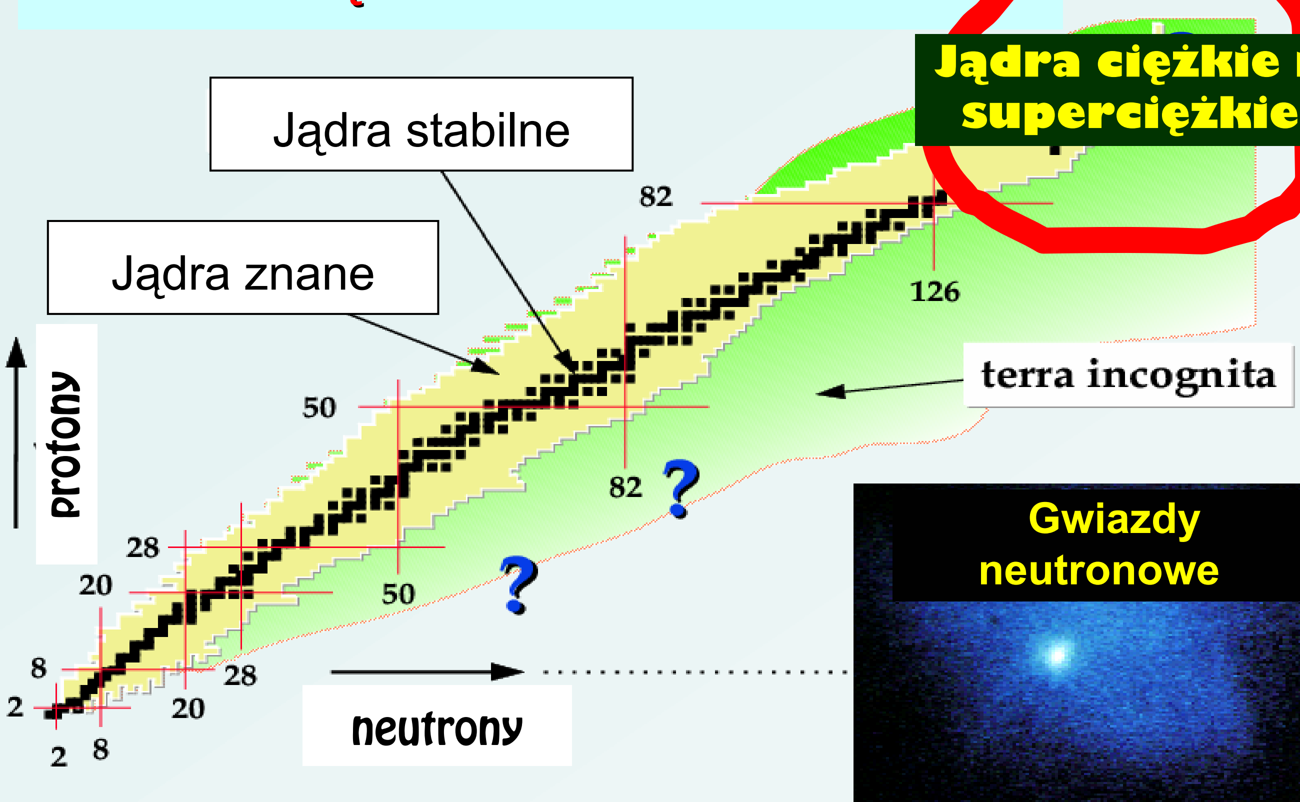
ur. 1933r. – Przeryty Bór k. Tarnowa.
Studia: Akademia Górniczo-Hutnicza w Krakowie,
Wydział Fizyki Uniwersytetu
Moskiewskiego im .Łomonosowa.
Doktorat: Wydział Fizyki Uniwersytetu Warszawskiego (1969)
Adiunkt: Wydział Fizyki Uniwersytetu Warszawskiego (1969-1973)
Od 1973: Instytut Fizyki Politechniki Warszawskiej
1998: tytuł Profesora Fizyki
Wieloletni z-ca Dyrektora Instytutu Fizyki PW i
Kierownik Zakładu Fizyki Jądrowej

Profesor Stefan Ćwiok
1933-2003

Badania naukowe: **struktura jądra atomowego**

JĄDRA ATOMOWE



ur. 1933r. – Przeryty Bór k. Tarnowa.
Studia: Akademia Górniczo-Hutnicza w Krakowie,
Wydział Fizyki Uniwersytetu
Moskiewskiego im .Łomonosowa.
Doktorat: Wydział Fizyki Uniwersytetu Warszawskiego (1969)
Adiunkt: Wydział Fizyki Uniwersytetu Warszawskiego (1969-1973)
Od 1973: Instytut Fizyki Politechniki Warszawskiej
1998: tytuł Profesora Fizyki
Wieloletni z-ca Dyrektora Instytutu Fizyki PW i
Kierownik Zakładu Fizyki Jądrowej

Badania naukowe: **struktura jądra atomowego**

Profesor Stefan Ćwiok
1933-2003

Osiągnięcia:

-hipoteza stabilności bardzo ciężkich jąder o N=162 – potwierdzona przez doświadczenia w GSI Darmstadt i Berkeley (USA).

-wyjaśnienie zagadki istnienia dwóch kanałów rozszczepienia w izotopach fermu.

-hipoteza istnienia (metastabilnych) stanów hiperzdeformowanych w aktynowcach - potwierdzona eksperymentalnie.

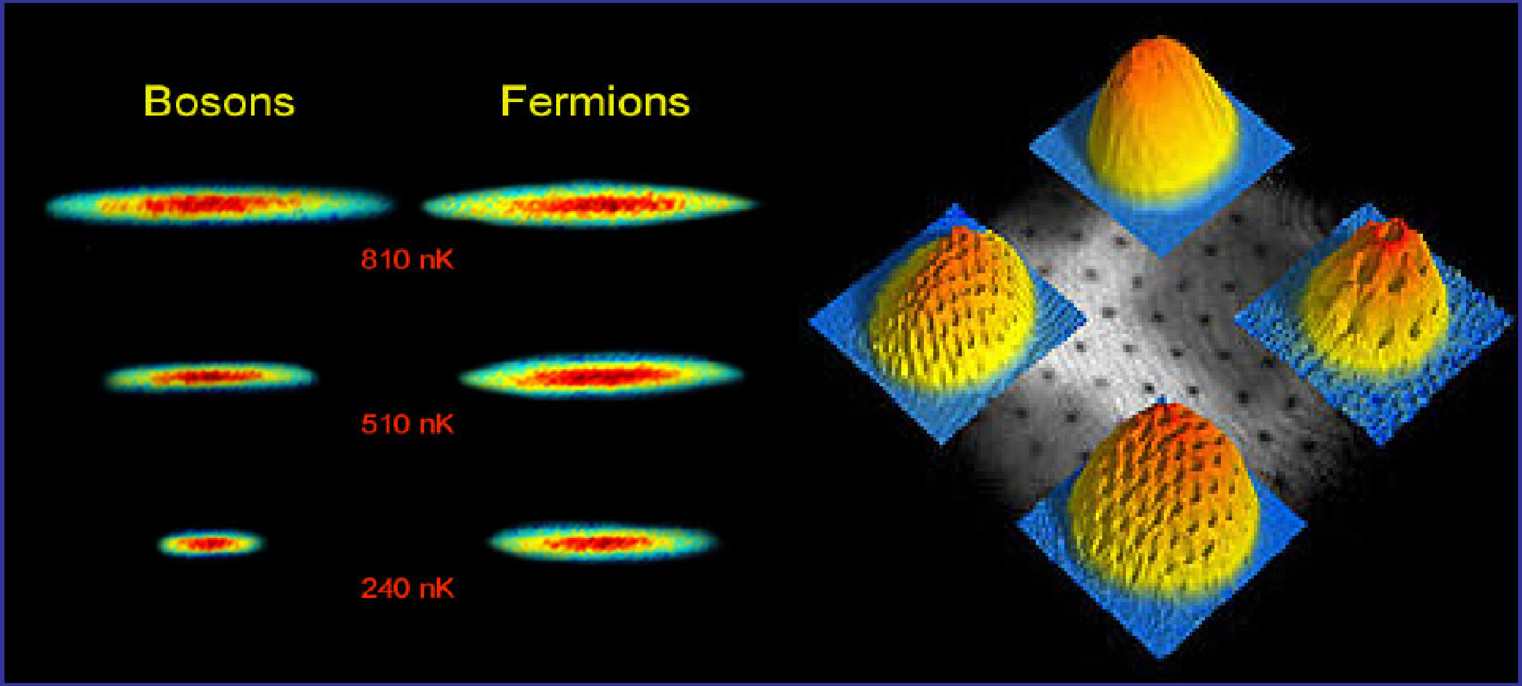
-systematyczne obliczenia dla nieparzystych jąder superciężkich (do dziś interpretacja wielu eksperymentów bazuje na tych wynikach).

-hipoteza istnienia liczby magicznej Z=126, a nie Z=114 jak dotychczas sądzono.

-hipoteza współistnienia kształtów w jądrach superciężkich (Nature, 433(2005)705)

www.if.pw.edu.pl/~zak7www/stefan/index_stefan.htm

O pewnych własnościach kwantowych gazów atomowych



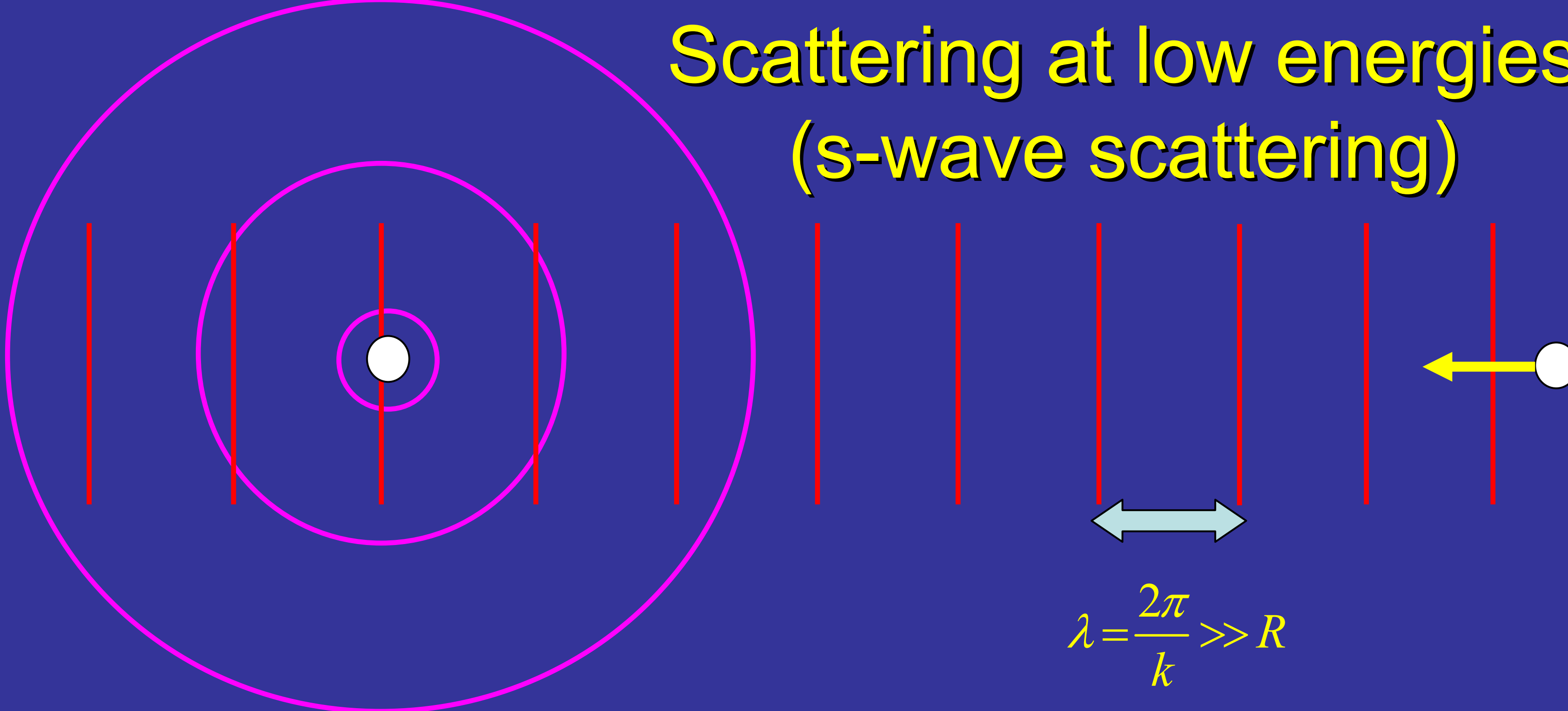
Piotr Magierski (WF PW)

Współpracownicy: **Aurel Bulgac, Joaquin E. Drut**
(University of Washington, Seattle)

Outline

- Particle scattering at low energies.
- BCS-BEC crossover. What is the unitary regime?
- How one can manipulate the two-body interaction in experiments with atomic gases?
- Theoretical approach: path integral description of strongly interacting Fermi gases.
- Equation of state for the Fermi gas in the unitary regime. Critical temperature.
- Conclusions.

Scattering at low energies (s-wave scattering)



$$\lambda = \frac{2\pi}{k} >> R$$

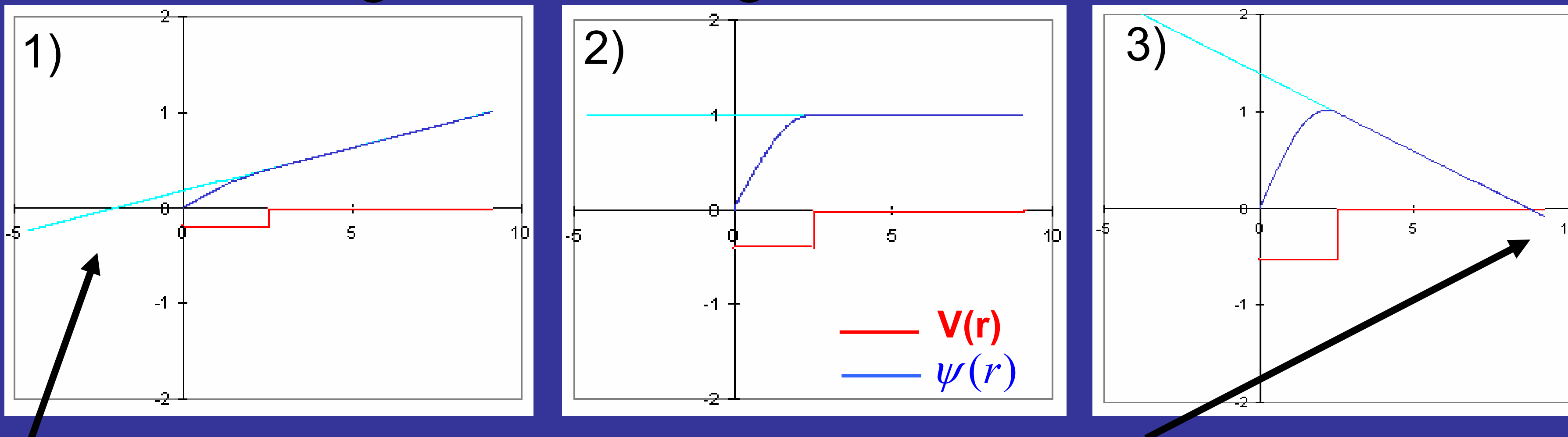
R - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f\frac{e^{ikr}}{r}; \quad f \text{ - scattering amplitude}$$

$$f = \frac{1}{-ik - {}^{1}\!/_{a} + \frac{1}{2} r_0 k^2},$$ a - scattering length, r_0 - effective range

If $k \to 0$ then the interaction is determined by the scattering length alone.

Scattering at low energies: attractive interaction



$a < 0$ there is no bound state $\qquad a = \pm\infty \qquad a > 0$ a bound state exists

What is the energy of the dilute Fermi gas? $\quad E(k_F a) = ?$

↑

$(k_F r_0 << 1)$ $\quad \varepsilon_F = \dfrac{\hbar^2 k_F^{\ 2}}{2m}; \quad n = \dfrac{k_F^{\ 3}}{3\pi^2}$ **- particle density**

Perturbation series:

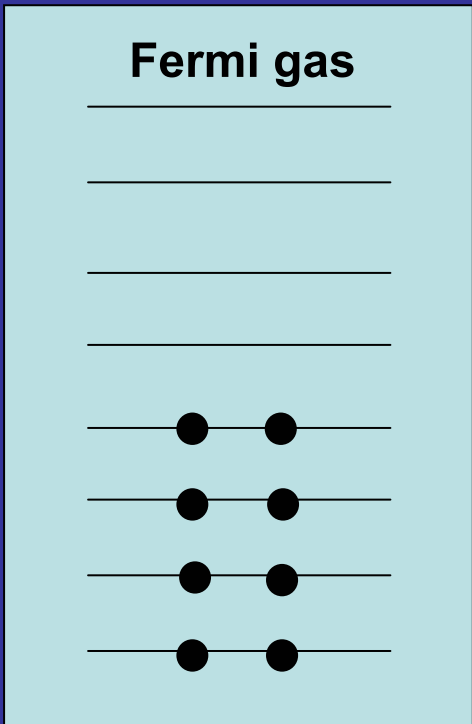
$$E/E_{FG} = 1 + \frac{10}{9\pi}(k_F a)\left[1 + \frac{6}{35\pi}(k_F a)(11 - 2\ln 2) + \ldots\right]$$

$$E_{FG} = \frac{3}{5}\varepsilon_F N$$ - Energy of the noninteracting Fermi gas

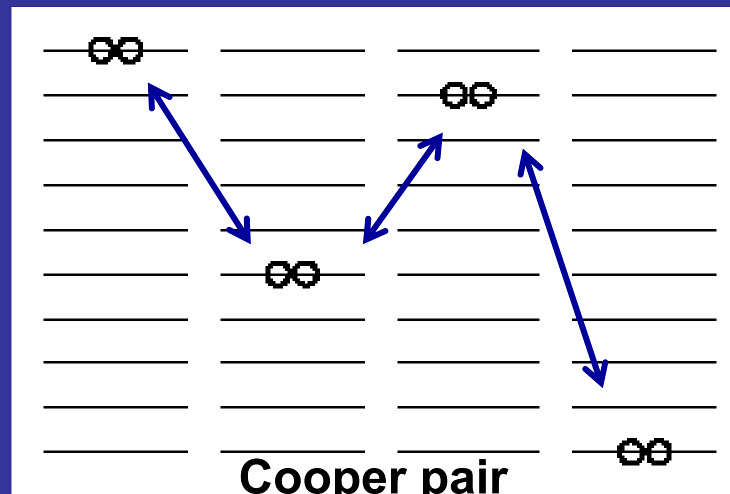
PAIRING NOT INCLUDED YET!

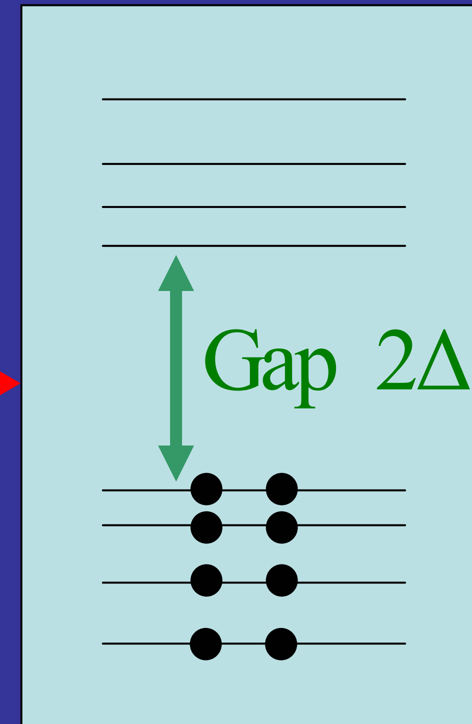
How pairing emerges?

$a < 0$



Cooper's argument (1956)





$$\Delta_{BCS} = \frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2k_F a}\right),$$

iff $k_F |a| \ll 1$ and $\frac{1}{k_F} \ll \eta = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta}$ - size of the Cooper pair

$$\frac{E_{HF+BCS}}{E_{FG}} = 1+\frac{10}{9\pi}(k_F a)+\ldots-\frac{5}{8}\left(\frac{\Delta_{BCS}}{\varepsilon_F}\right)^2 = 1+\frac{10}{9\pi}(k_F a)+\ldots-\frac{40}{e^4}\exp\left(\frac{\pi}{k_F a}\right)$$

Hartree-Fock term

BCS term

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ Dilute atomic Fermi gases $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ Liquid ^3He $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ Nuclei, neutron stars $T_c \approx 10^{5} - 10^{6}$ eV
- QCD color superconductivity $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ $\qquad$ $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$i.e.\ \ r_0 \to 0,\ a \to \pm\infty$$

NONPERTURBATIVE REGIME

System is dilute but strongly interacting!

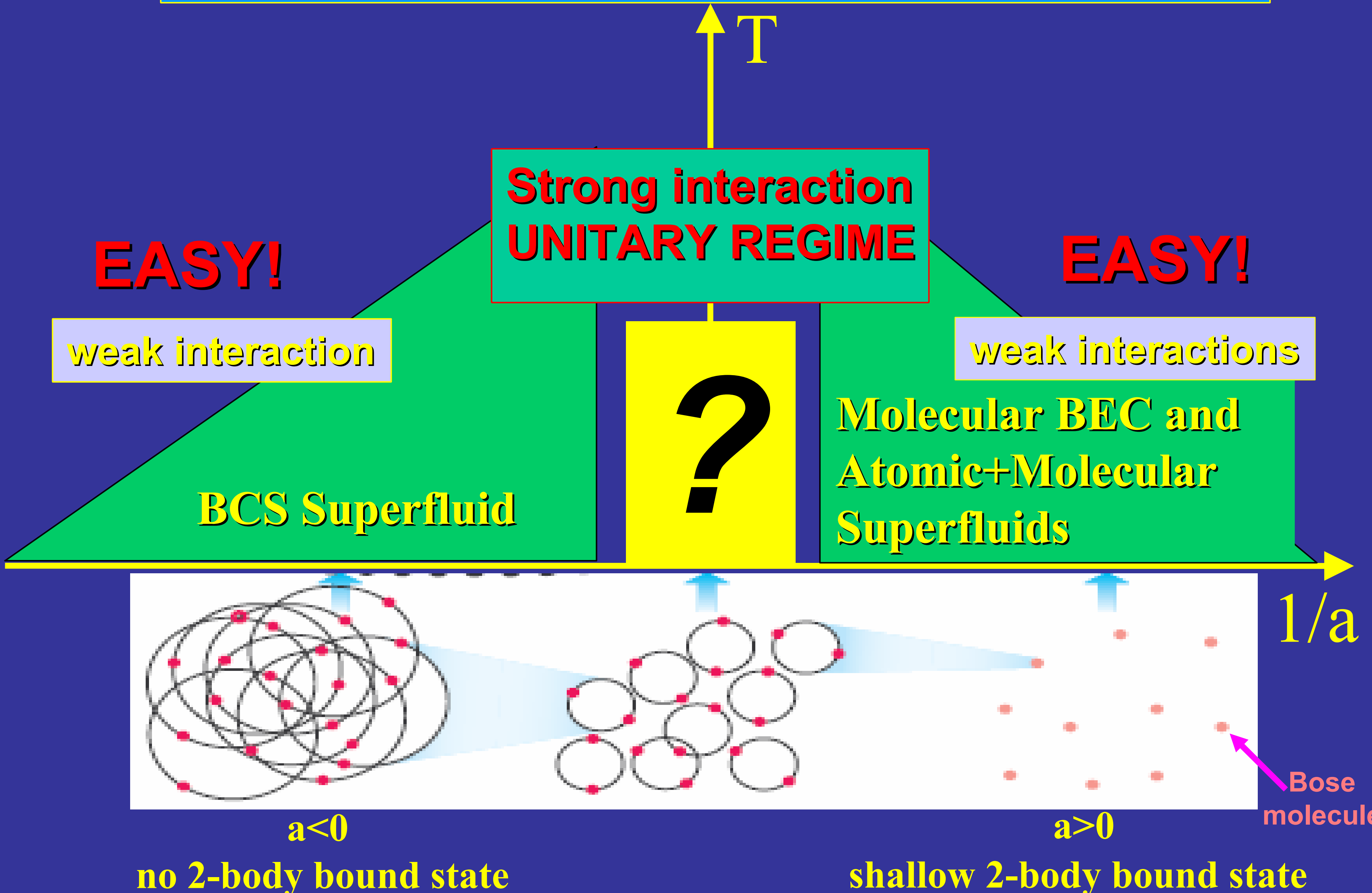
$$\sigma = 4\pi a^2$$

The only scale: $E_{FG}/N = \frac{3}{5}\varepsilon_F$

UNIVERSALITY: $E(T) = \xi\left(\frac{T}{\varepsilon_F}\right) E_{FG}$

QUESTIONS: What is the shape of $\xi(T/\varepsilon_F)$?
What is the critical temperature for the superfluid-to-normal transition?
...

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



A little bit of history

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

<u>In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.</u>

- ***systems of bosons are unstable (Efimov effect)***
- ***systems of three or more fermion species are unstable (Efimov effect)***

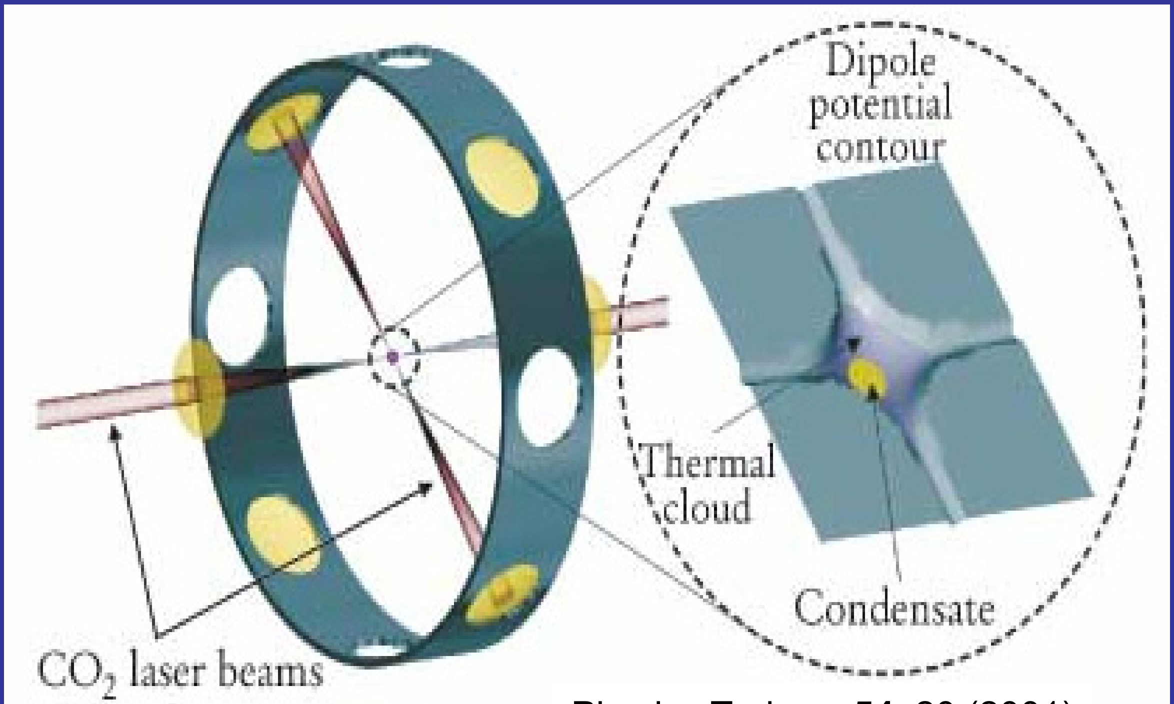
- **Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**

- **Carlson et al (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik et al. (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems:** $\xi(T=0) \approx 0.44$

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**

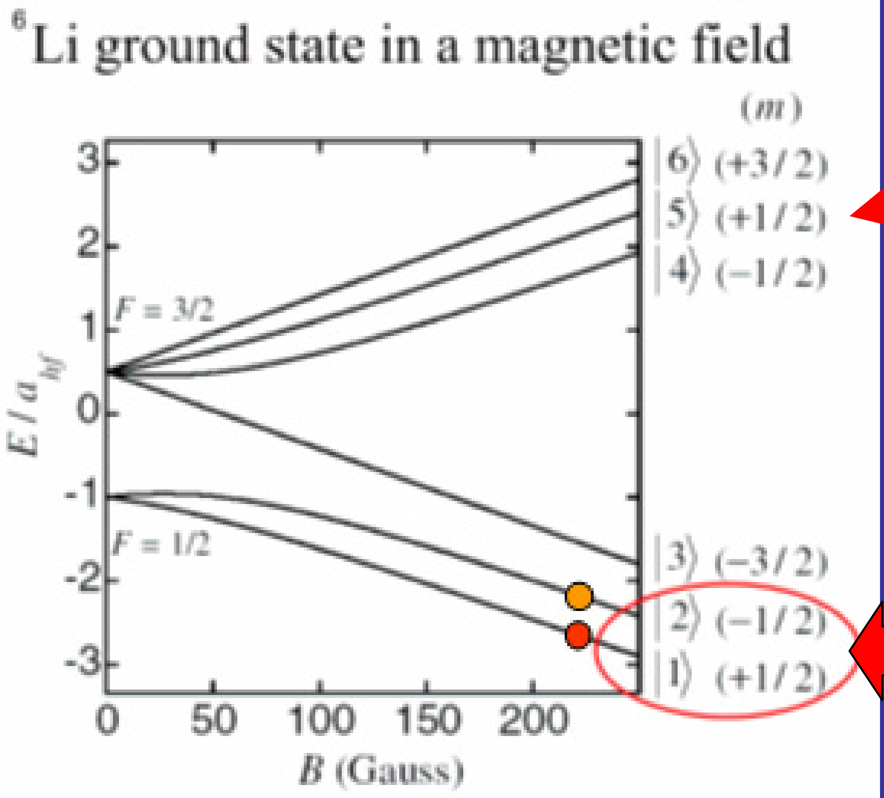


Physics Today, v**54**, 20 (2001)

Who does experiments?

- **Jin's group at Boulder**
- **Grimm's group in Innsbruck**
- **Thomas' group at Duke**
- **Ketterle's group at MIT**
- **Salomon's group in Paris**
- **Hulet's group at Rice**

One fermionic atom in magnetic field



$$\left| F\, m_F \right\rangle$$

$$\vec{F} = \vec{I} + \vec{J}\ ;\ \vec{J} = \vec{L} + \vec{S}$$

Nuclear spin **Electronic spin**

Two hypefine states are populated in the trap

Collision of two atoms:

At low energies (low density of atoms) only L=0 (s-wave) scattering is effective.

- **Due to the high diluteness atoms in the same hyperfine state do not interact with one another.**
- **Atoms in different hyperfine states experience interactions only in s-wave.**

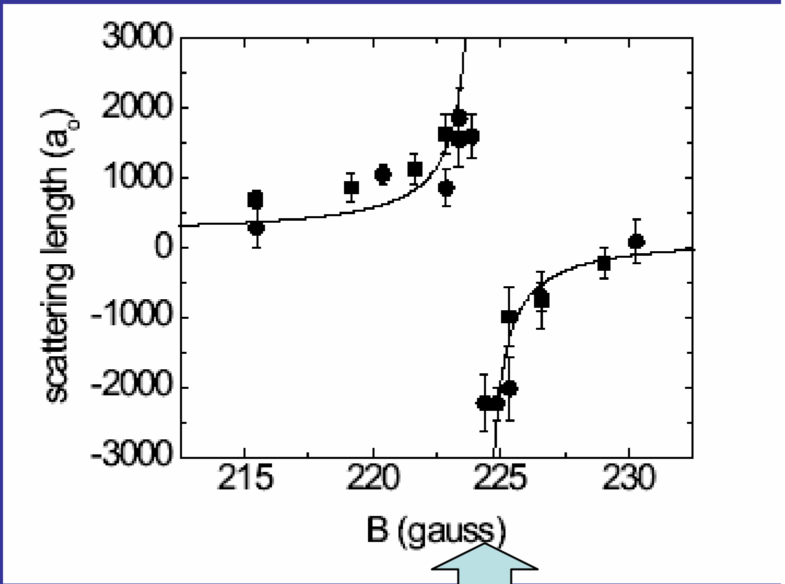
Feshbach resonance

$$H = \frac{\vec{p}^2}{2\mu} + \sum_{i=1}^{2}(V_i^{hf} + V_i^Z) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + V^d$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{I}\cdot\vec{J},\ V^Z = (\gamma_e J_z - \gamma_n I_z)B$$

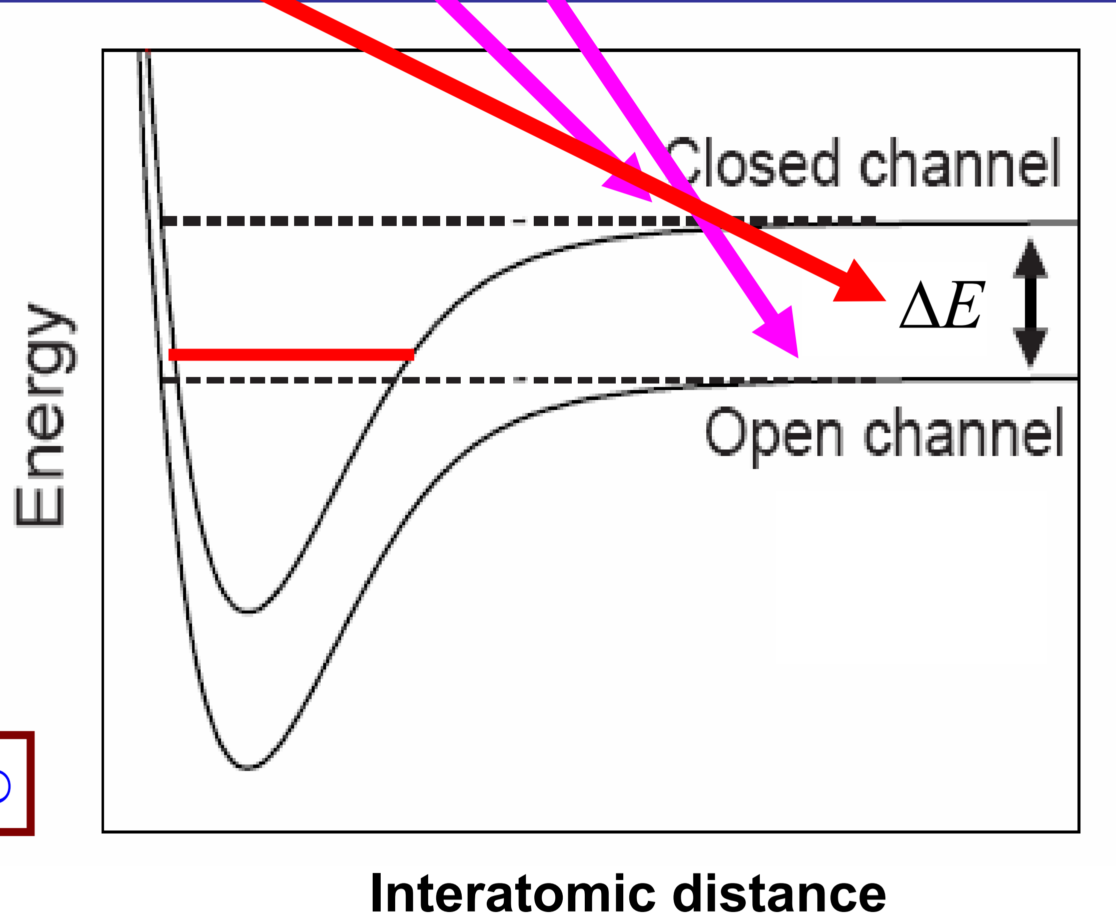
Tiesinga, Verhaar, Stoof, Phys. Rev. A47, 4114 (1993)

Channel coupling



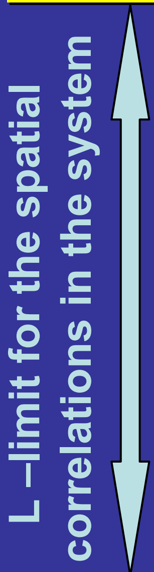
resonance: $a \to \pm\infty$

Regal and Jin, PRL 90, 230404 (2003)

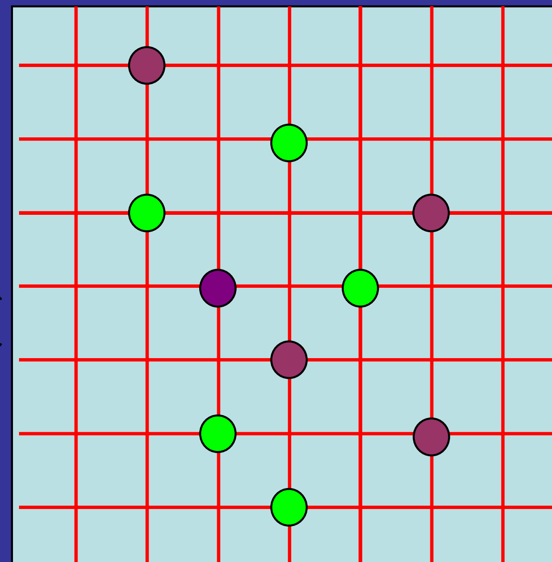


Theoretical approach: Fermions on 3D lattice

Coordinate space



$$k_{cut} = \frac{\pi}{\Delta x}\ ;\ \Delta x$$



- Spin up fermion: ↑

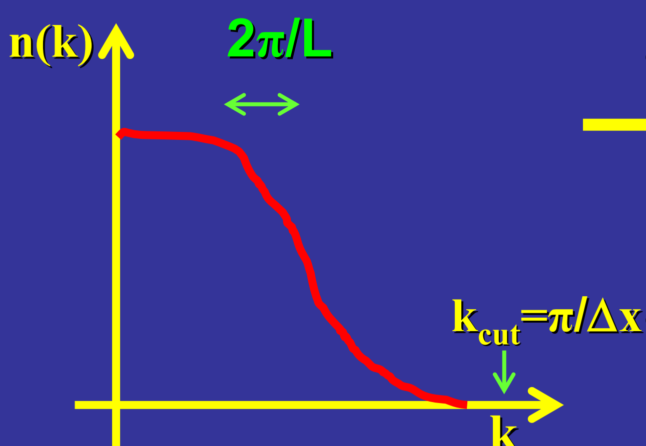
- Spin down fermion: ↓

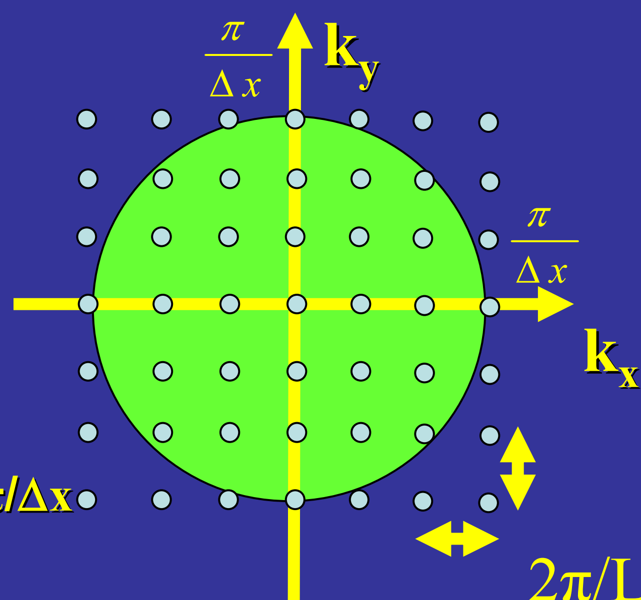
External conditions:

T - temperature

μ - chemical potential

Momentum space





$$\varepsilon_F,\ \Delta,\ kT << \frac{\hbar^2\pi^2}{2m(\Delta x)^2}$$

$$\delta\varepsilon > \frac{2\hbar^2\pi^2}{mL^2}$$

$$\delta p > \frac{2\pi\hbar}{L}$$

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) - g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{m k_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

$$\frac{1}{g} = \frac{m}{2\pi\hbar^2 \Delta x} \quad \text{- UNITARY LIMIT}$$

Grand-canonical ensemble:

$$E(T) = \langle \hat{H} \rangle = \frac{1}{Z(T)} Tr \left\{ \hat{H} \rho(\hat{H}, \hat{N}, T) \right\} = \frac{1}{Z} \sum_n E_n e^{-\frac{1}{kT}(E_n - \mu N_n)}$$

$$Z(T) = Tr \left\{ \rho(\hat{H}, \hat{N}, T) \right\} = \sum_n e^{-\frac{1}{kT}(E_n - \mu N_n)}; \quad \rho(\hat{H}, \hat{N}, T) = e^{-\frac{1}{kT}(\hat{H} - \mu\hat{N})}$$

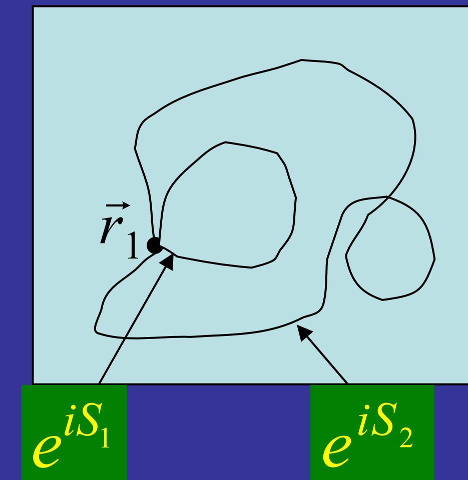
Eigenenergies of the Hamiltonian are unknown!

Path integral approach:

Single-particle quantum mechanics:

$$\left\langle \vec{r}_1 \left| e^{-i\hat{H}(t-t_0)} \right| \vec{r}_1 \right\rangle = \int D[r(t)] e^{i\int_{t_0}^{t} L\left(\vec{r}(t),\dot{\vec{r}}(t)\right)dt}$$

$$L\left(r(t),\dot{r}(t)\right) = \frac{m\dot{\vec{r}}(t)}{2} - V(\vec{r}); \quad e^{i\int_{t_0}^{t} L\left(\vec{r}(t),\dot{\vec{r}}(t)\right)dt} = e^{iS[\vec{r}(t)]}$$



Quantum statistical mechanics:

$$Z(\beta) = Tr\left\{\exp(-\beta(\hat{H}-\mu\hat{N})\right\} = \sum_{\substack{n-many \\ body\ states}} \left\langle n \right| \exp(-\beta(\hat{H}-\mu\hat{N}) \left| n \right\rangle$$

$$\beta = 1/kT\ ;\ \text{imaginary time: } \tau = it$$

$$Z(\beta) = \int D\left[\sigma(\vec{r},\tau)\right]\ e^{\ln\{\det[1+\hat{U}(\{\sigma\})]\}}$$

$$S[\sigma(\vec{r},\tau)] = -\ln\{\det[1+\hat{U}(\{\sigma\})]\} \text{ - action}$$

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\};\ \hat{h}(\{\sigma\}) - \text{one-body operator}$$

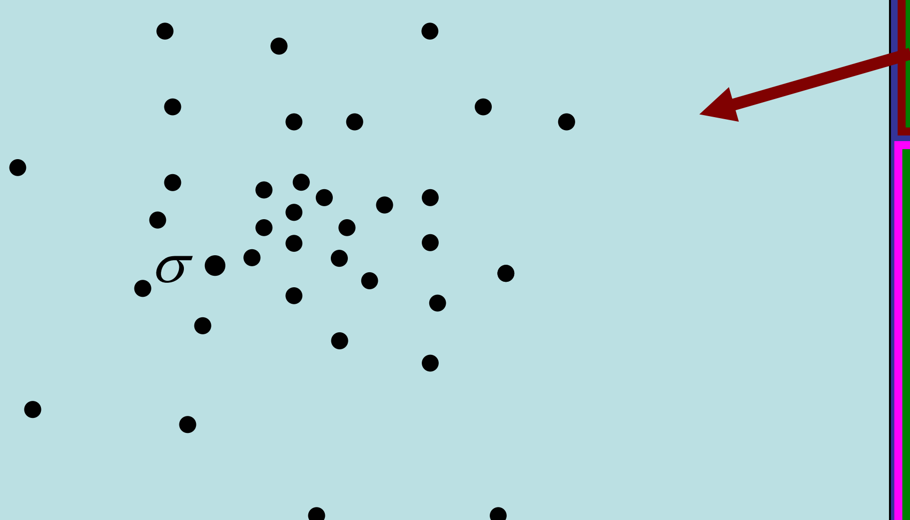
$$U(\{\sigma\})_{kl} = \left\langle \psi_k \middle| \hat{U}(\{\sigma\}) \middle| \psi_l \right\rangle; \quad \left|\psi_l\right\rangle \text{- single-particle wave function}$$

$$E(T) = \left\langle \hat{H} \right\rangle = \int \frac{D[\sigma(\vec{r},\tau)]e^{-S[\sigma]}}{Z(T)} E[U(\{\sigma\})]$$

$E[U(\{\sigma\})]$- energy associated with a given sigma field

Quantum Monte-Carlo:

Sigma space sampling



$$P(\sigma) \propto e^{-S[\sigma]}$$

$$\bar{E}(T) = \frac{1}{N_\sigma} \sum_{k=1}^{N_\sigma} E\left(U(\{\sigma_k\})\right)$$

$\bar{E}(T)$ - stochastic variable

$\left\langle \bar{E}(T) \right\rangle = E(T)$

$\sqrt{\left\langle \bar{E}(T)^2 \right\rangle - \left\langle \bar{E}(T) \right\rangle^2} \propto 1 / \sqrt{N_\sigma}$

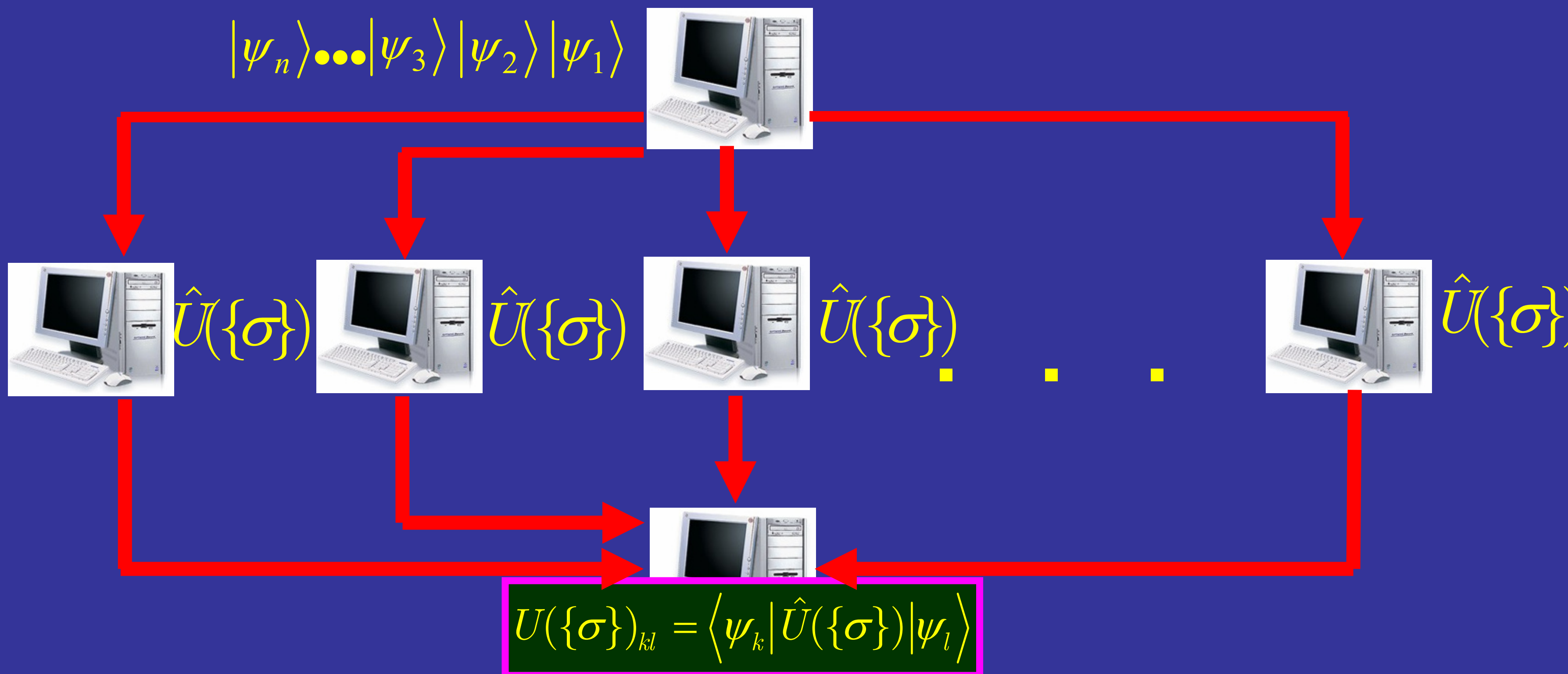
N_σ - number of uncorrelated samples

Quantum Monte-Carlo: parallel computing

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\};\ \hat{h}(\{\sigma\}) - \text{one-body operator}$$

$$U(\{\sigma\})_{kl} = \langle \psi_k | \hat{U}(\{\sigma\}) | \psi_l \rangle;\quad |\psi_l\rangle \text{- single-particle wave function}$$

For each sigma *n* single particle states have to be evolved.



More details of the calculations:

- Lattice sizes used from 8^3 x 257 (high Ts) to 8^3 x 1732 (low Ts), <N>=50, and 6^3 x 257 (high Ts) to 6^3 x 1361 (low Ts), <N>=30.

- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.

- Update field configurations using the Metropolis importance sampling algorithm.

- Change randomly at a fraction of all space and time sites the signs the auxiliary fields $\sigma(x,\tau)$ so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .

- Thermalize for 50,000 – 100,000 MC steps or/and use as a start-up field configuration a $\sigma(x,\tau)$-field configuration from a different T

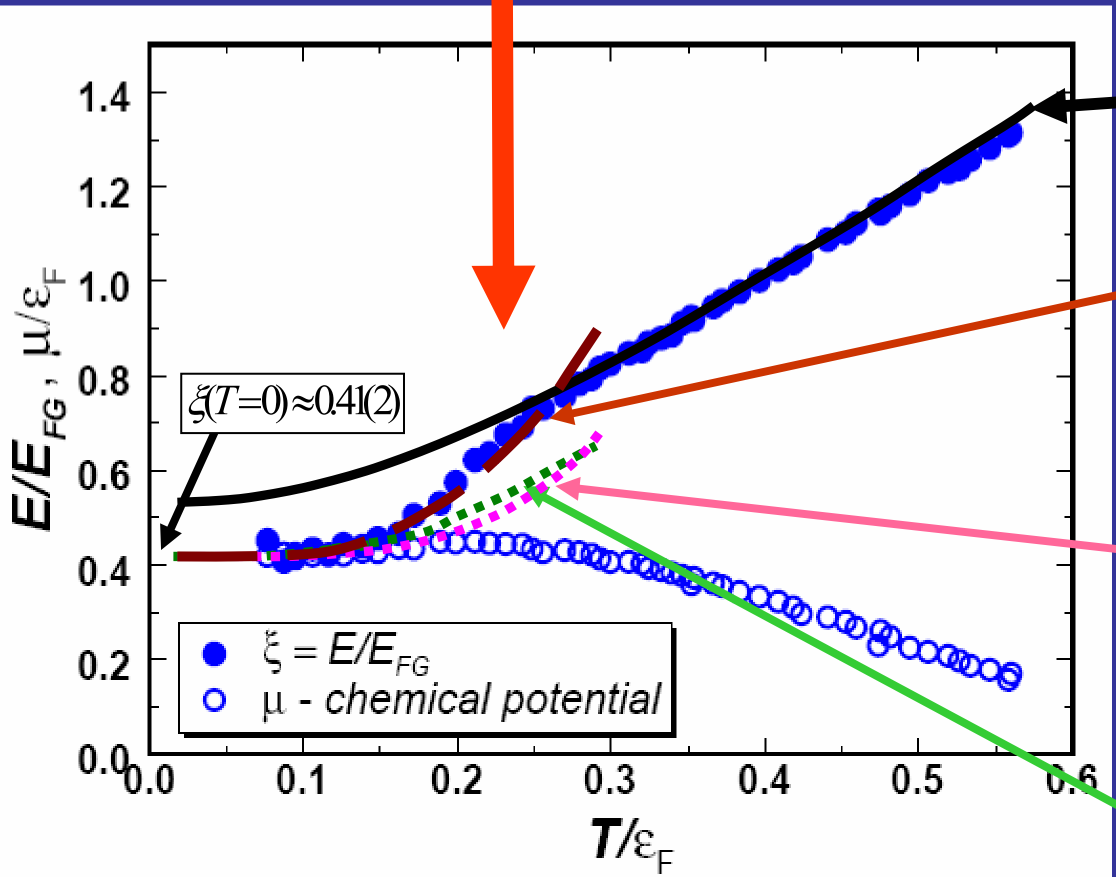
- At low temperatures use Singular Value Decomposition of the evolution operator U({σ}) to stabilize the numerics.

- Use 200,000-2,000,000 $\sigma(x,\tau)$- field configurations for calculations

- MC correlation "time" ≈ 150 – 200 time steps at $T \approx T_c$

a = ±∞

Superfluid to Normal Fermi Liquid Transition



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution
(dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)
People never consider this ???

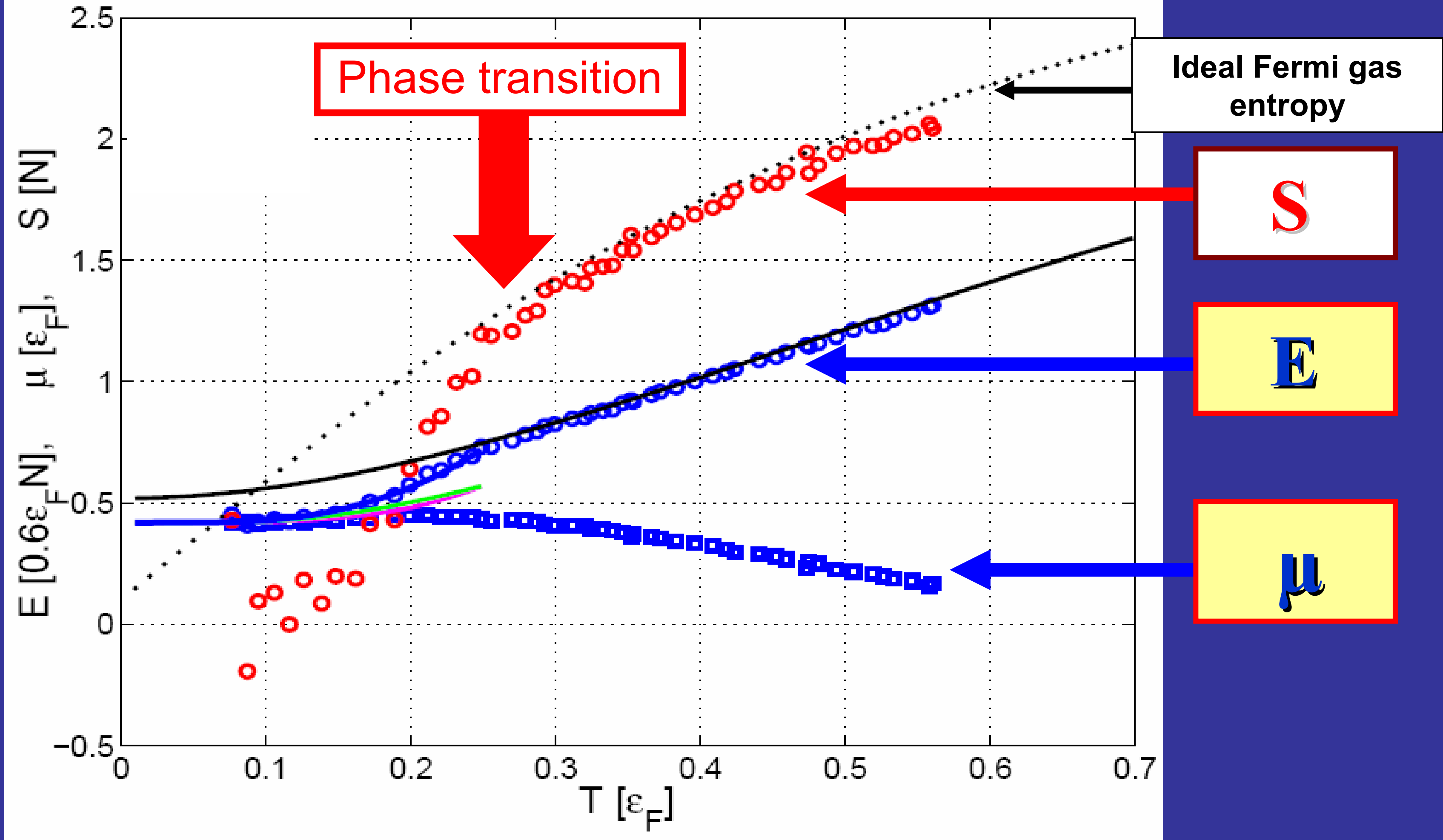
Quasi-particle contribution only
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N\ \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N\ \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

A. Bulgac, J.E. Drut, P. Magierski, cond-mat/0505374



$$E = \mu N - PV + TS = \frac{3}{5}\varepsilon_F(n)N\, e\left(\frac{T}{\varepsilon_F(n)}\right) = \varepsilon(n)nV$$

$$n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F(n) = \frac{\hbar^2 k_F^2}{2m}$$

$$S = \frac{\frac{5}{3}e(n) - \mu}{T}N = N\sigma\left(\frac{T}{\varepsilon_F(n)}\right), \qquad P = \frac{2}{3}e(n)n$$

Low temperature behaviour of a Fermi gas in the unitary regime

$$E(T) = \frac{3}{5}\varepsilon_F N \xi\left(\frac{T}{\varepsilon_F}\right) \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \text{ for } T < T_C$$

$$\mu(T) = \frac{dE(T)}{dN} = \varepsilon_F\left[\xi\left(\frac{T}{\varepsilon_F}\right) - \frac{2}{5}\frac{T}{\varepsilon_F}\xi'\left(\frac{T}{\varepsilon_F}\right)\right] \approx \varepsilon_F \xi_s$$

$$\xi\left(\frac{T}{\varepsilon_F}\right) = \xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^{5/2}, \qquad \varsigma_s \approx 11(1)$$

$$E(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^n\right]$$

Lattice results disfavor either $n \geq 3$ or $n \leq 2$ and suggest $n = 2.5(0.25)$

This is the same behavior as for a gas of <u>noninteracting</u> (!) bosons below the condensation temperature.

Conclusions

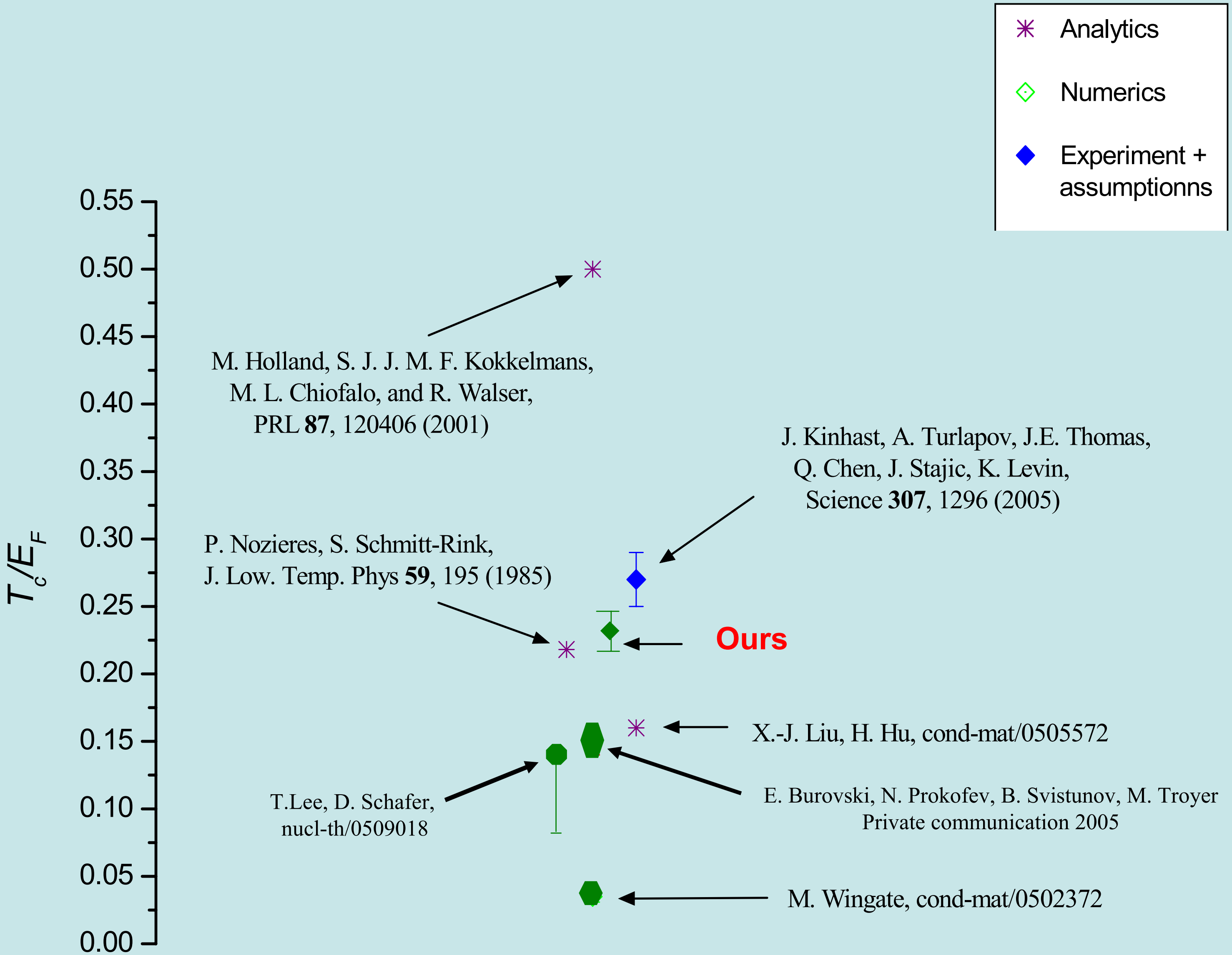
- ✓ Fully non-perturbative calculations for a spin ½ many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at $T_c = 0.23\ (2)\ \varepsilon_F$
(Exp: $T_c = 0.27(2)\ \varepsilon_F$, J. Kinast *et al.* Science, 307, 1296 (2005): Based on theoretical assumptions).

- ✓ Chemical potential is constant up to the critical temperature – note similarity with Bose systems!

- ✓ Below the transition temperature, both phonons and fermionic quasiparticles contribute almost equaly to the specific heat. In more than one way the system is at crossover between a Bose and Fermi systems.

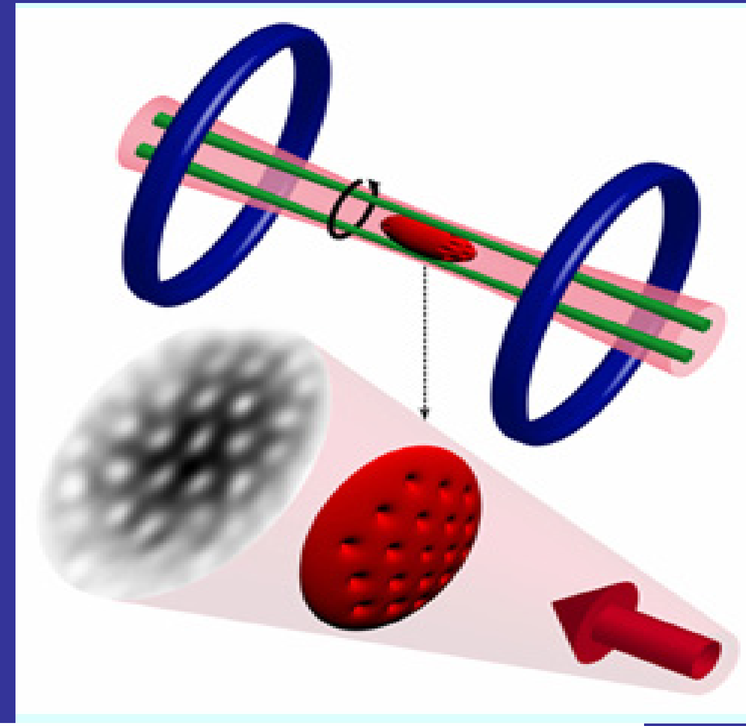
There are reasons to believe that below the critical temperature this system is a new type of fermionic superfluid, with unusual properties.

Quest for unitary point critical temperature



Boris Svistunov's talk (updated), Seattle 2005

Evidence for fermionic superfluidity: vortices!

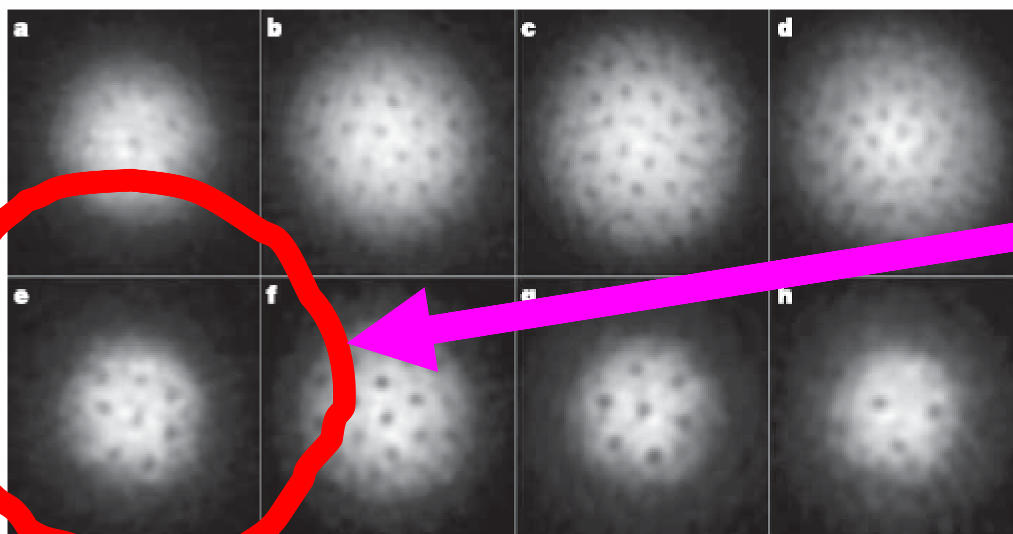


system of fermionic ^6Li atoms

Feshbach resonance: B=834G

BEC side: a>0

BCS side: a<0



UNITARY REGIME

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b–h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.