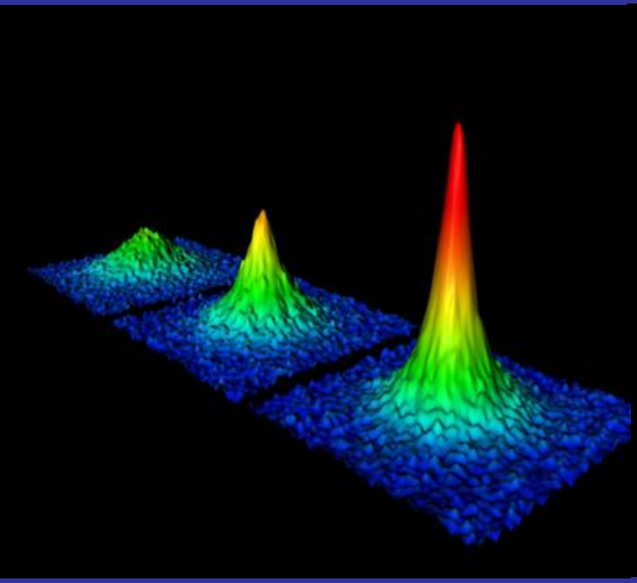
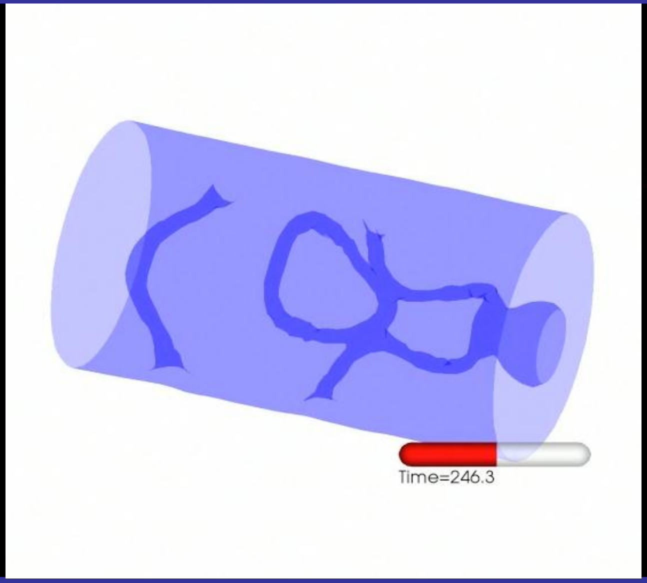
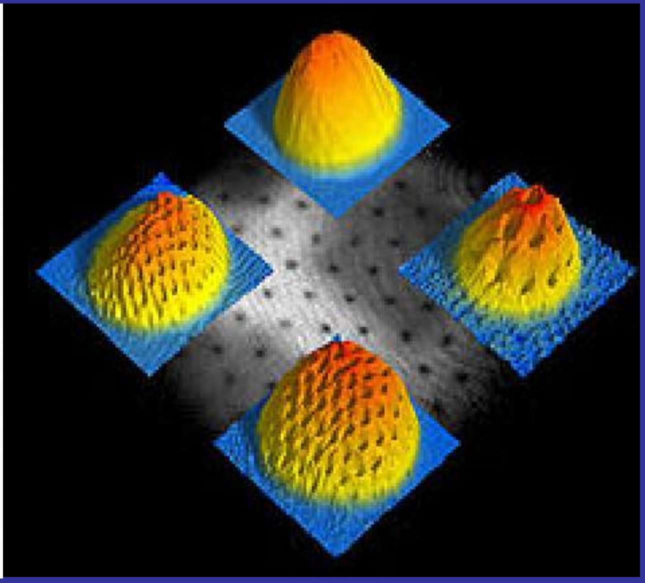
Pomiędzy nadprzewodnictwem a kondensacją Bosego-Einsteina

Piotr Magierski
(Wydział Fizyki Politechniki Warszawskiej)



100 years of superconductivity and superfluidity in Fermi systems

Discovery: H. Kamerlingh Onnes in 1911 cooled a metallic sample of mercury at T<4.2K

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

BCS – BEC crossover

Eagles (1969), Leggett (1980): Variational approach

$$|gs\rangle = \prod_k \left(\mathrm{u_k} + \mathrm{v_k} \hat{a}^\dagger_{k\uparrow} \hat{a}^\dagger_{-k\downarrow} \right) |vacuum\rangle \quad \text{BCS wave function}$$

BCS limit: $1/k_F a_s \to -\infty$

a_s - scattering length

$\mu \to \varepsilon_F$ chemical potential

$\Delta \to \frac{8}{\mathrm{e}^2} \varepsilon_F \exp\left(\frac{\pi}{2k_F a_s} \right)$ pairing gap

Usual BCS solution for small and negative scattering lengths, with exponentially small pairing gap describing the system of spatially overlapping Cooper pairs.

BEC limit: $1/k_F a_s \to +\infty$

$$\mu \to -\frac{\hbar^2}{2ma_s^2} = -\frac{E_b}{2}$$

$$\Delta \to \frac{4\varepsilon_F}{\sqrt{3\pi k_F a_s}}$$

Gas of weakly repelling molecules with binding energy E_b, essentially all at rest (almost pure BEC state)

No singularity within the whole range of scattering length!
Smooth crossover from spatially overlapping Cooper pairs to tightly bound difermionic molecules

Beyond mean field: Nozieres, Schmitt-Rink (1985), Randeria et al.(1993)

George Bertsch challenge (Many-Body X, Seattle, 1999):

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

Neutron-neutron scattering

Scattering length: $a \approx -18.5\ fm$

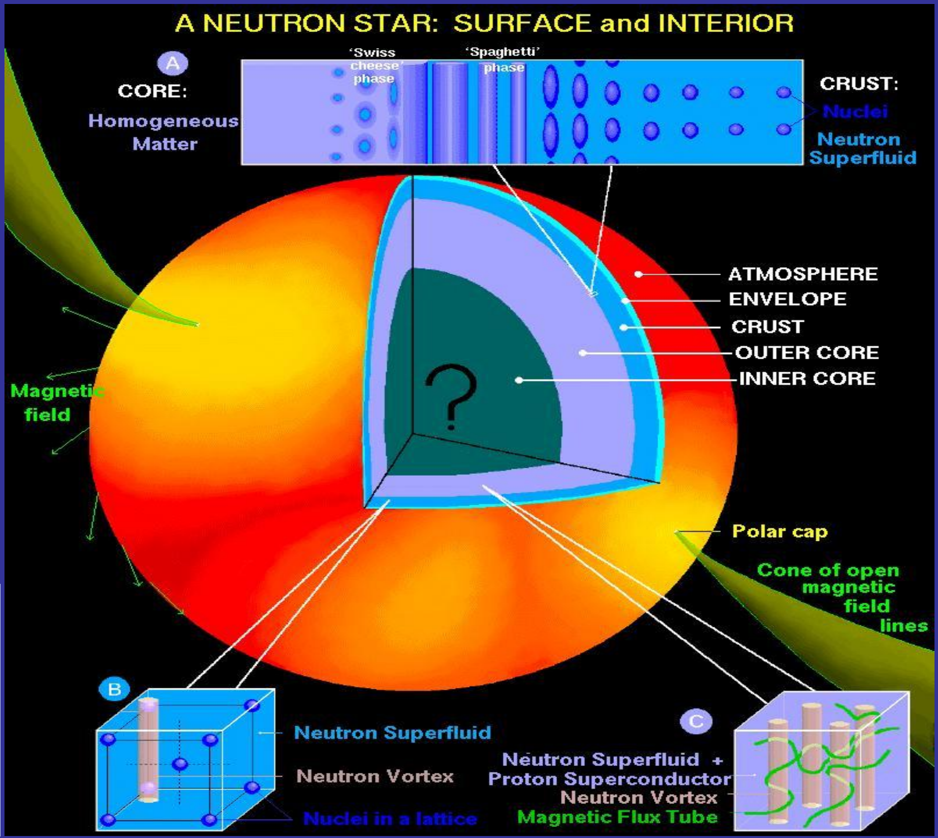
Effective range: $r_0 \approx 2.8\ fm$

$$n \approx 0.001\text{- } 0.01\ \text{fm}^{-3}$$

$$k_F \approx 0.3 - 0.7\ \text{fm}^{-1}$$

Inner crust of neutron stars

Important for understanding of thermal evolution of neutron stars and possibly a glitch phenomenon=a sudden speed up of rotation



What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$\boxed{n\, r_0^3 \ll 1}$ $\boxed{n\, |a|^3 \gg 1}$

n - particle density
a - scattering length
r_0 - effective range

$$\boxed{\text{i.e. } r_0 \to 0, a \to \pm\infty}$$

NONPERTURBATIVE REGIME

System is dilute but strongly interacting!

Universality: $E = \xi_0 E_{FG} \quad \text{for } T = 0$

$\xi_0 = 0.376(5)$ - **Exp. estimate**

E_{FG} - **Energy of noninteracting Fermi gas**

Expected phases of a two species dilute Fermi system
BCS-BEC crossover

T

Characteristic temperature:
T_c superfluid-normal phase transition

Strong interaction
UNITARY REGIME

Characteristic temperatures:
T_c superfluid-normal phase transition
T^* break up of Bose molecule
$T^* > T_c$

weak interaction

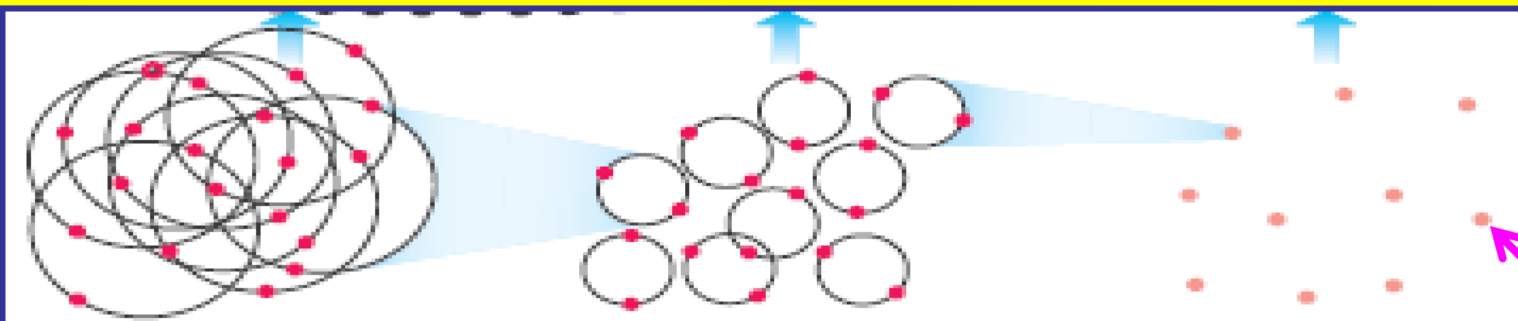
weak interactions

?

Molecular BEC and Atomic+Molecular Superfluids

BCS Superfluid

1/a



Bose molecule

a<0
no 2-body bound state

a>0
shallow 2-body bound state

Effective Hamiltonian of an atom-atom system

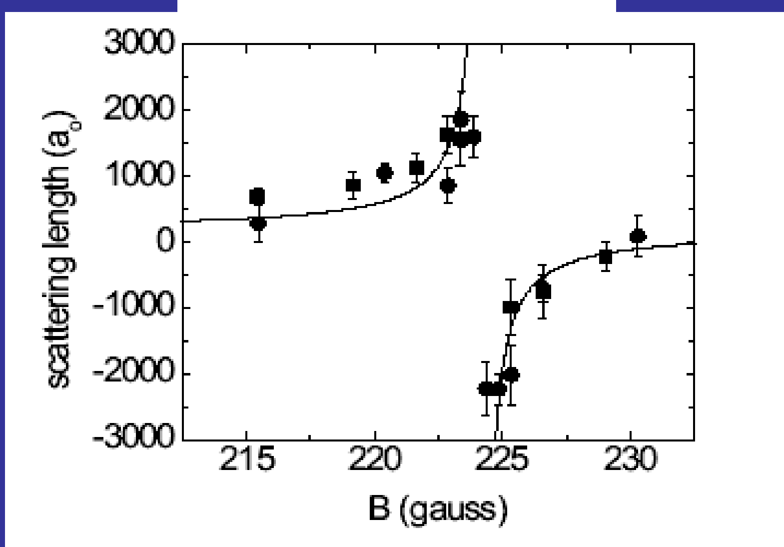
$$H = \frac{\vec{p}^{\,2}}{2\mu} + \sum_{i=1}^{2}(V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \ldots$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{I}\cdot\vec{J},\ V_i - \text{Coulomb term}$$

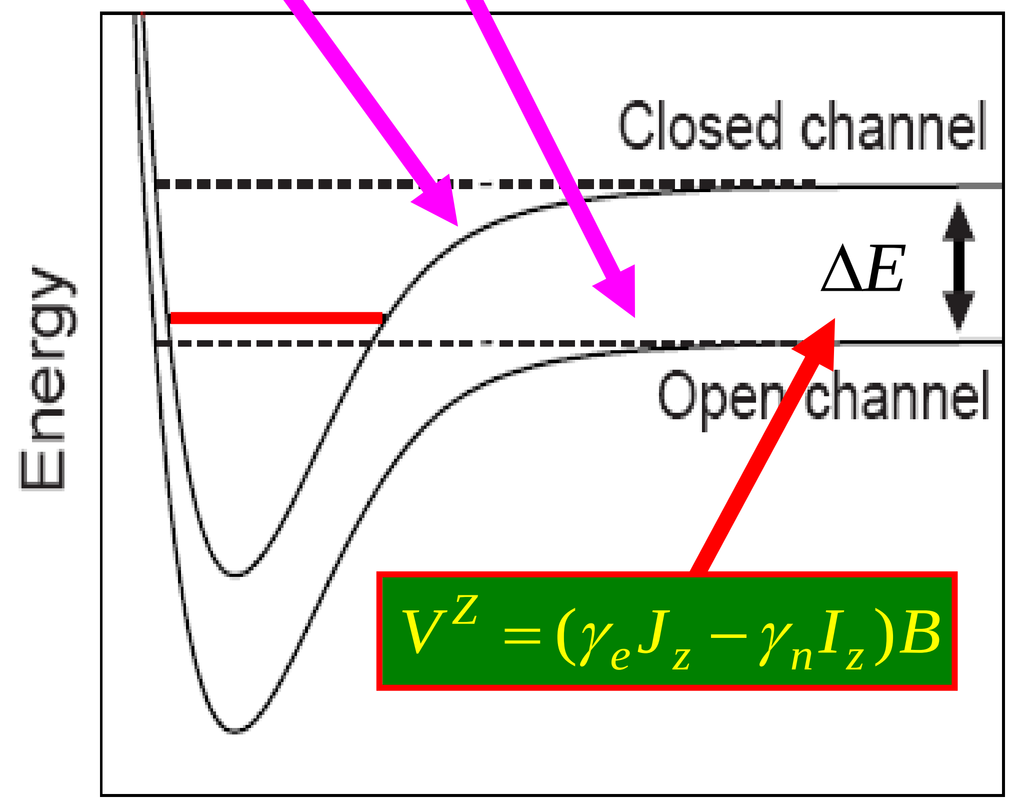
Tiesinga, Verhaar, Stoof, Phys. Rev. A47, 4114 (1993)

Channel coupling

Feshbach resonance



Regal and Jin, PRL 90, 230404 (2003)



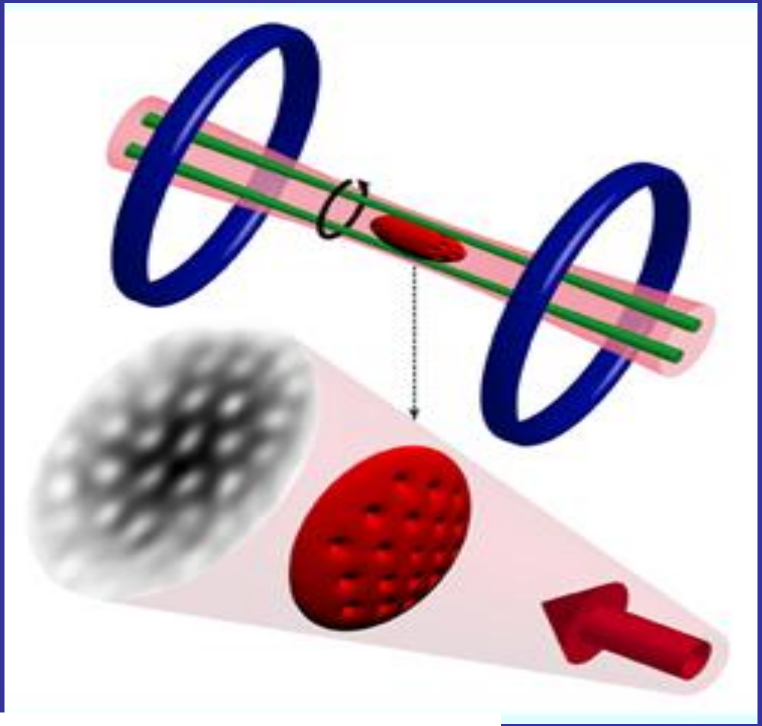
$$V^{Z} = (\gamma_e J_z - \gamma_n I_z)B$$

Interatomic distance

Short (selective) history:

✓In 1999 DeMarco and Jin created a degenerate atomic Fermi gas.

✓In 2005 Zwierlein/Ketterle group observed quantum vortices which survived when passing from BEC to unitarity – evidence for superfluidity!

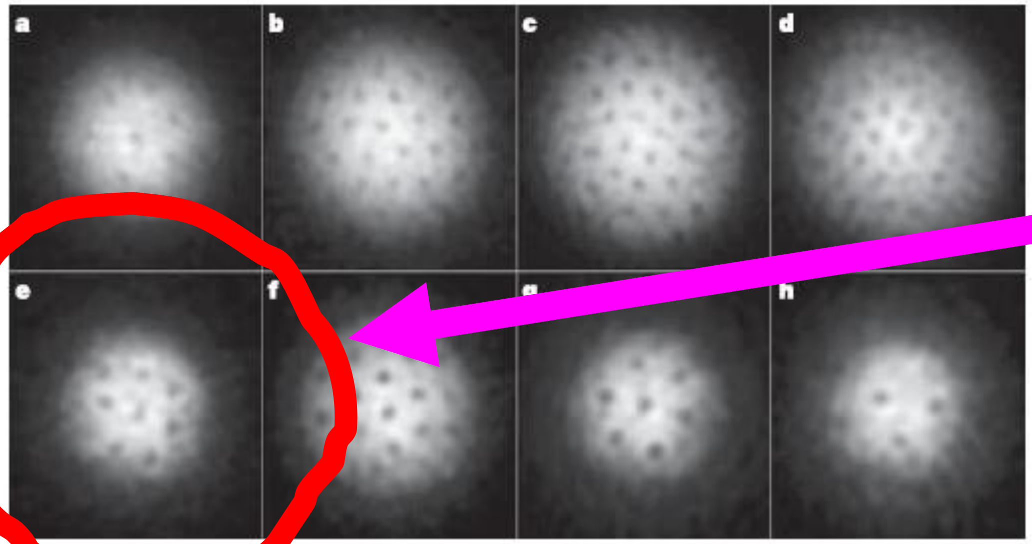


system of fermionic ^{6}Li atoms

Feshbach resonance: B=834G

BEC side: a>0

BCS side: a<0



UNITARY REGIME

Figure 2 | Vortices in a strongl[illegible] fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (s[illegible] magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**[illegible] 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of vie[illegible] 880 µm × 880 µm.

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Thermodynamics of the unitary Fermi gas

ENERGY: $$E(x) = \frac{3}{5}\xi(x)\varepsilon_F N; \quad x = \frac{T}{\varepsilon_F}$$

ENTROPY/PARTICLE: $$\sigma(x) = \frac{S(x)}{N} = \frac{3}{5}\int_0^x \frac{\xi'(y)}{y}dy$$

FREE ENERGY: $$F = E - TS = \frac{3}{5}\varphi(x)\varepsilon_F N$$

$$\varphi(x) = \xi(x) - x\sigma(x)$$

PRESSURE: $$P = -\frac{\partial E}{\partial V} = \frac{2}{5}\xi(x)\varepsilon_F \frac{N}{V}$$

$$PV = \frac{2}{3}E$$

Note the similarity to the ideal Fermi gas

Universal Tan relations

$$\lim_{k\to\infty} n(k) = \frac{C}{k^4}, \qquad 1/a >> k >> 1/r_0, \quad C \text{ - contact}$$

- Contact measures the probability that two fermions of opposite spins are close together.
- Extensive property of the system.

$$E = \sum_{\sigma=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m}\left(n_\sigma(k) - \frac{C}{k^4}\right) + \frac{\hbar^2}{4\pi m a_s} C$$

Total energy of the system

$$\left(\frac{dE}{d(1/a_s)}\right)_S = -\frac{\hbar^2}{4\pi m} C$$

Adiabatic relation

C and 1/a are conjugate thermodynamic variables
1/a – „generalized force”
C – „generalize displacement” – capture physics at short length scales.

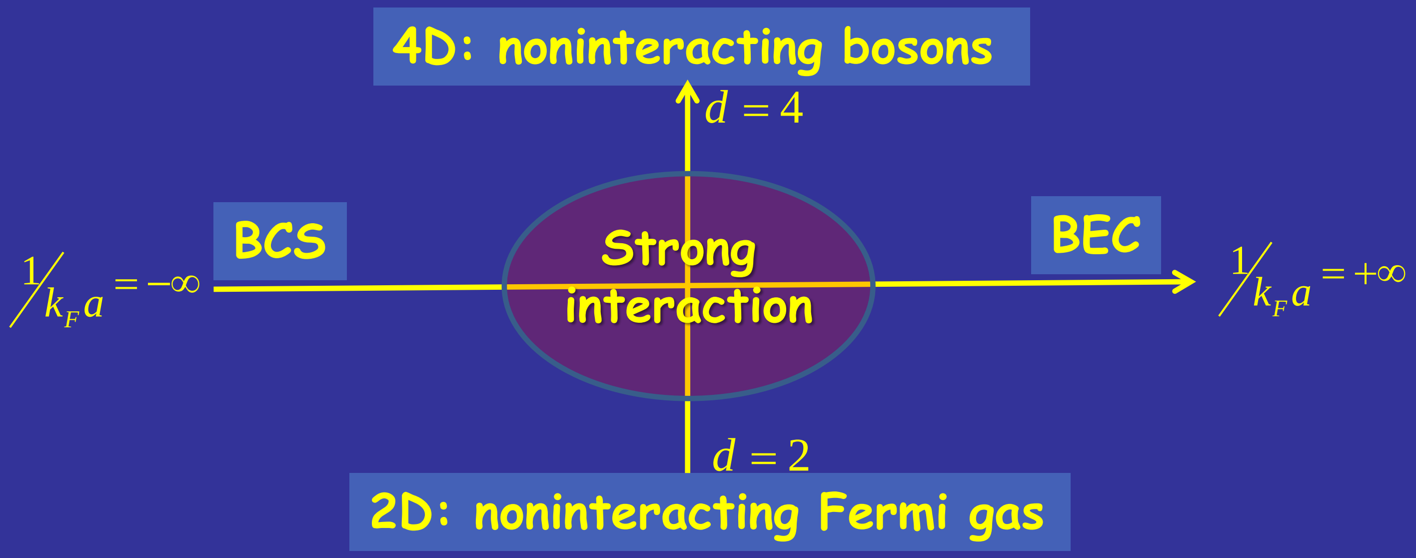
Shina Tan, Ann.Phys.323,2971(2008), Ann.Phys.323,2952(2008)
Other theory papers: Tan, Leggett, Braaten, Combescot, Baym, Blume, Werner, Castin, Randeria,Strinati,…

Unitary limit in 2 and 4 dimensions:

$$a \to \infty: \; R(r) \propto \frac{1}{r^{d-2}} + O(r^{4-d}), \quad \textit{Two body wave function for } r \to 0.$$

Intuitive arguments:

- For d=4 $\int R(r)^2 d^d r$ diverges at the origin
- For d=2 the singularity of the wave function disapears = interaction also disapears.



The only nontrivial case of unitary regime is in 3D

Nussinov,Nussinov, Phys.Rev. A74, 053622(2006)

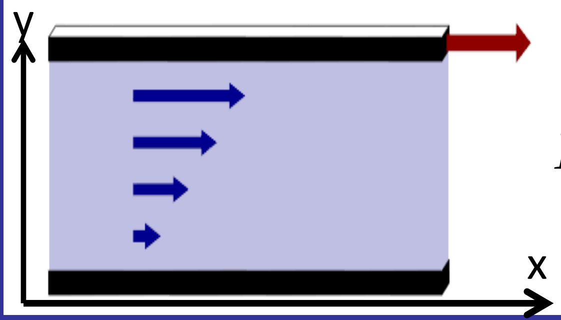
Hydrodynamics at unitarity

Scaling: $\psi_i(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \rightarrow \frac{1}{\lambda^{3N/2}} \psi_i\left({}^{\vec{r}_1}\!/_{\lambda}, {}^{\vec{r}_2}\!/_{\lambda}, \ldots, {}^{\vec{r}_N}\!/_{\lambda} \right); E_i \rightarrow {}^{E_i}\!/_{\lambda^2}$

No intrinsic length scale $\Rightarrow$ **Uniform expansion keeps the unitary gas in equilibrium**

Consequence:
uniform expansion does not produce entropy = bulk viscosity is zero!

Shear viscosity:



$$F = A\eta \frac{\partial v_x}{\partial y}$$

For any physical fluid:

$$\frac{\eta}{S} \geq \frac{\hbar}{4\pi k_B}$$

KSS conjecture
Kovtun,Son,Starinets, (2005) from AdS/CFT correspondence

Maxwell classical estimate: $\eta \sim$ mean free path

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: unitary Fermi gas, QGP

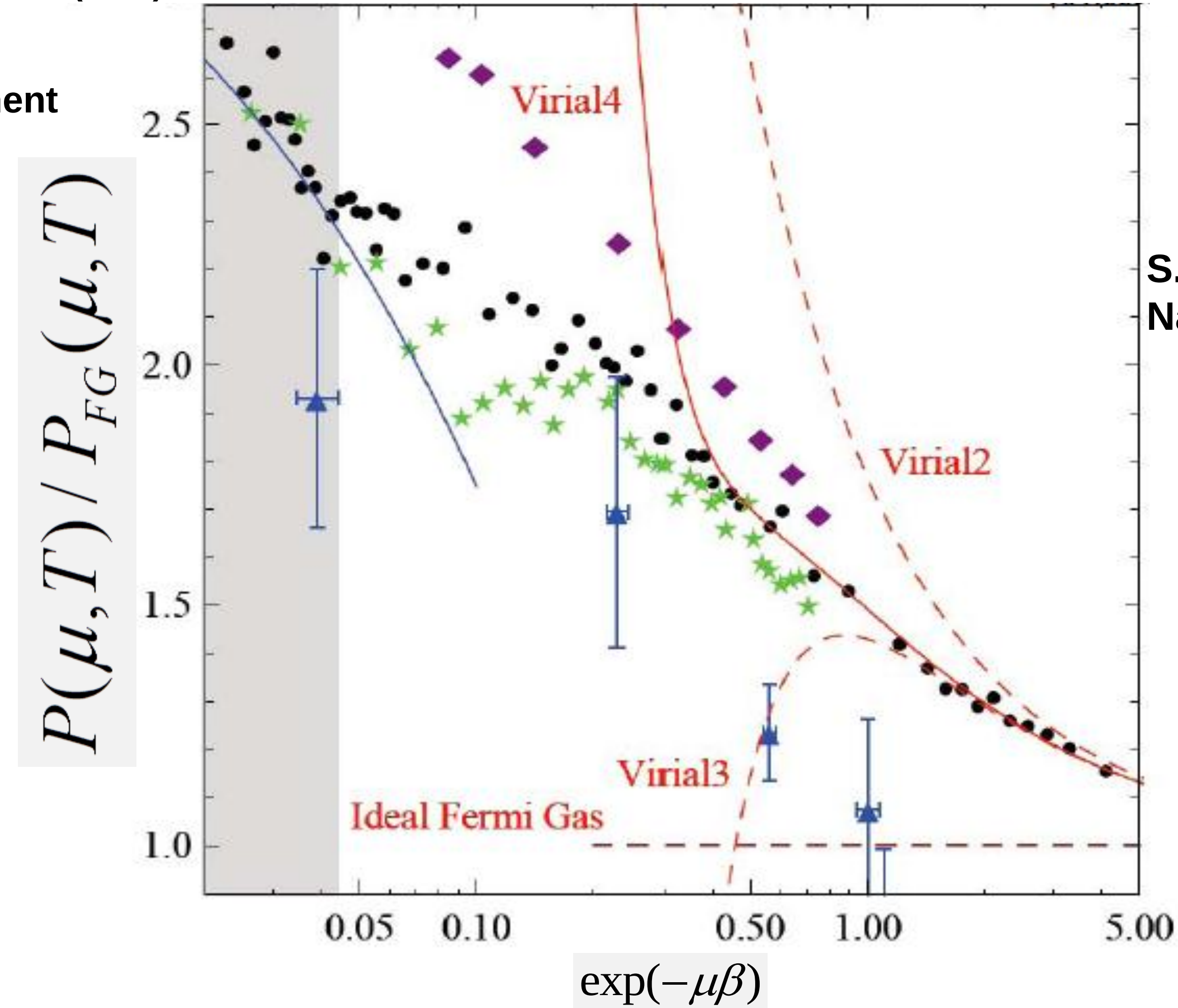
Comparison with Many-Body Theories (1)

Diagram. MC
Burovski et al.
PRL96, 160402(2006)

QMC
Bulgac, Drut, Magierski,
PRL99, 120401(2006)

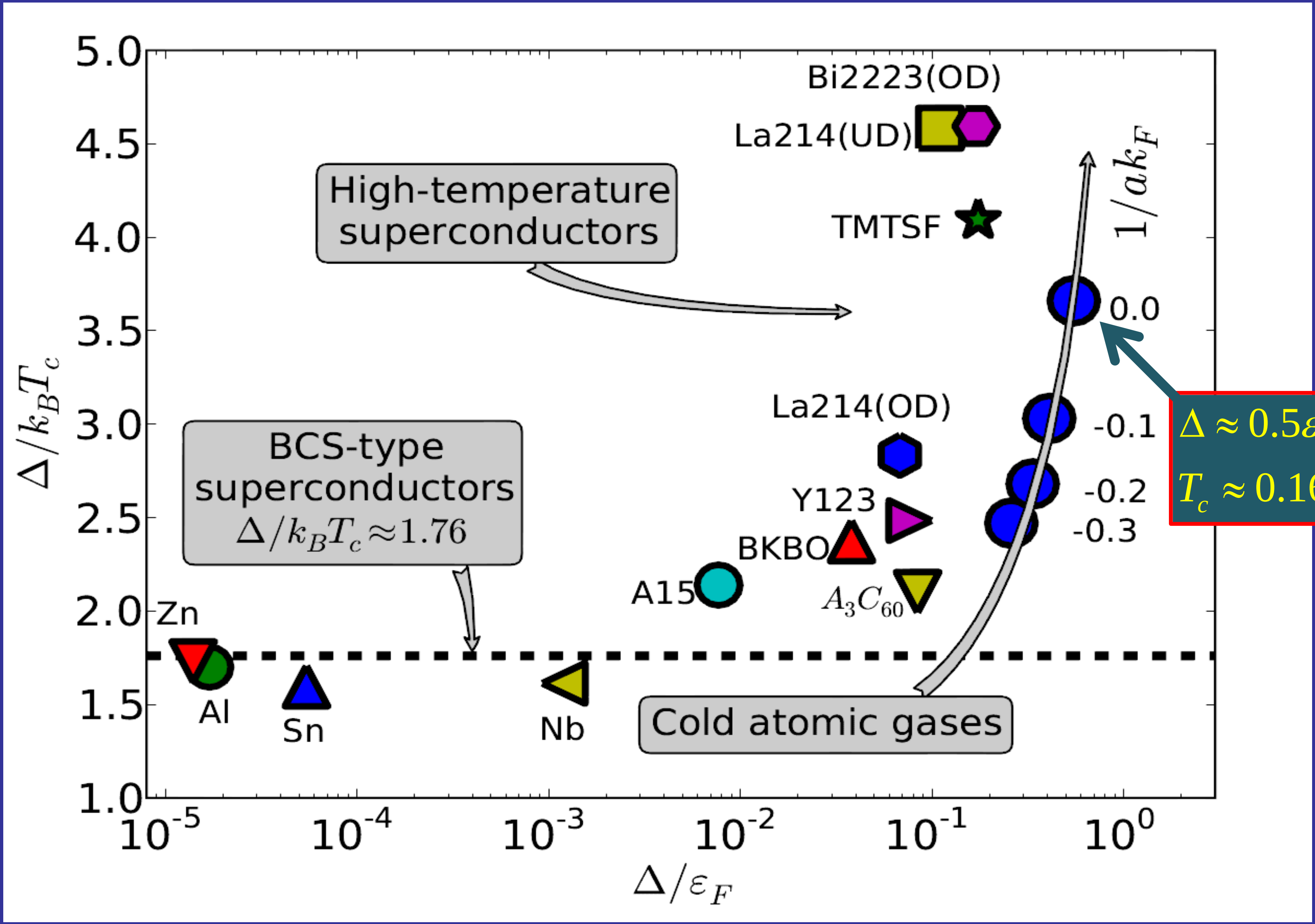
Diagram. + analytic
Haussmann et al.
PRA75, 023610(2007)

Experiment

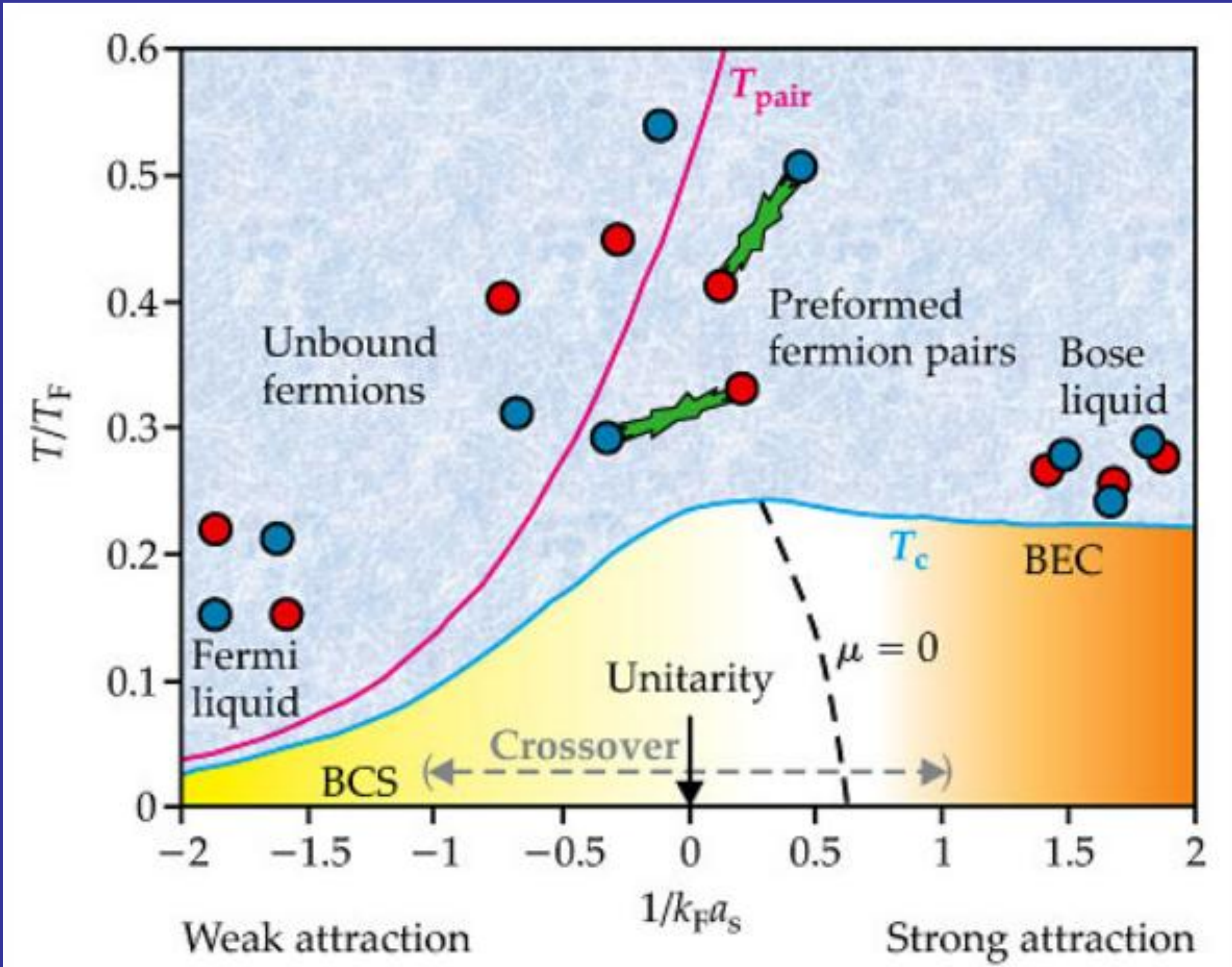


S. Nascimbene et al.
Nature 463, 1057 (2010)

Cold atomic gases and high Tc superconductors



From Magierski, Wlazłowski, Bulgac, Phys. Rev. Lett. 107, 145304 (2011)



From Sa de Melo, Physics Today (2008)

Pseudogap

Suppression of low-energy spectral weight function in the normal state ($T > T_c$)
= single particle density of states at the Fermi level is lowered

Origin of terminology:
Ding et al., „Spectroscopic evidence for a pseudogap in the normal state of underdoped high- T_c superconductors, Nature 382, 51 (1996)

Pairing pseudogap

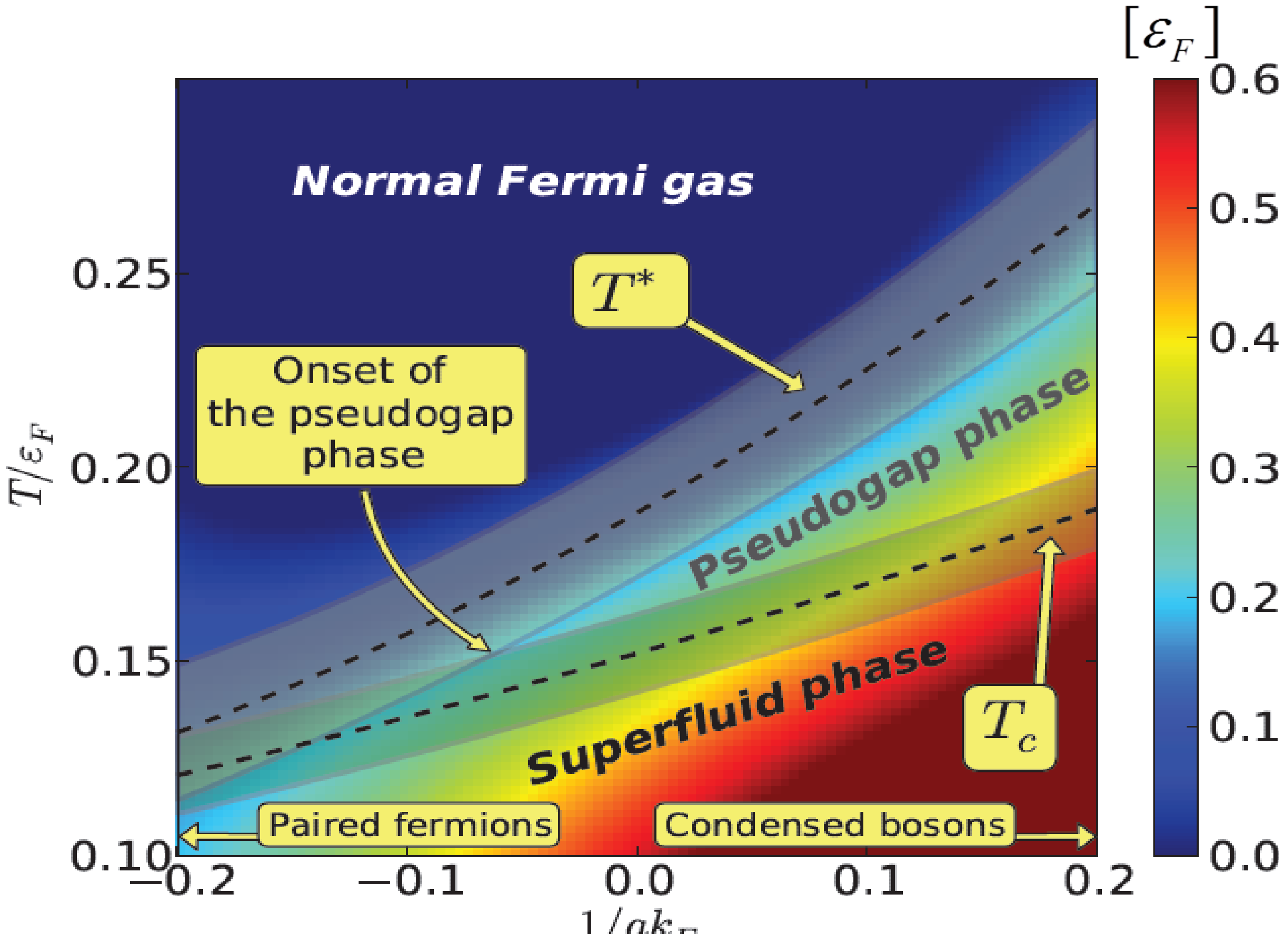
Suppression of low-energy spectral weight function due to incoherent pairing in the normal state ($T > T_c$)

Two important issues related to pairing pseudogap:

- What drives the transition to the superconducting state, amplitude (BCS theory) or phase fluctuations?

- Are there sharp gapless quasiparticles in a normal Fermi liquid (**YES**: Landau's Fermi liquid theory;
 NO: breakdown of Fermi liquid paradigm)

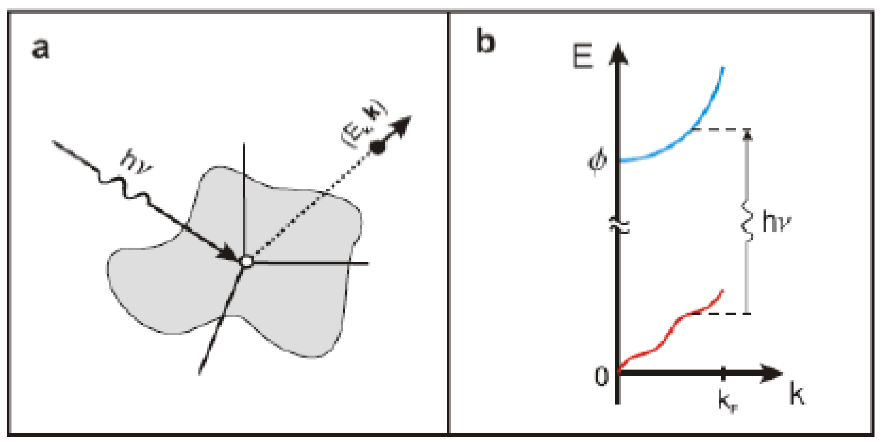
Gap in the single particle fermionic spectrum - theory



Ab initio result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.

Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.*107,145304(2011)
Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.*103,210403(2009)

RF spectroscopy in ultracold atomic gases

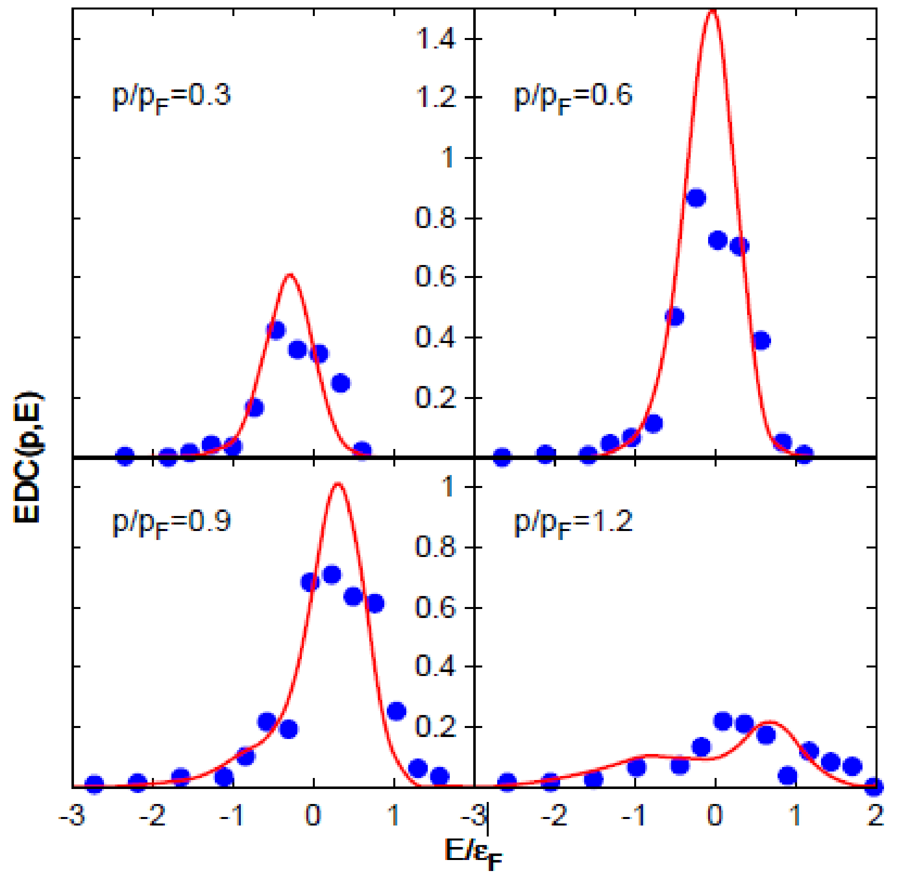


$$-E_s + h\nu = \frac{\hbar^2 k^2}{2m} + \phi$$

$$E(N) = E(N-1) + E_s$$

Stewart, Gaebler, Jin, Using photoemission spectroscopy to probe a strongly interacting Fermi gas, Nature, 454, 744 (2008)

$$\mathrm{EDC}(p,E,T) = \mathcal{C}p^2 \int_0^\infty dr\, r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E-\mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E-\mu(r)),$$



<u>Experiment (blue dots):</u> D. Jin's group
Gaebler et al. *Nature Physics 6, 569(2010)*
<u>Theory (red line):</u>
Magierski, Wlazłowski, Bulgac,
Phys.Rev.Lett.107,145304(2011)

Experimental status of pseudogap measurements around unitarity

Spectroscopy **YES**

(D. Jin's group, JILA)

Thermodynamics **NO**

(C.Salomon's group, Paris
M. Zwierlein's group, MIT)

Pseudogap at unitarity – theoretical predictions

Path Integral Monte Carlo - YES
Dynamic Mean Field - YES
Selfconsistent T-matrix - NO
Nonselfconsistent T-matrix - YES

Formalism for Time Dependent Phenomena

The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered.

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

http://www.tddft.org

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \underline{\vec{j}(\vec{r},t)}) + V_{ext}(\vec{r},t)n(\vec{r},t) + \ldots \right]$$

$$\begin{cases} [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{u}_i(\vec{r},t) + [\Delta(\vec{r},t) + \Delta_{ext}(\vec{r},t)]\mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{u}_i(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t) + \Delta^*_{ext}(\vec{r},t)]\mathrm{u}_i(\vec{r},t) - [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu]\mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{v}_i(\vec{r},t)}{\partial t} \end{cases}$$

In each case we solved on JaguarPf or Franklin the TDSLDA equations for a 32^3 , 48^3 and 32^2 *96 spatial lattices (approximately for 30k to 40k quasiparticle wavefunctions) for about 10k to 100k time steps using from about 30k to 40k PEs

Fully symmetry unrestricted calculations!

Road to quantum turbulence

Classical turbulence: *energy is transfered from large scales to small scales where it eventually dissipates.*

Kolmogorov spectrum: $E(k)=C\,\varepsilon^{2/3}\,k^{-5/3}$

E – kinetic energy per unit mass associated with the scale 1/k
ε - energy rate (per unit mass) transfered to the system at large scales.
k - wave number (from Fourier transformation of the velocity field).
C – dimensionless constant.

Superfluid turbulence (quantum turbulence): disordered set of quantized vortices. The friction between the superfluid and normal part of the fluid serves as a source of energy dissipation.

Problem: how the energy is dissipated in the superfluid system at small scales at T=0? - „pure" quantum turbulence

Possibility: vortex reconnections → Kelvin waves → phonon radiation

Vortex reconnections

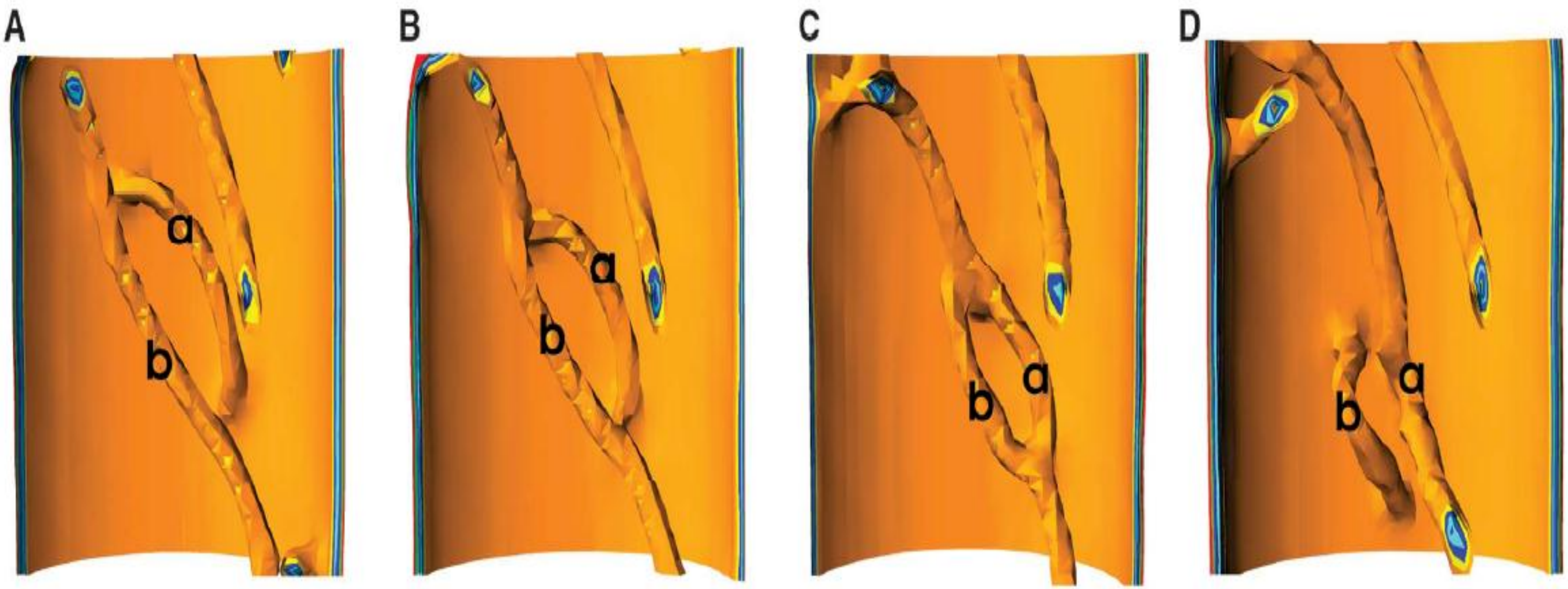


Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

We created a set of accurate and efficient tools for studies of large, superfluid Fermi systems.

They have been successfully implemented on leadership class computers (Franklin, JaguarPF)

- **Currently capable of treating large volumes with up to 40,000-50,000 fermions , and for long times, fully self-consistently and with no symmetry restrictions under the action of complex spatio-temporal external probes.**

- **There is a clear path towards exascale applications and implementation of Stochastic TD(A)SLDA**

APPLICATIONS: - Dynamics of unitary Fermi gas
- Dynamics of atomic nuclei:
  neutron capture, induced fission, fussion,
  low energy nuclear reaction,
  dynamics of vortices in neutron star crust, etc.

Real Time Dynamics of Quantum Gases and Nuclear Systems.

Collaborators:

A. Bulgac - Seattle
Y-L. (Alan) Luo - Seattle
K.J. Roche – PNNL
I. Stetcu - LANL
Y. Yu - Wuhan

Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart[A)]. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

Robert B. Laughlin, Nobel Lecture, December 8, 1998