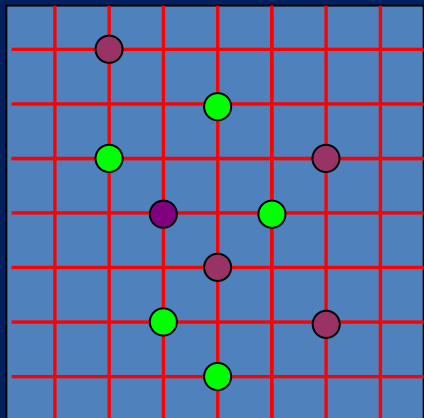


Pseudogap from Path Integral Monte Carlo (PIMC) on the Lattice

Coordinate space

L-limit for the spatial correlations in the system

$$k_{cut} = \frac{\pi}{\Delta x}; \Delta x$$



Volume = L^3
lattice spacing = Δx

- - Spin up fermion
- - Spin down fermion

External conditions:

T - temperature

μ - chemical potential

Periodic boundary conditions imposed

UV momentum cutoff $\Lambda_{UV} = \frac{\pi}{\Delta x}$
 IR momentum cutoff $\Lambda_{IR} = \frac{2\pi}{L}$
 $\frac{\hbar^2 \Lambda_{IR}^2}{2m} \ll \epsilon_F, \Delta \ll \frac{\hbar^2 \Lambda_{UV}^2}{2m}$

$$\hat{H} = \hat{T} + \hat{V} = \int d^3 r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta^2}{2m} \right) \hat{\psi}_s(\vec{r}) - g \int d^3 r \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3 r (\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})); \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

Pairing gap

Spectral weight function: $A(\vec{p}, \omega)$

$$G^{\text{ret/adv}}(\vec{p}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p}, \omega')}{\omega - \omega' \pm i0^+}$$

$$G(\vec{p}, \tau) = -\frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega A(\vec{p}, \omega) \frac{e^{-\omega\tau}}{1 + e^{-\beta\omega}}$$

From Monte Carlo calcs.

$$G(\vec{p}, \tau) = \frac{1}{Z} \text{Tr} \{ e^{-(\beta-\tau)(\hat{H}-\mu\hat{N})} \hat{\psi}_{\uparrow}(\vec{p}) e^{-\tau(\hat{H}-\mu\hat{N})} \hat{\psi}_{\uparrow}^{\dagger}(\vec{p}) \}$$

Constraints:

$$A(\mathbf{p}, \omega) \geq 0, \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(\mathbf{p}, \omega) = 1, \quad \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(\mathbf{p}, \omega) \frac{1}{1 + \exp(\omega\beta)} = n(\mathbf{p}),$$

Maximum entropy method

From Bayes' theorem:

$$P(A | G) \propto P(G | A)P(A)$$

A priori probability:

$$P(A) \propto \exp(\alpha S)$$

Relative entropy:

$$S(\mathcal{M}) = \sum_{k=1}^{n_A} \Delta\omega \left[A(\omega_k) - \mathcal{M}(\omega_k) - A(\omega_k) \ln \left(\frac{A(\omega_k)}{\mathcal{M}(\omega_k)} \right) \right]$$

Likelihood function:

$$P(G | A) \propto \exp\left(-\frac{1}{2} \chi^2\right) \quad \chi^2 = \sum_{i=1}^{n_\tau} \left(\frac{\tilde{\mathcal{G}}_{\tau_i} - \mathcal{G}(\tau_i)}{\sigma_{\tau_i}} \right)^2 \quad \mathcal{G}(\tau_i) = \sum_{k=1}^{n_A} \frac{e^{-\omega_k \tau_i}}{1 + e^{-\omega_k \beta}} A_k \Delta\omega.$$

Maximum entropy method:

$$\min_{A(\omega)} \left(\frac{1}{2} \chi^2 - \alpha S \right)$$

SVD method

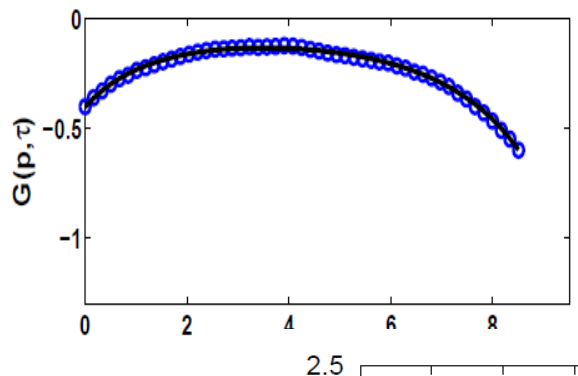
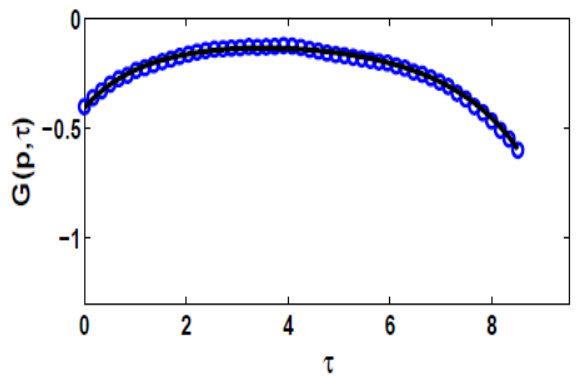
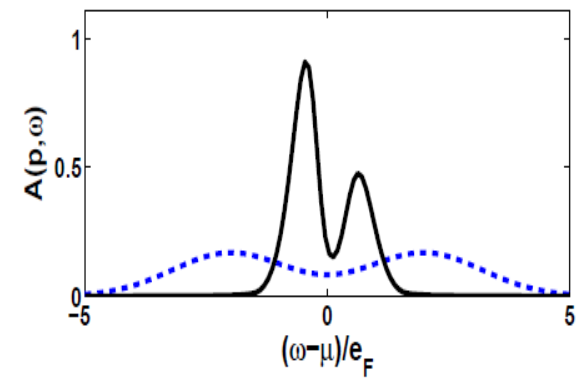
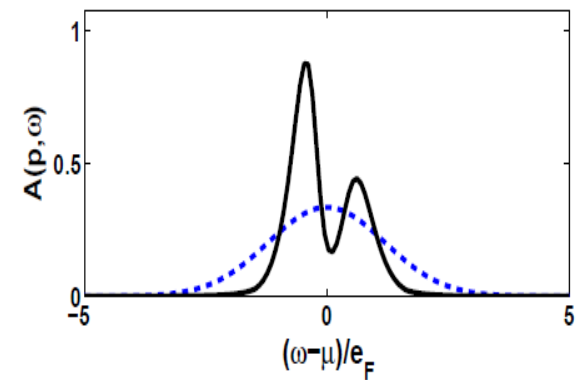
$$\mathcal{G}(\mathbf{p}, \tau_i) = (\mathcal{K}A)(\mathbf{p}, \tau_i).$$

$$\mathcal{K}u_i = \lambda_i \vec{v}_i, \quad \mathcal{K}^* \vec{v}_i = \lambda_i u_i,$$

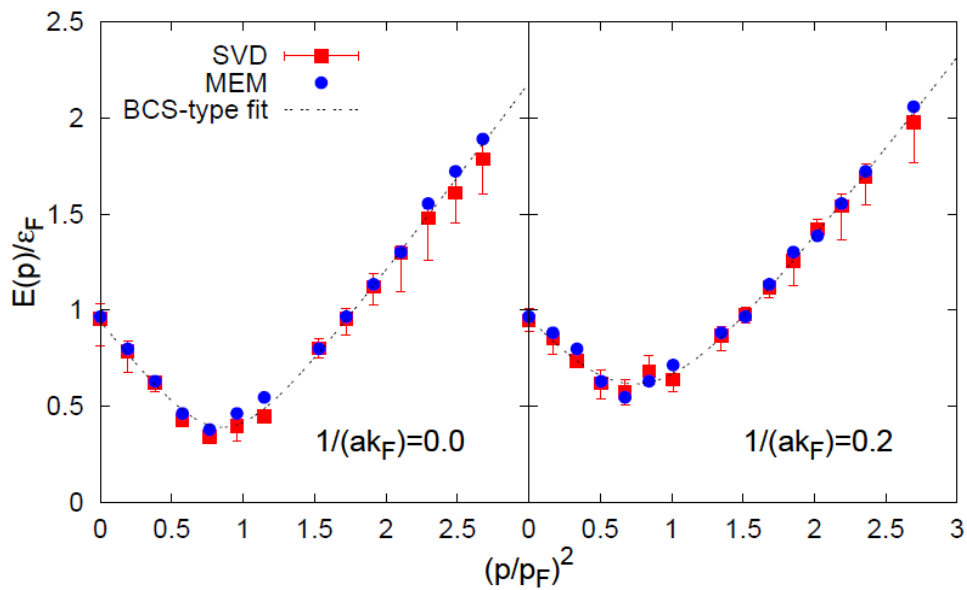
$$u_i(\omega) = \frac{1}{\sigma_i} \sum_{k=1}^{n_\tau} (\vec{v}_i)_k \phi_{\tau_k}(\omega) = -\frac{1}{2\pi\sigma_i} \sum_{k=1}^{n_\tau} (\vec{v}_i)_k \frac{e^{-\omega\tau_k}}{1 + e^{-\omega\beta}}.$$

$$A(\mathbf{p}, \omega) = \sum_{i=1}^r b_i(\mathbf{p}) u_i(\omega), \quad b_i(\mathbf{p}) = \frac{1}{\lambda_i} (\vec{\mathcal{G}}(\mathbf{p}) \cdot \vec{v}_i),$$

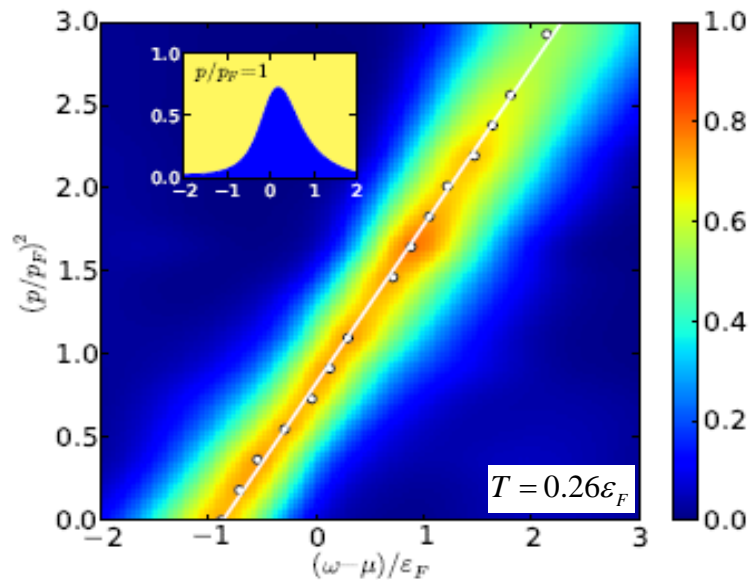
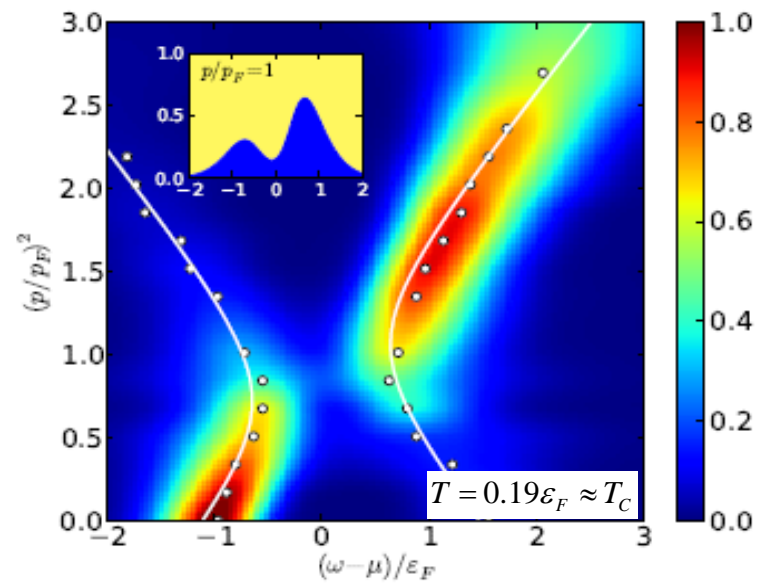
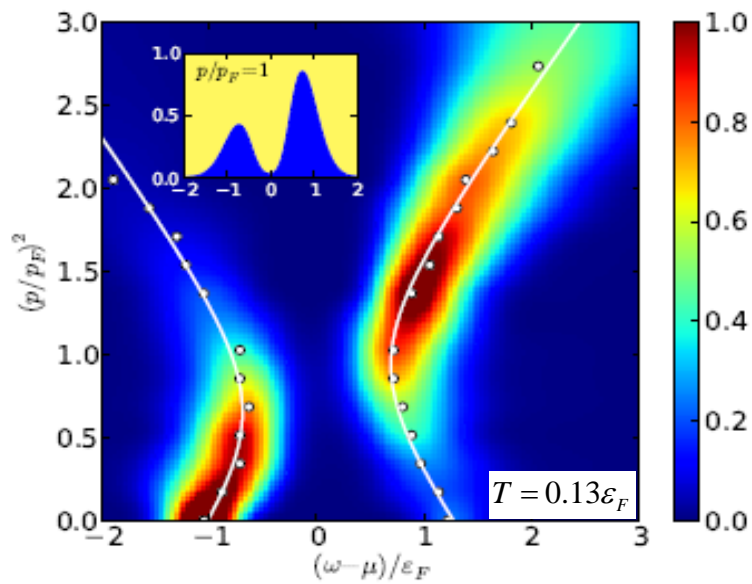
Spectral weight function from MEM



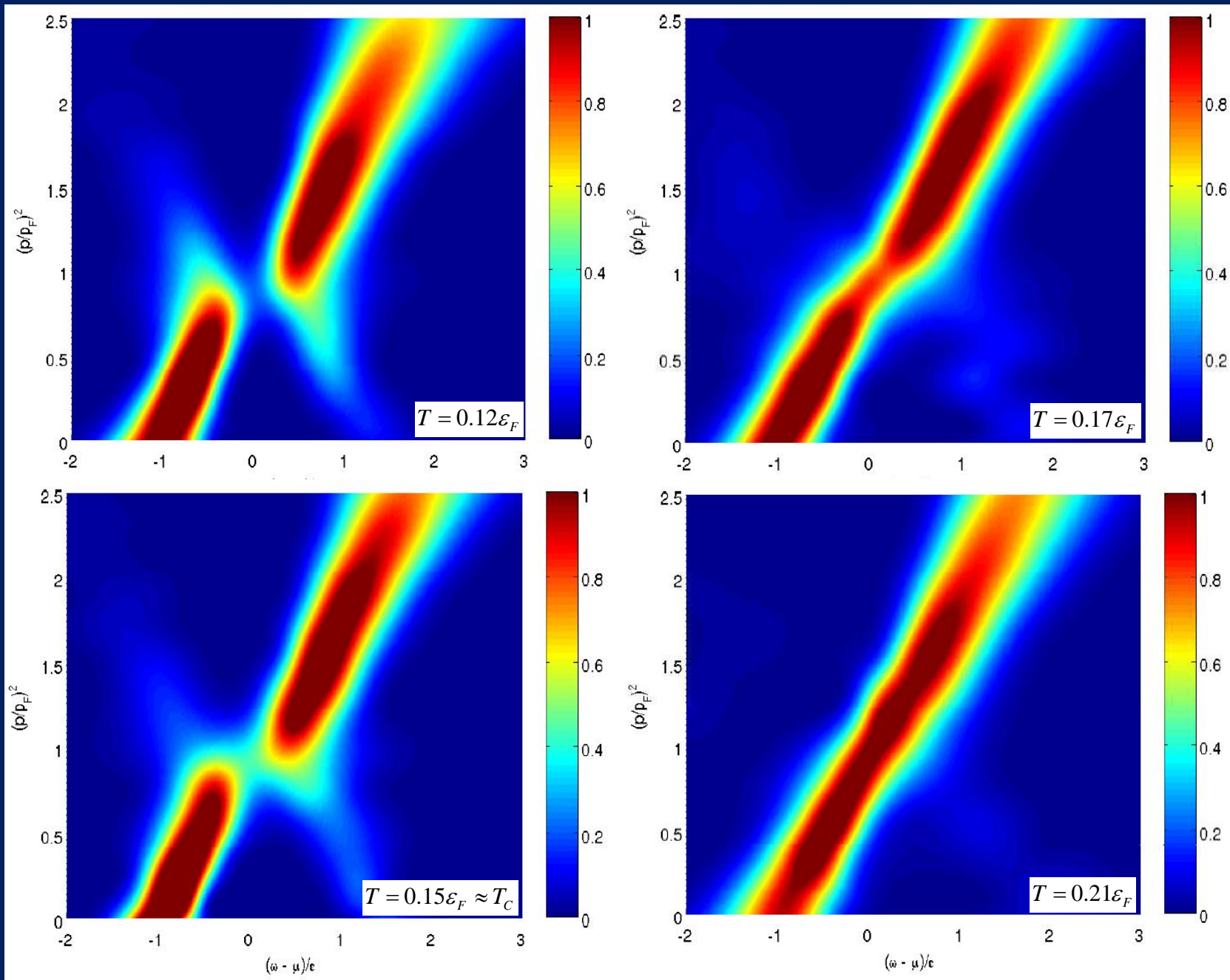
MEM vs SVD



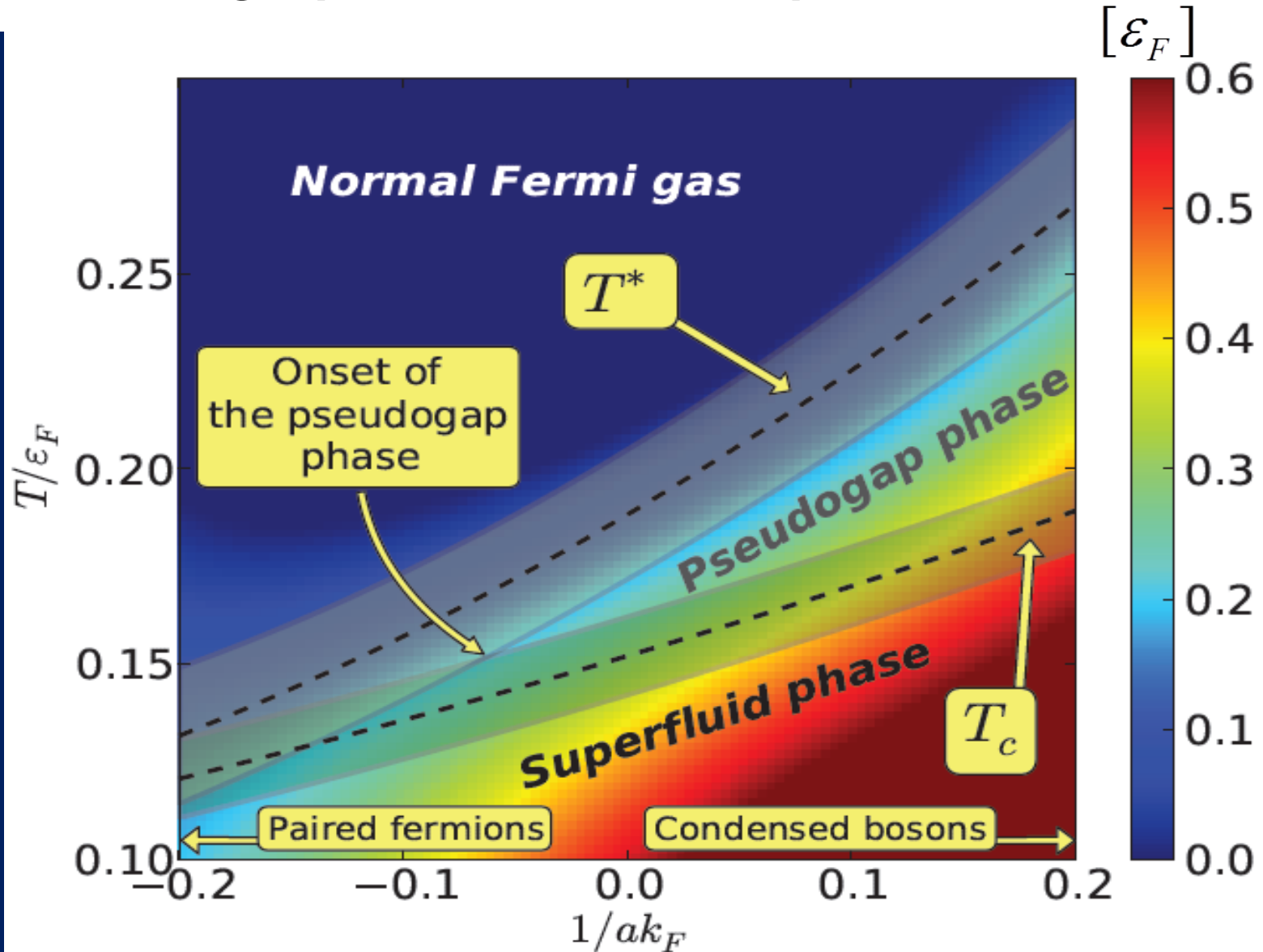
Spectral weight function at the BEC side: $(k_F a)^{-1} = 0.2$



Spectral weight function at unitarity: $(k_F a)^{-1} = 0$



Gap in the single particle fermionic spectrum from MC calcs.

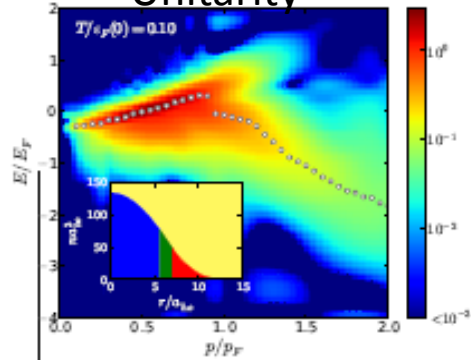


Ab initio result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.

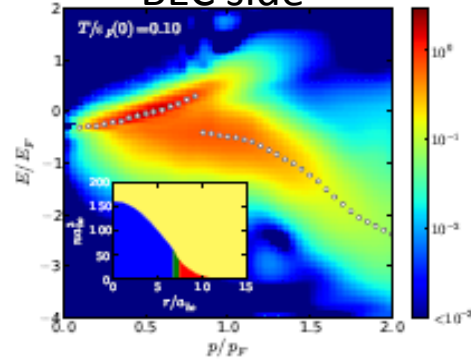
Energy distribution curves (EDC) from the spectral weight function

$$\text{EDC}(p, E, T) = C p^2 \int_0^\infty dr r^2 \frac{1}{\varepsilon_F(r)} A \left[\frac{p}{p_F(r)}, \frac{E - \mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)} \right] f(E - \mu(r)),$$

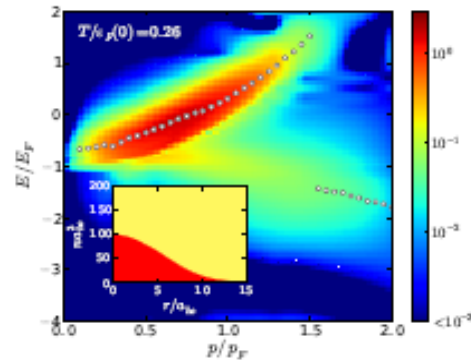
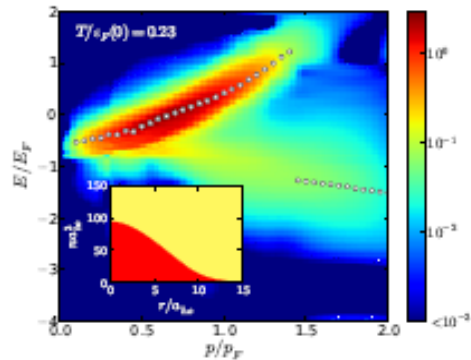
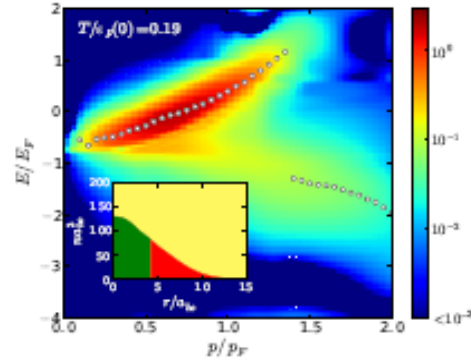
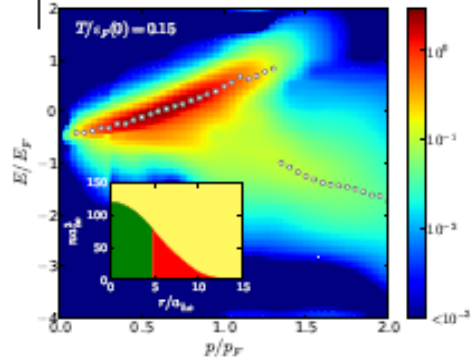
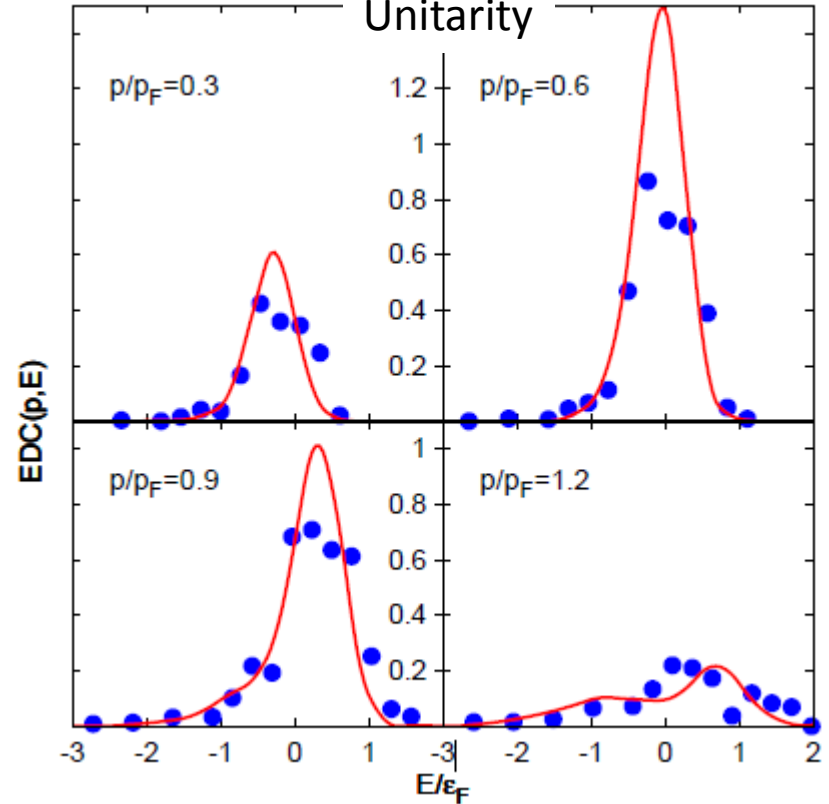
Unitarity



BEC side



Unitarity

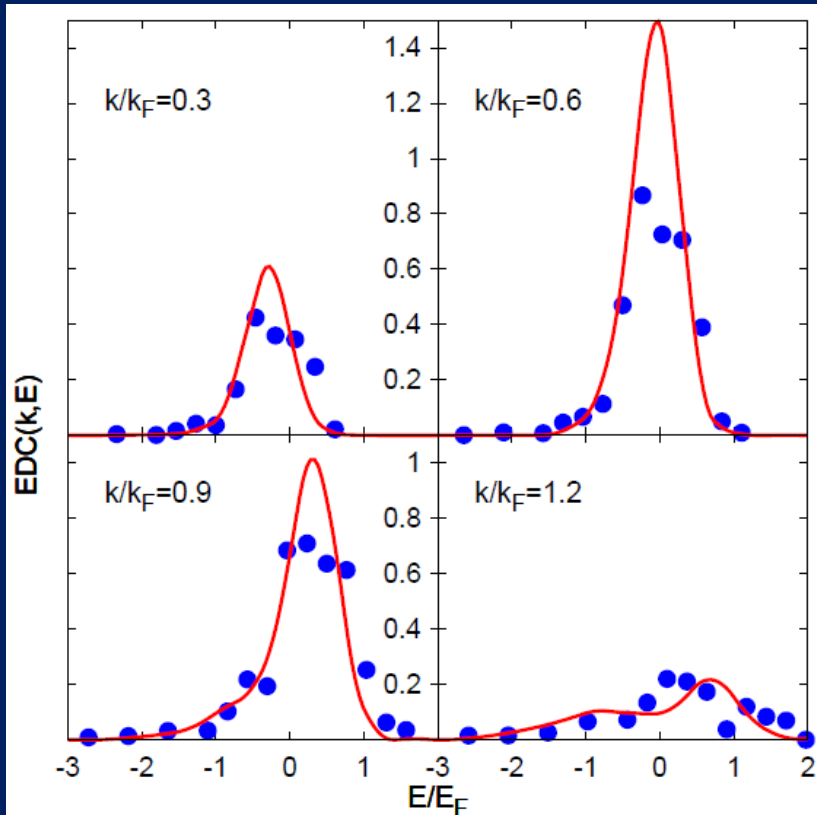


Experiment (blue dots): D. Jin's group
 Theory (red line) PIMC:
 Magierski, Wlazłowski, Bulgac,
 arXiv1103.4382

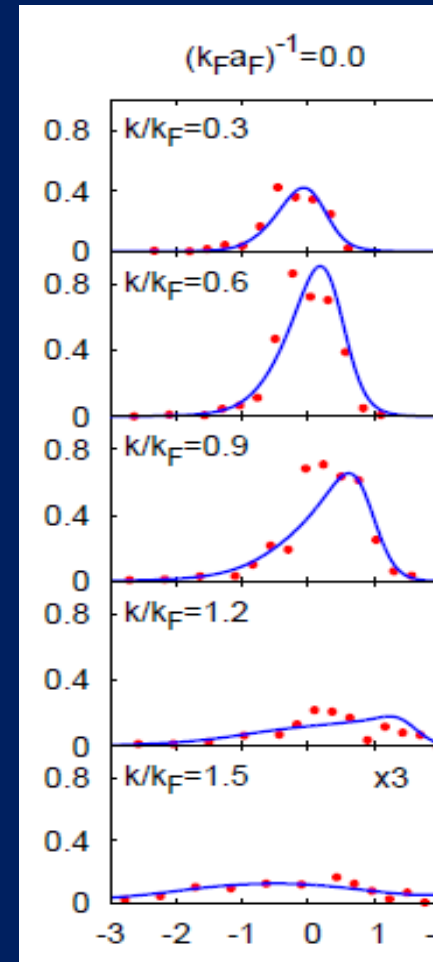
Theory vs Experiment (photoemission spectr.)

$$EDC(k, E) \sim A(k, \omega) f(\omega)$$

PIMC



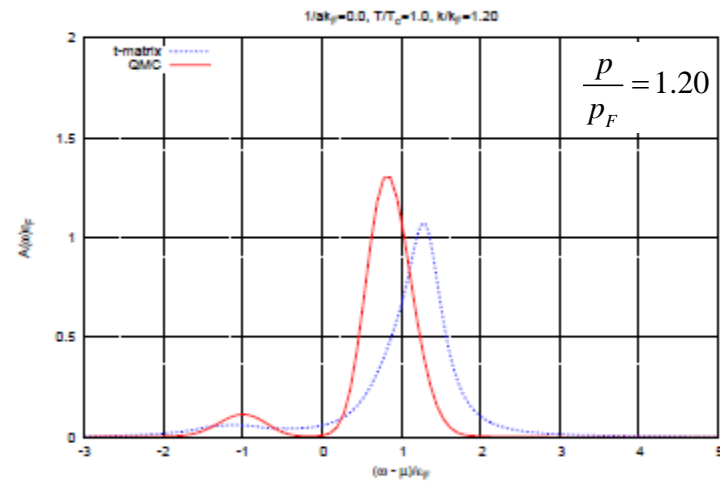
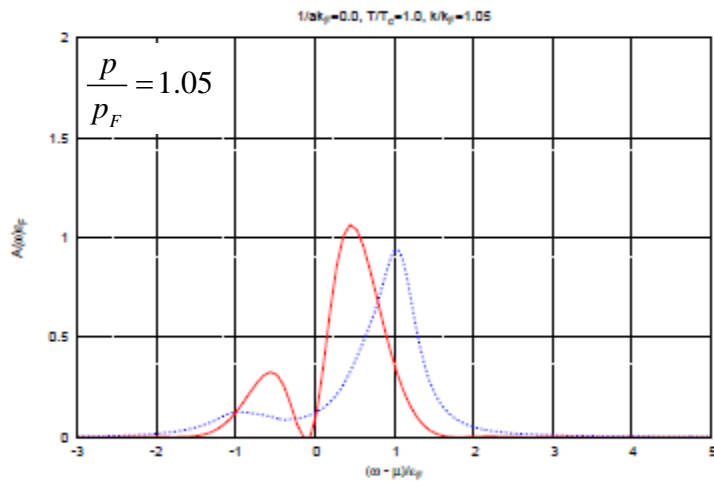
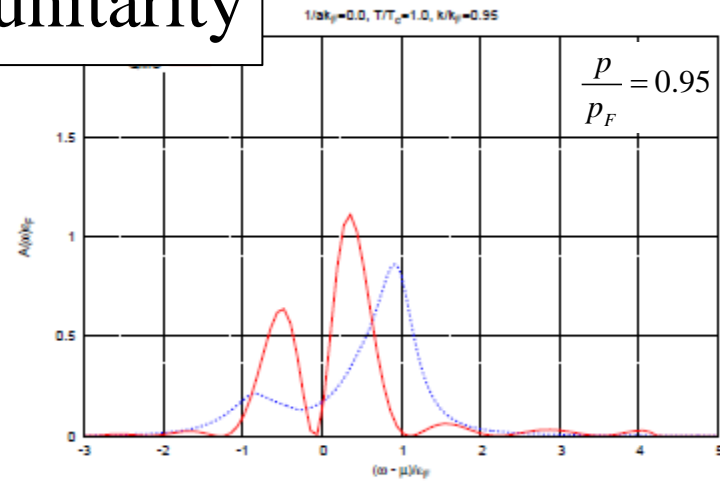
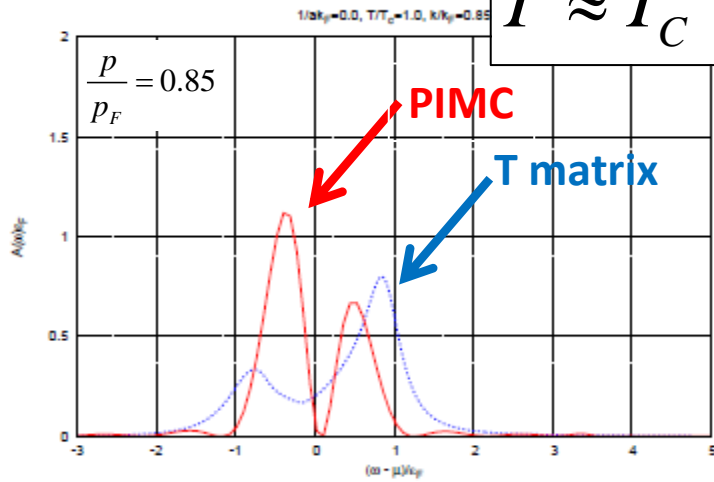
Non selfconsistent t-matrix approx.



PIMC vs T-matrix approach (preliminary results)

$T \approx T_C$ at unitarity

$A(p, \omega)$



$$\frac{\omega - \mu}{\epsilon_F}$$

T matrix gap at T_c is about twice larger than the *ab initio* gap!