Nuclear structure and dynamics in the neutron star crust

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The existence of neutron stars was predicted by Landau (1932), Baade & Zwicky (1934) and Oppenheimer & Volkoff (1939).

On November 28, 1967, Cambridge graduate student Jocelyn Bell (now Burnell) and her advisor, Anthony Hewish discovered a source with an exceptionally regular pattern of radio flashes. These radio flashes occurred every 1 1/3 seconds like clockwork. After a few weeks, however, three more rapidly pulsating sources were detected, all with different periods. They were dubbed "pulsars."

### Nature of the pulsars

- **Pulse rate**: 30 per second
- **Slowing down rate**: 38 nanoseconds per day

Calculated energy loss due to rotation of a possible neutron star

\[ \approx \text{Energy radiated} \]

**Conclusion**: the pulses are produced by rotation!
Basic facts about neutron stars:

- Radius: \( \sim 10 \text{ km} \)
- Mass: \( \sim 1-2 \text{ solar masses} \)
- Average density: \( \sim 10^{14} \text{ g/cm}^3 \)
- Magnetic field: \( \sim 10^8 - 10^{12} \text{ G} \)
- Magnetars: \( \sim 10^{15} \text{ G} \)
- Rotation period: 1.5 msec. – 5 sec.

Number of known pulsars: > 1000
Number of pulsars in our Galaxy: \( \sim 10^8 \)

Gravitational energy of a nucleon at the surface of neutron star: \( \sim 100 \text{ MeV} \)

Binding energy per nucleon in an atomic nucleus: \( \sim 8 \text{ MeV} \)

Neutron star is bound by gravitational force
Birth of a neutron star

Supernova explosion and formation of proto-neutron star

- Core collapse
  \[ t_{\text{collapse}} \approx 100 \text{ ms} \]
- B. E. \[ 2-3 \times 10^{53} \text{ ergs} \]
- Shock wave
  \[ E_{\text{shock}} \approx 10^{51} \text{ ergs} \]
- 1500 km
- 3 \times 10^7 \text{ km}
- 10 km
- Proton-dense Proto-neutron Star: \[ t \approx 1-2 \text{ s} \]
- 100 km

Summary: End Points of Stellar Evolution

<table>
<thead>
<tr>
<th>Remnant</th>
<th>Progenitor Mass</th>
<th>Remnant Mass</th>
<th>Size</th>
<th>Density</th>
<th>Means of Support</th>
<th>Final Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Dwarf</td>
<td>( M_* &lt; 8M_\odot )</td>
<td>( M_{WD} &lt; 1.4M_\odot )</td>
<td>( R_{WD} \sim R_{\text{earth}} )</td>
<td>1 ton/cm(^3)</td>
<td>( e^- ) degeneracy</td>
<td>Planetary Nebula</td>
</tr>
<tr>
<td>Neutron Star</td>
<td>( 8M_\odot &lt; M_* &lt; 20M_\odot )</td>
<td>( M_{NS} &lt; 3M_\odot )</td>
<td>( R_{NS} \sim 10 \text{ km} )</td>
<td>200 million ton/cm(^3)</td>
<td>( n ) degeneracy</td>
<td>Supernova</td>
</tr>
<tr>
<td>Black Hole</td>
<td>( M_* &gt; 20M_\odot )</td>
<td>( M_{BH} &gt; 3M_\odot )</td>
<td>( R_{\text{grav}} = 2GM/c^2 )</td>
<td>( \infty )</td>
<td>none</td>
<td>?</td>
</tr>
</tbody>
</table>
Thermal evolution of a neutron star:

Temperature: 50 MeV $\rightarrow$ 0.1 MeV ($t \sim$ min.)

URCA process:

\[
p + e \rightarrow n + \nu_e \\
n \rightarrow p + e + \bar{\nu}_e
\]

Temperature: 0.1 MeV $\rightarrow$ 100eV ($t \sim 10^5$ yr.)

MURCA process:

\[
p + p + e \rightarrow p + n + \nu_e \\
n + p + e \rightarrow n + n + \nu_e \\
p + n \rightarrow p + p + e + \bar{\nu}_e \\
n + n \rightarrow n + p + e + \bar{\nu}_e
\]

Energy transfer between core and surface:

\[
\frac{\partial T}{\partial t} = D \nabla^2 T; \quad D = \frac{\kappa}{C_v}
\]

For $\tau < 100$ years:

$T_{\text{core}} < T_{\text{surf}}$
Relaxation time: a typical time for the cooling wave to reach the surface

Structure of a neutron star

A NEUTRON STAR: SURFACE and INTERIOR

- CORE: Homogeneous Matter
- CRUST: Nuclei Neutron Superfluid
- ATMOSPHERE ENVELOPE CRUST OUTER CORE INNER CORE
- Polar cap Cone of open magnetic field lines

- Neutron Superfluid Neutron Vortex Proton Superconductor

- Nuclei in a lattice Magnetic Flux Tube

~ 0.01 - 0.02 $M_{\text{NS}}$
Structure of the inner crust

Neutron density distribution

Proton density distribution

Proton fraction

Neutron Cooper pairs

Unbound neutrons

s-wave pairing gap in infinite neutron matter with realistic NN-interactions

\[ \Delta_F \] [MeV]
\[ k_F \] [fm\(^{-1}\)]

Chen et al., NPA 451, 509 (1986)
Ainsworth et al., PLB 222, 173 (1989)
Chen et al., NPA 555, 59 (1993)
Wambach et al., NPA 555, 128 (1993)
Schulze et al., PLB 375, 1 (1996)

K. Klimaszewski, diploma thesis, 2004
Dynamics of a nucleus immersed in a neutron superfluid

\[ H = \sum_m \left( \frac{\pi m}{2M} \right)^2 + \frac{C}{2} |\alpha m|^2 \]

\[ M = m \rho_{\text{in}} \frac{(\gamma - 1)^2}{2(\gamma + 1)} R_N^5; \quad \gamma = \frac{\rho_{\text{out}}}{\rho_{\text{in}}} \]

\[ C = C^{\text{surf}} + C^{\text{coul}} \]

- \( R_N \): nuclear radius
- \( \rho_{\text{in}} \): nuclear density
- \( \rho_{\text{out}} \): density of unbound neutrons

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Dynamics of a nucleus immersed in a neutron superfluid: correction due to the coupling to the lattice

Vibrational energy depends on the orientation with respect to the lattice

Splitting of a quadrupole vibrational multiplet:

\[ \Gamma_{tot} \sim V_{Coul} \times V_{WS}^{-1} \times \alpha \]

- \( V_{Coul} \): Coulomb interaction energy of the nucleus with the lattice
- \( V_{WS} \): Wigner-Seitz cell volume
- \( \alpha \): Amplitude of vibration
Spherical symmetry breaking due to the coupling between lattice and nuclear vibrations.

Nuclear quadrupole excitation energy in the inner crust.
At densities of $10^{13}$ - $10^{14}$ g/cm$^3$ competition between nuclear attraction and Coulomb repulsion leads to a very complex ground state that involve round (meat ball), rod (spaghetti), plate (lasagna), and other shapes.

This „nuclear pasta” is expected to have unusual properties and dynamics. It may be important for radio, x-ray, and neutrino radiation.

Simple semiclassical model predict a sequence of phase transitions:
First fully microscopic 3D calculations of pasta phases:

Hartree-Fock calcs. In coordinate space for 1000 nucleons in the Wigner-Seitz (WS) cell!

Coulomb interaction treated beyond the WS approximation!

‘Lasagna’ phase

Energy difference between the spherical phase and the ‘spaghetti’ phase:

Energy difference between the spherical phase and the ‘lasagna’ phase:

What is the origin of these oscillations?

H.B.G. Casimir (1948): two parallel uncharged metallic plates attract each other in vacuum

\[
\frac{F^\parallel(L)}{A} = -\frac{\hbar c}{L^4} \frac{\pi^2}{240} \approx -1.3 \times 10^{-7} \frac{1}{L^4} \text{N} \mu\text{m}^4\cm^{-2}
\]

\[
\varepsilon^\parallel(L) = -\frac{\hbar c}{L^3} \frac{\pi^2}{720} A
\]

- Origin: zero-point fluctuations of e.m. field modified by the addition of the two plates relative to free case

\[\Rightarrow \text{change in the energy of the vacuum: } \sum \hbar \omega_k \big|_{\text{plates}(L)} - \sum \hbar \omega_k \big|_{\text{free}}\]

Casimir effect: Mesoscopic manifestation of quantum fluctuations of the vacuum

- Experimental confirmation in the last decade (for the sphere-plate system!)
  - S. Lamoureux, Phys. Rev. Lett. 78 (1997);
  - 9-Feb-2001 issue of New York Times about Casimir effect in MicroElectro Mechanical Systems; etc.
Original Casimir effect:

1) “box” geometry
2) EM b.c.’s: \( n \cdot B = 0 \) & \( n \wedge E = 0 \) \)
\[
\phi^D(x, y, z, t)|_{z=0} = \phi^D(x, y, z, t)|_{z=L} = 0 \quad \text{(Dirichlet b.c.’s)}
\]
\[
\phi_{nk_\perp}^D(x, y, z, t) = \sin(k_z z) e^{i(k_x x + k_y y)} e^{-i\omega_{n,k_x,k_y} t}
\]
\[
k_z = \left( \frac{\pi}{L} \right) n, \quad n = 1, 2, 3, \ldots \; ; \; \omega_{n,k_x,k_y} = c \sqrt{k_x^2 + k_y^2 + \left( \frac{n \pi}{L} \right)^2}
\]

TM:
\[
\frac{\partial}{\partial z} \phi^N(x, y, z, t)|_{z=0} = \frac{\partial}{\partial z} \phi^N(x, y, z, t)|_{z=L} = 0 \quad \text{(Neumann b.c.’s)}
\]
\[
\phi_{nk_\perp}^N(x, y, z, t) = \cos(k_z z) e^{i(k_x x + k_y y)} e^{-i\omega_{n,k_x,k_y} t}
\]
\[
k_z = \left( \frac{\pi}{L} \right) n, \quad n = 0, 1, 2, 3, \ldots .
\]

\[
\mathcal{E}_{\text{EM}}(L) = \lim_{\Lambda \to \infty} A \int \int \frac{dk_x dk_y}{(2\pi)^2} \left( \sum_{n=1}^{\infty} \frac{1}{2} \hbar \omega_{n,k_x,k_y} + \sum_{n=0}^{\infty} \frac{1}{2} \hbar \omega_{n,k_x,k_y} \right) e^{-\frac{\hbar \omega_{n,k_x,k_y}}{\Lambda}}
\]
\[
= \lim_{\Lambda \to \infty} \hbar c A L \left[ \frac{3(\Lambda/\hbar c)^4}{\pi^2} + (-1+1) \frac{(\Lambda/\hbar c)^3}{4\pi L} - \frac{\pi^2}{720 L^4} + O((\hbar c)^2/\Lambda^2 L^6) \right]
\]
“Generalization” of concept of Casimir energy:

1) geometry dependence

**Casimir energy** $\equiv$ vacuum energy from the *geometry-dependent* part of the density of states (d.o.s)

$(\leftrightarrow$ *shell* correction energy in Nucl. Phys.)

$$\text{d.o.s.: } \rho(E) \equiv \sum_{E_k} \delta(E - E_k) = \rho_0(E) + \rho_{\text{bulk}}(E) + \delta \rho_C(E, \text{geom.-dep.})$$

$$\text{N.o.s.: } N(E) \equiv \sum_{E_k} \Theta(E - E_k) = \int_0^E dE' \rho(E')$$

**Casimir Energy:** $E_C \equiv \int dE \int dE' \delta \rho_C(E, \text{geom.-dep.}) = \int dE N_C(E, \text{geom.-dep.})$

2) matter fields

- **Here**: space not “filled” with *fluctuating* EM modes, but with gas of *non-interacting* (non-relativistic) fermions.

- **Similarity**: $\exists$ mode sums $\sum \hbar \omega_k$ with *constant* degeneracy factor, (because of Pauli’s exclusion principle).

- **Difference**: $\exists$ of second scale: fermi energy = *chemical potential* $\mu$ (at $T \approx 0$) in addition to geometric size and distance scale(s).

- **Concrete**: Matter fields (fermions) in the space between voids build up a quantum pressure on the voids

$\exists$ *effective interaction* between empty regions of space in the background of *non-interacting* fermions
Let me create a caricature of the “pasta phase” in the crust of a neutron star.

**Question:** What is the most favorable arrangement of these two spheres?

**Casimir Interaction among Objects Immersed in a Fermionic Environment**

$$E_C \approx \frac{\hbar^2 k_F}{8\pi m r^3} a^2 \cos\left(2k_F(r - 2a)\right)$$

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The Casimir energy for the displacement of a single void in the lattice

A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)
What are the basic degrees of freedom of the neutron-proton-electron matter at subnuclear densities?

Estimates of various contributions to the specific heat in the crust

Characteristic temperature: $T \sim 0.1\ MeV$

- **Electrons:** $C_V \approx T/\varepsilon_F$
- **Superfluid uniform nuclear liquid:** $C_V \approx (\Delta/T)^{3/2} \Delta \exp(-\Delta/T)$
- **Lattice vibrations:** $C_V \approx 3N$
- **Nuclear shape vibrations:** $C_V \approx (2l+1)N$
Contributions to the specific heat of the inner crust
Conclusions

- There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.

- **Thermal and electric conductivities** of the inner crust are expected to be modified. In particular, the contributions coming from Umklapp processes have to be recalculated using the renormalized ion masses.

- Due to the coupling between the nuclear surface vibrations and the ion lattice part of the crust is filled with non-spherical nuclei. The phase transition takes place at densities far lower than the predicted density for the transition to the exotic „pasta phases“.

- The contribution to the **specific heat associated with nuclear shape vibrations** seems to be important at densities around 0.02 fm$^{-3}$ where the pairing correlations are predicted to reach their maximum.

- **Quantum corrections (Casimir energy)** to the ground state energy of an inhomogeneous neutron matter at the bottom of the crust are of the same magnitude or larger than the energy differences between spherical, „spaghetti”, and „lasagna” phases.

- The “pasta phase” might have a rather complex structure, various shape can coexist, and at the same time significant lattice distortions are likely and the bottom of the neutron star crust could be on the verge of a disordered phase.
Open questions:

- Basic degrees of freedom of the „pasta phase“?
- Influence on the cooling curve of neutron stars?
- The role of isovector nuclear modes?
- Mechanical properties of the crust? Liquid crystal?
- The physics of superfluid vortices in the inner crust?