Nuclear structure and dynamics in the neutron star crust

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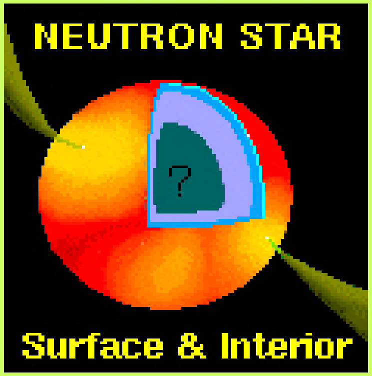
The 8th International Conference on
Clustering Aspects of Nuclear Structure and Dynamics
Nara, Japan

Content:



- Structure of the neutron star crust.
- Nuclear hydrodynamics in the inner crust
- In-medium ion mass renormalization and lattice vibrations of the Coulomb crystal.
- Collective excitations of nuclei immersed in a superfluid neutron environment.
- Spherical symmetry breaking in the inner crust.
- Specific heat of the inner crust.
- Nuclear clustering in the bottom of the inner crust: selfconsistent description of exotic ‘pasta’ phases. Fermionic Casimir effect.
- Conclusions and open questions.

Cooling stages of a neutron star:

1) Temperature: 50 MeV $\to$ 0.1 MeV

Dominant process:

URCA process: $p + e \to n + \nu_e$

$n \to p + e + \bar{\nu}_e$

2) Temperature: 0.1 MeV $\to$ 100 eV

Dominant process:

Modified URCA process: $p + n \to p + p + e + \bar{\nu}_e$

$n + n \to n + p + e + \bar{\nu}_e$

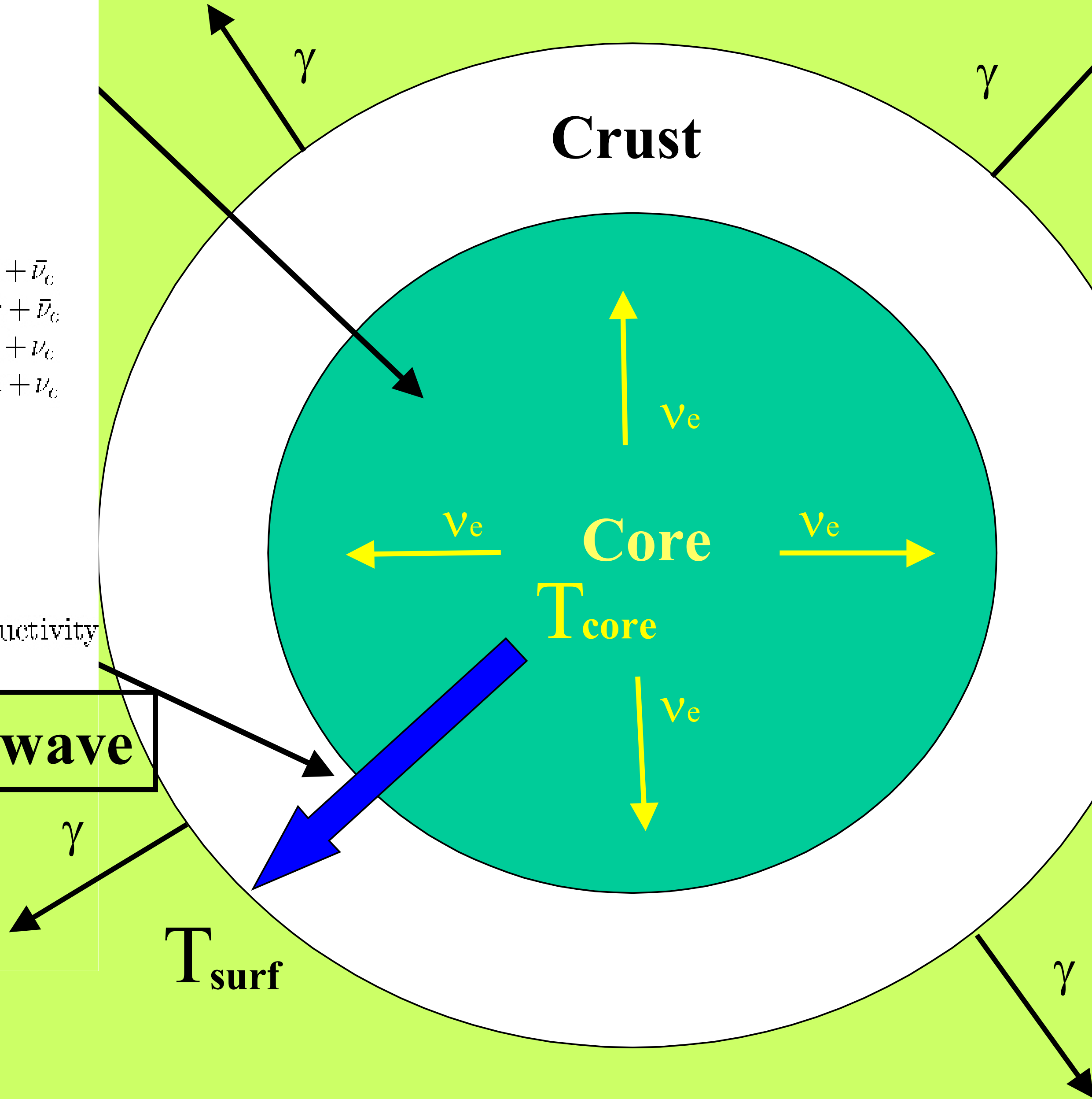
$p + p + e \to p + n + \nu_e$

$n + p + e \to n + n + \nu_e$

Heat conduction: $\frac{\partial T}{\partial t} = D \nabla^2 T$

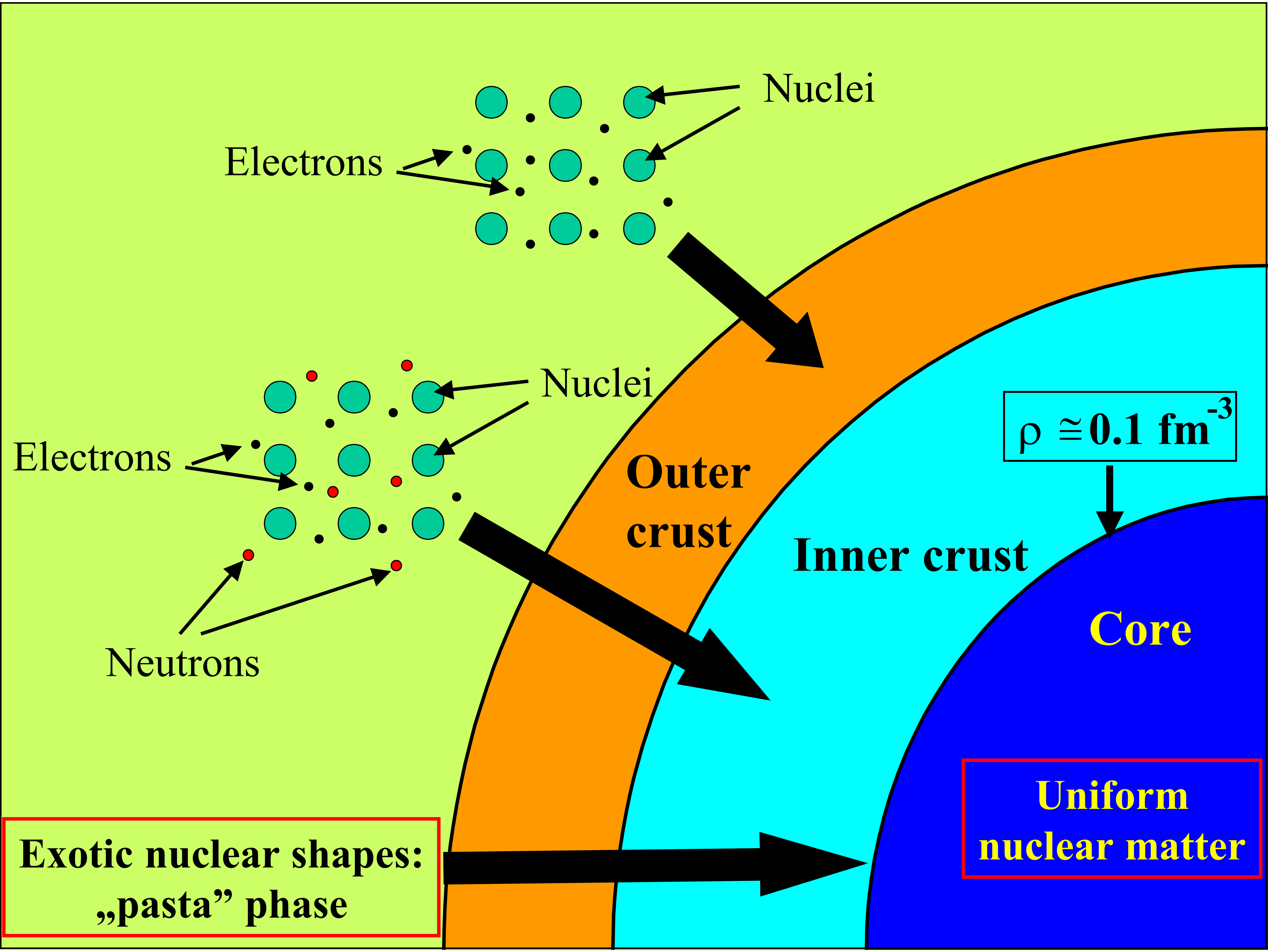
$D = \frac{\kappa}{C_v}$, κ - conductivity

Cooling wave



For τ < 100 years:

$T_{core} < T_{surf}$


Nuclei
Electrons
Nuclei
Electrons
Neutrons
Outer crust
Inner crust
ρ ≅ 0.1 fm-3
Core
Uniform nuclear matter
Exotic nuclear shapes: „pasta” phase

Estimations of various contributions to the specific heat

Characteristic temperature: T $\sim$ 0.1 MeV ($k_B = 1$)

Electrons (ultrarelativistic at densities $\rho > 10^6 \frac{g}{cm^3}$):

$$C_v \sim \frac{T}{\varepsilon_F} \qquad (\text{for } T \ll \varepsilon_F)$$

Superfluid uniform nuclear liquid:

$$C_v \sim (\frac{\Delta}{T})^{\frac{3}{2}} \Delta e^{-\frac{\Delta}{T}} \quad (\text{for } T \ll \Delta)$$

Lattice vibrations:

$$\frac{C_v}{\mathbf{N k_B}} \longrightarrow 3 \quad (\text{for } T \gg h\omega_p)$$

$$\omega_p = \sqrt{\frac{3(Ze)^2}{R_c^3 M}}$$ – plasma frequency, R_c - radius of a WS cell

Nuclear shape vibrations (multipolarity l):

$$\boxed{\frac{C_v}{\mathbf{N k_B}} \; \rangle \; 2l + 1 \; (\text{for } T \gg h\omega_l)}$$

$h\omega_l$ - excitation energy of a multipole vibration

Question:
What is a typical value of $h\omega_l$ in the inner crust of a neutron star?

What are the basic degrees of freedom of the neutron-proton-electron matter at subnuclear densities?

Nuclear collective excitations in the hydrodynamic approximation

Assumptions: incompressibility of a nuclear liquid
superfluidity (potential flow)

The motion is described by the function:

$$\Phi(\vec{r})$$

defined as: $\vec{u}(\vec{r}) = -\nabla\Phi(\vec{r})$,

where $\vec{u}(\vec{r})$ is a velocity field (stationary)

The continuity equation: $\nabla^2\Phi(\vec{r}) = 0$

has to be solved inside ($\Phi_{\rm in}$) and outside ($\Phi_{\rm out}$) **a nucleus.**

The boundary conditions:

$$\Phi_{\rm in}(\vec{r})|_{r=R} = \Phi_{\rm out}(\vec{r})|_{r=R}$$

$$\rho_{\rm in}\left(\frac{\partial\Phi_{\rm in}}{\partial r} - \vec{n}\cdot\vec{u}\right)\Big|_{r=R} = \rho_{\rm out}\left(\frac{\partial\Phi_{\rm out}}{\partial r} - \vec{n}\cdot\vec{u}\right)\Big|_{r=R}$$

$$\Phi_{\rm out}(\vec{r}) \rightarrow 0 \;\; \text{for } r \rightarrow \infty$$

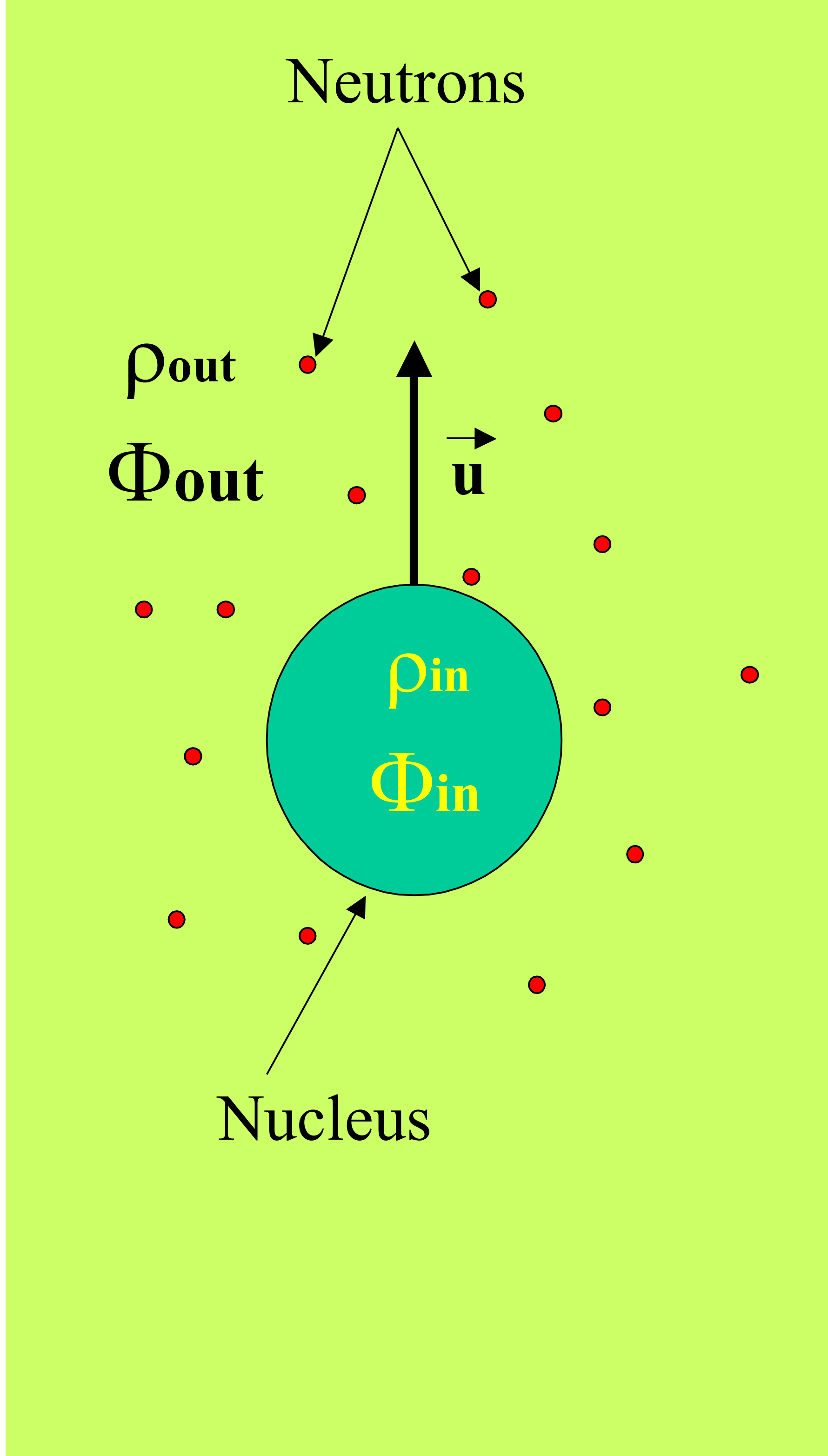
where:

$\rho_{\rm in}$, $\rho_{\rm out}$ - densities inside and outside a nucleus

$\vec{n}$ - normal unit vector

R - nuclear radius

P.Magierski, A. Bulgac, Acta Phys Pol. B, submitted



In-medium ion mass renormalization and lattice vibrations

$$\boxed{M^{ren} = M \frac{(1-\gamma)^2}{2\gamma+1}} \qquad \gamma = \frac{\rho_{out}}{\rho_{in}}$$

M - bare nuclear mass

M^{ren} - renormalized mass

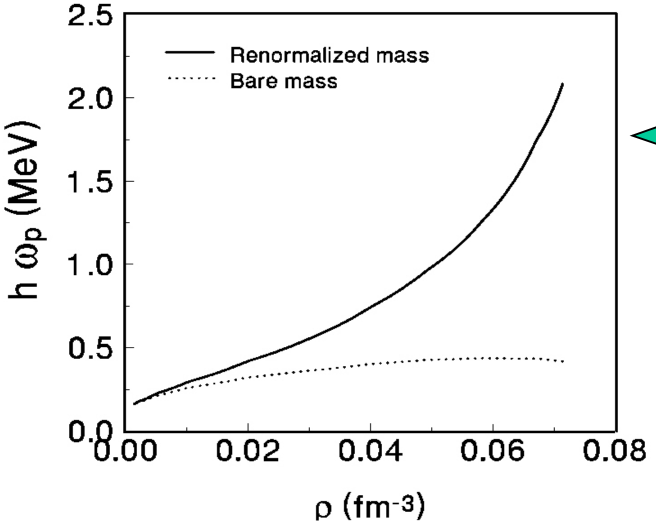
Limiting cases:

1) $M^{ren} \longrightarrow M$ (nucleus in a vacuum)
2) $M^{ren} \longrightarrow 0$ (uniform system)
3) $M^{ren} \longrightarrow \gamma/2\, M$ (bubble)

Plasma frequency:

$$\boxed{\frac{\omega_p}{\omega_p^{ren}} = \frac{|1-\gamma|}{(2\gamma+1)^{1/2}}}$$

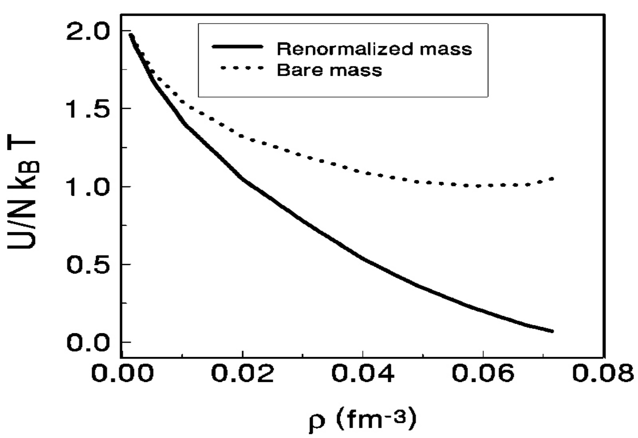
P.Magierski, A. Bulgac, Acta Phys Pol. B, submitted

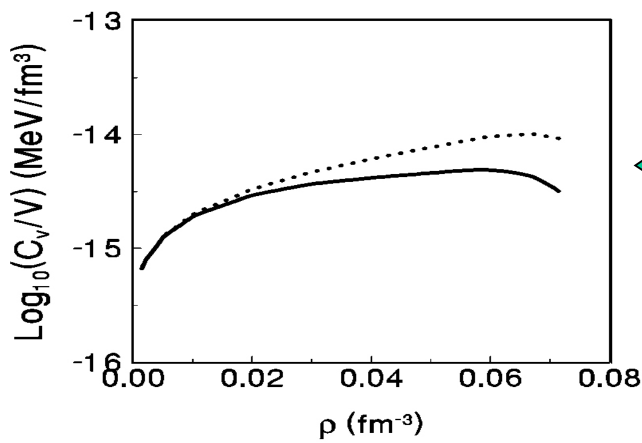


Plasma frequency

Lattice thermal energy per ion/nucleus

BCC Coulomb crystal in the inner crust





Lattice specific heat per unit volume

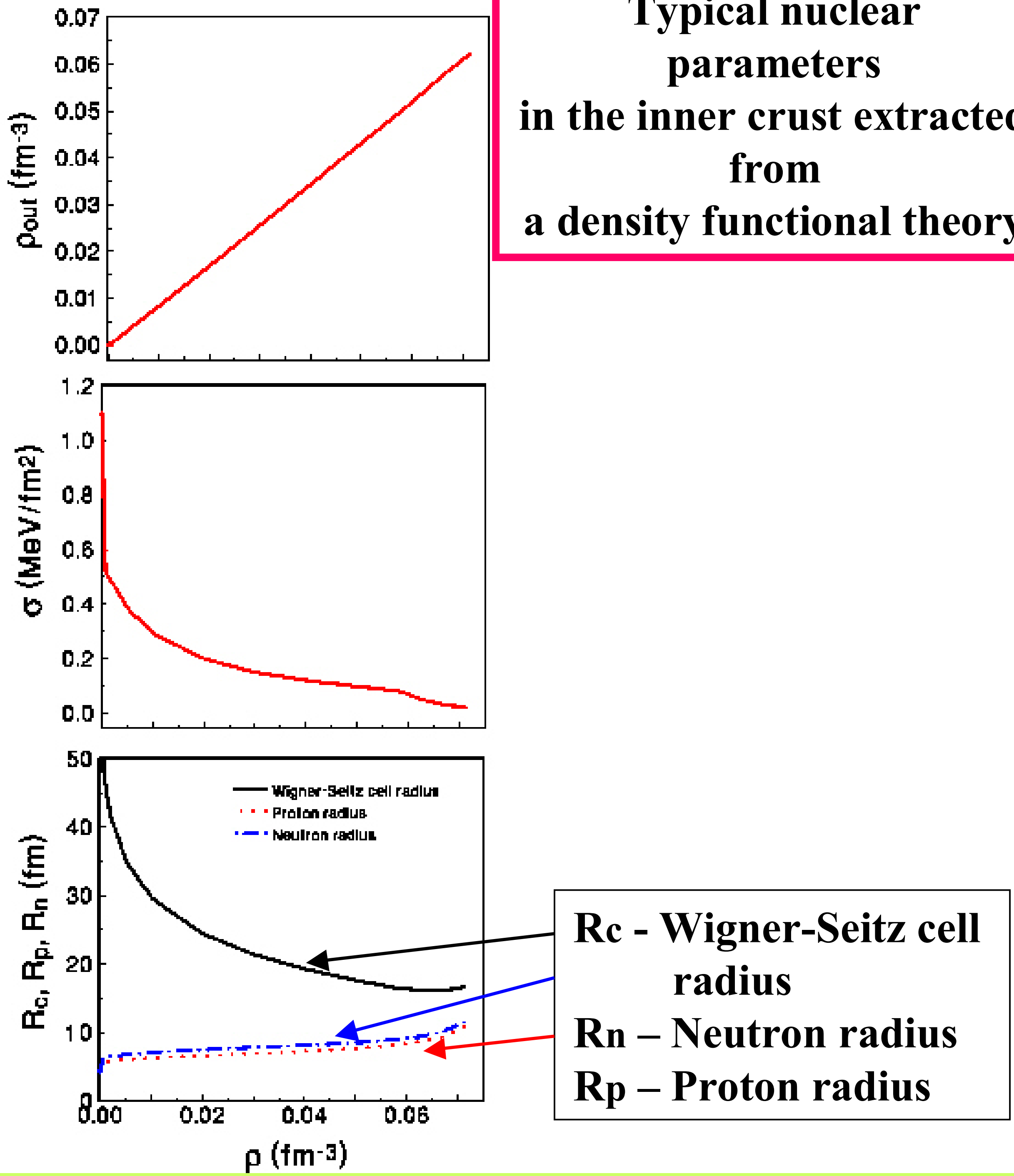
P.Magierski, Int.J.Mod.Phys.E, in press

Typical nuclear parameters in the inner crust extracted from a density functional theory

ρ_{out} - density of outside neutrons

σ- surface tension of a nucleus

From P. Haensel, J.L. Zdunik, J. Dobaczewski, Astron. Astrophys. 222 (1989) 353
From F. Douchin, P. Haensel, J. Meyer, Nucl. Phys. A665 (2000) 419



Hamiltonian of a nucleus immersed in a neutron gas

(harmonic approximation):

$$H = \frac{1}{2}\sum_{l,m}\left(\frac{|P_{lm}|^2}{M_l} + C_l|\alpha_{lm}|^2\right)$$

Mass parameters:

$$\boxed{M_l = m\rho_{\rm in}\frac{(\gamma-1)^2}{\gamma(l+1)+l}R^5} \qquad \gamma = \frac{\rho_{\rm out}}{\rho_{\rm in}}$$

Note that:

$M_l \to \rho_{\rm in}\frac{R^5}{l}$ for $\rho_{\rm out}=0$ - nucleus in a vacuum

$M_l \to 0$ for $\rho_{\rm out}=\rho_{\rm in}$ - uniform system

$M_l \to \rho_{\rm out}\frac{R^5}{l+1}$ for $\rho_{\rm in}=0$ - bubble nucleus

Stiffness

$$C_l = C_l^{\rm surf} + C_l^{\rm coul}$$

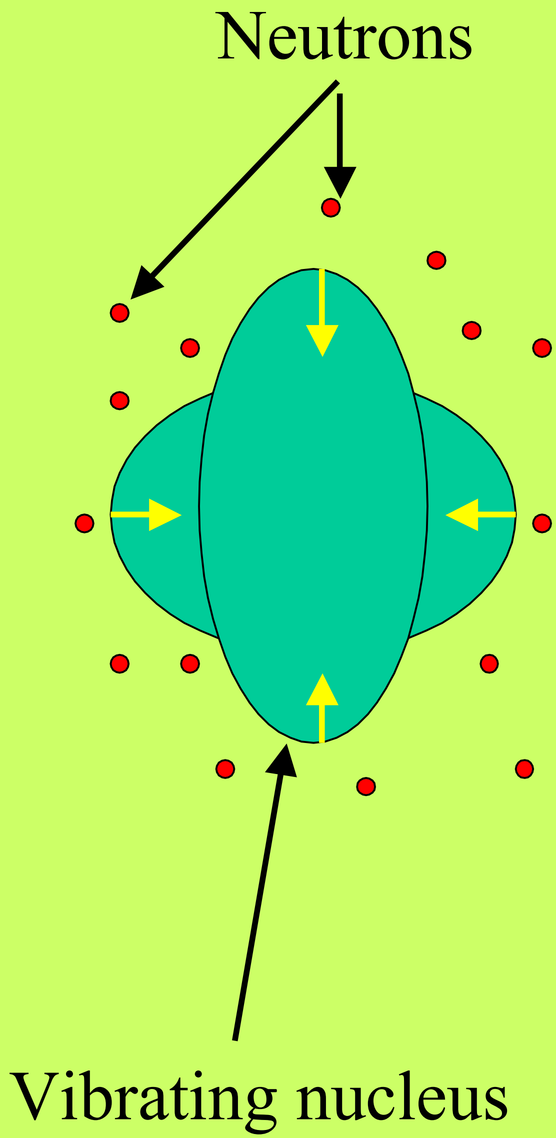
$$C_l^{\rm surf} = R^2\sigma(l-1)(l+2);\ \ \sigma - \text{surface tension}$$

$$C_l^{\rm coul} = -\frac{3}{2\pi}\frac{(Ze)^2}{R_p}\frac{l-1}{2l+1} + \frac{3}{4}\frac{(Ze)^2}{R_c}\left(\frac{R_p}{R_c}\right)^2$$

R_p - proton radius

R_c - Wigner-Seitz cell radius

Interaction with a lattice



Spreading width of a quadrupole vibrational state

Static width (Coulomb):

$$\Gamma^{\rm coul}_{\rm stat} \cong 0.169\,\frac{(Zc)^2}{R_c}\left(\frac{R_p}{R_c}\right)^2\left(\frac{3\hbar\omega_2}{2C_2}\right)^{\frac{1}{2}}$$

Dynamic width (Coulomb):

$$\Gamma^{\rm coul}_{\rm dyn} \cong 0.248\,\frac{(Zc)^2}{R_c}\left(\frac{R_p}{R_c}\right)^3\left(\frac{3\hbar\omega_2}{2C_2}\frac{U(T)}{C_1}\right)^{\frac{1}{2}}$$

Dynamic width (hydrodynamical):

$$\Gamma^{\rm hydr}_{\rm dyn} \cong 0.396\,m\rho_{\rm out}\frac{(\gamma-1)^2}{(2\gamma-1)(3\gamma-3)}\left(\frac{R_n}{R_c}\right)^4 R_n^5\left(N\,\frac{\hbar\omega_2}{2M_2}\frac{U(T)}{M_1}\right)^{\frac{1}{2}}$$

Note that:

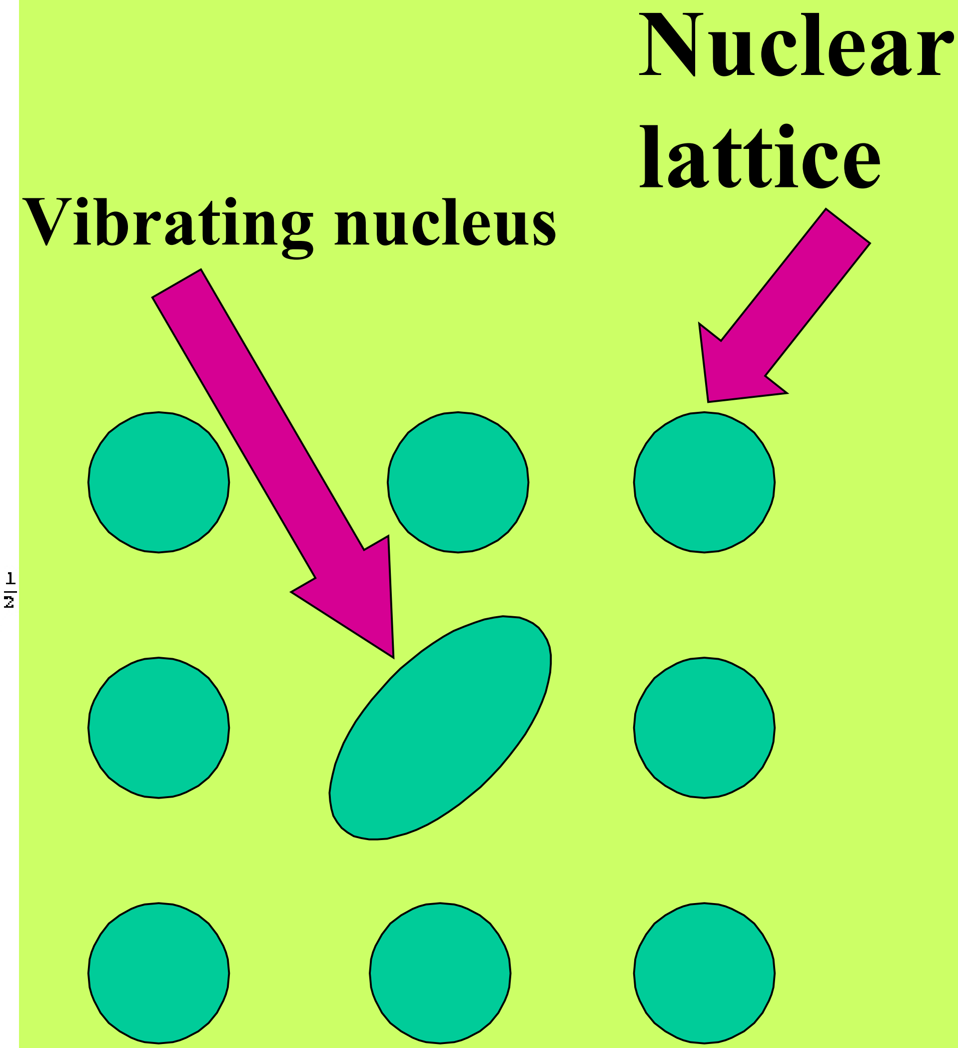
$\Gamma^{\rm coul}_{\rm dyn}(T)$ and $\Gamma^{\rm hydr}_{\rm dyn}(T)$ **- depends on temperature**

U(T) - the internal energy per ion associated with lattice vibrations.

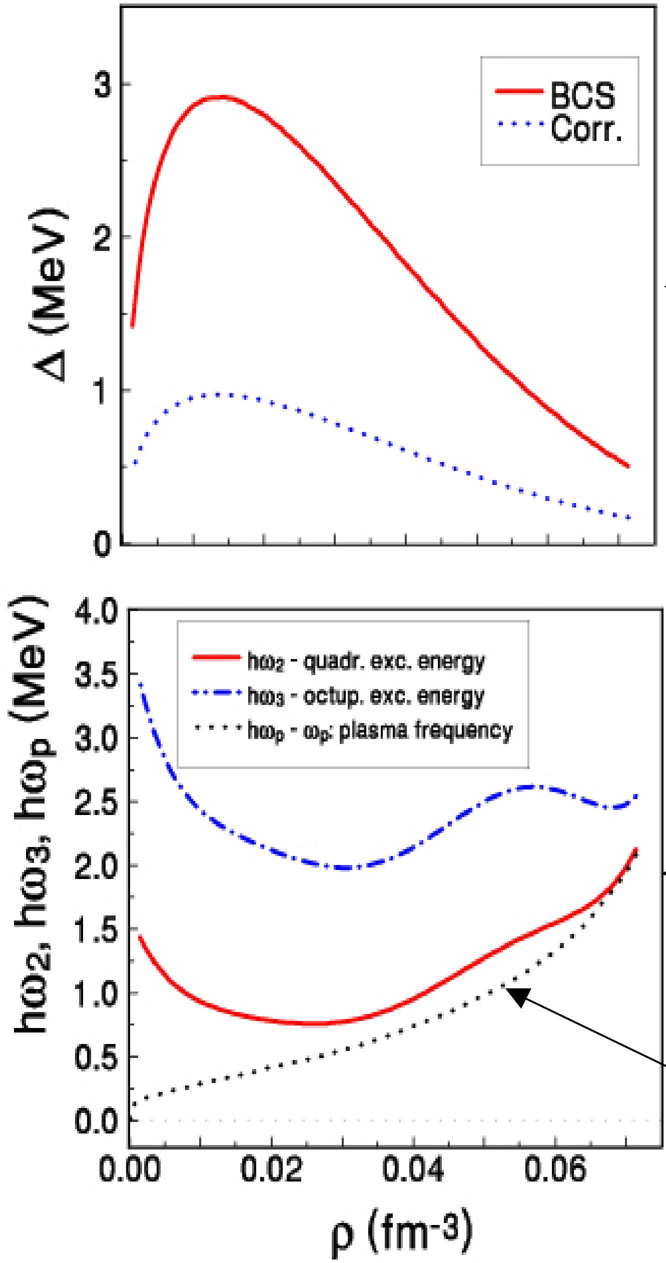
N - number of nearest neighbours

R_n - neutron radius

M_1, C_1 - mass parameter and stiffness for a dipole motion



Energy depends on the orientation with respect to the lattice vectors

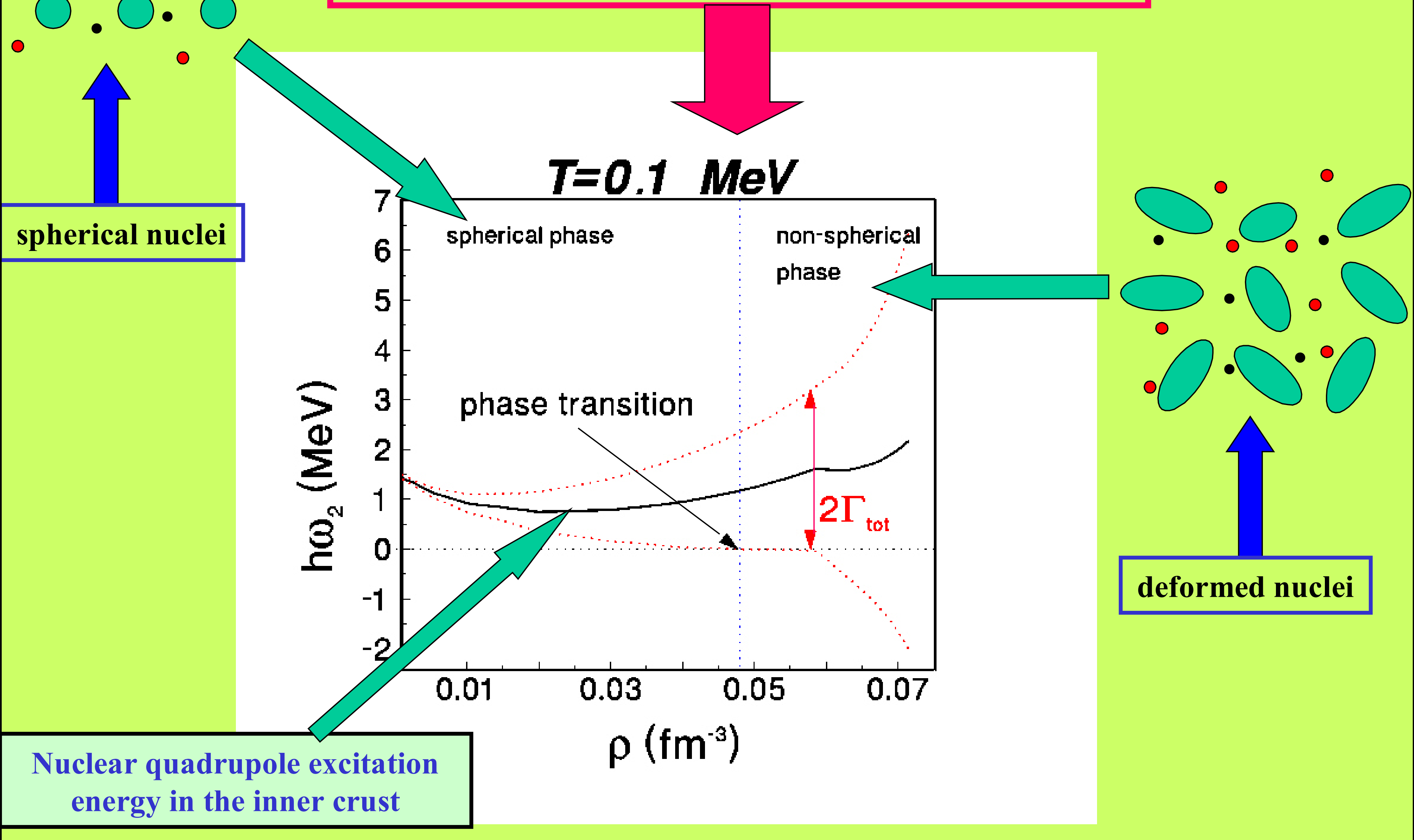


Typical values of pairing gap

Quadrupole and octupole excitation energies in the inner crust

Plasma frequency

Spherical symmetry breaking due to the coupling between lattice and nuclear vibrations



Contribution to the specific heat coming from the nuclear vibrations

$$C_v = \frac{1}{(2T)^2} \sum_{l,m} (2l+1)\left(\frac{h\omega_l}{\mathrm{Sinh}\frac{h\omega_l}{2T}}\right)^2$$

In the case when $\Gamma_{l\,\mathrm{tot}} \neq 0$, where

$$\Gamma_{l\,\mathrm{tot}} = \left((\Gamma^{\mathrm{coul}}_{l\,\mathrm{stat}})^2 + (\Gamma^{\mathrm{coul}}_{l\,\mathrm{dyn}})^2 + (\Gamma^{\mathrm{hydr}}_{l\,\mathrm{dyn}})^2 \right)^{\frac{1}{2}}$$

one obtains:

$$C_v = \frac{1}{(2T)^2} \int g(\omega)\left(\frac{h\omega}{\mathrm{Sinh}\frac{h\omega}{2T}}\right)^2 d(h\omega)$$

Specific heat per ion

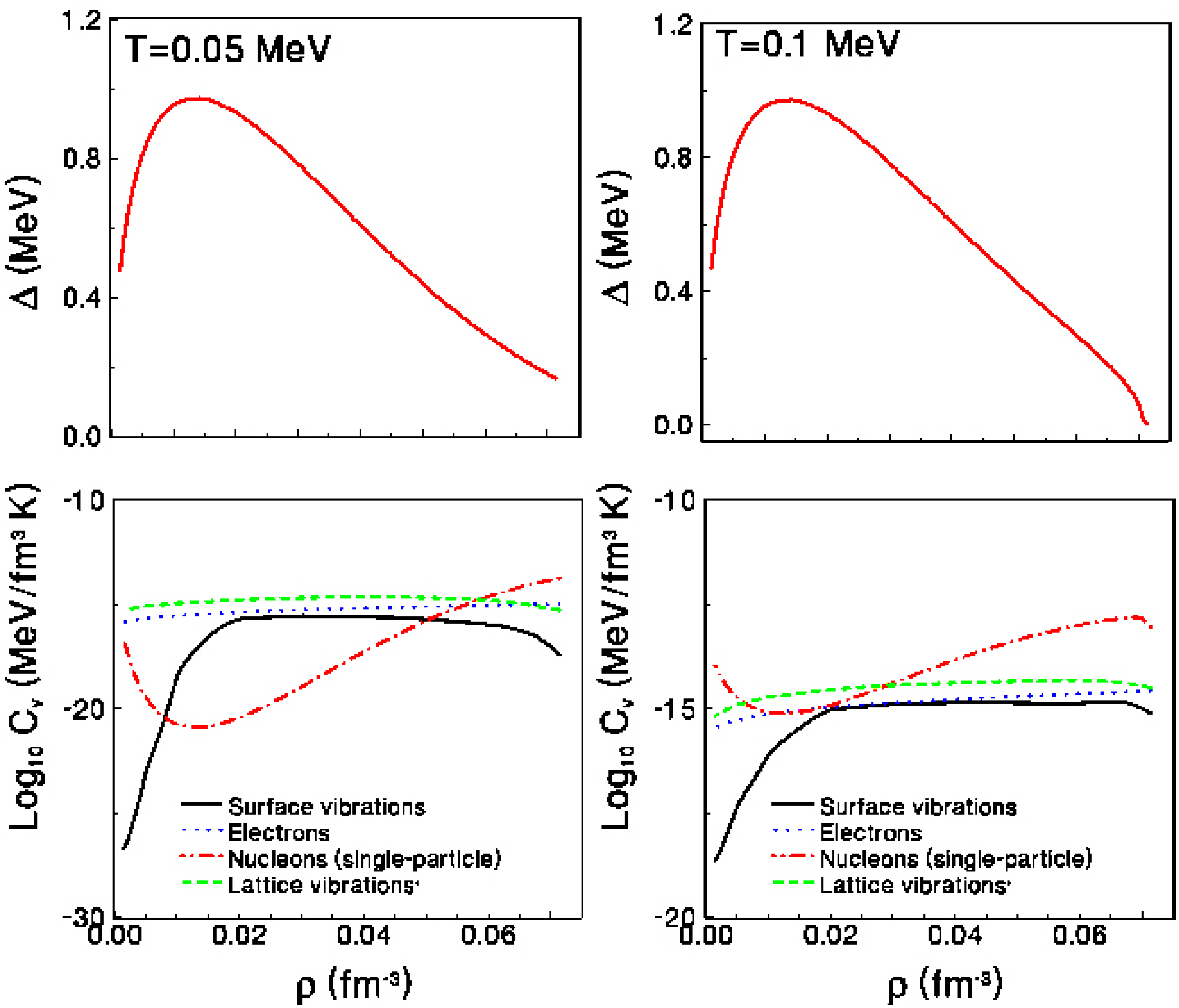
where

$$g(\omega) = N_l(2l+1)e^{-\frac{(h\omega_l - h\omega)^2}{2(\Gamma_{l\,\mathrm{tot}})^2}}$$

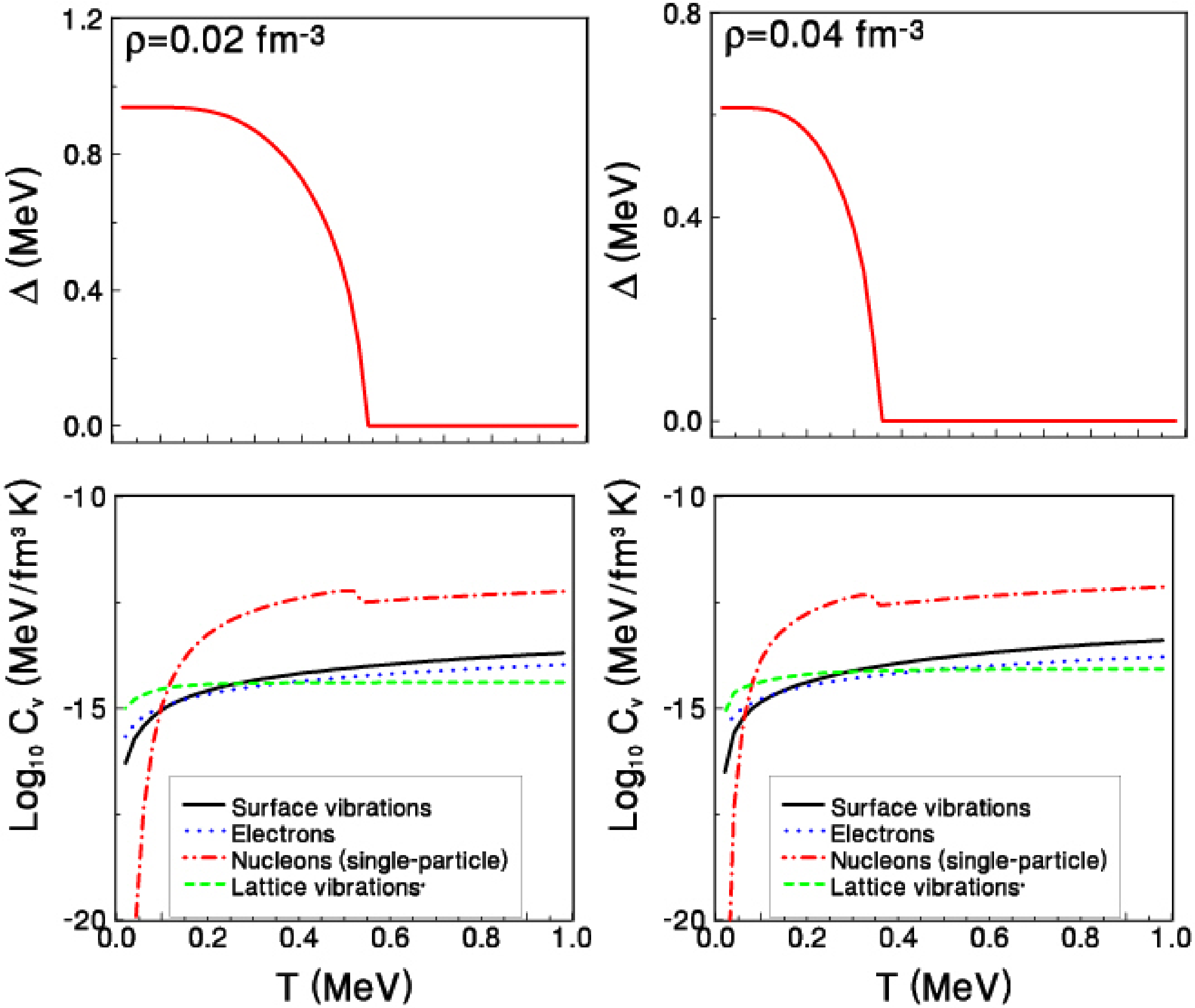
$$\int g(\omega)d(h\omega) = (2l+1)$$

The dominant contribution comes from quadrupole vibrations

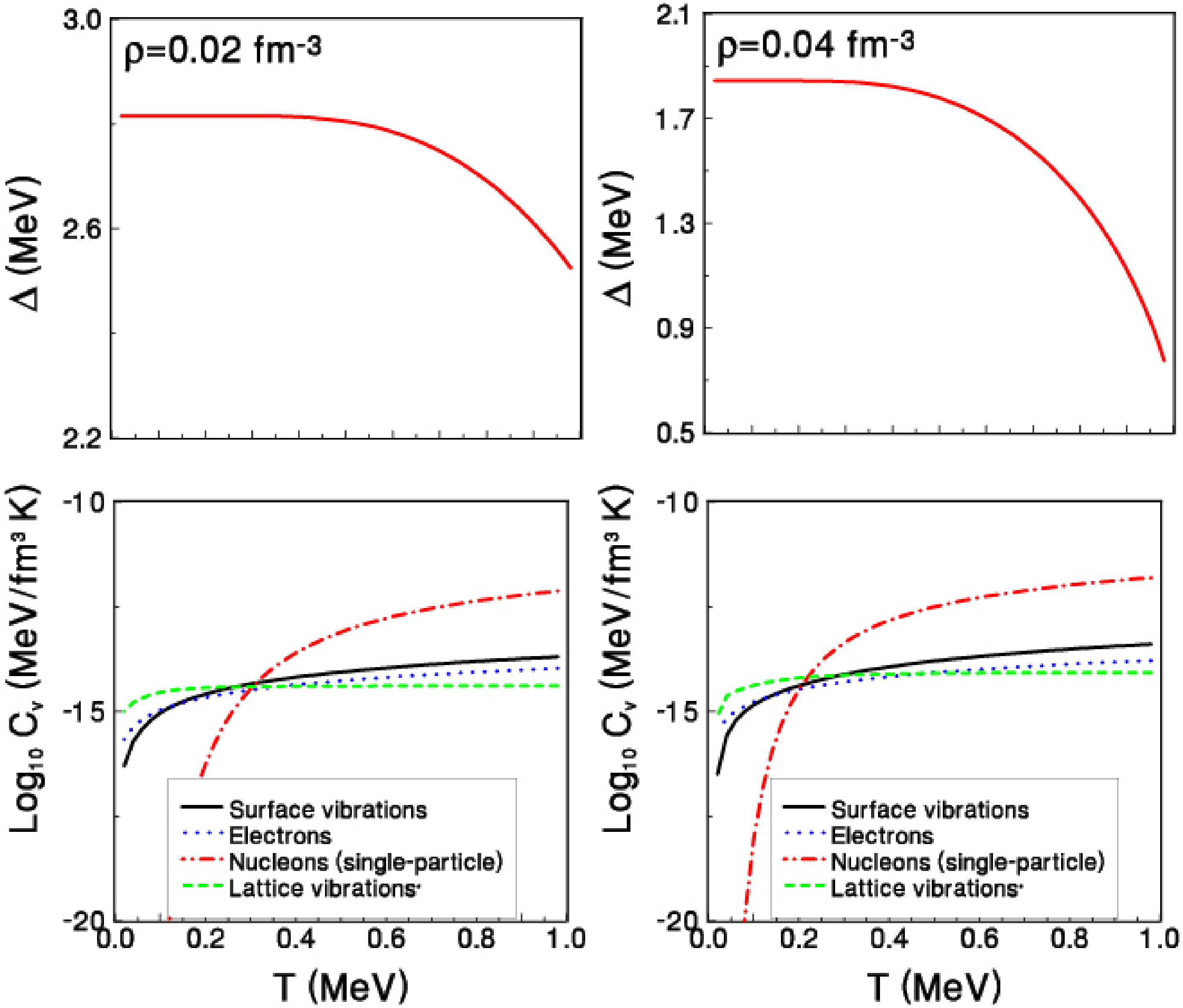
Contributions to the specific heat of the inner crust



Contributions to the specific heat of the inner crust

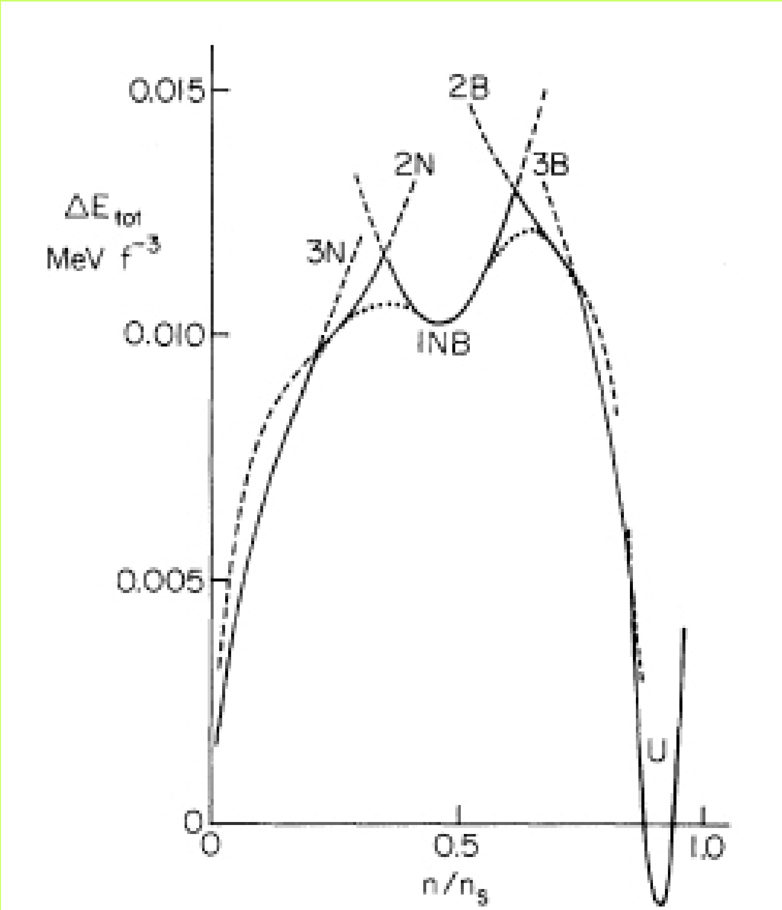


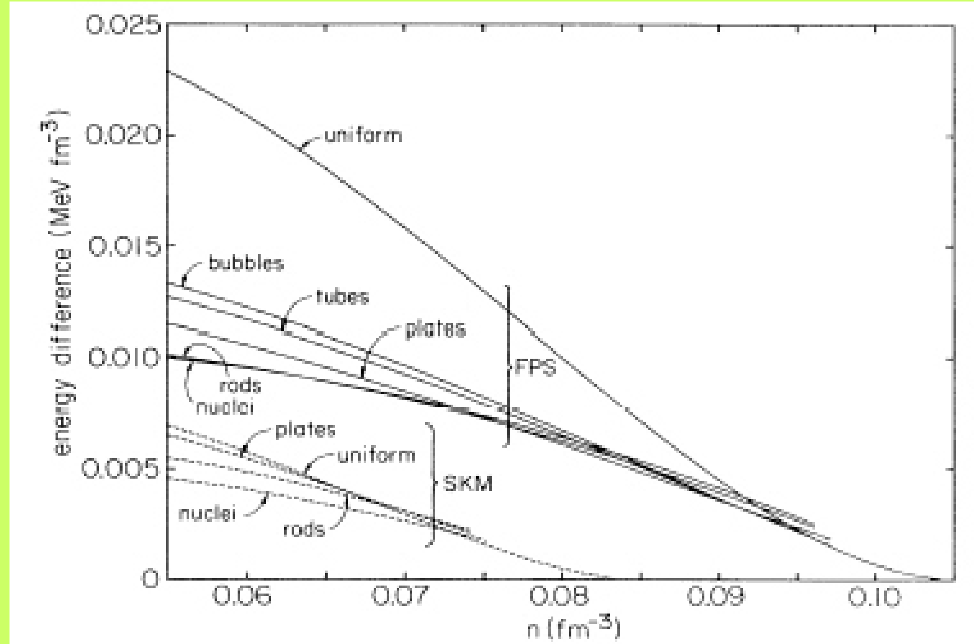
BCS pairing



ρ=0.02 fm-3
ρ=0.04 fm-3
Δ (MeV)
3.0
2.6
2.2
2.1
1.7
1.3
0.9
0.5
Log10 Cv (MeV/fm3 K)
-10
-15
-20
0.0
0.2
0.4
0.6
0.8
1.0
T (MeV)
Surface vibrations
Electrons
Nucleons (single-particle)
Lattice vibrations

Structure of the bottom of the inner crust: „pasta” phases



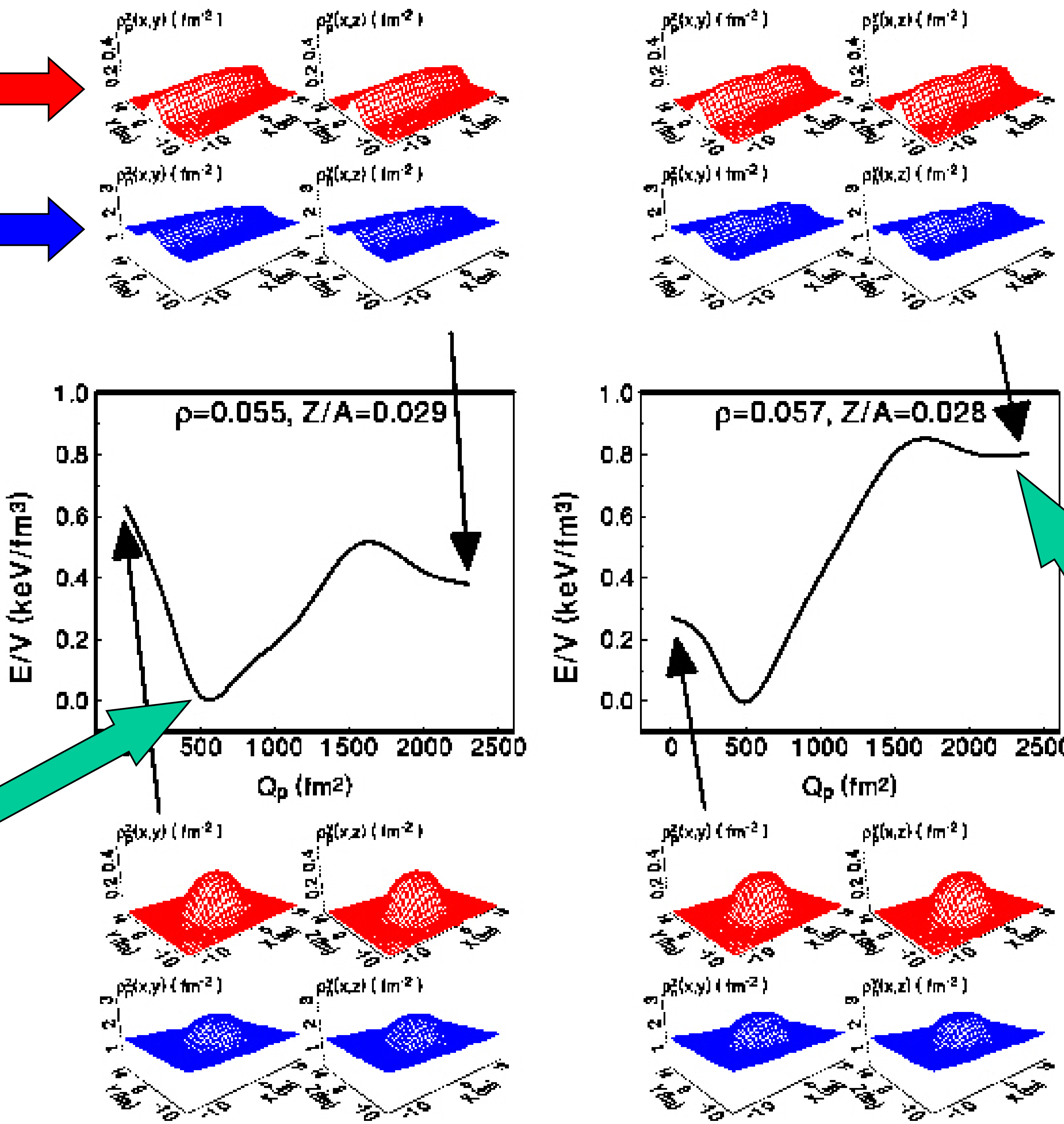


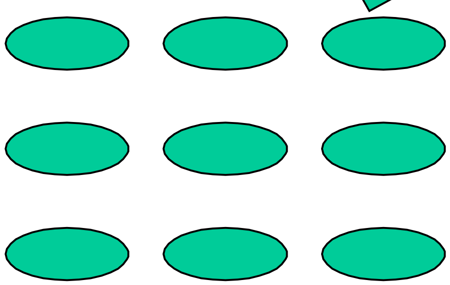
Lorenz, Ravenhall and Pethick
Phys. Rev. Lett. 70, 379 (1993)

Ravenhall, Pethick and Wilson
Phys. Rev. Lett. 50, 2066 (1983)

Proton density distribution

Neutron density distribution





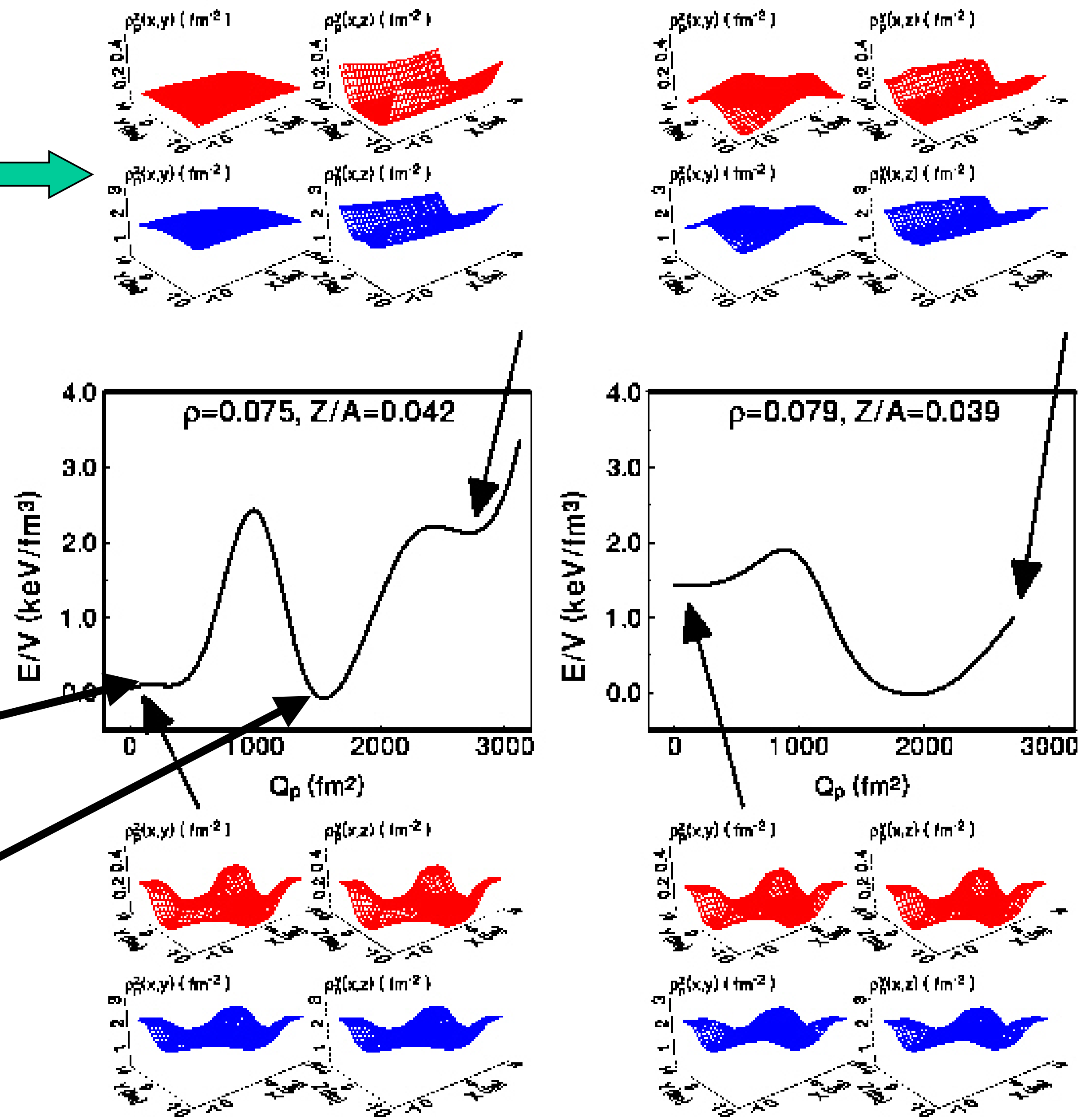
'Spaghetti' phase

Skyrme HF with SLy4, P. Magierski and P.-H. Heenen, Phys.Rev.C65,045804 (2002)

‘Lasagna’ phase

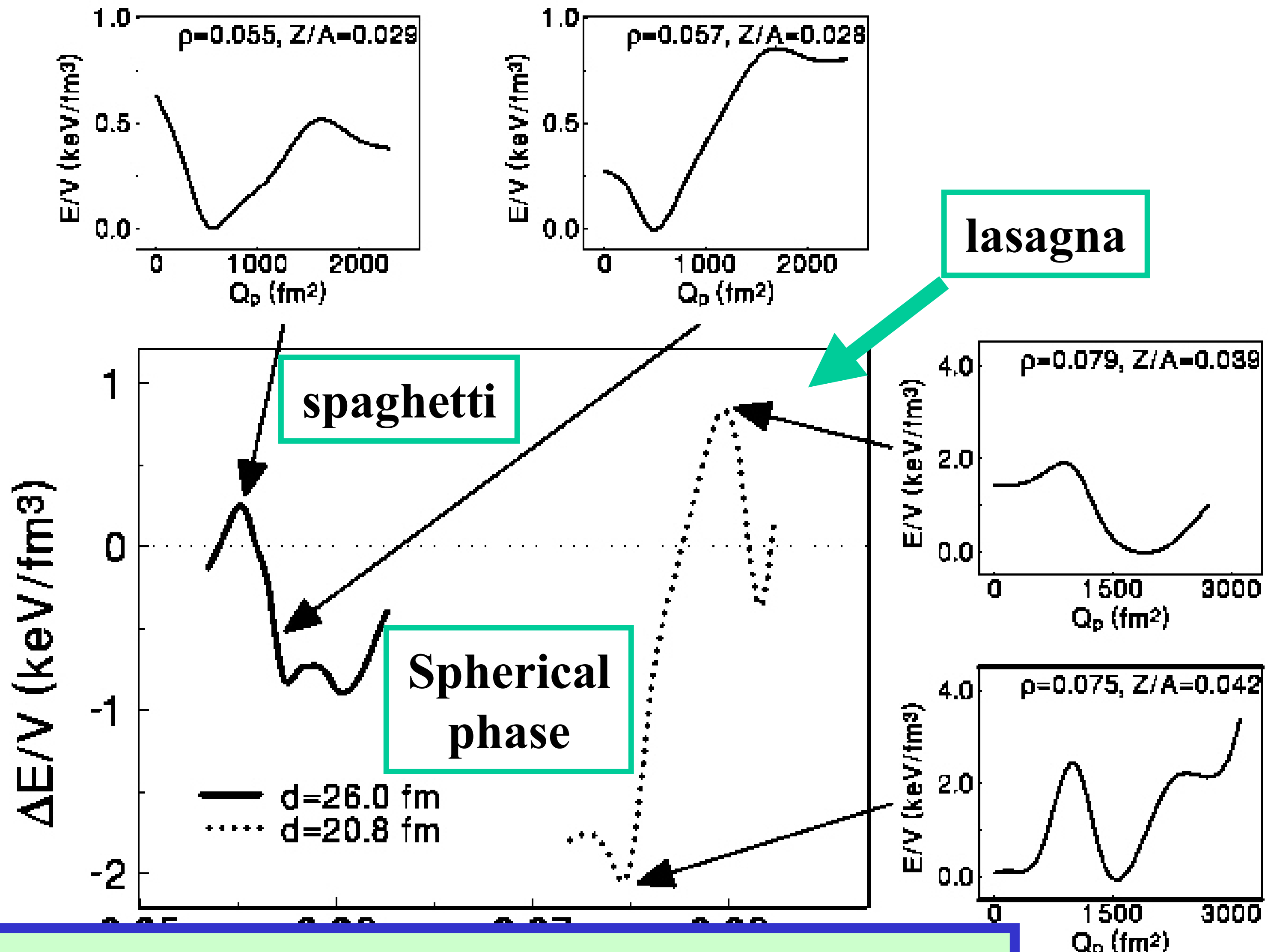
scc lattice

bcc lattice



Skyrme HF with SLy4, P.Magierski and P.-H.Heenen, Phys.Rev.C 65, 045804 (2002)

Energy difference between the spherical phase and the 'spaghetti' phase: ———
Energy difference between the spherical phase and the 'lasagna' phase: ....

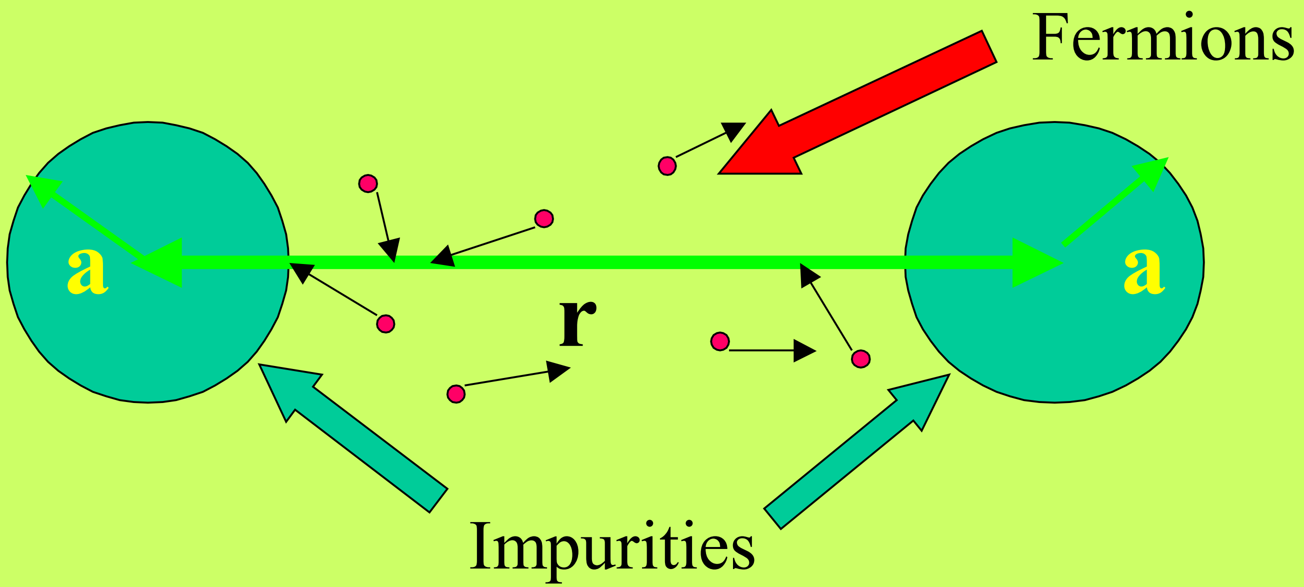


What is the origin of these oscillations?

Skyrme HF with SLy4, P.Magierski and P.-H.Heenen, Phys.Rev.C 65, 045804 (2002)

Shell effects

Let me create a caricature of a the "pasta phase" in the crust of a neutron star.



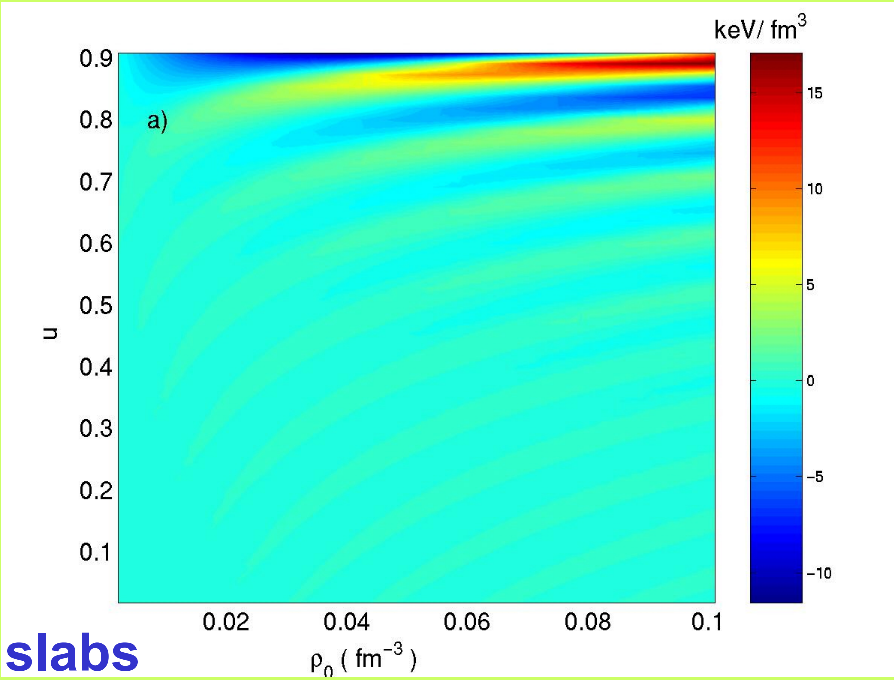
Question: **What is the most favorable arrangement of these two spheres?**

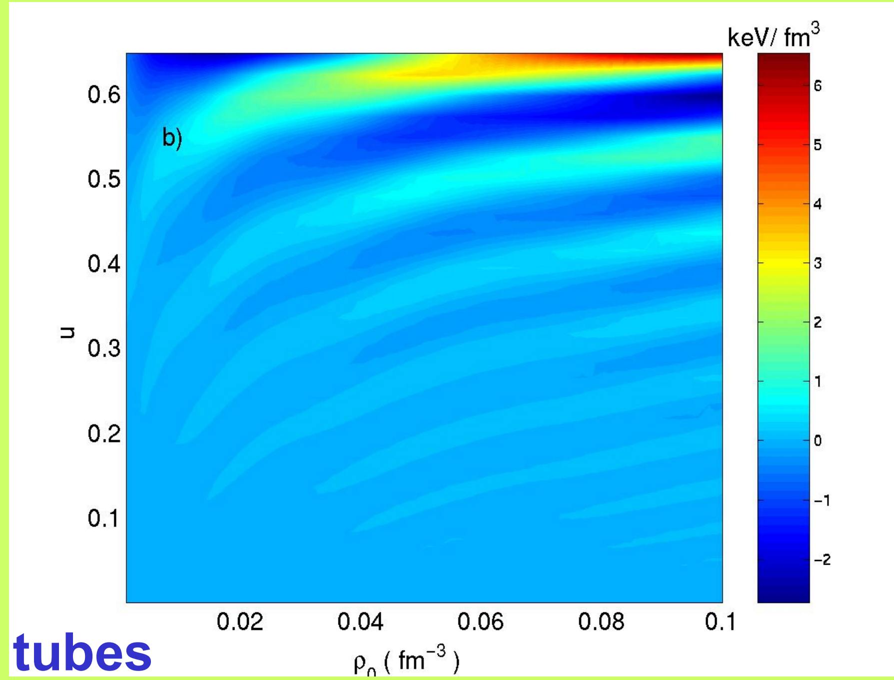
Casimir Interaction among Objects Immersed in a Fermionic Environment

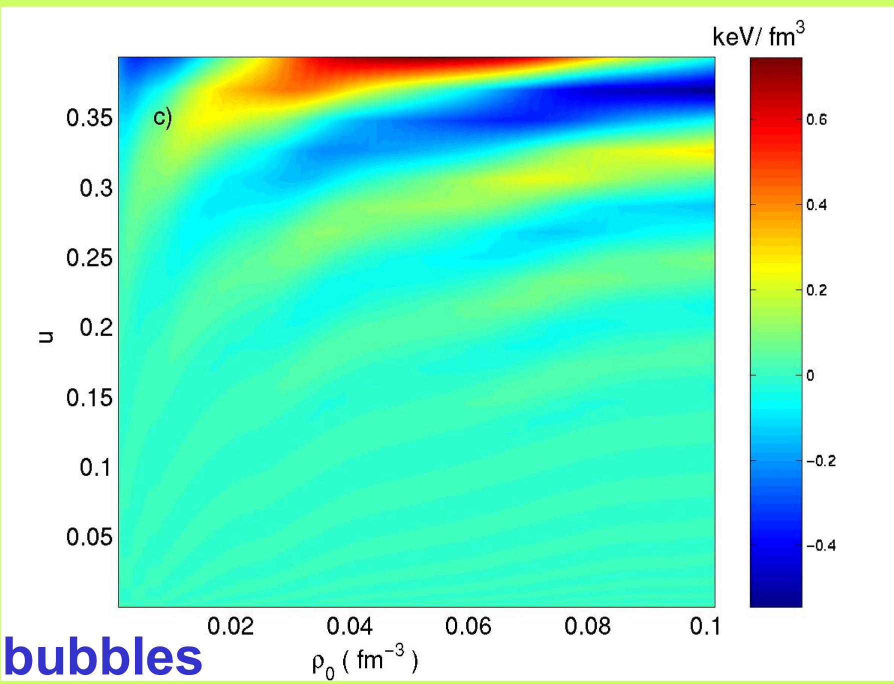
Semiclassical approximation:

$$E_C \approx \frac{\hbar^2 k_F a^2}{8\pi m r^3} \cos(2k_F(r-2a)) \quad ; \textit{ for } r>>a$$

Quantum Corrections to the GS Energy of Inhomogeneous NM



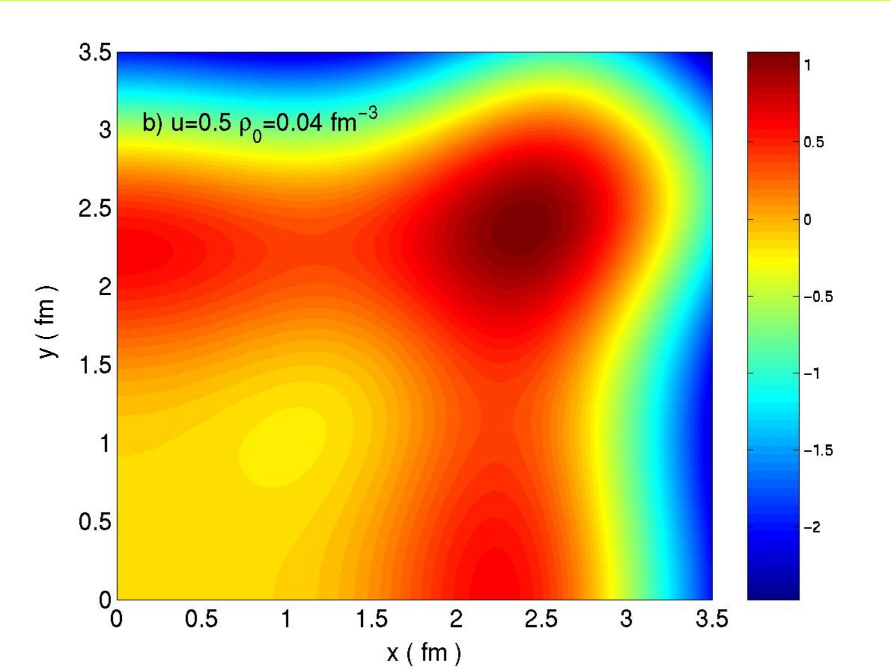




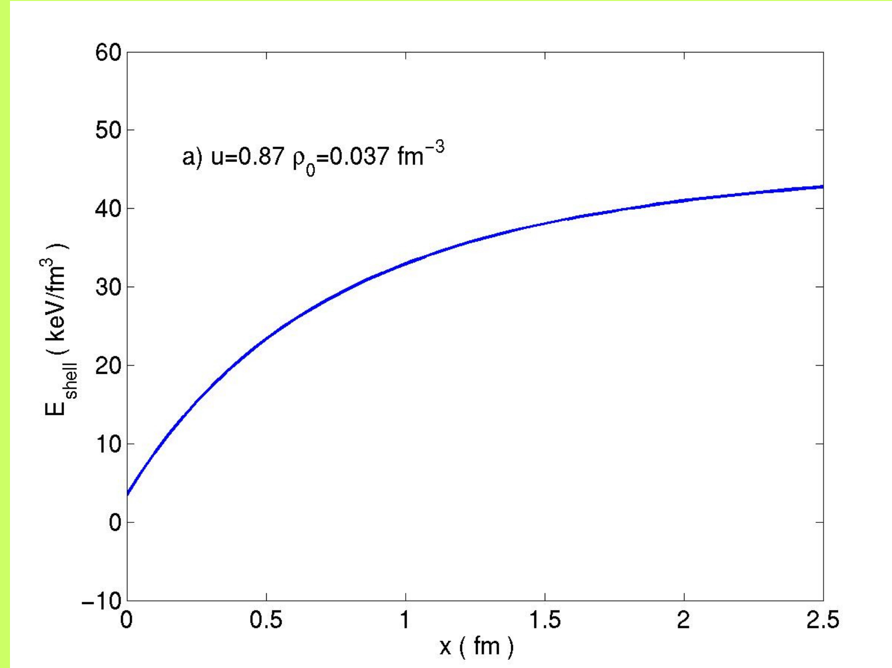
The Casimir energy for various phases.
The lattice constants are:
L=23, 25 and 28 fm respectively.
u — anti-filling factor

A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)
Nucl. Phys, 703, 892 (2002) (E)

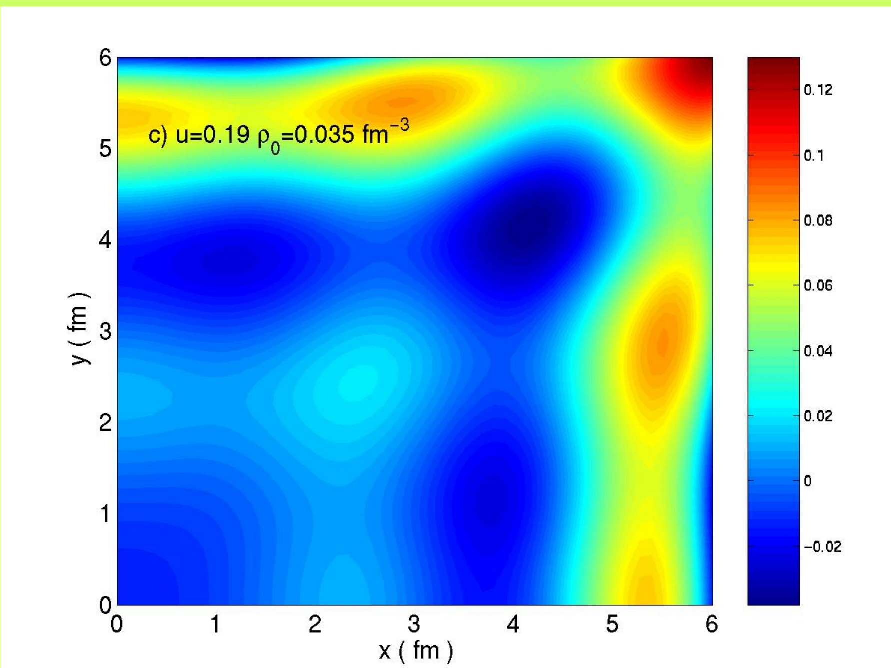
The Casimir energy for the displacement of a single void in the lattice



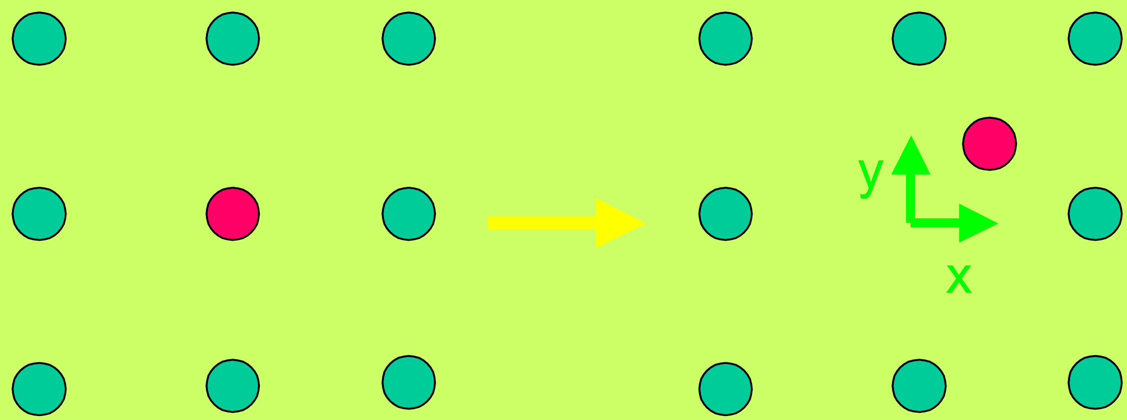
Rod phase



Slab phase



Bubble phase



A. Bulgac and P. Magierski
Nucl. Phys. 683, 695 (2001)
Nucl. Phys, 703, 892 (2002) (E)

Conclusions

- **There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.**
- **Thermal and electric conductivities of the inner crust are expected to be modified. In particular, the contributions coming from Umklapp processes have to be recalculated using the renormalized ion masses.**
- **Due to the coupling between the nuclear surface vibrations and the ion lattice part of the crust is filled with non-spherical nuclei. The phase transition takes place at densities far lower than the predicted density for the transition to the exotic „pasta phases”.**
- **The contribution to the specific heat associated with nuclear shape vibrations seems to be important at densities around 0.02 fm $^{-3}$ where the pairing correlations are predicted to reach their maximum.**
- **Quantum corrections (Casimir energy) to the ground state energy of an` inhomogeneous neutron matter at the bottom of the crust are of the same magnitude or larger than the energy differencies between spherical, „spaghetti”, and „lasagna” phases.**
- **The “pasta phase” might have a rather complex structure, various shape can coexist, and at the same time significant lattice distortions are likely and the bottom of the neutron star crust could be on the verge of a disordered phase.**

Open questions:

- **Basic degrees of freedom of the „pasta phase”?**
- **Influence on the cooling curve of neutron stars?**
- **The role of isovector nuclear modes?**
- **Mechanical properties of the crust?**
- **The role of superfluid vortices in the inner crust?**