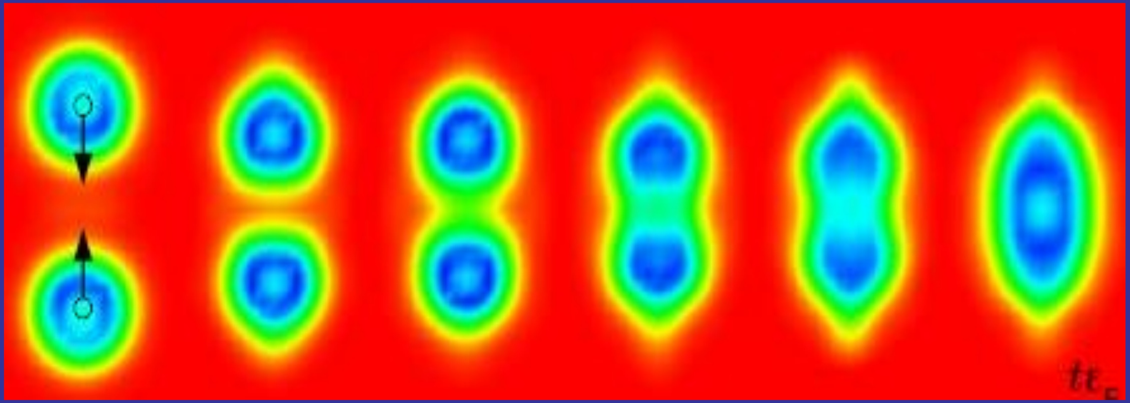
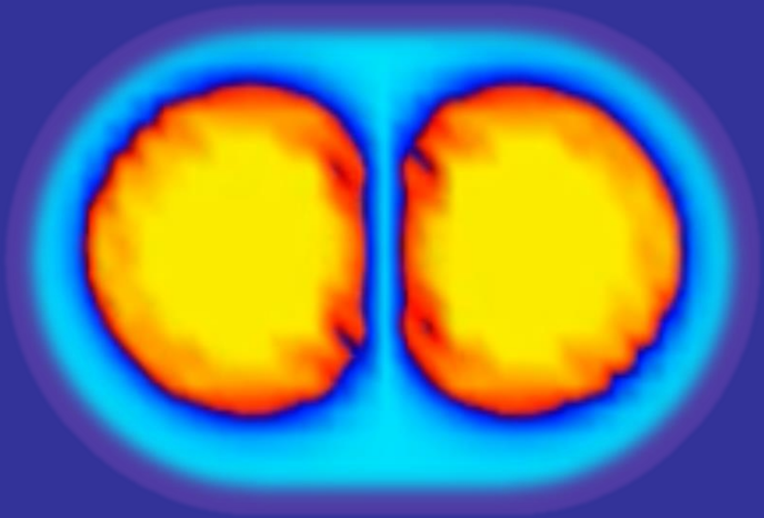
Solitonic excitations in heavy-ion collisions and ultracold atoms





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Gabriel Wlazłowski (WUT)

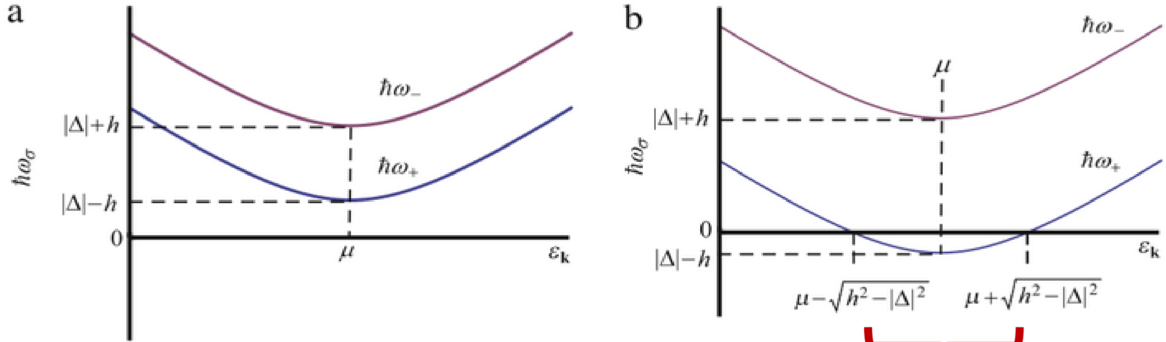
OUTLINE

- Spin-polarized excitations in ultracold Fermi gas
  - FFLO droplets
- Solitonic excitations in heavy-ion collisions
  at the Coulomb barrier

Pairing in spin imbalanced superfluids

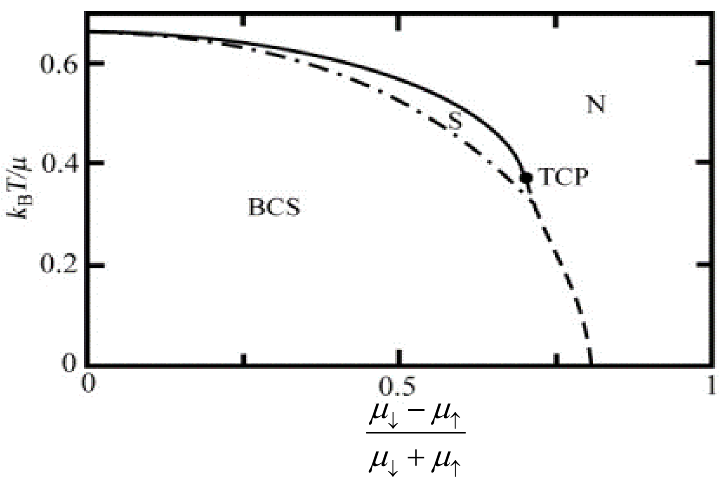
Clogston-Chandrasekhar condition sets the limit for the chemical potential difference at which superfluidity is lost:

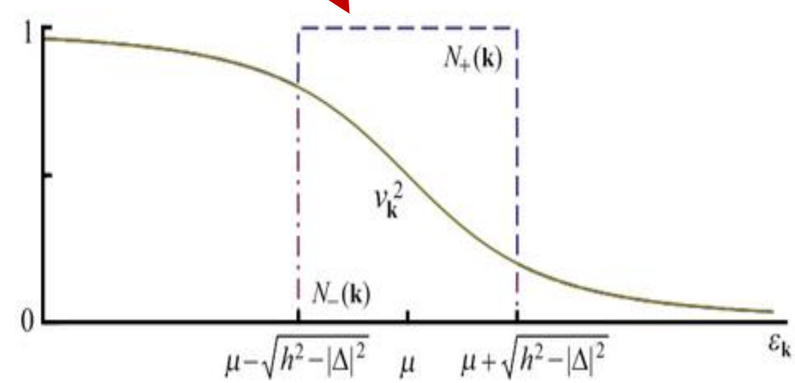
$$\left|\mu_{\downarrow} - \mu_{\uparrow}\right| \propto \Delta$$



Sarma phase (interior gap) phase

G. Sarma, J. Phys. Chem. Solids 24 (1963) 1029.
W.V. Liu, F. Wilczek, Phys. Rev. Lett. 90 (2003) 047002.





Unstable for balanced masses at T=0

Phase separation in momentum space

K.B. Gubbels, H.T.C. Stoof / Physics Reports 525 (2013) 255–313

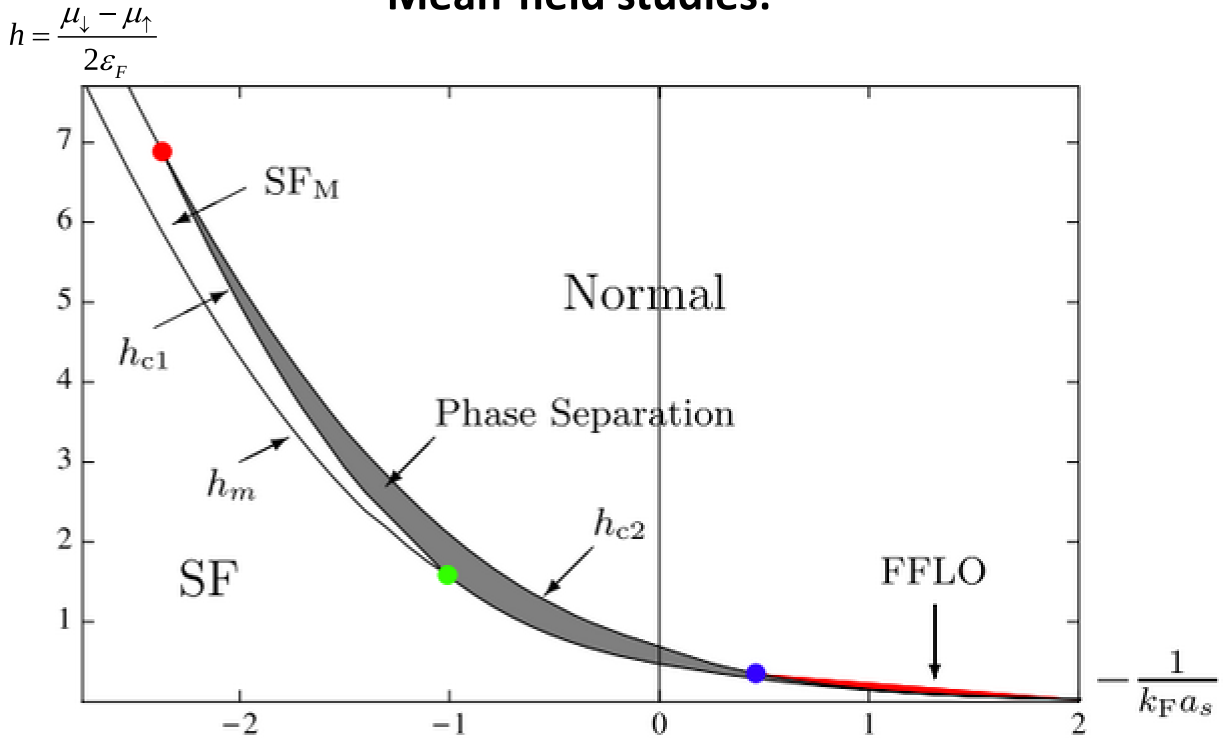
Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

Larkin-Ovchinnikov: $\Delta(r) \sim cos(qr)$

Fulde-Ferrell: $\Delta(r) \sim \exp(iqr)$

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

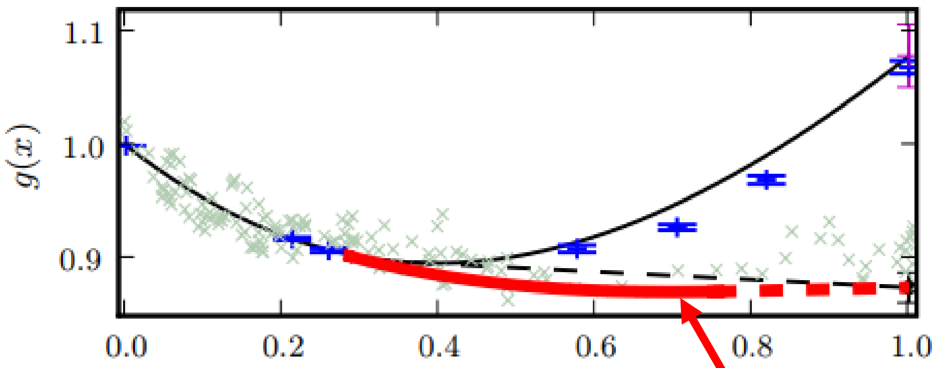
Mean-field studies:



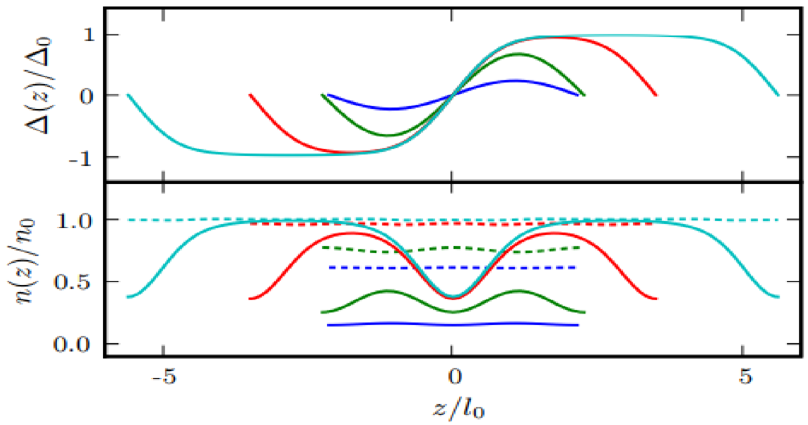
BEC - BCS

Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG)



$E \sim [n_a g(x)]^{5/3}$ $x = n_b/n_a$



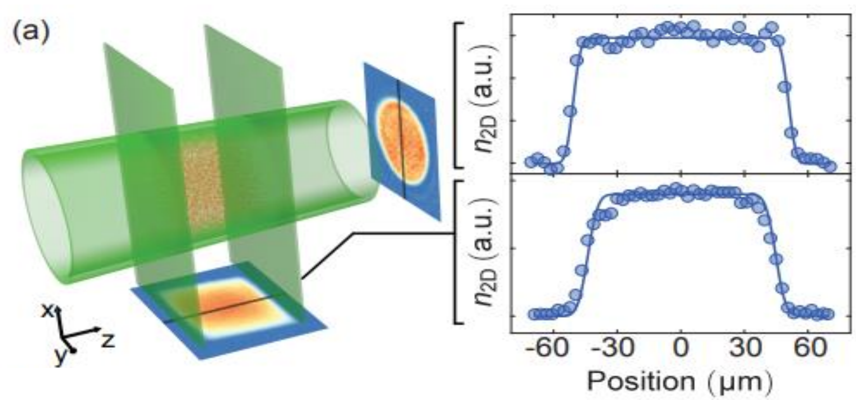
LO configuration – supersolid state

A. Bulgac, M.M.Forbes, PRL101,215301 (2008)

Ultracold atoms in a uniform potential

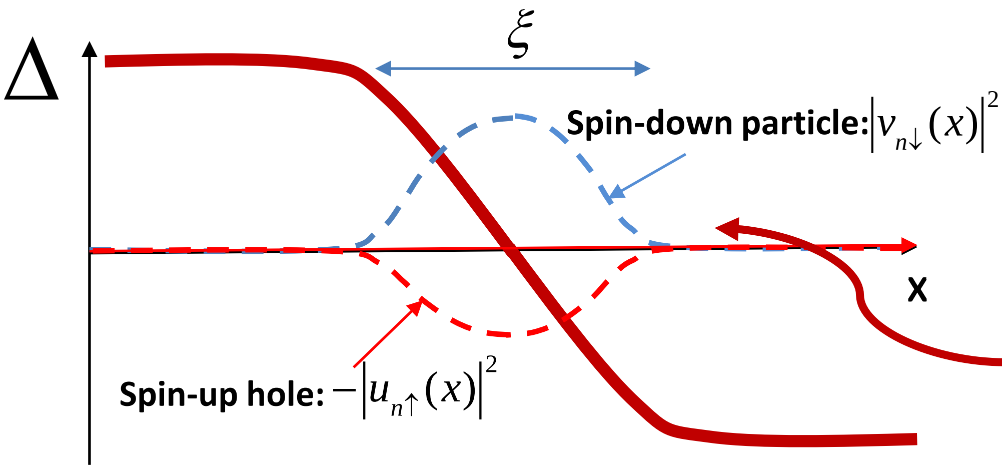
The problem:
In the trap the volume where LOFF phase may be created is relatively small .

unless



B. Mukherjee etal. Phys. Rev. Lett. 118, 123401 (2017)

Can we induce a stable spin-polarized region locally?

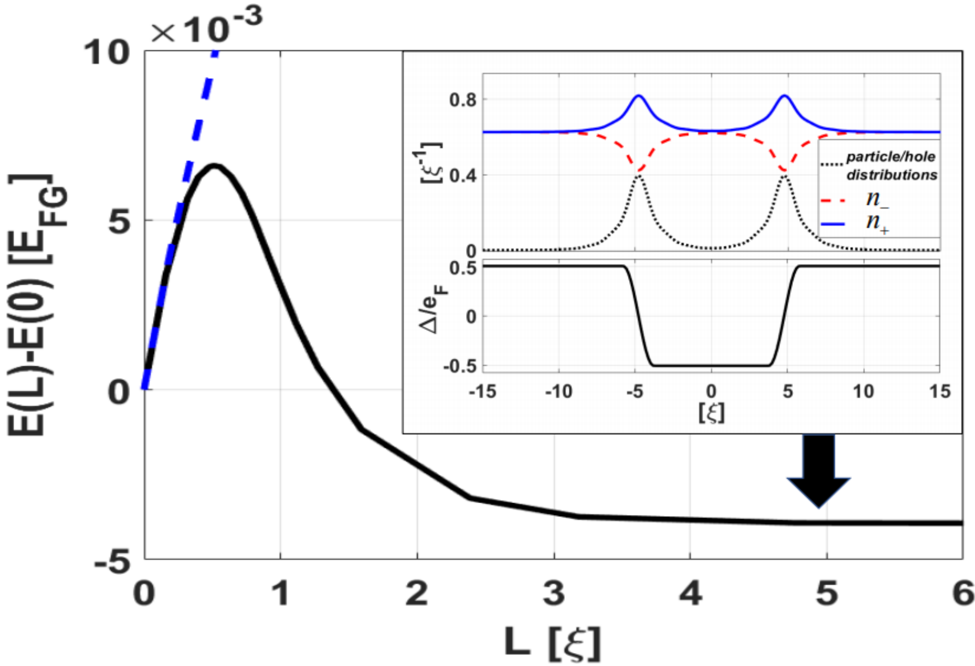


Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

BdG in the Andreev approx. ($\Delta \ll k_F^2$)

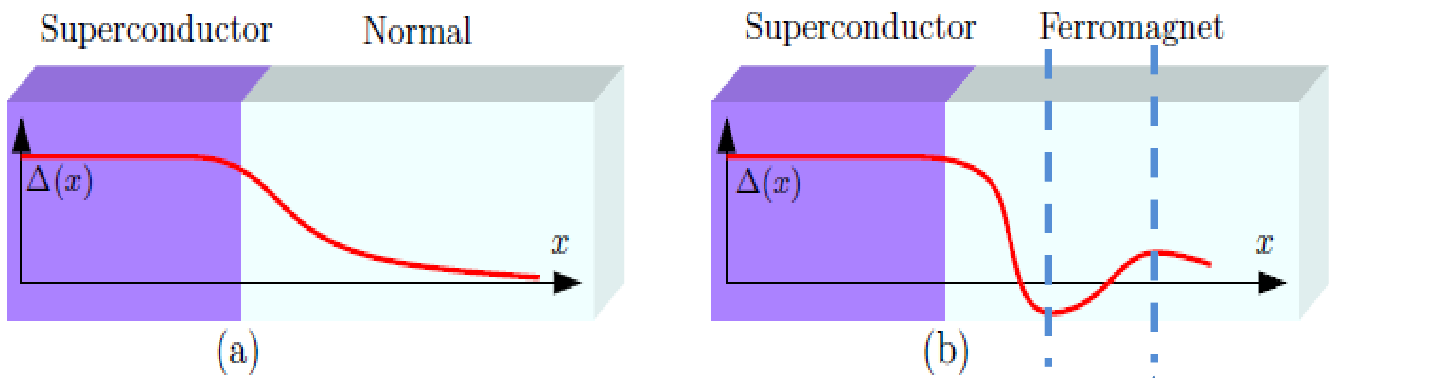
$$\begin{bmatrix} -2ik_F \dfrac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \dfrac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

Energetics of two nodal points in 1D system



Two nodal points repel each other.

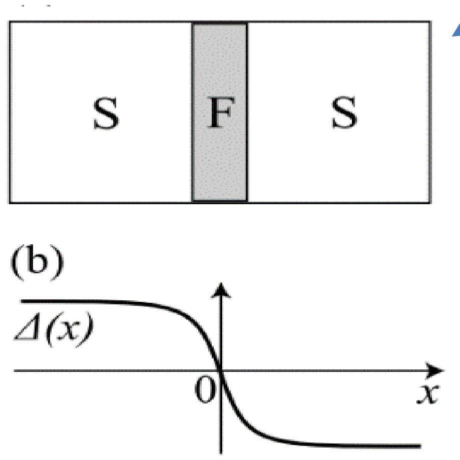
Another perspective: superconductor-ferromagnet junction

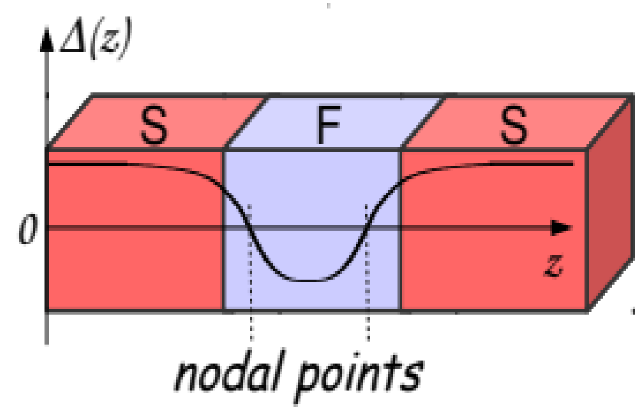


Due to the difference between Fermi momenta of spin-up and spin-down particles:

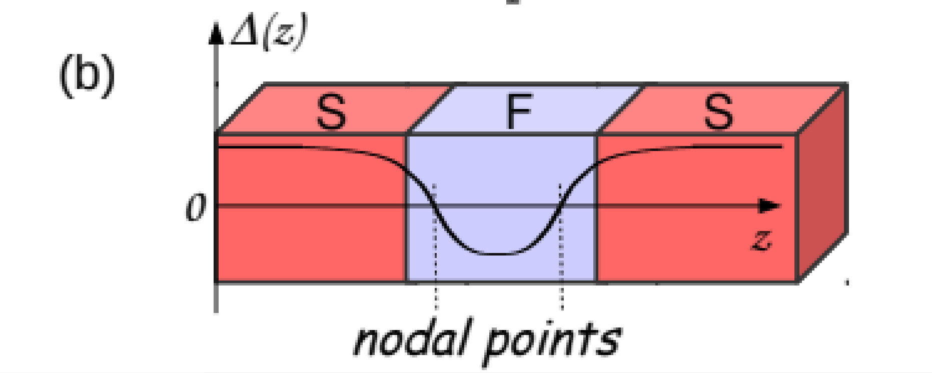
$$\left.\begin{aligned} k_{F\uparrow} &= k_F + \delta k_F \\ k_{F\downarrow} &= k_F - \delta k_F \end{aligned}\right\}$$

Induces spatial modulation of the order parameter of the period: $\dfrac{\pi}{\delta k_F}$

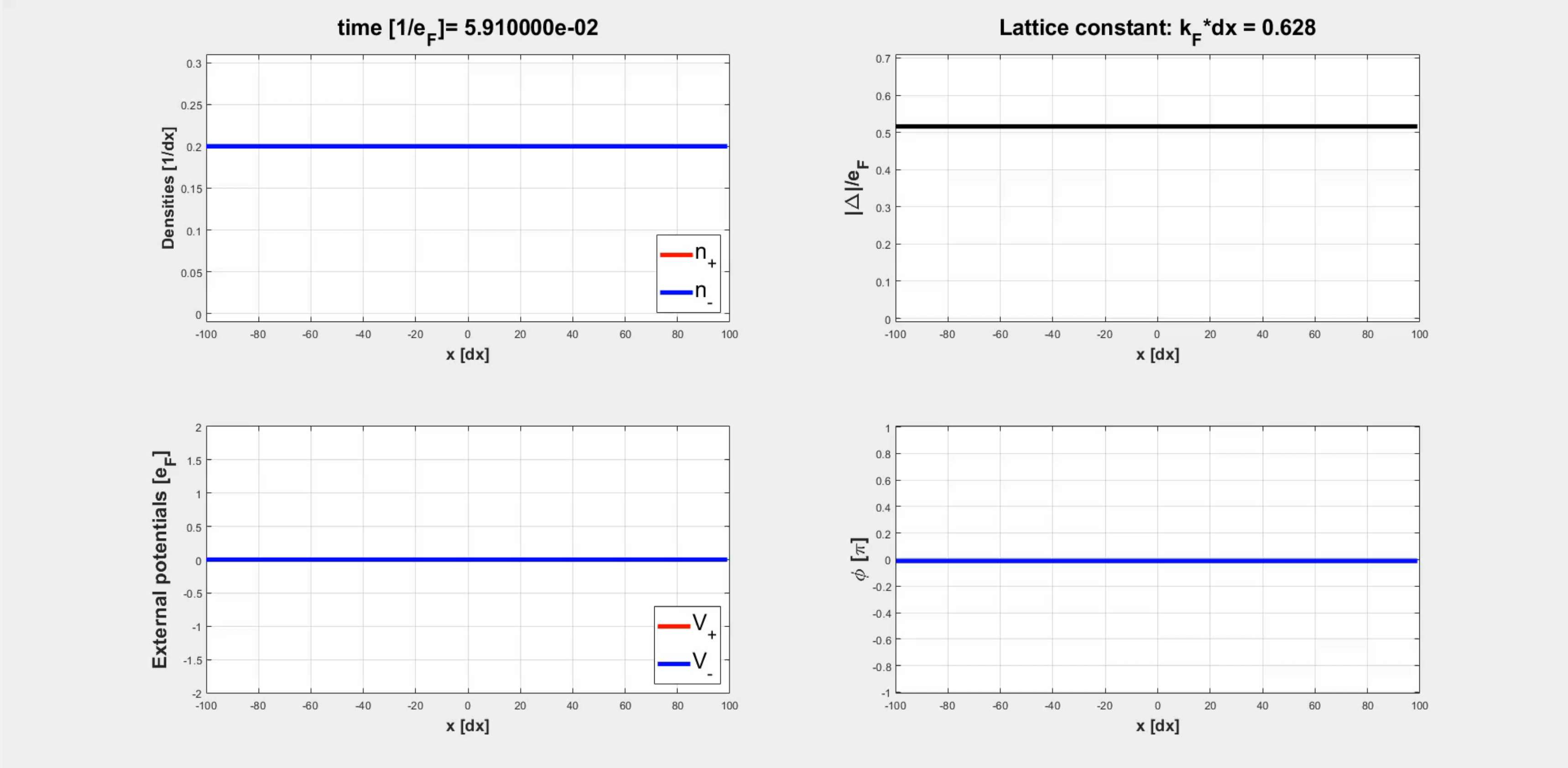




Josephson-Pi junction



Dynamic creation in 1D of two nodal points superfluid Fermi gas



The nodal points repel each other: unstable structure in 1D

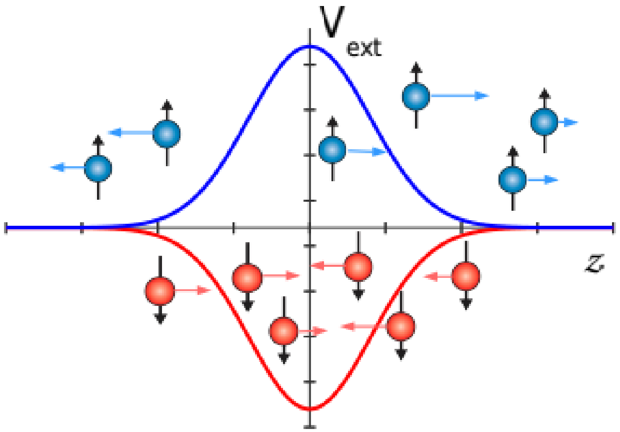
$$V_\lambda(x,t) = 1.8 f(t)\lambda\varepsilon_F \exp(-\frac{x^2}{2\sigma^2}),$$

$k_F\sigma = 4.441$ time scales: $\varepsilon_F \Delta t \sim 10$

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

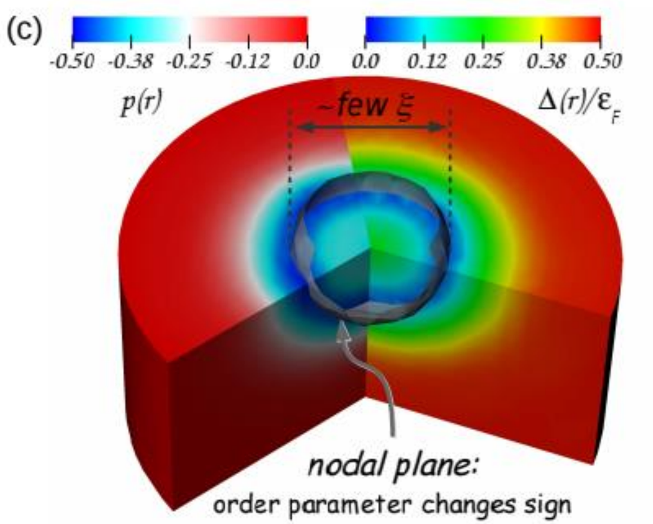
Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:



Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

For example the spherical nodal structure:



$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r}) \nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$+ g(n_\uparrow, n_\downarrow)\nu$$

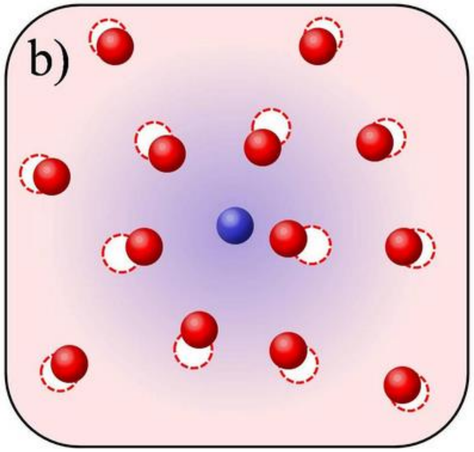
$$+ [1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

A. Bulgac, M.M. Forbes, P. Magierski,
Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities: $$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<}$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{I}$$



EDF:

$$\mathcal{H} = \boxed{\alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}}$$

$$+D(n_\uparrow, n_\downarrow)$$

$$+g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$+[1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

Kinetic term:
Effective mass α_σ of the particle depends on local polarization

$$p(\boldsymbol{r}) = \frac{n_\uparrow(\boldsymbol{r}) - n_\downarrow(\boldsymbol{r})}{n_\uparrow(\boldsymbol{r}) + n_\downarrow(\boldsymbol{r})}$$

and guarantees that correct limit is attained for $n_\uparrow >> n_\downarrow$, where the problem reduces to the *polaron* problem

A. Bulgac, M.M. Forbes, P. Magierski,
Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r})\nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$\boxed{+D(n_\uparrow, n_\downarrow)}$$

$$+g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$+[1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

Normal interaction energy:

$$D(n_\uparrow, n_\downarrow) \sim (n_\uparrow + n_\downarrow)^{5/3}\beta(p)$$

in order to get the proper scaling:

$$E = \xi E_{FFG}$$

A. Bulgac, M.M. Forbes, P. Magierski,
Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r})\nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+ D(n_\uparrow, n_\downarrow)$$

$$\boxed{+ g(n_\uparrow, n_\downarrow)\nu^\dagger\nu}$$

$$+ [1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}$$

Pairing energy:

$$g(n_\uparrow, n_\downarrow) = \frac{\gamma(p)}{(n_\uparrow + n_\downarrow)^{1/3}}$$

in order to get proper scaling:

$$\Delta/\varepsilon_F = \text{const} \approx 0.5$$

A. Bulgac, M.M. Forbes, P. Magierski,
Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

$$\phi_n \longrightarrow (u_{n,\uparrow}, u_{n,\downarrow}, v_{n,\uparrow}, v_{n,\downarrow})$$

Densities:

$$n_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2, \qquad \tau_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2,$$

$$\nu(\boldsymbol{r}) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r}) v^*_{n,\downarrow}(\boldsymbol{r}), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r})\nabla v_{n,\sigma}(\boldsymbol{r})],$$

EDF:

$$\mathcal{H} = \alpha_\uparrow(p)\frac{\hbar^2\tau_\uparrow}{2m_\uparrow} + \alpha_\downarrow(p)\frac{\hbar^2\tau_\downarrow}{2m_\downarrow}$$

$$+D(n_\uparrow, n_\downarrow)$$

$$+g(n_\uparrow, n_\downarrow)\nu^\dagger\nu$$

$$\boxed{+[1-\alpha_\uparrow(p)]\frac{j_\uparrow^2}{2n_\uparrow} + [1-\alpha_\downarrow(p)]\frac{j_\downarrow^2}{2n_\downarrow}}$$

In order to restore Galilean invariance of the functional

More details:
A. Bulgac, M.M. Forbes, P. Magierski,
The Unitary Fermi Gas: From Monte Carlo to Density Functionals,
Lecture Notes in Physics 836
ed. W. Zwerger, Springer (2011).

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\boldsymbol{\nabla}^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\boldsymbol{\nabla} + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix}$$

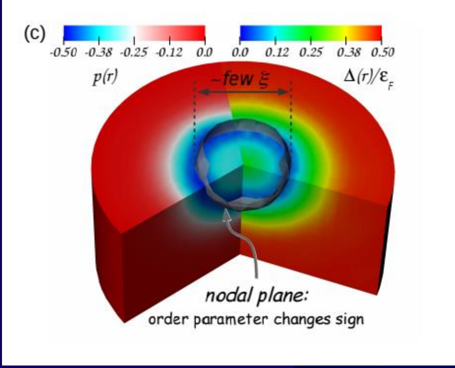
We explicitly track fermionic degrees of freedom!

where *h* and Δ depends on “densities”:

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

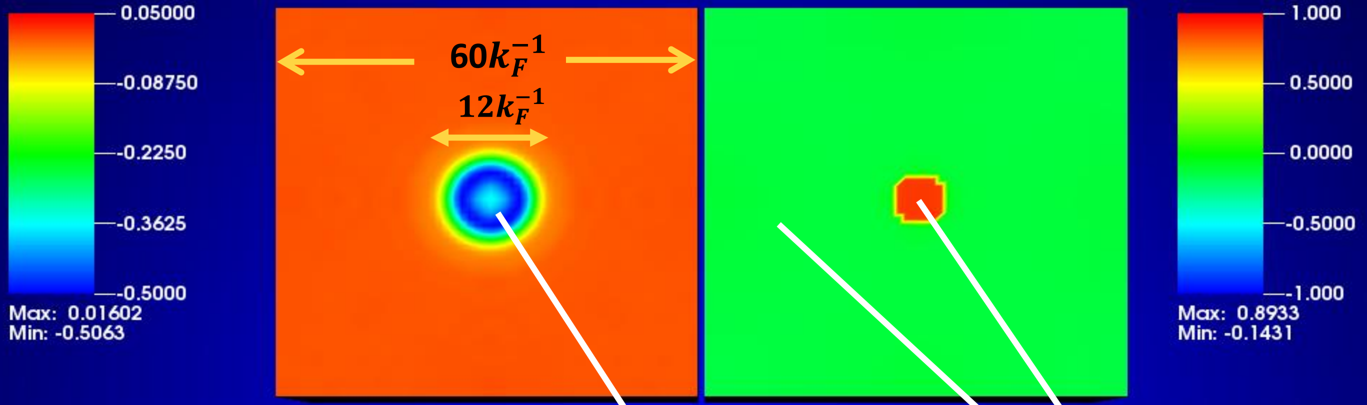
$$\nu(\boldsymbol{r},t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

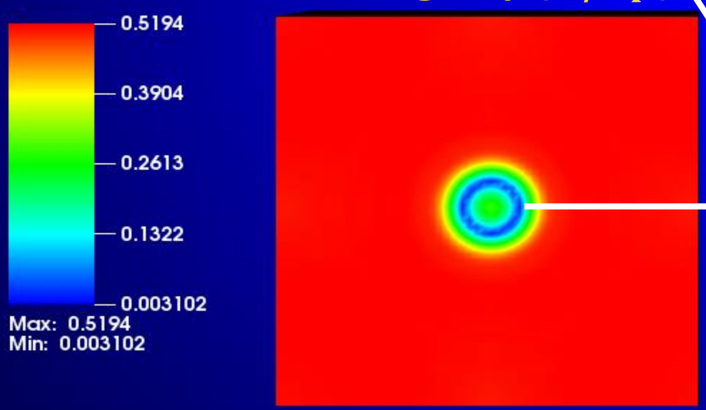


Forming a spherical nodal surface

Polarization $p(r)$ **Phase of Pairing $[\pi]$**



Pairing Gap $|\Delta/\varepsilon_F|$

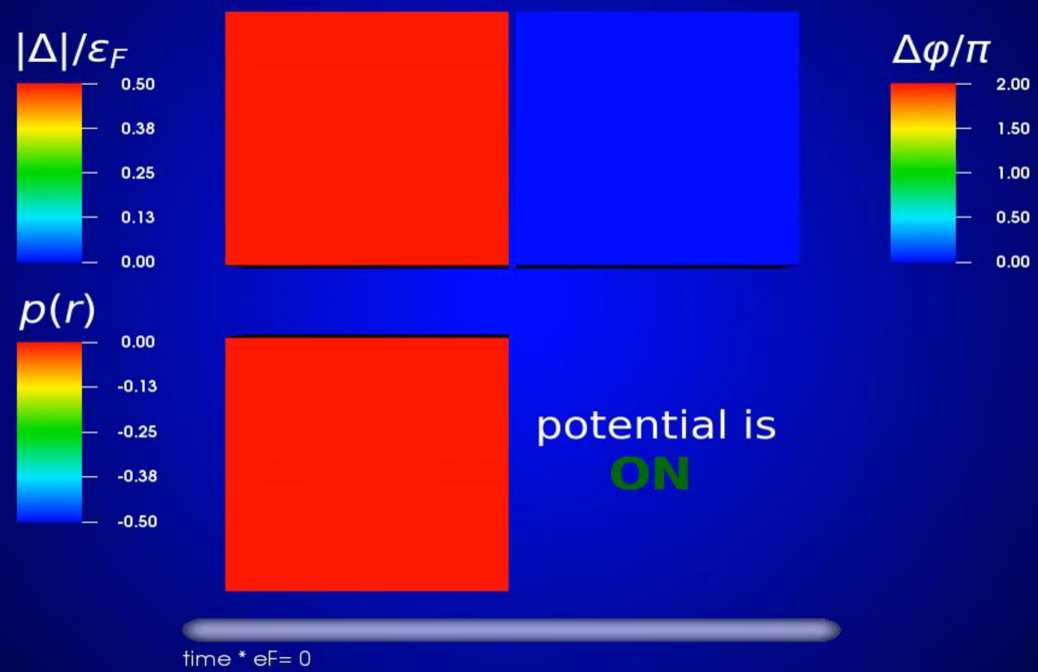


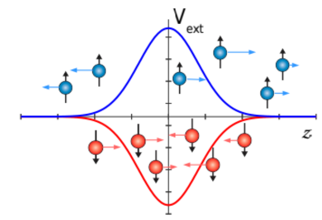
Phase difference is π

Maximum polarization occurs within a shell where the pairing field vanishes.

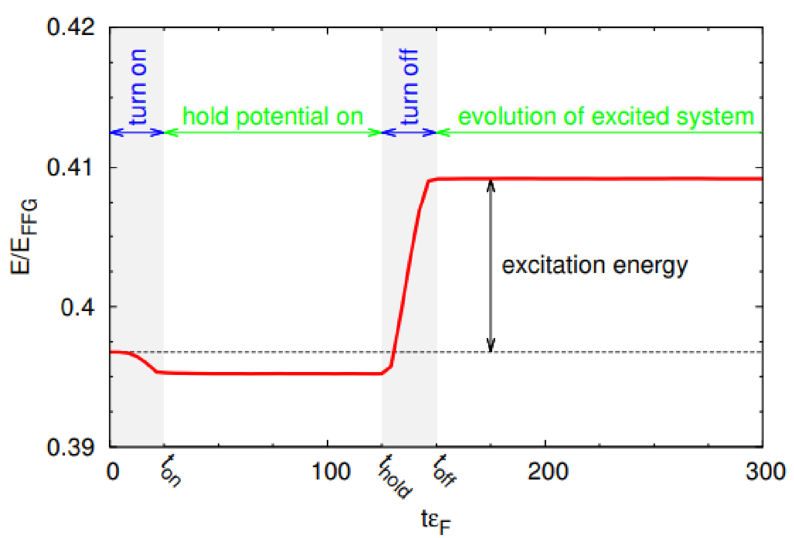
Potential: $A = 2\varepsilon_F,\ \sigma = 3.14\xi$

$$V_p(\boldsymbol{r},t) = \lambda A(t)\exp\left[-\frac{x^2 + (1-\epsilon_y)y^2 + (1-\epsilon_z)z^2}{2\sigma^2}\right]$$





time scales: $\varepsilon_F \Delta t \sim 100$



Energy of the system during the procedure of the *impurity* creation

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

The origin of stability of the polarized impurity (at T=0): $E_{imp} \approx E_{\mathrm{int}} + E_{shell}$

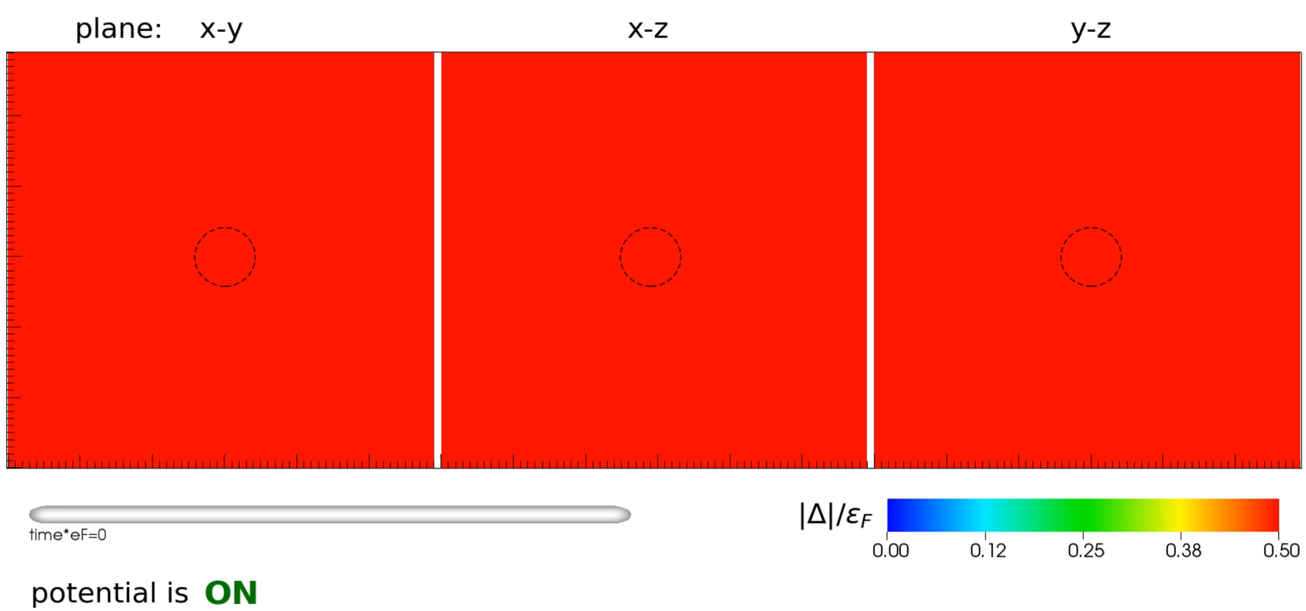
E_{int} - Energy associated with the volume
E_{shell} - Energy associated with the polarized shell located at the surface

Contraction of the nodal sphere is prevented by the pairing potential barrier.
Expansion of the nodal sphere will cost the energy due to the polarization shell expansion.

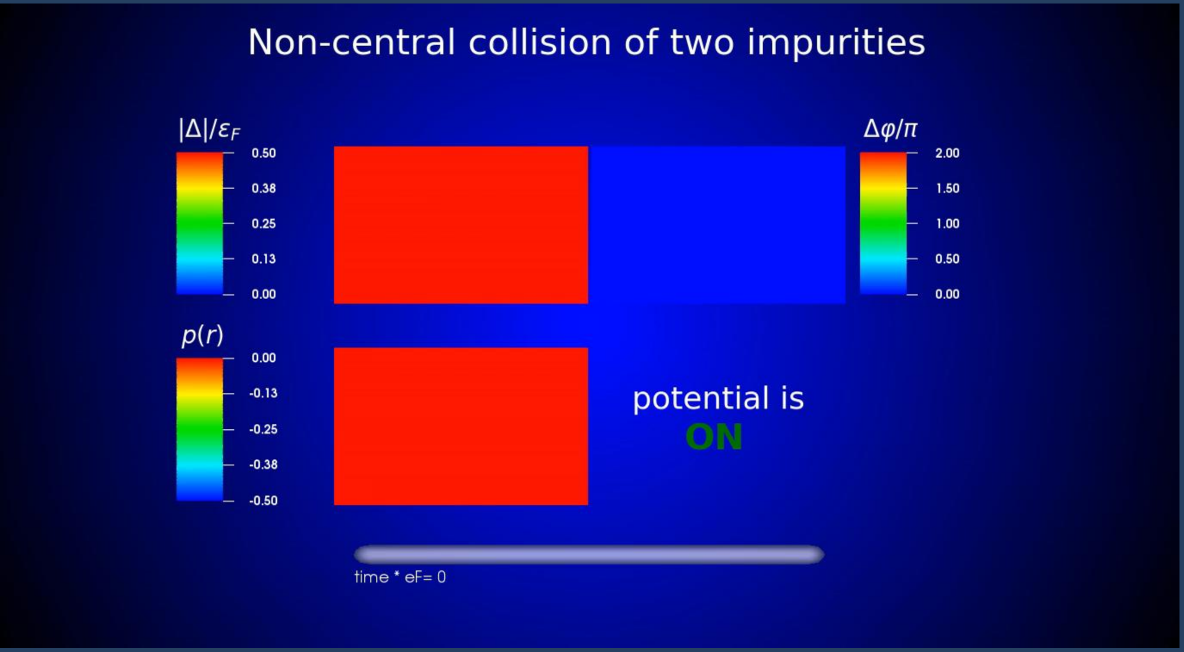
As a result of the interplay between volume and surface energies keeps the impurity stable

Evolution of the deformed impurity

Potential: $A = 2\varepsilon_F$, $\sigma_x = 4.71\xi$, $\sigma_y = 6.28\xi$, $\sigma_z = 7.85\xi$



Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

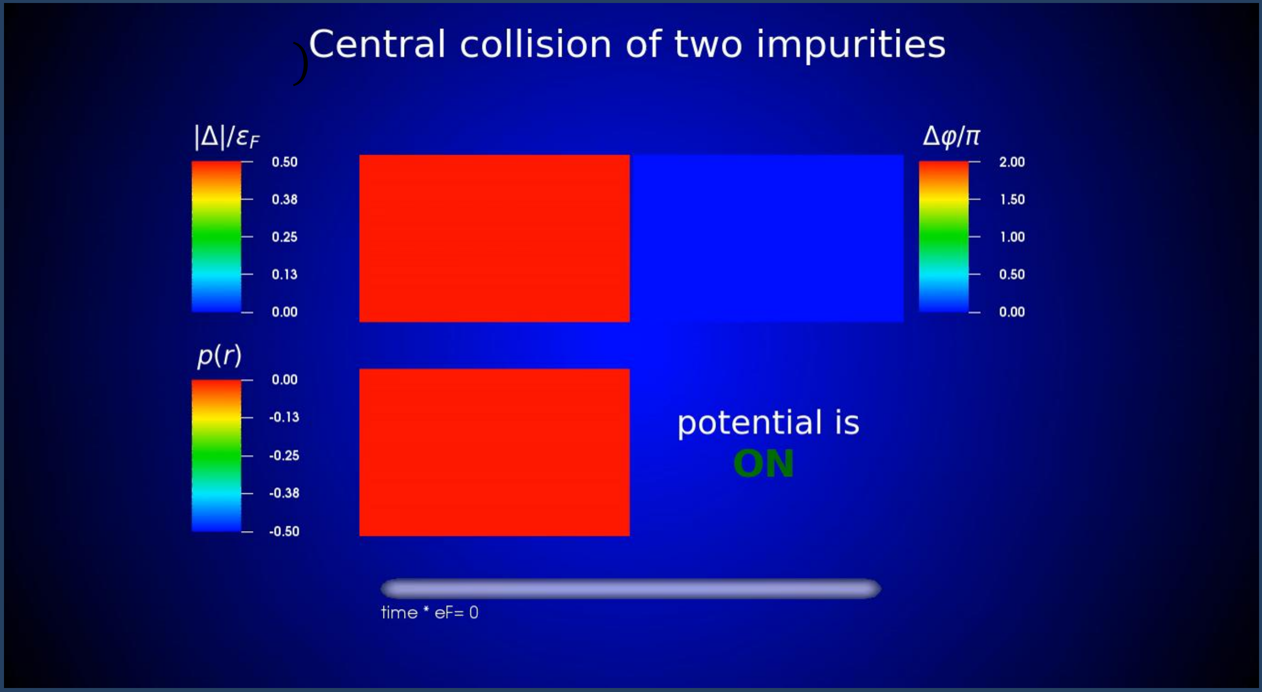


Moving impurity:

From Larkin-Ovchinnikov
towards
Fulde-Ferrell limit

$$\Delta(r) : cos(qr) \Rightarrow \exp(iqr)$$

The velocities of impurites are about 30% of the velocity of sound



Note the rigidity of
the structure of impurities
during collision

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

Creation of more complex nodal surface structures:

Concentric nodal spheres

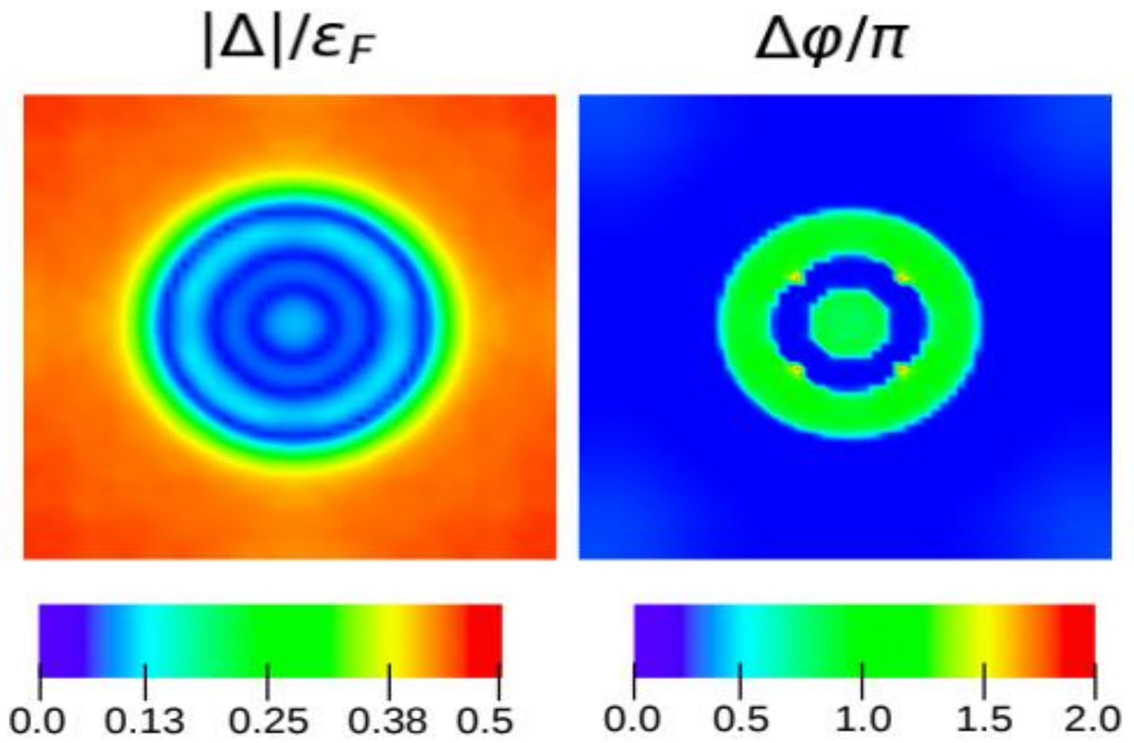
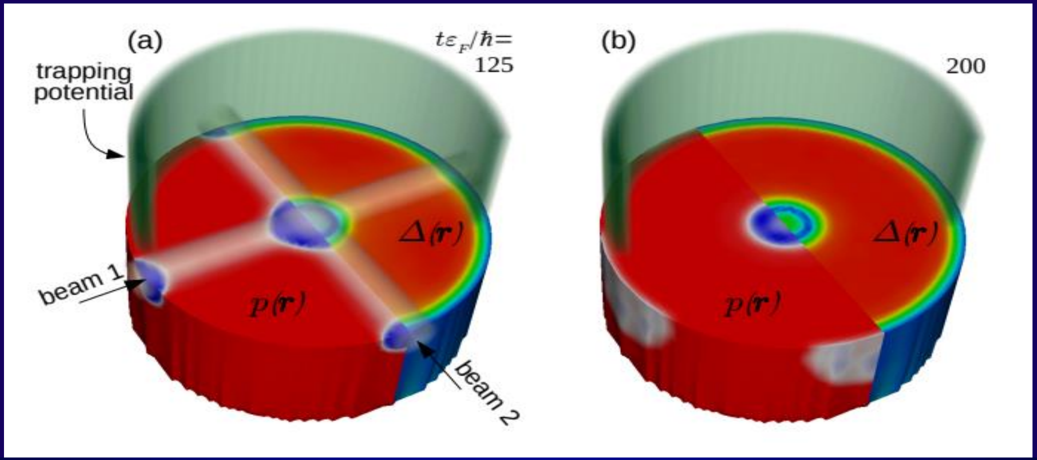
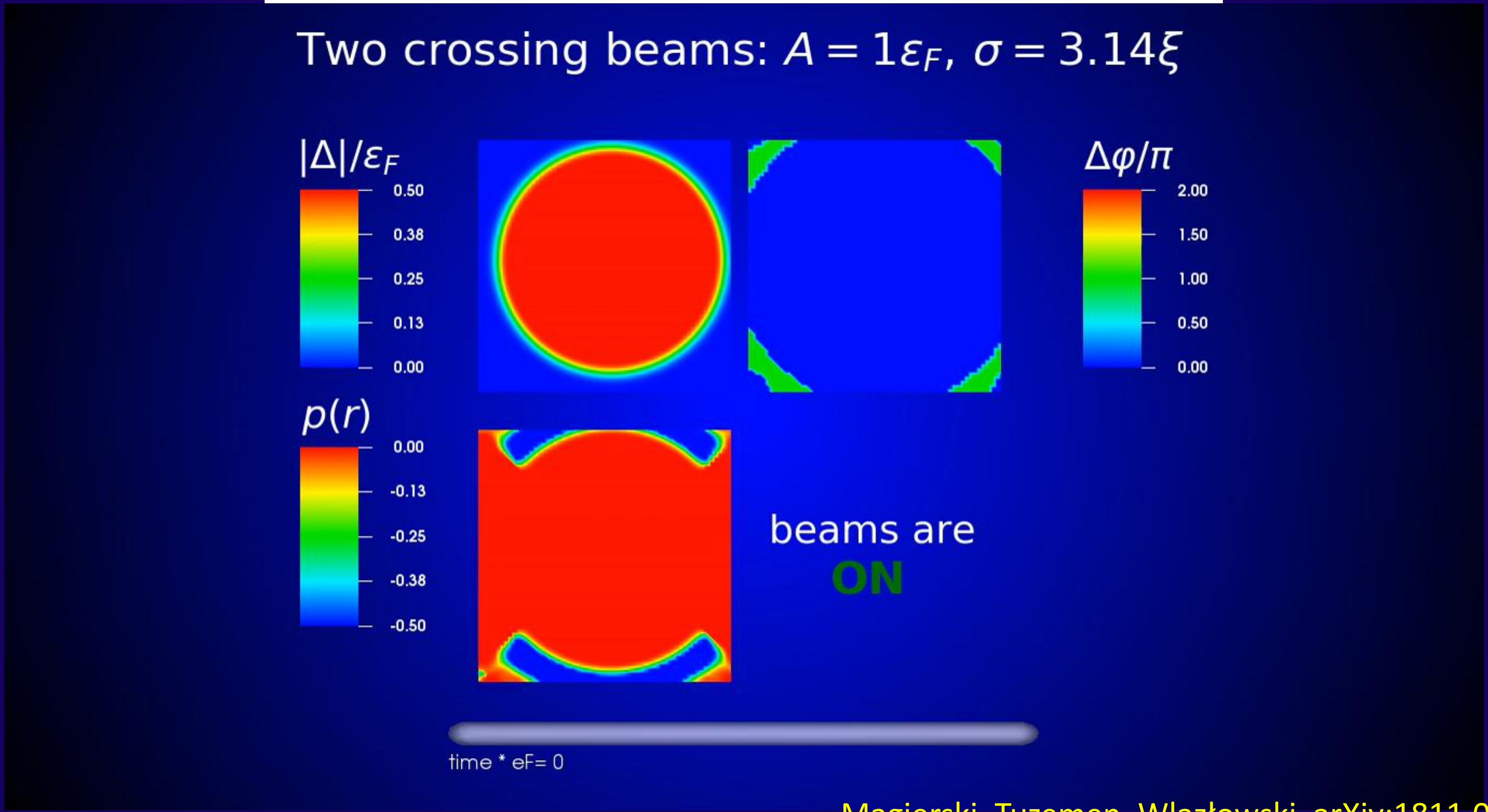


FIG. 13: Snapshot from simulation demonstrating the internal structure of the large ferron-like excitation taken after the time $\Delta t \approx 220\varepsilon_F^{-1}$ with respect to the moment when the potential was removed. It is clearly seen that the phase changes sign three times as we proceed towards the center of the impurity. For full movie see *Movie 13*.

Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

Suggestion for experimental protocol





Magierski, Tuzemen, Wlazłowski, arXiv:1811.00446

Conclusions

It is possible to create dynamically stable, locally spin-polarized region in the ultracold Fermi gas.
The stability is due to the peculiar pairing structure characteristic for the FFLO phase.

The conditions of stability:

- do not depend on details of the functional (simple BdG approach predicts qualitatively the same results)
- do not depend whether we are on the BEC-side (a>0) or on the BCS-side (a<0), although UFG may be the best system for experimental realization.

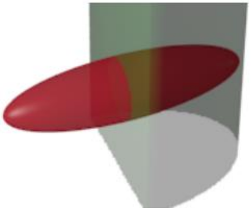
The effect can be viewed as:

- long-lived, spin-polarized excitation mode of UFG
- FFLO droplet (although its size is of few coherence lengths only)

We dubbed it ***ferron***

Open problems

- Experimental realization
- Collision effects beyond the mean-field picture (impact on stability)
- Stability as a function of temperature (for T<<Tc ferron is unaffected)
- ...

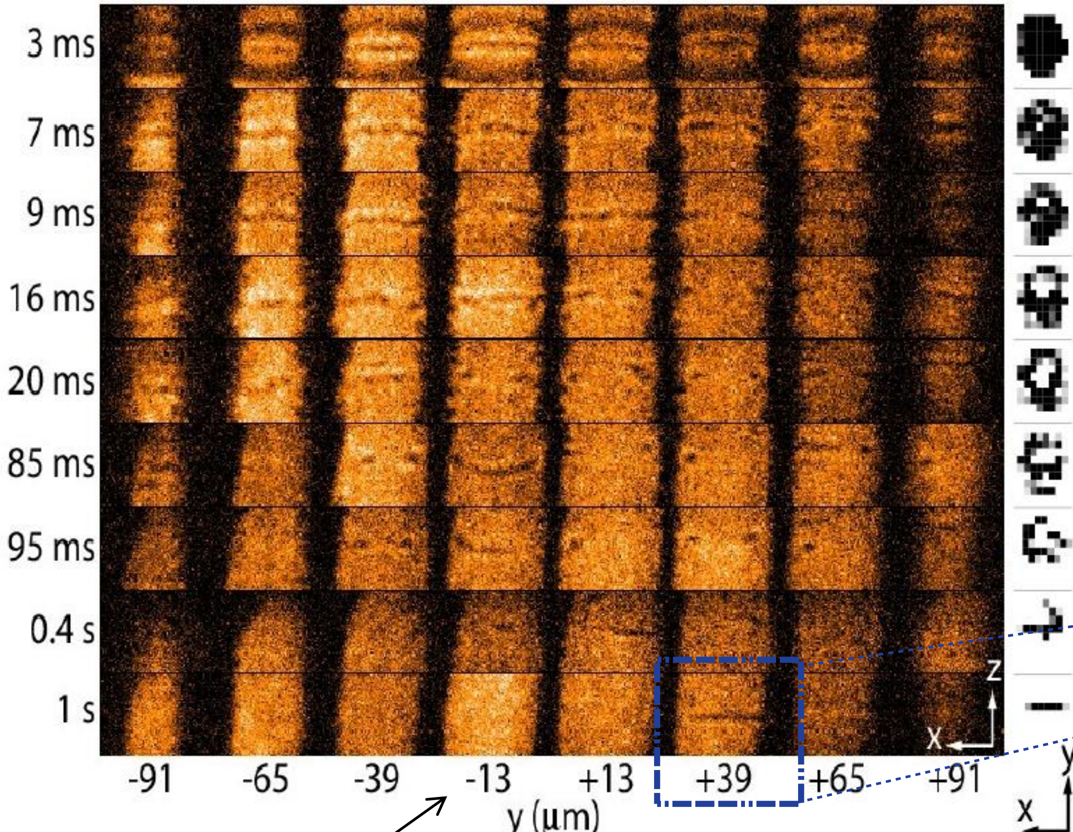


(a)

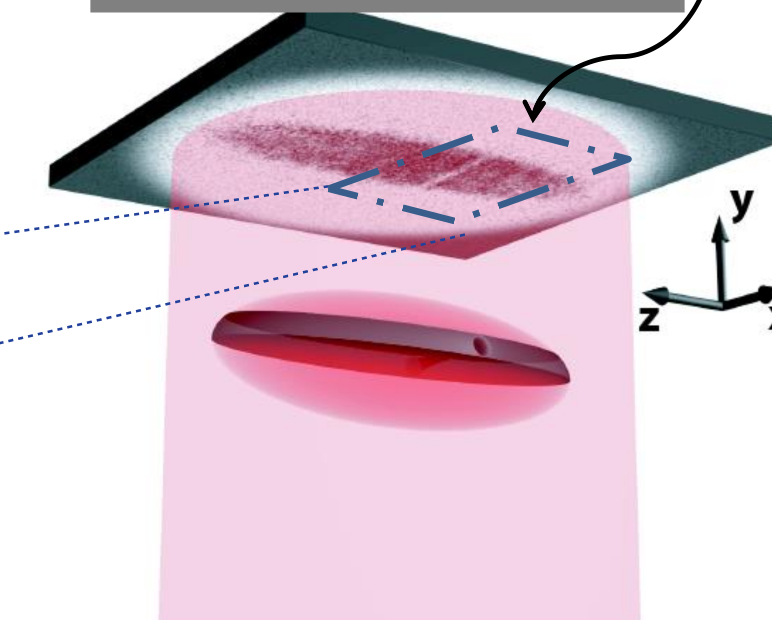
experiment

Phys. Rev. Lett. 116, 045304 (2016)

Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line

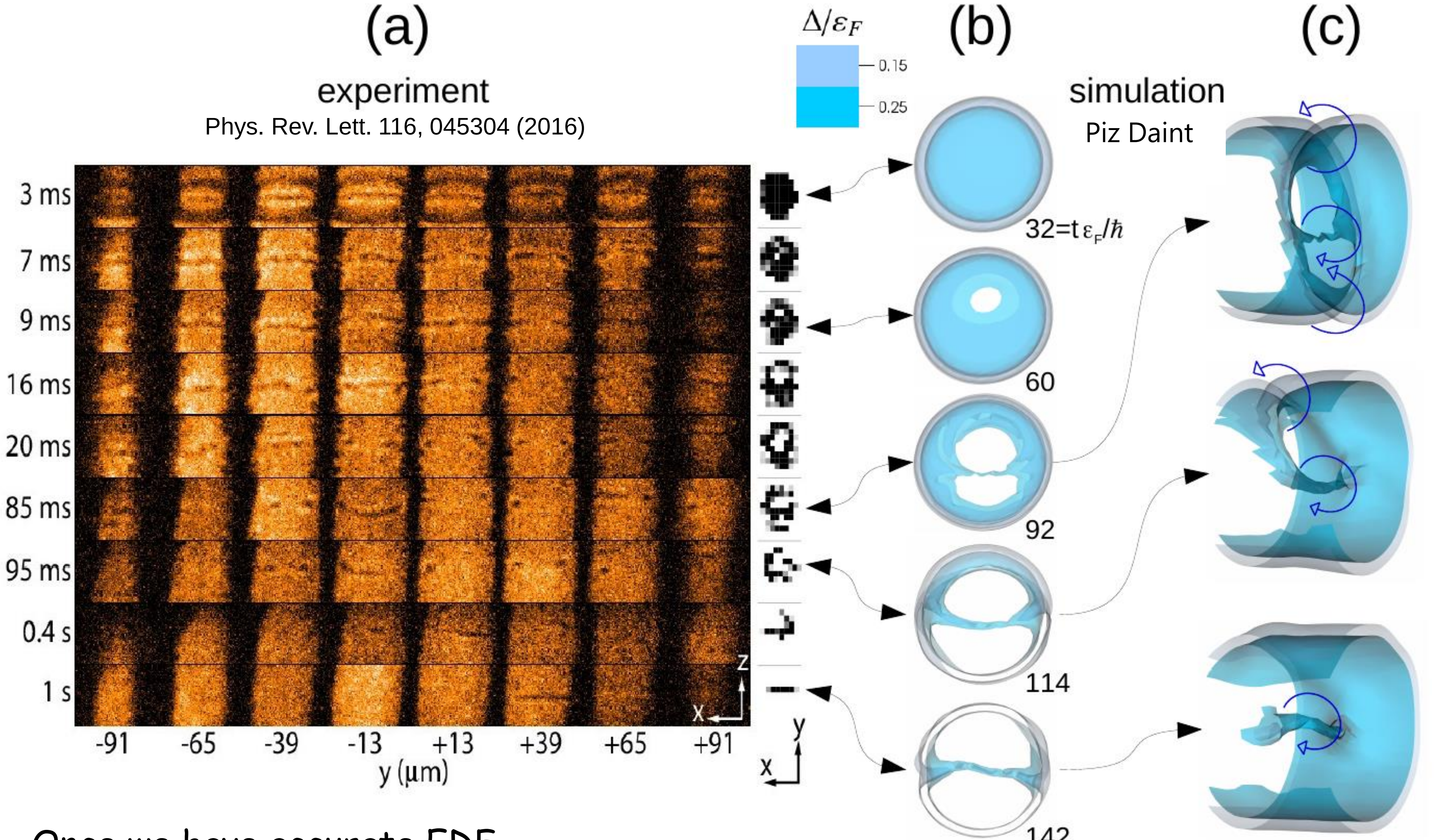


Note:
here we observe directly time evolution of ***density n(r,t)*** *for quantum system*



unitary Fermi gas

(superfluid properties manifest here in the form of topological defects)



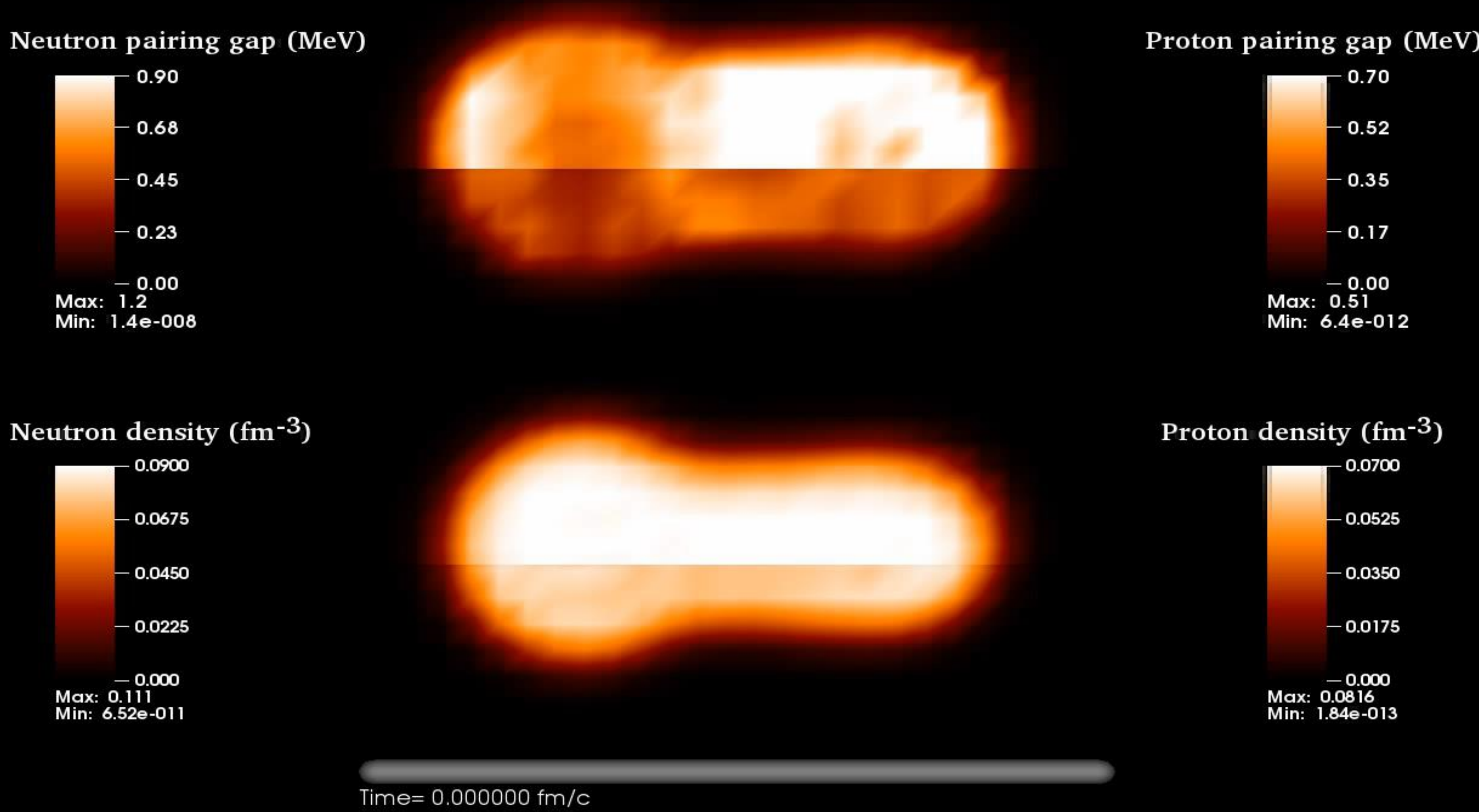
Once we have accurate EDF
→ remarkable agreement between theory and data!

No adjusting parameters to the experiment!

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, Phys. Rev. Lett. **120**, 253002 (2018)

Pairing dynamics

Fission of ^{240}Pu at excitation energy E_x = 8.08 MeV

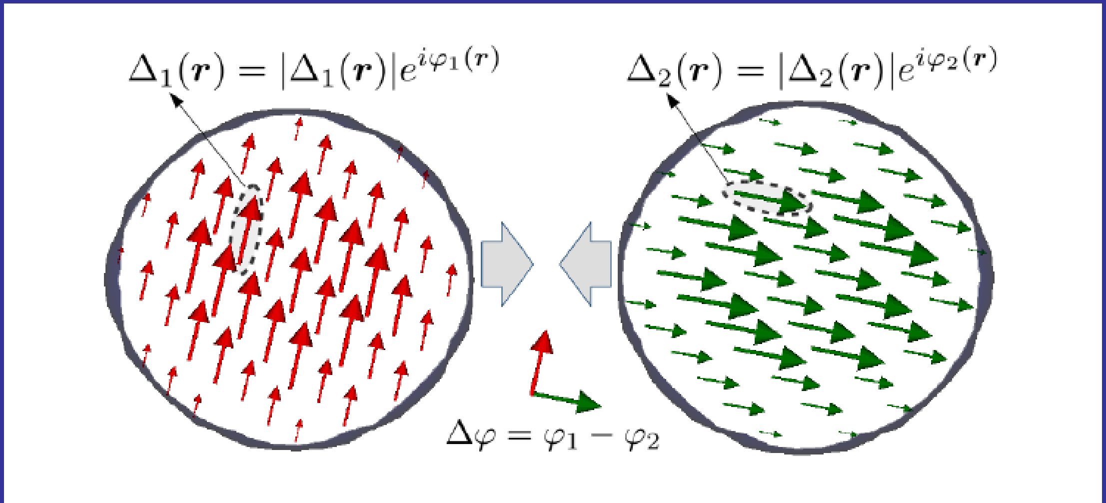


A. Bulgac, P.Magierski, K.J. Roche, and I. Stetcu, Phys. Rev. Lett. 116, 122504 (2016)

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:
-how a possible solitonic structure can be manifested in nuclear system?
-what observable effect it may have on heavy ion reaction:
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.

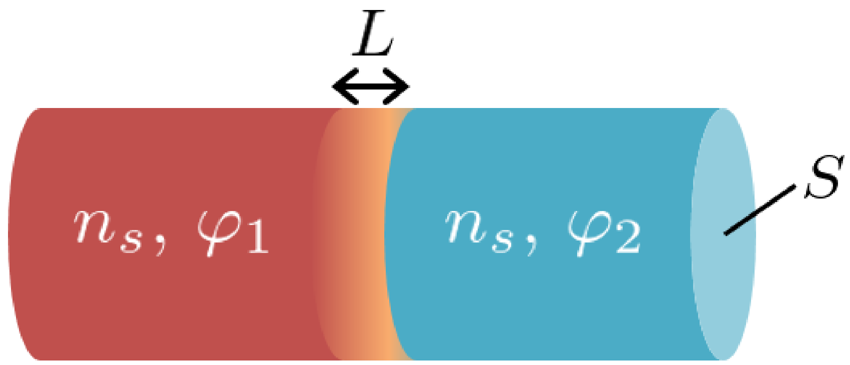


Estimates for the magnitude of the effect

At first one may think that the magnitude of the effect is determined by the nuclear pairing energy which is of the order of MeV's in atomic nuclei (according to the expression):

$$\frac{1}{2} g(\varepsilon_F)|\Delta|^2; \quad g(\varepsilon_F) \text{ - density of states}$$

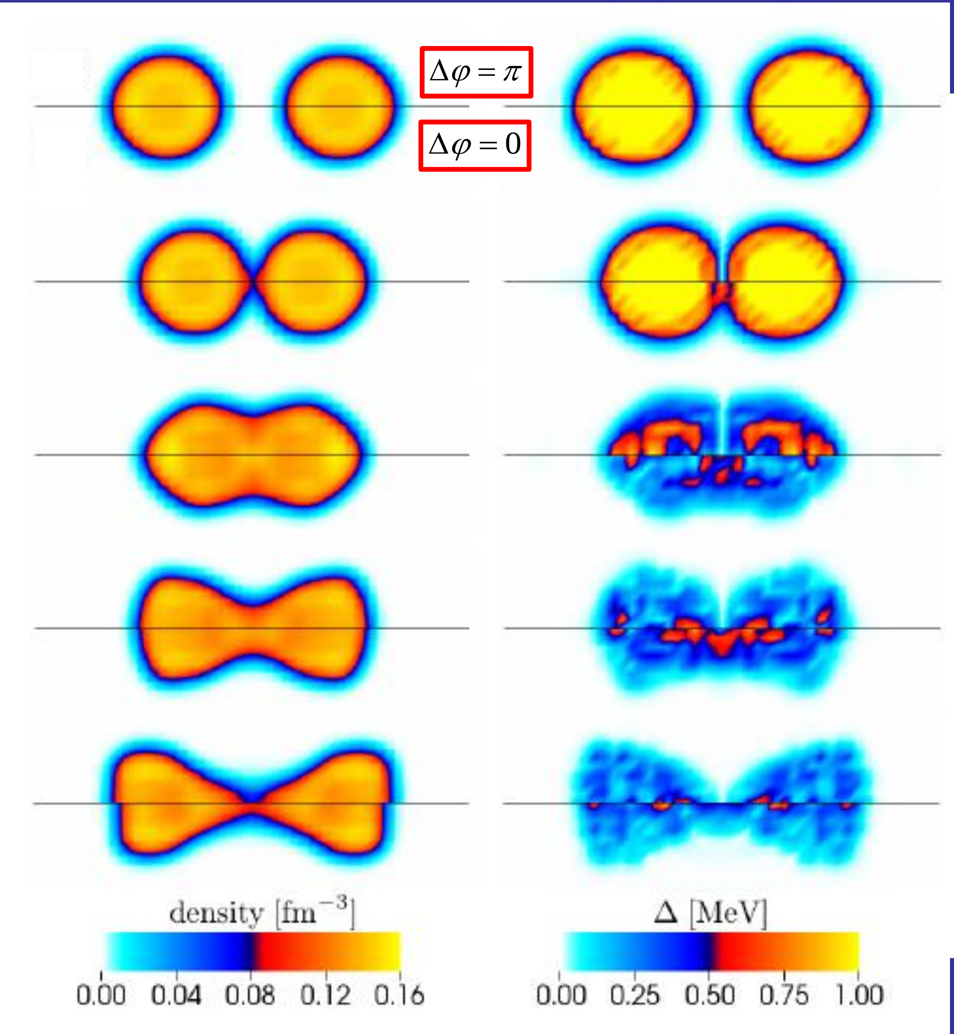
On the other hand the energy stored in the junction can be estimated from Ginzburg-Landau (G-L) approach:



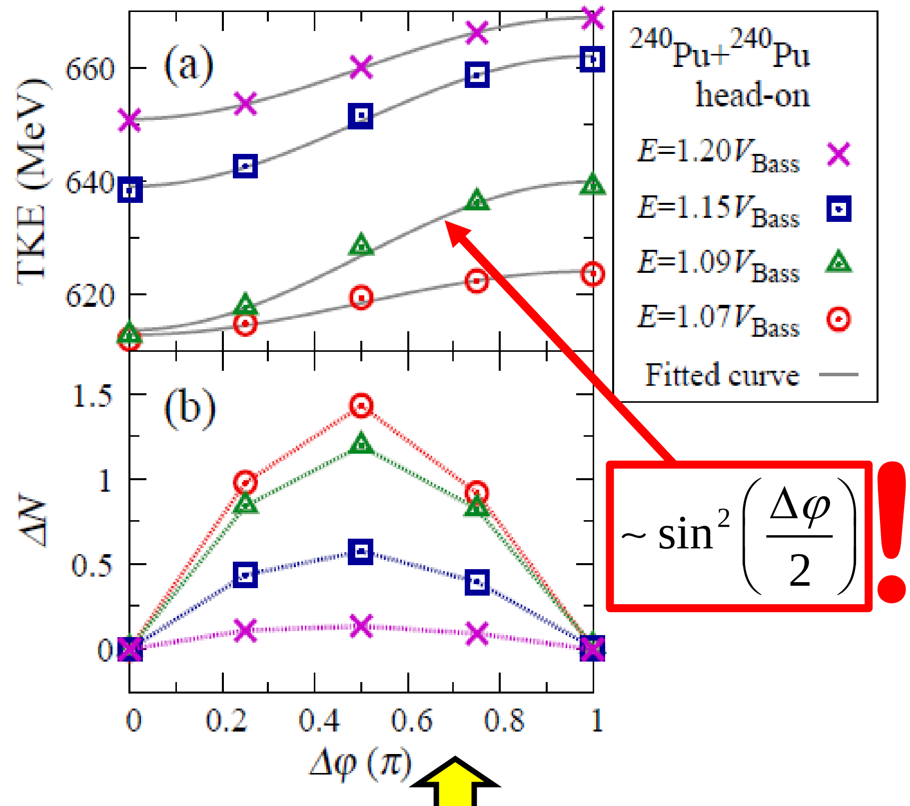
$$\Delta\varphi \ (\equiv \varphi_1 - \varphi_2)$$

$$\boxed{E_j = \frac{S}{L}\frac{\hbar^2}{2m} n_s \sin^2\frac{\Delta\varphi}{2}}$$

For typical values characteristic for two medium nuclei: $E_j \approx 30 MeV$



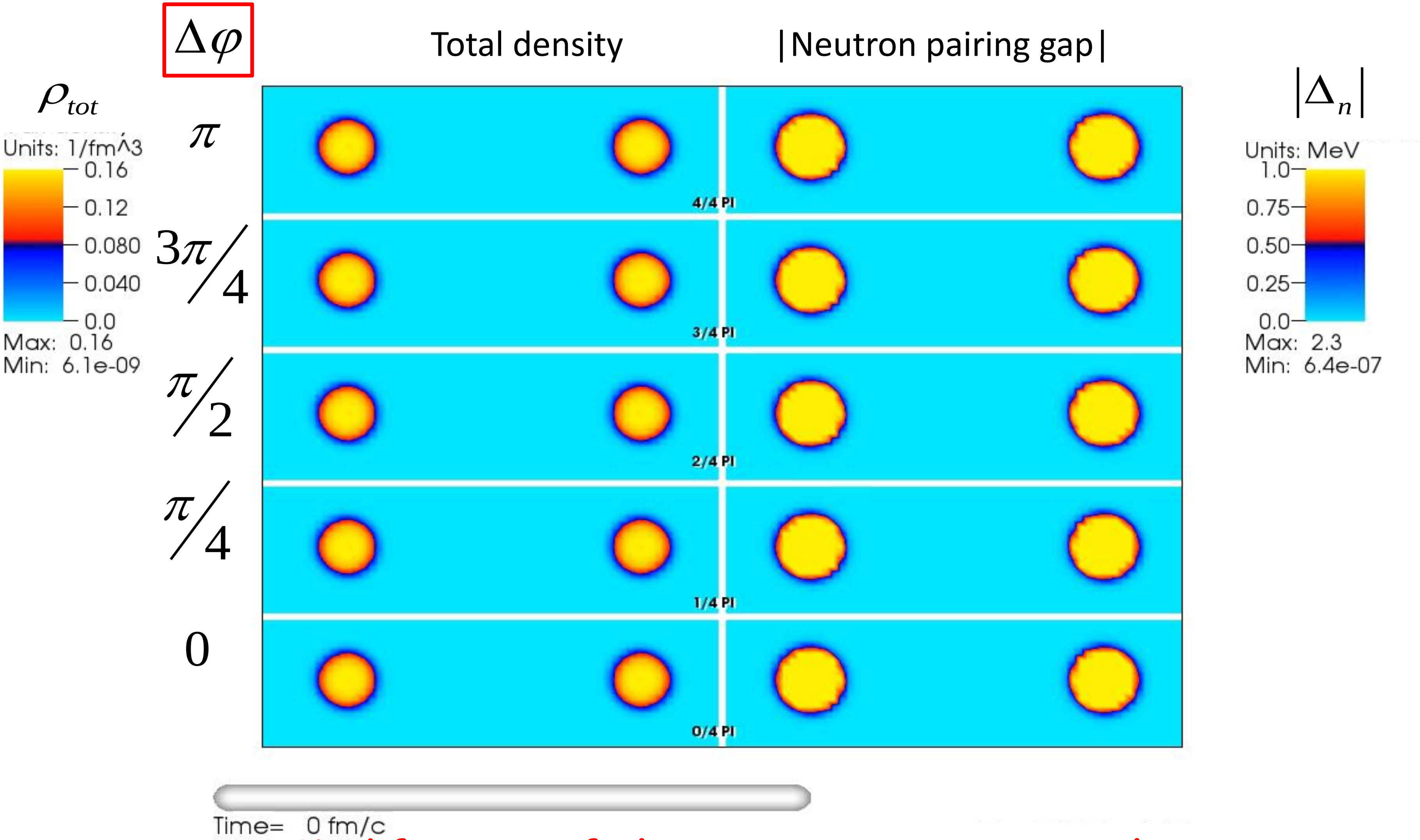
Total kinetic energy of the fragments (TKE)



Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.
Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

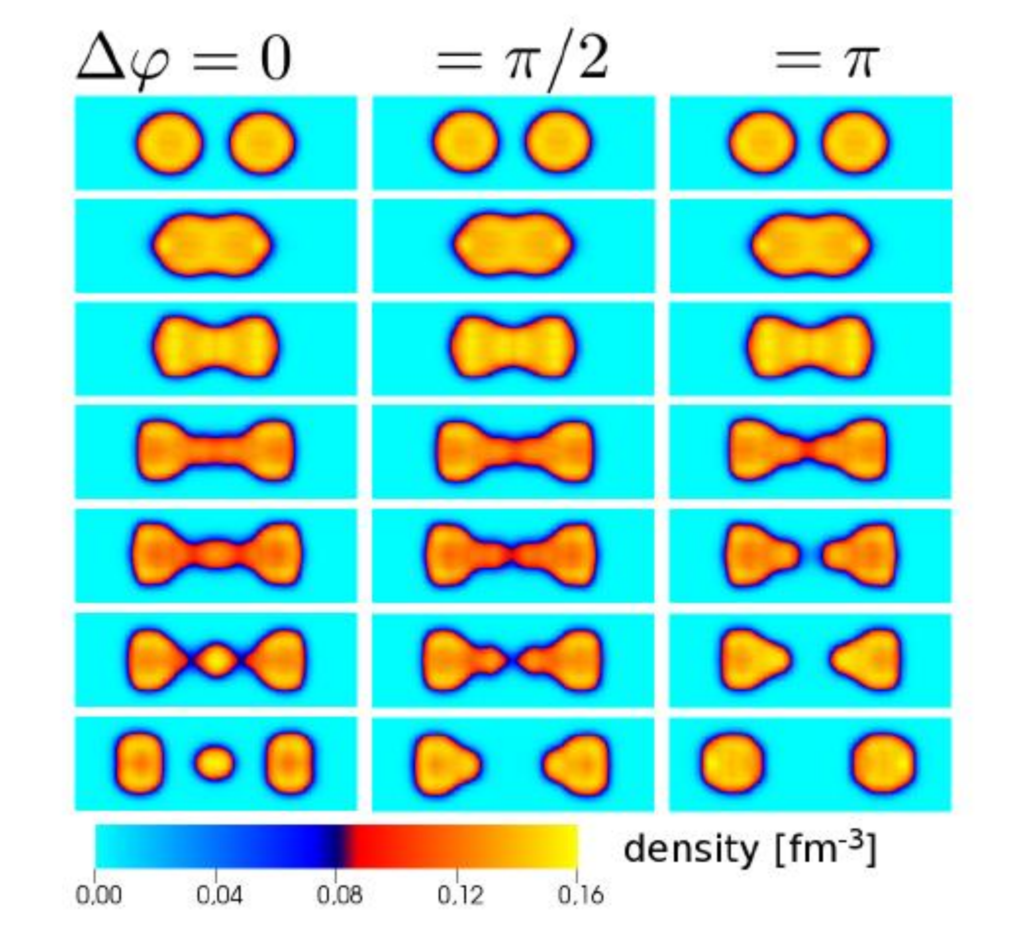
$^{90}Zr + {}^{90}$Zr at energy $E \simeq V_{Bass}$

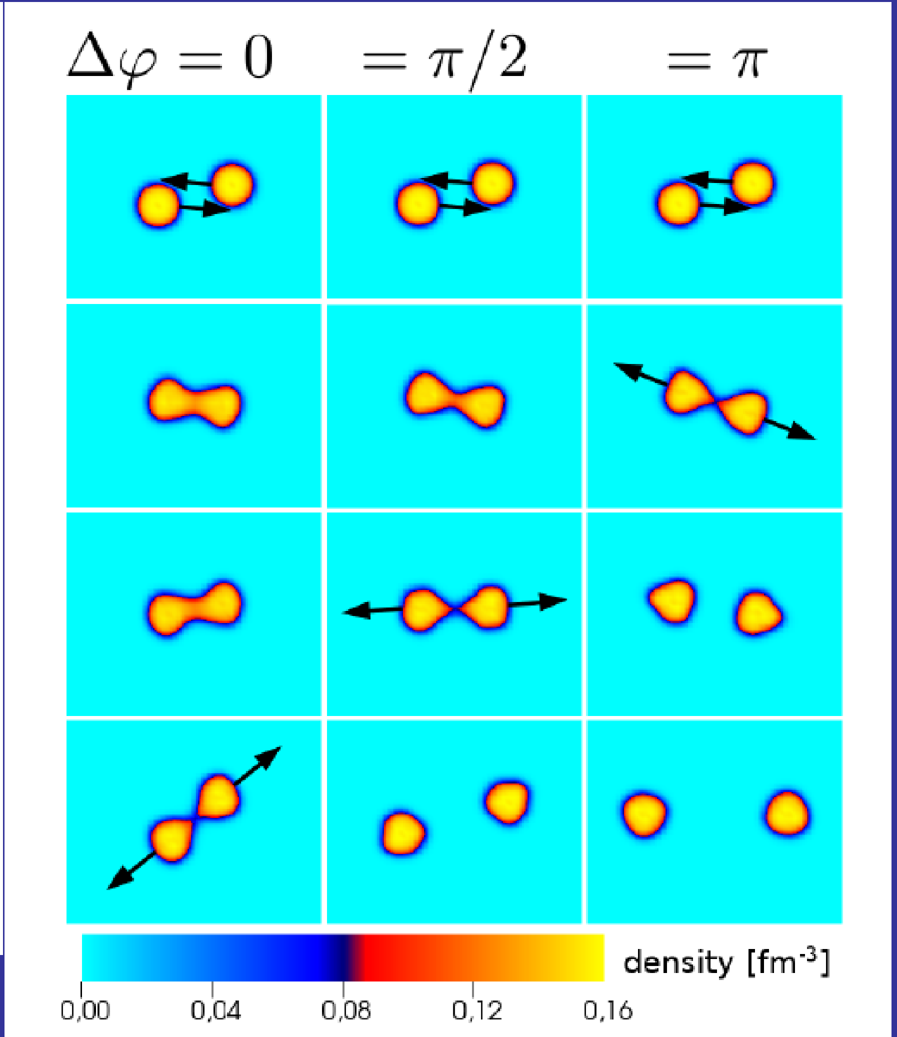


Modification of the capture cross section!

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)
See also for light nuclei: Y. Hashimoto, G. Scamps, Phys. Rev. C94, 014610 2016)

Noncentral collisions



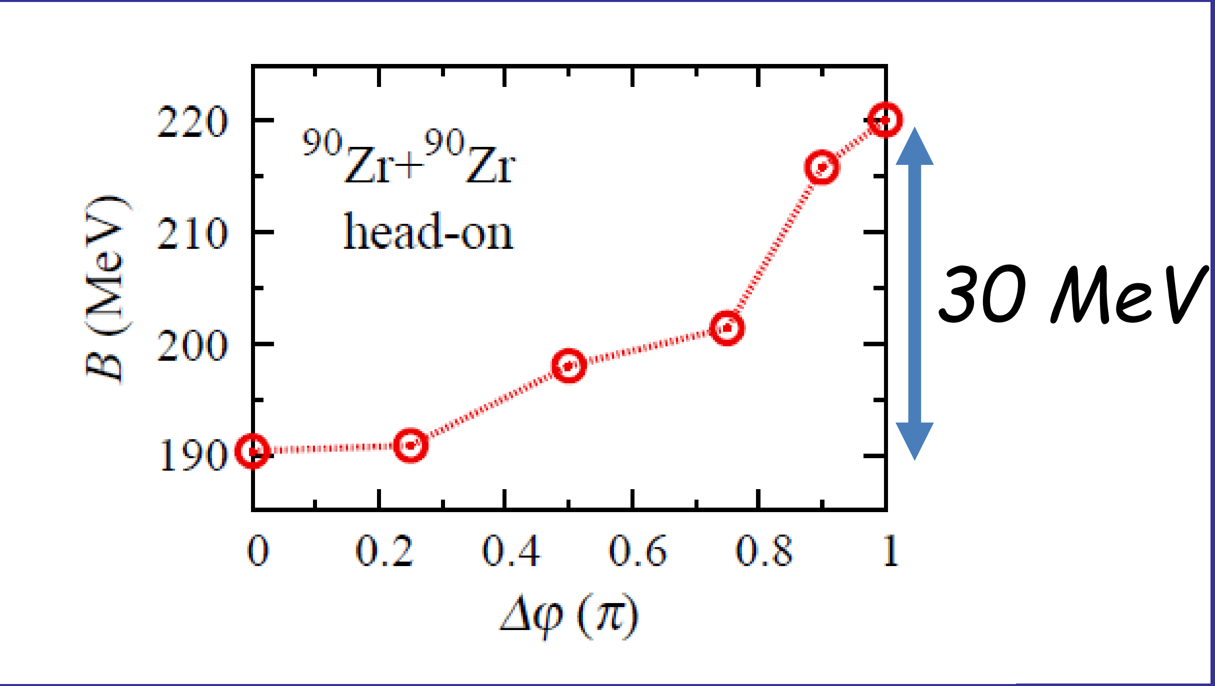


At higher energies (1.3-1.5 of the barrier height) the phase difference modifies the reaction outcomes suppressing the reaction channel leading to 3 fragments.

For noncentral collisions the trajectories of outgoing nuclei are affected due to the shorter contact time for larger phase differences.

P. Magierski, K. Sekizawa, G. Wlazłowski, arXiv:1611.10261

Effective barrier height for fusion as a function of the phase difference



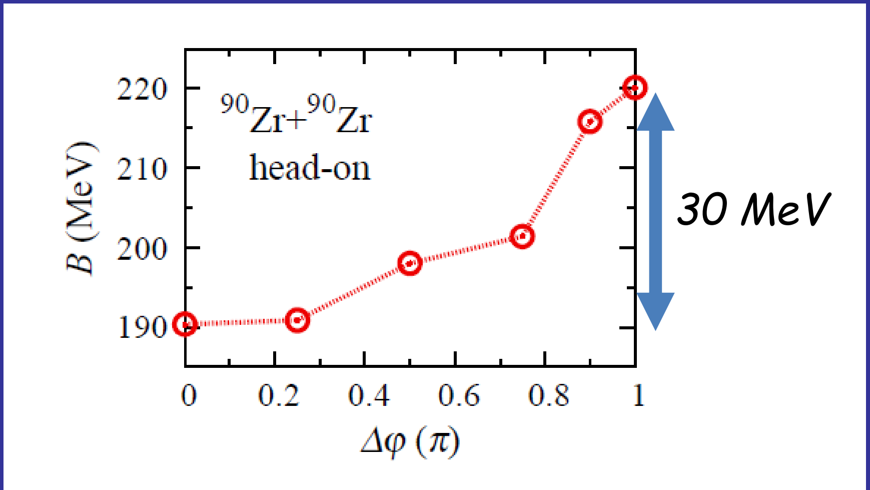
What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi}\int_0^{\pi}\left(B(\Delta\varphi) - V_{Bass}\right) d(\Delta\varphi) \approx 10 MeV$$

The phase difference of the pairing fields of colliding medium or heavy nuclei produces a similar <u>solitonic structure</u> as the system of two merging atomic clouds.
The energy stored in the created junction is subsequently released giving rise to an increased kinetic energy of the fragments. The effect is found to be of the order of <u>30MeV</u> for medium nuclei and occur for <u>energies up to 20-30% of the barrier height</u>.

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

Effective barrier height for fusion as a function of the phase difference



What is an average extra energy needed for the capture?

$$\boxed{E_{extra} = \frac{1}{\pi}\int_0^{\pi}\left(B(\Delta\varphi) - V_{Bass}\right)d(\Delta\varphi) \approx 10MeV}$$

The effect is found (within TDDFT) to be of the order of <u>30MeV</u> for medium nuclei and occur for <u>energies up to 20-30% of the barrier height</u>.

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

It raises an interesting question:
to what extent systems of hundreds of particles can be described using the concept of pairing field?

G. Scamps, Phys. Rev. C 97, 044611 (2018): barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT