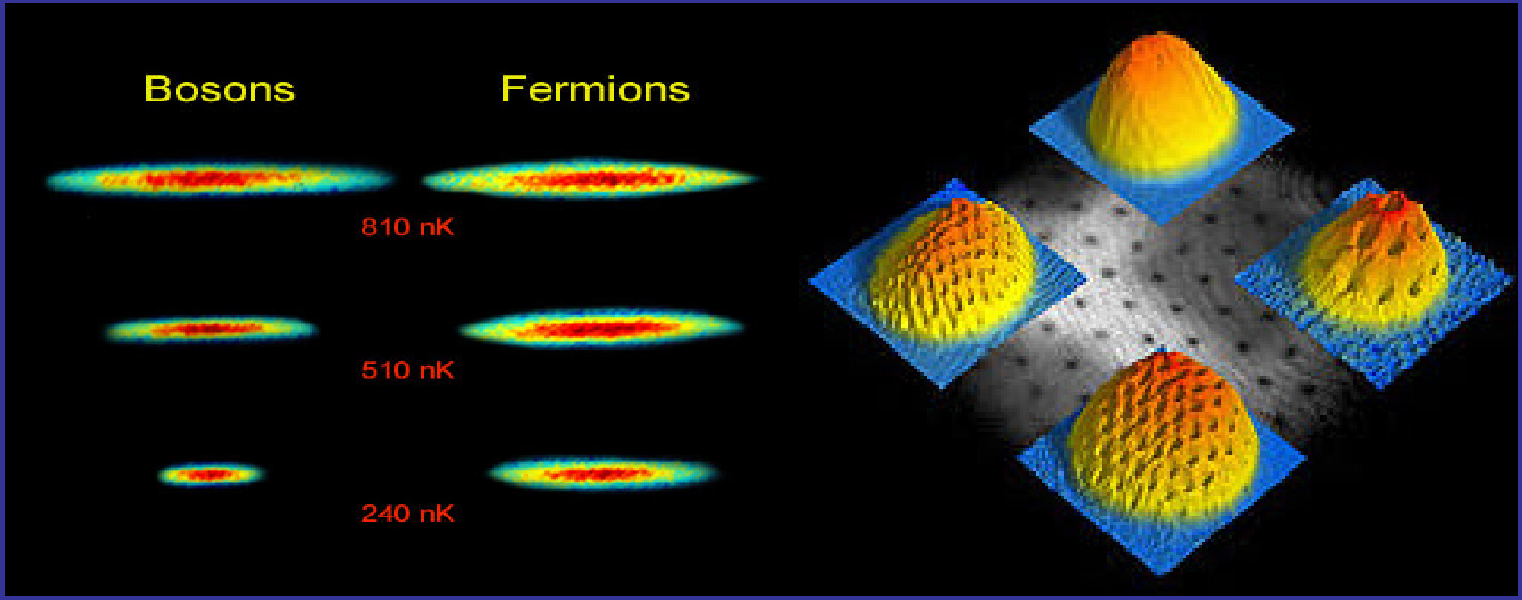
Pairing properties of a Fermi gas with infinite scattering length



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$$E/E_{FG} = 1 + \frac{10}{9\pi}(k_F a)\left[1 + \frac{6}{35\pi}(k_F a)(11 - 2ln2) + \ldots\right] + \text{pairing}$$

Perturbation series

$$E_{FG} = \frac{3}{5}\varepsilon_F N$$ - Energy of the noninteracting Fermi gas

➤ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ $\quad$ $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$i.e.\;\; r_0 \to 0,\; a \to \pm\infty$$

⇐ **NONPERTURBATIVE REGIME**

System is dilute but strongly interacting!

UNIVERSALITY: $E = \xi_0 E_{FG}$

AT FINITE TEMPERATURE: $E(T) = \xi\left(\frac{T}{\varepsilon_F}\right)E_{FG},\;\; \xi(0) = \xi_0$

Dilute neutron matter:

Effective range: $r_0 \approx 2.8$ fm
Scattering length: $a \approx -18.5$ fm

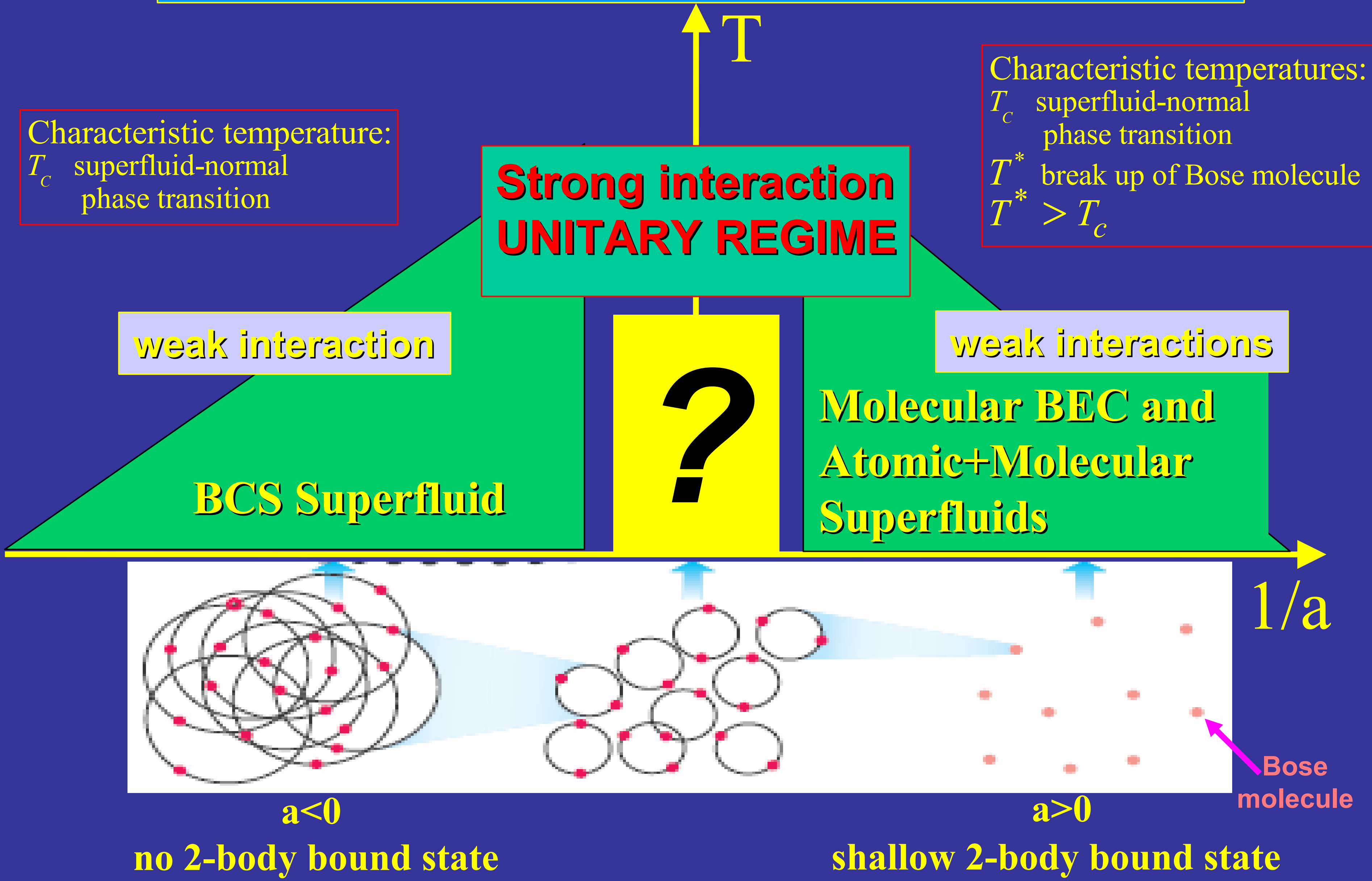
Unitary gas:

Effective range: $r_0 \approx 0$
Scattering length: $a \approx \pm\infty$

Physical realization eg.:

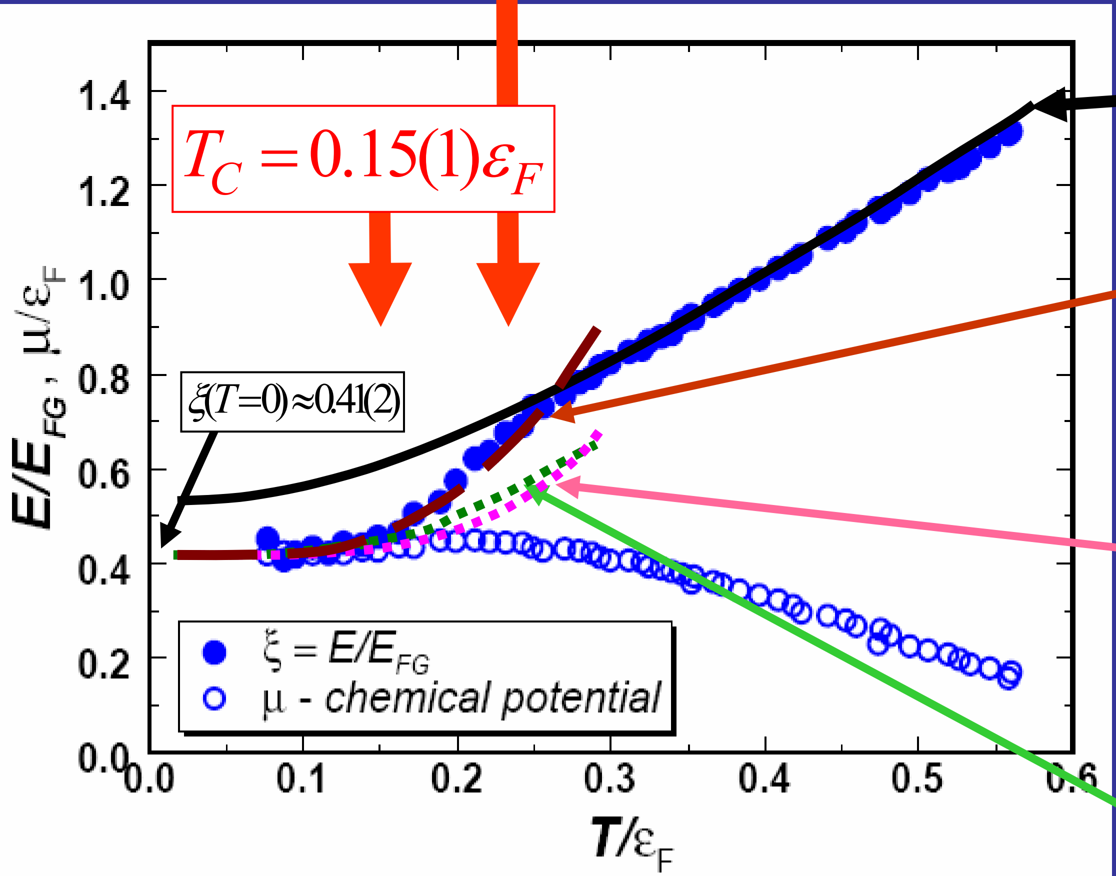
dilute gas of ^6Li atoms

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



$a = \pm\infty$

Deviation from Normal Fermi Gas



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution
(dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

Quasi-particle contribution only
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \ \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \ \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.41$$

A. Bulgac, J.E. Drut, P. Magierski,PRL96,090404(2006)

Low temperature behaviour of a Fermi gas in the unitary regime

$$F(T) = \frac{3}{5}\varepsilon_F N \varphi\left(\frac{T}{\varepsilon_F}\right) = E - TS \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \text{ for } T < T_C$$

$$\mu(T) = \frac{dF(T)}{dN} = \varepsilon_F\left[\varphi\left(\frac{T}{\varepsilon_F}\right) - \frac{2}{5}\frac{T}{\varepsilon_F}\varphi'\left(\frac{T}{\varepsilon_F}\right)\right] \approx \varepsilon_F \xi_s$$

$$\varphi\left(\frac{T}{\varepsilon_F}\right) = \varphi_0 + \varphi_1\left(\frac{T}{\varepsilon_F}\right)^{5/2}$$

$$E(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^n\right]$$

Lattice results disfavor either $n \geq 3$ or $n \leq 2$ and suggest $n = 2.5(0.25)$

This is the same behavior as for a gas of noninteracting (!) bosons below the condensation temperature.

Experiment

John Thomas' group at Duke University,

L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)

Dilute system of fermionic ^6Li atoms in a harmonic trap

- The number of atoms in the trap: N=1.3(0.2) x 10^5 atoms divided 50-50 among the lowest two hyperfine states.
- Fermi energy: $\varepsilon_F^{ho} = \hbar\Omega(3N)^{1/3}; \ \Omega = \left(\omega_x\omega_y\omega_z\right)^{1/3}$

$$\varepsilon_F^{ho} / k_B \approx 1\mu K$$

- Depth of the potential: $U_0 \approx 10\varepsilon_F^{ho}$
- How they measure: energy, entropy and temperature?

$$\left.\begin{aligned} PV &= \frac{2}{3}E \\ \vec{\nabla}P &= -n(\vec{r})\vec{\nabla}U \end{aligned}\right\} \Rightarrow N\langle U\rangle = \frac{E}{2} \text{ - virial theorem}$$

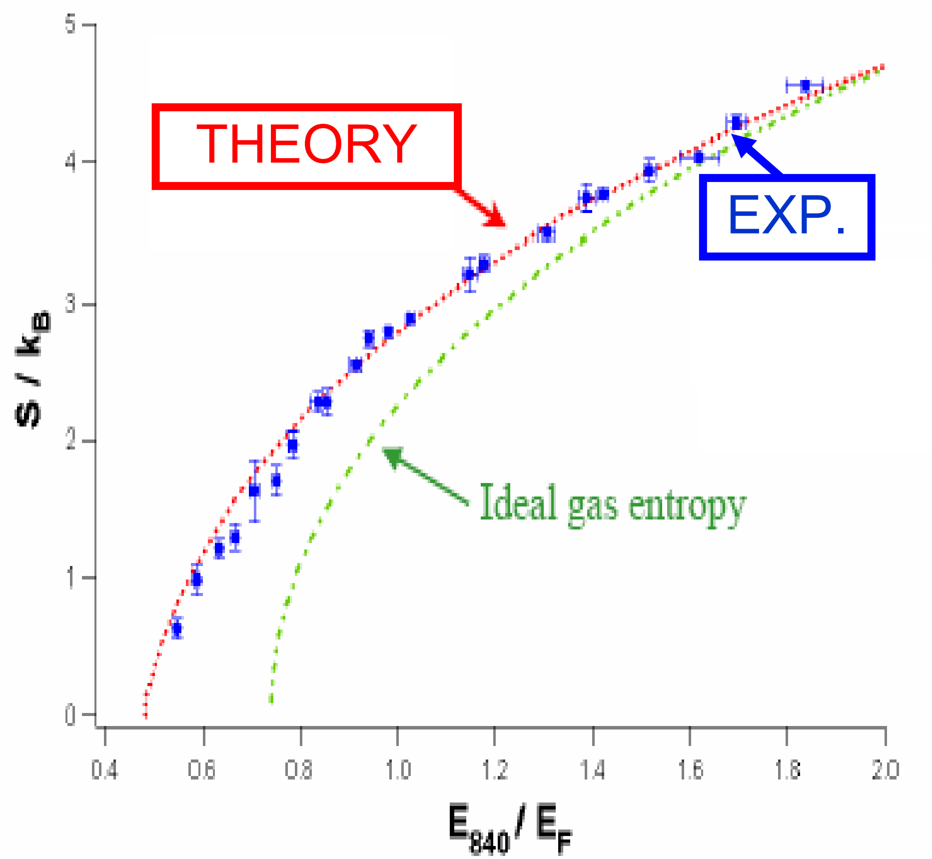
$n(\vec{r})$ - local density

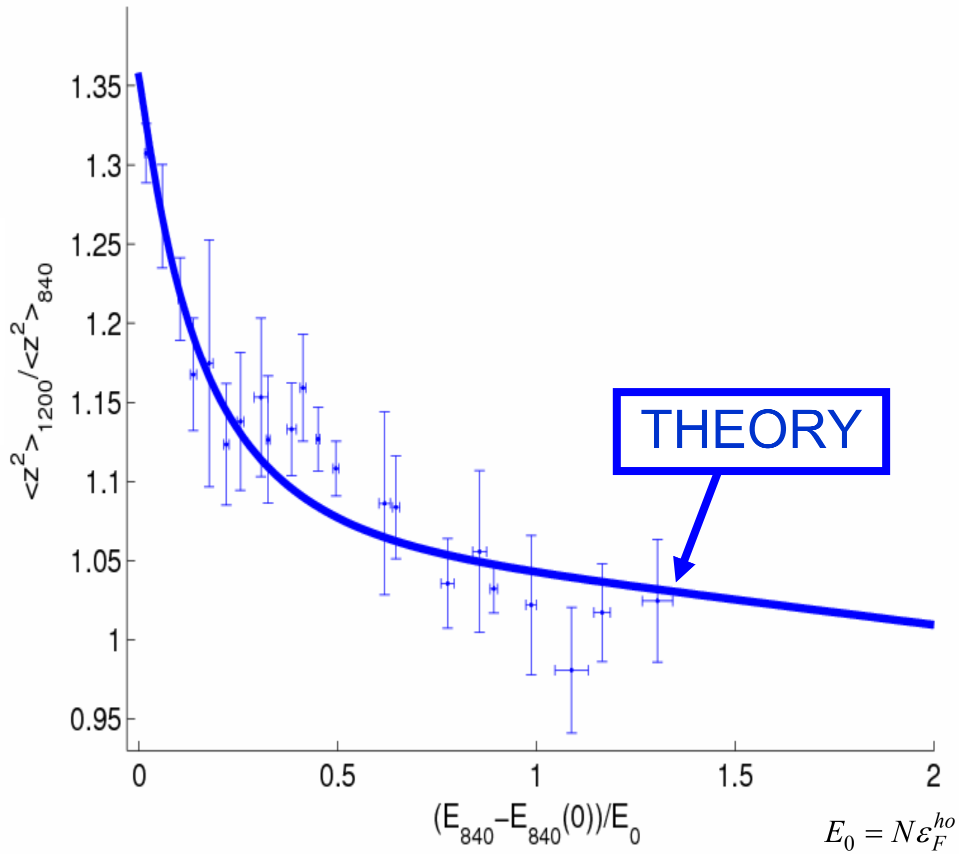
Holds at unitarity and for noninteracting Fermi gas

Comparison with experiment

John Thomas' group at Duke University,

L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)





$E_0 = N\varepsilon_F^{ho}$

Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.

Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$B = 1200G \Rightarrow 1/k_F a \approx -0.75$

Theory: **Bulgac, Drut, and Magierski PRL 99, 120401 (2007)**

Pairing gap

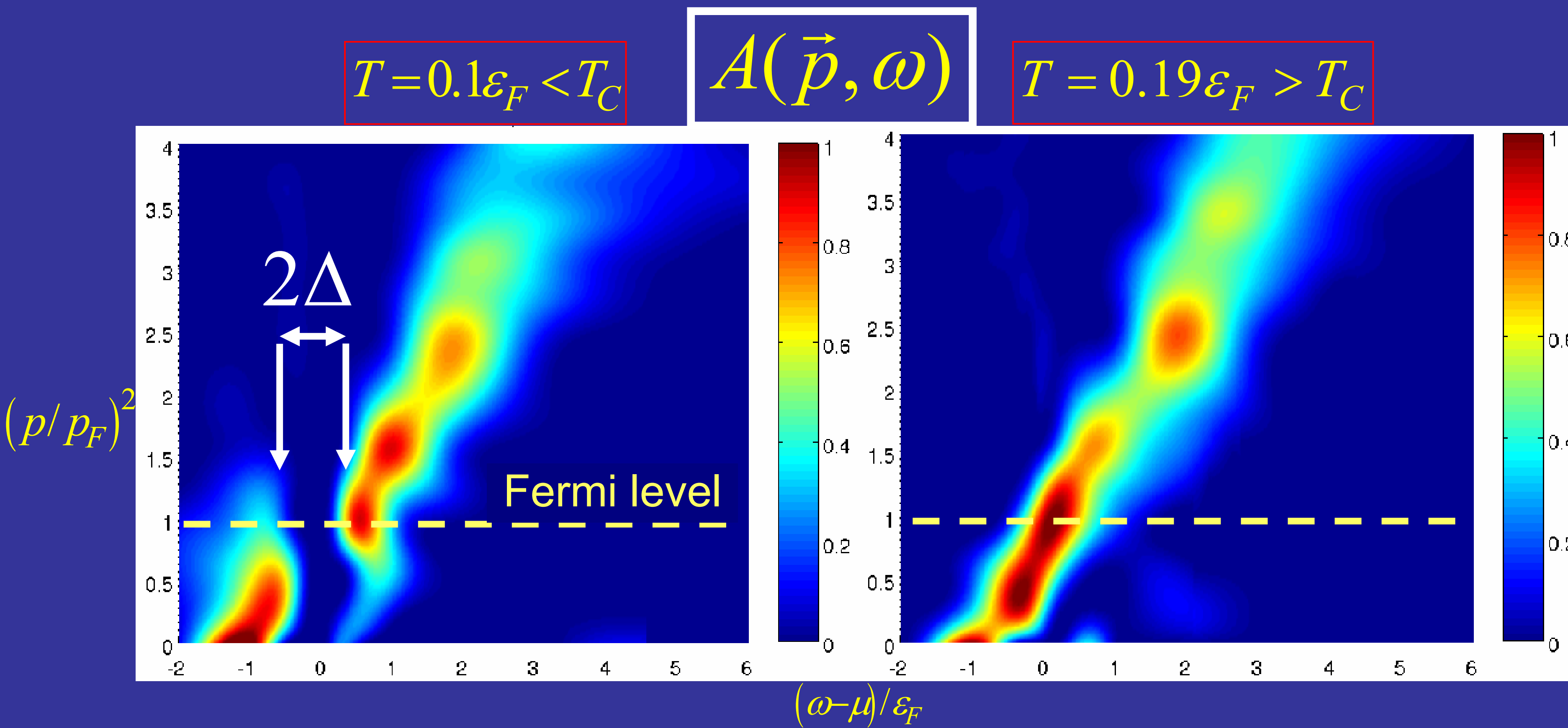
Spectral weight function: $A(\vec{p},\omega)$

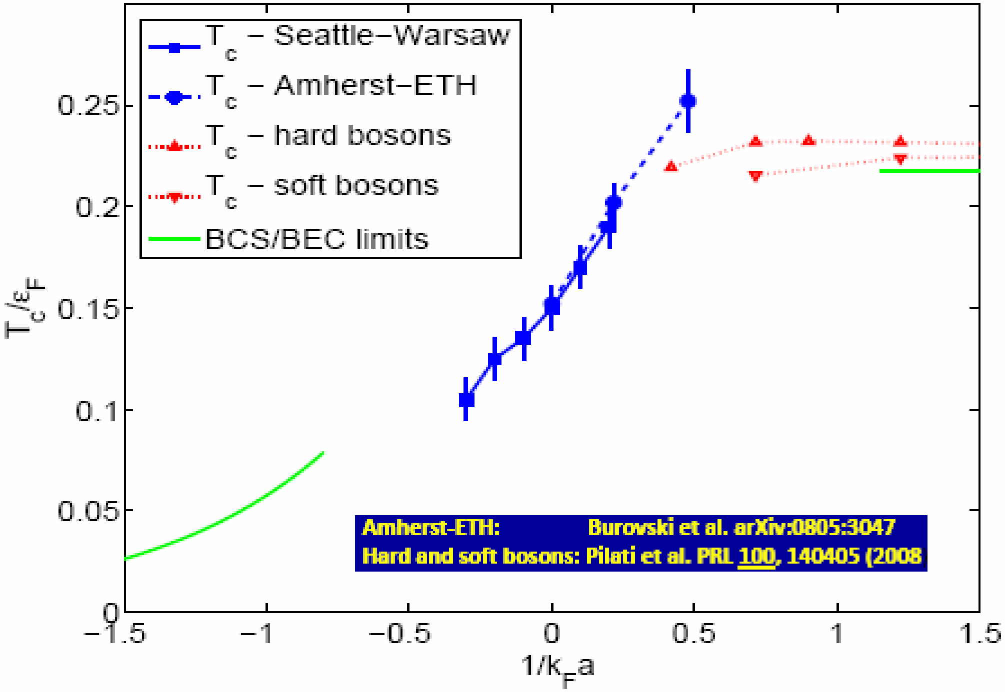
$$G^{ret/adv}(\vec{p},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p},\omega')}{\omega-\omega'\pm i0^+}$$

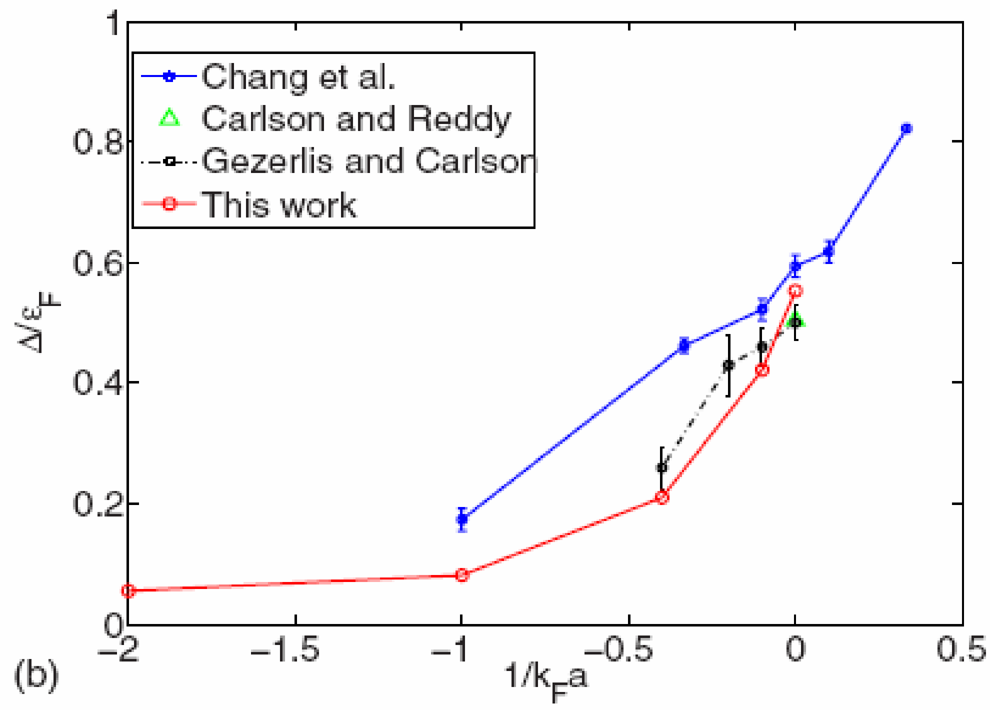
$$G(\vec{p},\tau) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega A(\vec{p},\omega)\frac{e^{-\omega\tau}}{1+e^{-\beta\omega}}$$

From Monte Carlo calcs.

In the limit of independent quasiparticles: $A(\vec{p},\omega) = 2\pi\delta(\omega - E(p))$







Results in the vicinity of the unitary limit:
- Critical temperature
- Pairing gap at T=0

Note that
- at unitarity: $\Delta/\varepsilon_F \approx 0.5$
- for atomic nucleus: $\Delta/\varepsilon_F \approx 0.03$

BCS theory predicts:

$$\Delta(T=0)/T_C \approx 1.7$$

At unitarity:

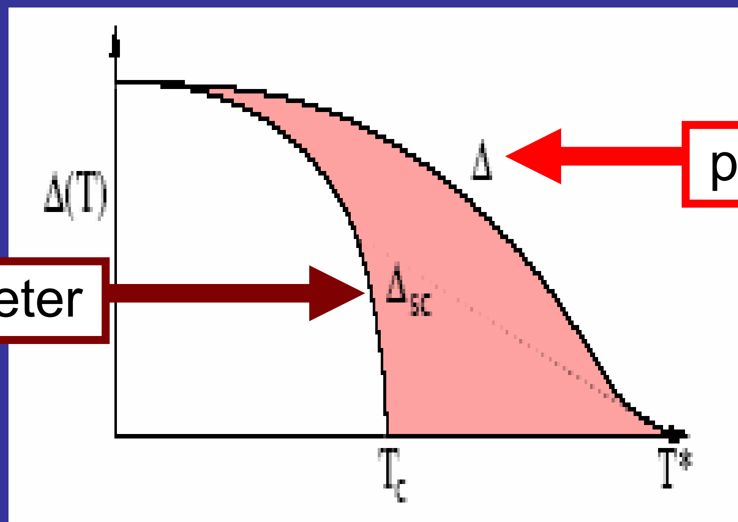
$$\Delta(T=0)/T_C \approx 3.3$$

This is NOT a BCS superfluid!

Bulgac, Drut, Magierski, PRA78, 023625(2008)

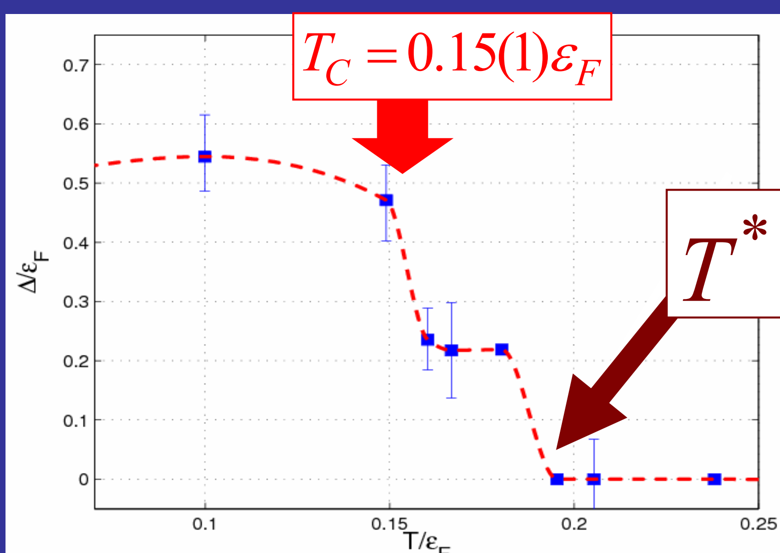
Pairing gap and pseudogap

Outside the BCS regime close to the unitary limit, but still before BEC, superconductivity/superfluidity emerge out of a very exotic, non-Fermi liquid normal state



pseudogap

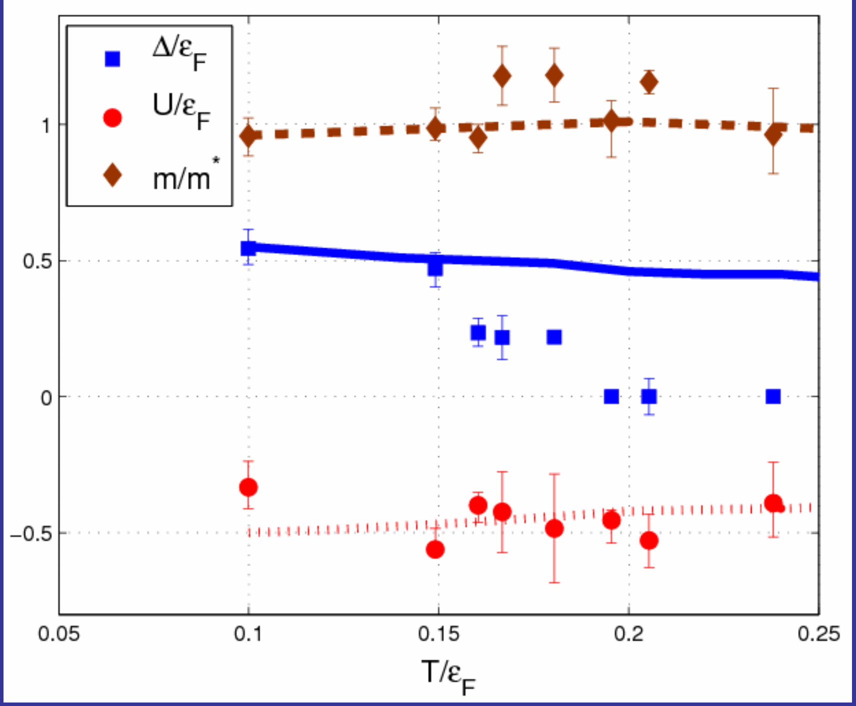
order parameter



Monte Carlo calculations

The onset of superconductivity occurs in the presence of fermionic pairs!

Single-particle properties



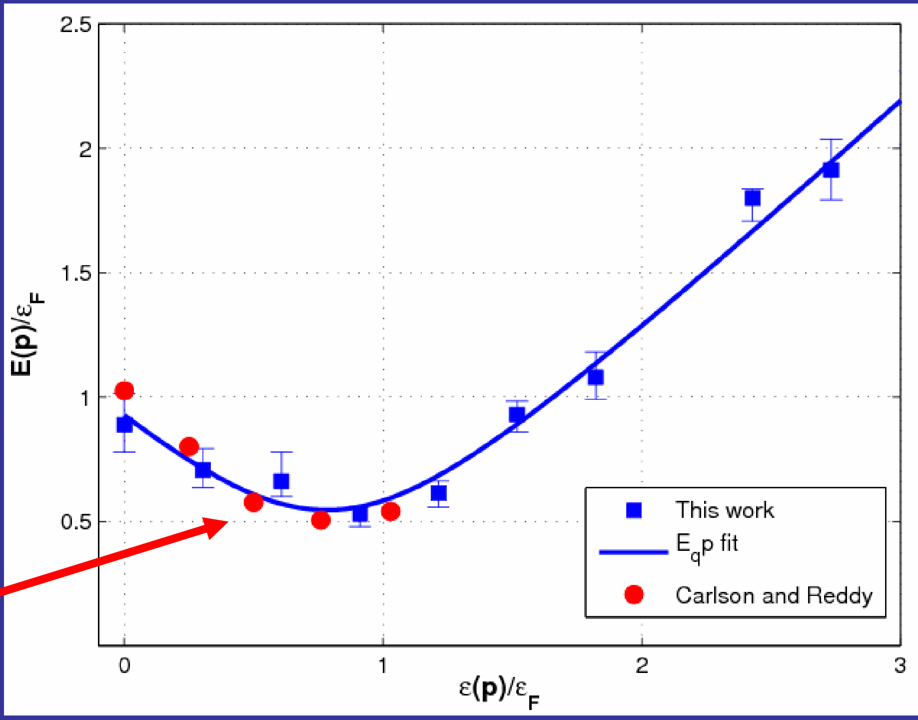
Effective mass: $m^* = (1.0 \pm 0.2)m$

Mean-field potential: $U = (-0.5 \pm 0.2)\varepsilon_F$

Weak temperature dependence!

Quasiparticle spectrum extracted from spectral weight function at $T = 0.1\varepsilon_F$

Fixed node MC calcs. at T=0



Conclusions

- ✓ Fully non-perturbative calculations for a spin ½ many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at $T_c = 0.15\,(1)\,\varepsilon_F$.

- ✓ Between T_c and T_0 =0.23(2) ε_F the system is neither superfluid nor follows the normal Fermi gas behavior. Possibly due to pairing effects.

- ✓ Results (energy, entropy vs temperature) agree with recent measurments: L. Luo et al., PRL 98, 080402 (2007)

- ✓ The system at unitarity is NOT a BCS superfluid. There is an evidence for the existence of *pseudogap* at unitarity (similarity with high-Tc supeconductors).

- ✓ Description of the system at finite temperatures will pose a challenge for the density functional theory (two temperature scales are present).

- ✓ Surprisingly at low temperatures the gap extracted from the response function within the independent quasiparticle model accurately reproduce the one obtained from the spectral weight function.

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

More details of the calculations:

- Lattice sizes used: $6^3 - 10^3$.
  Imaginary time steps: 8^3 x 300 (high Ts) to 8^3 x 1800 (low Ts)

- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.

- Update field configurations using the Metropolis importance sampling algorithm.

- Change randomly at a fraction of all space and time sites the signs the auxiliary fields $\sigma(r,\tau)$ so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .

- Thermalize for 50,000 – 100,000 MC steps or/and use as a start-up field configuration a $\sigma(x,\tau)$-field configuration from a different T

- At low temperatures use Singular Value Decomposition of the evolution operator $U(\{\sigma\})$ to stabilize the numerics.

- Use 200,000-2,000,000 $\sigma(x,\tau)$- field configurations for calculations

- MC correlation "time" ≈ 250 – 300 time steps at $T \approx T_c$