Nuclear structure and dynamics in the neutron star crust

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Content:

- Collective excitations of nuclei immersed in a superfluid neutron environment. Specific heat of the inner crust.

- Spherical symmetry breaking of nuclei in the inner crust.

- Nuclear clustering in the bottom of the inner crust: selfconsistent description of exotic ‘pasta’ phases.

- Fermionic Casimir effect.

- Neutron localization induced by the pairing field in the inner crust.
Exotic nuclear shapes: „pasta” phase.
Most likely disordered system:
A. Bulgac, P. Magierski, NPA683(2001)695

Uniform nuclear matter

Outer crust

Inner crust

Core

\[ \rho \approx 0.1 \text{ fm}^{-3} \]
Hamiltonian of a nucleus immersed in a neutron superfluid ($E_{\text{exc}} < \Delta$):

$$H = \sum_{l,m} \left( \frac{\hat{\pi}_{lm}}{M_l} \right)^2 + C_l \left| \hat{\alpha}_{lm} \right|^2$$

$$M_l = m \rho_{\text{in}} \frac{(\gamma - 1)^2}{\gamma (l+1) + l} R_N^5; \quad \gamma = \frac{\rho_{\text{out}}}{\rho_{\text{in}}}$$

$$C_l = C_l^{\text{surf}} + C_l^{\text{coul}}$$

Spreading width of a quadrupole vibrational multiplet ($l=2$):

$$\Gamma_{\text{tot}} \approx 0.169 \left( \frac{Ze}{R_C} \right)^2 \left( \frac{R_p}{R_C} \right)^2 \left( \frac{3\hbar \omega_2}{2C_2} \right)^{1/2}$$

$R_p$ – proton radius

$R_C$ – Wigner–Seitz cell radius

Energy depends on the orientation with respect to the lattice vectors
Spherical symmetry breaking due to the coupling between lattice and nuclear vibrations.
Andreev approximation:
\[
\begin{pmatrix}
h - \mu & \Delta(\vec{r}) \\
\Delta^*(\vec{r}) & -h + \mu
\end{pmatrix}
\begin{pmatrix}
u(\vec{r}) \\
v(\vec{r})
\end{pmatrix}
= E
\begin{pmatrix}
u(\vec{r}) \\
v(\vec{r})
\end{pmatrix}
\]

BdG eqs.

\[h = -\frac{\hbar^2}{2m} \nabla^2; \quad \Delta(\vec{r} + \vec{a}) = \Delta(\vec{r})\]

1-dimensional problem:

\[\Delta(\vec{r})\]

Bound state

Andreev approximation:
\[
\begin{pmatrix}
u(r) \\
v(r)
\end{pmatrix}
= \begin{pmatrix}
\bar{u}(r) \\
\bar{v}(r)
\end{pmatrix} e^{i k_F r}
\]
Quantization condition: \[ A(\phi, \psi) e^{2iqL} = 1 \]

\[ A(\phi, \psi) = \frac{(e^{-\phi} - e^{-\dot{\psi}})(e^{\phi} - e^{\dot{\psi}})}{(e^{-\phi} - e^{\dot{\psi}})(e^{\phi} - e^{-\dot{\psi}})} ; \]

\[ \cos \psi = \frac{E}{\Delta_+} \]
\[ \cosh \phi = \frac{E}{\Delta_-} \]

\[ q = \frac{m}{\hbar^2 k_F} \sqrt{E^2 - \Delta_-^2} \]

There is always at least one bound state!

Penetration length inside a barrier \( \Delta_+ \)

\[ \xi = \hbar^2 k_F / (m \sqrt{\Delta_+^2 - E^2}) \]
Localization condition: $\xi < R_C - R_N$

- $R_C$ – Wigner-Seitz cell radius
- $R_N$ – Nuclear radius

Localization condition: $F(\rho) > 1$

where:

$$F(\rho) = \frac{1}{2} k_F R_N \sqrt{\left(\frac{\Delta_+}{\mu}\right)^2 - \left(\frac{E}{\mu}\right)^2 \left(\frac{R_C}{R_N} - 1\right)}$$
\[ \Delta(r + a) = \Delta(r) \]
\[ a = 80 \text{ fm} \]

\[ F(\rho) > 1 \]
$\Delta(r+a) = \Delta(r)$

$a = 40 \text{ fm}$
Conclusions

• Due to the coupling between the nuclear surface vibrations and the ion lattice part of the crust is filled with non-spherical nuclei. The phase transition takes place at densities far lower than the predicted density for the transition to the exotic „pasta phases”.

• There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.

• The contribution to the specific heat associated with nuclear shape vibrations seems to be important at densities around 0.02 $fm^{-3}$ where the pairing correlations are predicted to reach their maximum.

• At low densities in the inner crust neutrons around the Fermi level are localized due to the inhomogeneity of the pairing field.

Summarizing, due to these effects the transport properties (thermal and electric conductivities) across the crust are expected to be modified.