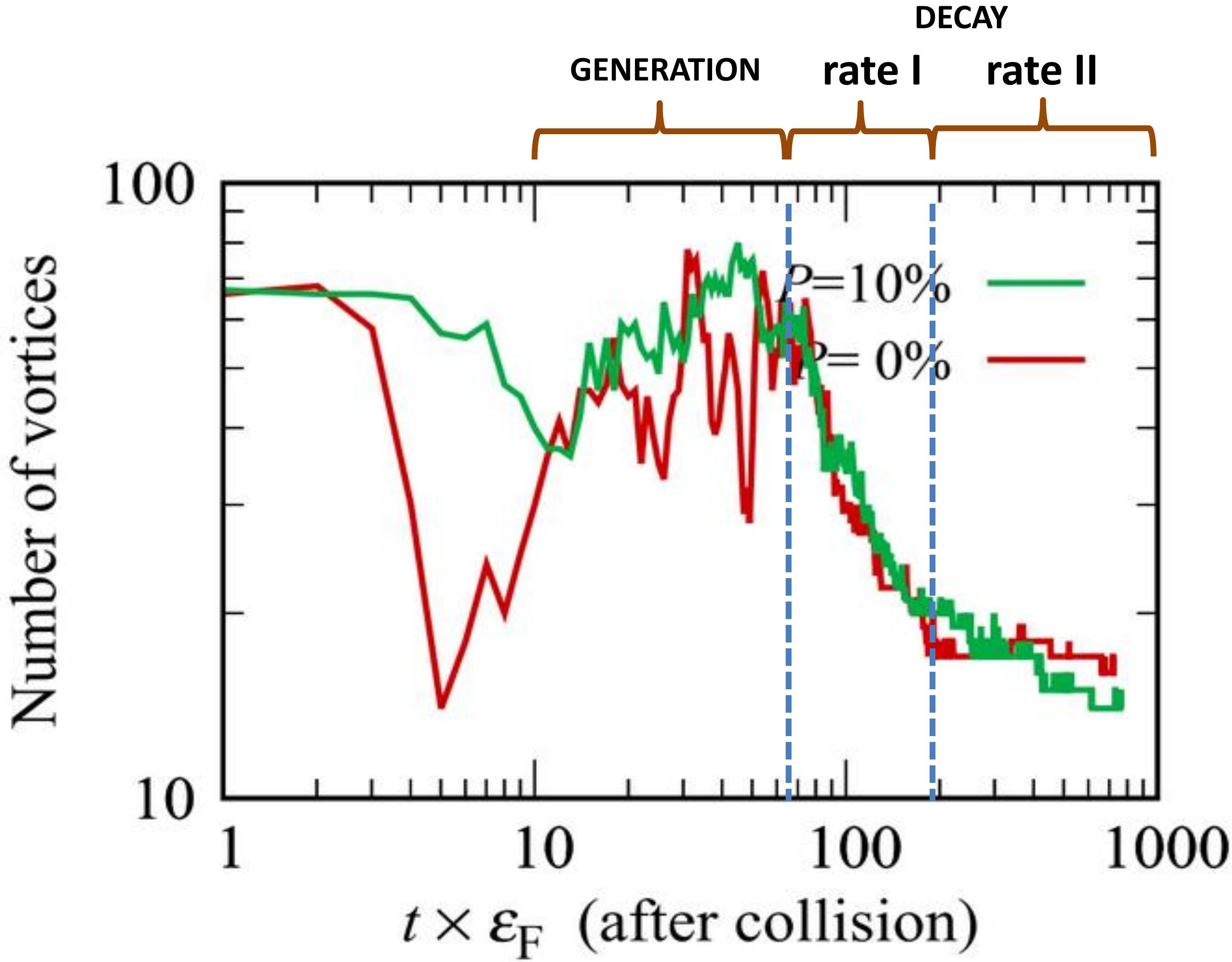
Decay rate of the first stage is almost insensitive to the spin-polarization of the system. It affects only the second stage of the decay process.

GENERATION



$t\varepsilon_F = 0$

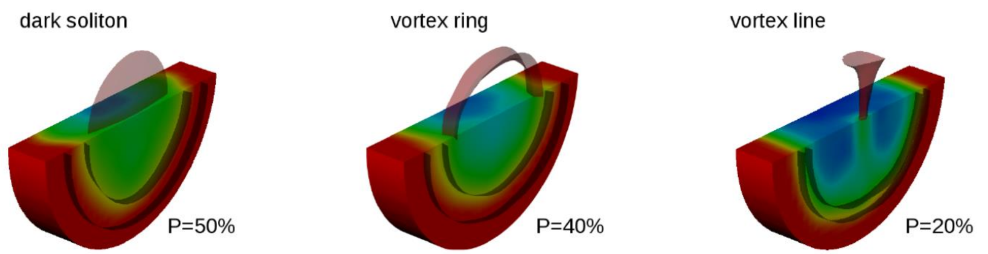

total lenght of vortices [Δx]
position of reconnection on z-axis [Δx]
t
mean tangent vector
⟨t̂x⟩
⟨t̂y⟩
⟨t̂z⟩
<z component of tangent vector>


DECAY
GENERATION
rate I
rate II
100
10
1
10
100
1000
Number of vortices
$t \times \varepsilon_F$ (after collision)
P=10%
P= 0%

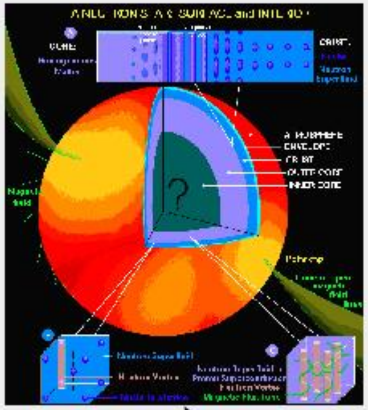
CONCLUSIONS

- (TD)DFT – route for unified description of static and dynamic properties of large strongly correlated Fermi systems → studies of the quantum turbulence in the reach

- Spin-imbalanced system may provide new platform for studies of QT
  - *Coexistence of superfluid (paired) and normal (unpaired) components even at zero temperature limit*
  - *Significant changes of internal structure of topological defects*
  - *Changes of stability and rigidity of topological defects due to polarization effects*
  - *...*

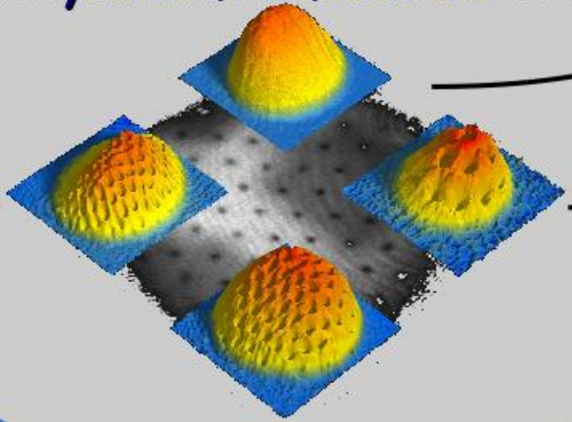


Superfluid effects in neutron stars (glitches, turbulence)



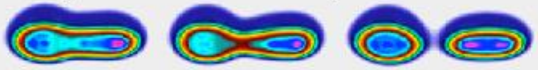
Quantum turbulence

Ultracold fermionic gases as *quantum simulators* of...

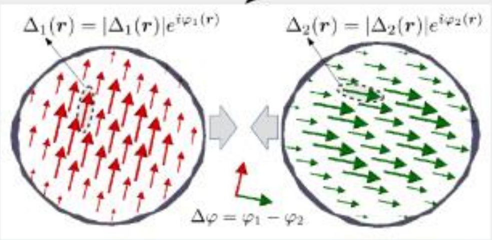


Supercomputing

Impact of superfluidity on fission dynamics of heavy nucleus



Collisions of two superfluid nuclei



Open call for PhD student position.

Open call for summer internship for student

Thank you