Solitonic excitations in heavy-ion collisions and ultracold atoms

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OUTLINE

• Spin-polarized excitations in ultracold Fermi gas - FFLO droplets

• Solitonic excitations in heavy-ion collisions at the Coulomb barrier
Pairing in spin imbalanced superfluids

Clogston-Chandrasekhar condition sets the limit for the chemical potential difference at which superfluidity is lost:

$$\left| \mu_\downarrow - \mu_\uparrow \right| \propto \Delta$$

Unstable for balanced masses at $T=0$

Phase separation in momentum space

Sarma phase (interior gap) phase

Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

Larkin-Ovchinnikov: \( \Delta(r) \sim \cos(qr) \)

Fulde-Ferrell: \( \Delta(r) \sim \exp(iqr) \)

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

Mean-field studies:

\[
h = \frac{\mu_s - \mu_t}{2\epsilon_f}
\]

Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG)

\[ E \sim \left[ n_a g(x) \right]^{5/3} \]

\[ x = n_b / n_a \]

LO configuration – supersolid state


The problem: In the trap the volume where LOFF phase may be created is relatively small.

Ultracold atoms in a uniform potential

Can we induce a stable spin-polarized region locally?

Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

\[ B\text{dG in the Andreev approx. (} \Delta \ll k_F^2 \) \]

\[
\begin{pmatrix}
-2ik_F \frac{d}{dx} & \Delta(x) \\
\Delta^*(x) & 2ik_F \frac{d}{dx}
\end{pmatrix}
\begin{pmatrix}
\begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}
\end{pmatrix}
= E_n
\begin{pmatrix}
\begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}
\end{pmatrix}
\]

Energetics of two nodal points in 1D system

Two nodal points repel each other.
Another perspective: superconductor-ferromagnet junction

Due to the difference between Fermi momenta of spin-up and spin-down particles:

\[
\begin{align*}
k_{F\uparrow} &= k_F + \delta k_F \\
k_{F\downarrow} &= k_F - \delta k_F
\end{align*}
\]

Induces spatial modulation of the order parameter of the period:

\[
\frac{\pi}{\delta k_F}
\]
The nodal points repel each other: unstable structure in 1D

$$V_\lambda(x, t) = 1.8 f(t) \lambda \varepsilon F \exp\left(-\frac{x^2}{2\sigma^2}\right), \quad k_F \sigma = 4.441 \quad \text{time scales: } \varepsilon_F \Delta t \sim 10$$

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446
Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:

Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

For example the spherical nodal structure:
\[ \phi_n \rightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow}) \]

**Densities:**

\[ n_\sigma(r) = \sum_{E_n < E_c} |v_{n,\sigma}(r)|^2, \quad \tau_\sigma(r) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(r)|^2, \]

\[ \nu(r) = \sum_{E_n < E_c} u_{n,\uparrow}(r)v_{n,\downarrow}^*(r), \quad j_\sigma(r) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}(r)\nabla v_{n,\sigma}(r)], \]

**EDF:**

\[ \mathcal{H} = \alpha_{\uparrow}(p) \frac{\hbar^2 \tau_{\uparrow}}{2m_{\uparrow}} + \alpha_{\downarrow}(p) \frac{\hbar^2 \tau_{\downarrow}}{2m_{\downarrow}} \]

\[ + D(n_{\uparrow}, n_{\downarrow}) \]

\[ + g(n_{\uparrow}, n_{\downarrow}) \nu \]

\[ + [1 - \alpha_{\uparrow}(p)] \frac{j_{\uparrow}^2}{2n_{\uparrow}} + [1 - \alpha_{\downarrow}(p)] \frac{j_{\downarrow}^2}{2n_{\downarrow}} \]

A. Bulgac, M.M. Forbes, P. Magierski,
\( \phi_n \rightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow}) \)

**Densities:**

\[
n_\sigma(r) = \sum_{E_n < E_c} |v_{n,\sigma}(r)|^2, \quad \tau_\sigma(r) = \sum_{E_n < E_c} 1
\]

\[
v(r) = \sum_{E_n < E_c} u_{n,\uparrow}(r)v_{n,\downarrow}^*(r), \quad j_\sigma(r) = \sum_{E_n < E_c} 1
\]

**EDF:**

\[
\mathcal{H} = \alpha_{\uparrow}(p) \frac{\hbar^2 \tau_{\uparrow}}{2m_{\uparrow}} + \alpha_{\downarrow}(p) \frac{\hbar^2 \tau_{\downarrow}}{2m_{\downarrow}}
+ \frac{g(n_{\uparrow}, n_{\downarrow})}{2} \nabla \cdot \nabla
+ [1 - \alpha_{\uparrow}(p)] \frac{j_{\uparrow}^2}{2n_{\uparrow}} + [1 - \alpha_{\downarrow}(p)] \frac{j_{\downarrow}^2}{2n_{\downarrow}}
\]

**Kinetic term:**

Effective mass \( \alpha_\sigma \) of the particle depends on local polarization

\[
p(r) = \frac{n_{\uparrow}(r) - n_{\downarrow}(r)}{n_{\uparrow}(r) + n_{\downarrow}(r)}
\]

and guarantees that correct limit is attained for \( n_{\uparrow} \gg n_{\downarrow} \), where the problem reduces to the *polaron* problem

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A. Bulgac, M.M. Forbes, P. Magierski,
\[
\phi_n \rightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})
\]

**Densities:** \[ n_\sigma(r) = \sum_{E_n < E_c} |v_{n,\sigma}(r)|^2, \quad \tau_\sigma(r) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(r)|^2, \]

\[ v(r) = \sum_{E_n < E_c} u_{n,\uparrow}(r) v_{n,\downarrow}^*(r), \quad j_\sigma(r) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(r) \nabla v_{n,\sigma}(r)], \]

**EDF:**
\[
\mathcal{H} = \alpha_{\uparrow}(p) \frac{\hbar^2 \tau_{\uparrow}}{2m_{\uparrow}} + \alpha_{\downarrow}(p) \frac{\hbar^2 \tau_{\downarrow}}{2m_{\downarrow}} + D(n_{\uparrow}, n_{\downarrow}) + g(n_{\uparrow}, n_{\downarrow}) v^\dagger v
\]

\[ + [1 - \alpha_{\uparrow}(p)] \frac{j_{\uparrow}^2}{2n_{\uparrow}} + [1 - \alpha_{\downarrow}(p)] \frac{j_{\downarrow}^2}{2n_{\downarrow}} \]

**Normal interaction energy:**
\[
D(n_{\uparrow}, n_{\downarrow}) \sim (n_{\uparrow} + n_{\downarrow})^{5/3} \beta(p)
\]

in order to get the proper scaling:
\[
E = \xi E_{FFG}
\]

A. Bulgac, M.M. Forbes, P. Magierski,
\[ \phi_n \rightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow}) \]

**Densities:**

\[ n_\sigma(r) = \sum_{E_n < E_c} |v_{n,\sigma}(r)|^2, \quad \tau_\sigma(r) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(r)|^2, \]

\[ \nu(r) = \sum_{E_n < E_c} u_{n,\uparrow}(r)v_{n,\downarrow}^*(r), \quad j_\sigma(r) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(r)\nabla v_{n,\sigma}(r)], \]

**EDF:**

\[ \mathcal{H} = \alpha_\uparrow(p) \frac{\hbar^2 \tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p) \frac{\hbar^2 \tau_\downarrow}{2m_\downarrow} \]

\[ + T(n_\uparrow, n_\downarrow) \]

\[ + g(n_\uparrow, n_\downarrow) \nu^\dagger \nu \]

\[ + [1 - \alpha_\uparrow(p)] \frac{j^2_\uparrow}{2n_\uparrow} + [1 - \alpha_\downarrow(p)] \frac{j^2_\downarrow}{2n_\downarrow} \]

---

**Pairing energy:**

\[ g(n_\uparrow, n_\downarrow) = \frac{\gamma(p)}{(n_\uparrow + n_\downarrow)^{1/3}} \]

in order to get proper scaling:

\[ \Delta/\varepsilon_F = \text{const} \approx 0.5 \]

A. Bulgac, M.M. Forbes, P. Magierski,
In order to restore Galilean invariance of the functional

Densities: \[ n_\sigma(r) = \sum_{E_n < E_c} |v_{n,\sigma}(r)|^2, \quad \tau_\sigma(r) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(r)|^2, \]

\[ \nu(r) = \sum_{E_n < E_c} u_{n,\uparrow}(r)v_{n,\downarrow}^*(r), \quad j_\sigma(r) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(r)\nabla v_{n,\sigma}(r)], \]

EDF:

\[ \mathcal{H} = \alpha_\uparrow(p) \frac{\hbar^2 \tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p) \frac{\hbar^2 \tau_\downarrow}{2m_\downarrow} + D(n_\uparrow, n_\downarrow) + g(n_\uparrow, n_\downarrow)\nu^\dagger \nu \]

\[ + [1 - \alpha_\uparrow(p)] \frac{j_\uparrow^2}{2n_\uparrow} + [1 - \alpha_\downarrow(p)] \frac{j_\downarrow^2}{2n_\downarrow} \]

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations.

\[
h \sim f_1(n, \nu, \ldots) \nabla^2 + f_2(n, \nu, \ldots) \cdot \nabla + f_3(n, \nu, \ldots)
\]

\[
h \hbar \frac{\partial}{\partial t} \begin{pmatrix}
  u_{n,a}(r, t) \\
  u_{n,b}(r, t) \\
  v_{n,a}(r, t) \\
  v_{n,b}(r, t)
\end{pmatrix} =
\begin{pmatrix}
  h_a(r, t) & 0 & 0 & \Delta(r, t) \\
  0 & h_b(r, t) & -\Delta(r, t) & 0 \\
  0 & -\Delta^*(r, t) & -h_a^*(r, t) & 0 \\
  \Delta^*(r, t) & 0 & -h_b^*(r, t)
\end{pmatrix}
\begin{pmatrix}
  u_{n,a}(r, t) \\
  u_{n,b}(r, t) \\
  v_{n,a}(r, t) \\
  v_{n,b}(r, t)
\end{pmatrix}
\]

where \( h \) and \( \Delta \) depends on “densities”:

\[
n_\sigma(r, t) = \sum_{E_n \leq E_c} |v_{n,\sigma}(r, t)|^2,
\]

\[
\tau_\sigma(r, t) = \sum_{E_n \leq E_c} |\nabla v_{n,\sigma}(r, t)|^2,
\]

\[
v(r, t) = \sum_{E_n \leq E_c} u_{n,\uparrow}(r, t) v_{n,\downarrow}^*(r, t),
\]

\[
j_\sigma(r, t) = \sum_{E_n \leq E_c} \text{Im}[v_{n,\sigma}(r, t) \nabla v_{n,\sigma}(r, t)],
\]

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice \( n = 1, 2, \ldots, 10^5 - 10^6 \))
Phase difference is $\pi$

Maximum polarization occurs within a shell where the pairing field vanishes.

Forming a spherical nodal surface
Energy of the system during the procedure of the impurity creation

$V_p(r, t) = \lambda A(t) \exp \left[ -\frac{x^2 + (1 - \epsilon_y)y^2 + (1 - \epsilon_z)z^2}{2\sigma^2} \right]$
The origin of stability of the polarized impurity (at T=0): $E_{imp} \approx E_{int} + E_{shell}$

$E_{int}$ - Energy associated with the volume
$E_{shell}$ - Energy associated with the polarized shell located at the surface

Contraction of the nodal sphere is prevented by the pairing potential barrier. Expansion of the nodal sphere will cost the energy due to the polarization shell expansion.

As a result of the interplay between volume and surface energies keeps the impurity stable

**Evolution of the deformed impurity**

Potential: $A = 2\epsilon_F$, $\sigma_x = 4.71\xi$, $\sigma_y = 6.28\xi$, $\sigma_z = 7.85\xi$

Plane: x-y, x-z, y-z

Time*EF=0

Potential is **ON**
Moving impurity:

From Larkin-Ovchinnikov towards Fulde-Ferrell limit

$\Delta(r) : \cos(qr) \Rightarrow \exp(iqr)$

The velocities of impurities are about 30% of the velocity of sound.

Note the rigidity of the structure of impurities during collision.

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446
Creation of more complex nodal surface structures:

**Concentric nodal spheres**

*FIG. 13:* Snapshot from simulation demonstrating the internal structure of the large ferron-like excitation taken after the time $\Delta t \approx 220\varepsilon_F^{-1}$ with respect to the moment when the potential was removed. It is clearly seen that the phase changes sign three times as we proceed towards the center of the impurity. For full movie see *Movie 13.*
Suggestion for experimental protocol

Two crossing beams: $A = 1\varepsilon_F$, $\sigma = 3.14\xi$

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446
Conclusions

It is possible to create dynamically stable, locally spin-polarized region in the ultracold Fermi gas. The stability is due to the peculiar pairing structure characteristic for the FFLO phase.

The conditions of stability:
- do not depend on details of the functional (simple BdG approach predicts qualitatively the same results)
- do not depend whether we are on the BEC-side ($a>0$) or on the BCS-side ($a<0$), although UFG may be the best system for experimental realization.

The effect can be viewed as:
- long-lived, spin-polarized excitation mode of UFG
- FFLO droplet (although its size is of few coherence lengths only)

We dubbed it **ferron**

Open problems

- Experimental realization
- Collision effects beyond the mean-field picture (impact on stability)
- Stability as a function of temperature (for $T \ll T_c$ ferron is unaffected)
- ...
Unitary Fermi gas
(superfluid properties manifest here in the form of topological defects)
Once we have accurate EDF
→ remarkable agreement between theory and data!

Pairing dynamics

Fission of $^{240}\text{Pu}$ at excitation energy $E_x = 8.08$ MeV

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:
- How a possible solitonic structure can be manifested in nuclear system?
- What observable effect it may have on heavy ion reaction: kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.
Estimates for the magnitude of the effect

At first one may think that the magnitude of the effect is determined by the nuclear pairing energy which is of the order of MeV’s in atomic nuclei (according to the expression):

\[ \frac{1}{2} g(\varepsilon_F) |\Delta|^2 ; \quad g(\varepsilon_F) \text{- density of states} \]

On the other hand the energy stored in the junction can be estimated from Ginzburg-Landau (G-L) approach:

For typical values characteristic for two medium nuclei:

\[ E_j \approx 30 \text{MeV} \]
Total kinetic energy of the fragments (TKE)

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments. Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!
Modification of the capture cross section!

At higher energies (1.3-1.5 of the barrier height) the phase difference modifies the reaction outcomes suppressing the reaction channel leading to 3 fragments.

For noncentral collisions the trajectories of outgoing nuclei are affected due to the shorter contact time for larger phase differences.

The phase difference of the pairing fields of colliding medium or heavy nuclei produces a similar solitonic structure as the system of two merging atomic clouds. The energy stored in the created junction is subsequently released giving rise to an increased kinetic energy of the fragments. The effect is found to be of the order of $30 \text{MeV}$ for medium nuclei and occur for energies up to 20-30% of the barrier height.

Effective barrier height for fusion as a function of the phase difference

What is an average extra energy needed for the capture?

\[ E_{\text{extra}} = \frac{1}{\pi} \int_{0}^{\pi} \left( B(\Delta \varphi) - V_{\text{Bass}} \right) d(\Delta \varphi) \approx 10 \text{MeV} \]

The effect is found (within TDDFT) to be of the order of 30 MeV for medium nuclei and occur for energies up to 20-30% of the barrier height.


It raises an interesting question: **to what extent systems of hundreds of particles can be described using the concept of pairing field?**

G. Scamps, Phys. Rev. C 97, 044611 (2018): barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT