

# Computing Time Dependent Density Functional Theory for Superfluid Systems

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# Theory

- thousands of nuclei , tens of interactions (?)
- 10K to 100K time steps per nucleus
- multiple densities per time step
  - bilinear products of complex four-component single quasiparticle wavefunctions, gradients and time derivatives
- evaluate observables and spectrum

$$Q(\omega) = \sum_{\sigma} \int d^3r dt Q(\mathbf{r}, \sigma, t) \rho(\mathbf{r}, \sigma, t) \exp(i\omega t)$$

# Implementation

- Fortran 90 , C
  - GNU Compiler Collection, Portland Group , Intel
- Discretization (MPI dependence)
  - 3D coordinate and momentum lattice representation
    - quasiparticles represented in plane wave basis
    - canonical index distribution algorithm to map quasiparticles to processes
  - Adams-Bashforth-Milne time stencil ,  $O(h^5)$ 
    - expensive but preserves variance in number density and total energy to  $1.e^{-6}$  over thousands of time steps
- Major Operations per time step (FFTW dependence)
  - (Local) gradients, time derivatives, potential evaluations, (Global) reductions

# Observed Scaling Features

- Strong scaling examples (fix problem complexity, add factor of k processes, reduce runtime by factor of k)

$N_x=N_y=N_z=30$

PEs (jaguarcl)	576	1152	1728	2304
<NWF / PE>	48	24	16	12
[s]/ts	56.2	28.8	19.3	14.92 (14.05)

$N_x=N_y=N_z=40$

PEs (jaguar)	942 (x4)	1884(x4)	2826(x4)	3768(x4)
<NWF/PE>	70	36	24	17
[s] / 10 time steps	296.15	153.31	103.17	77.02 (74.03)
Total INS	7.87635E+14	8.15353E+14	8.15732E+14	8.10577E+14
max(INS(PE))	8.38278E+11	4.25906E+11	2.83689E+11	2.13813E+11
Total FLOP	1.84227E+14	1.84997E+14	1.85766E+14	1.86536E+14
max(INS(FLOP))	1.9578E+11	99655061696	66696994580	50218692684

# Observed Scaling Features

- Weak scaling example (scale problem complexity by factor k, scale processes by factor k, fix runtime)

$N^3$	$30^3$	$40^3$	$50^3$
quasiparticles	28288	66796	130528
PEs	168(x4)	942(x4)	3626(x4)
[s] / 10 time steps	297.31	296.15	319.03
Total INS	1.41538E+14	7.87635E+14	3.13385E+15
Total FLOP	3.3787E+13	1.84227E+14	8.22772E+14
Total BYTES	5.37701E+11	3.00957E+12	1.14865E+13

$t(50^3)/t(40^3) = 1.077$	PE/PE=3.849	INS/INS=3.97	FLOP/FLOP=4.46
$t(50^3)/t(30^3) = 1.073$	PE/PE=21.583	INS/INS=22.14	FLOP/FLOP=24.35
$t(40^3)/t(30^3) = .996$	PE/PE=5.607	INS/INS=5.564	FLOP/FLOP=5.45

# Memory discussion

$N^3$	Quasiparticles	BYTES	BYTES (OOC)
$30^3$	28288	5.37701E+11	44280000
$40^3$	66796	3.00957E+12	104960000
$50^3$	130528	1.14865E+13	205000000
$60^3$	226156	3.43902E+13	354240000
$70^3$	359056	8.6702E+13	562520000
$80^3$	535516	1.93026E+14	839680000
$90^3$	763824	3.92007E+14	1195560000
$100^3$	1046604	7.36809E+14	1640000000
$110^3$	1393008	1.30528E+15	2182840000
$120^3$	1808172	2.19966E+15	2833920000
$130^3$	2299056	3.55592E+15	3603080000

threaded , buffered swapping

```
touch_kfil();  
kfil_wr();  
kfil_rd();  
delete_kfil();  
kwr_seek_r();  
krd_seek_r();
```

These routines are quite general and have some nice features .

# Scalable Data Structures using Disk and Network Memory (MPI) for Arbitrary Rank Object Storage and Manipulation

```

8 100 100 400 400
/tmp/525740.dat.0 0 33554431
/tmp/525740.dat.1 33554432 67108863
/tmp/525740.dat.2 67108864 100663295
/tmp/525740.dat.3 100663296 134217727
/tmp/525740.dat.4 134217728 167772159
/tmp/525740.dat.5 167772160 201326591
/tmp/525740.dat.6 201326592 234881023
...
/tmp/525740.dat.377 12650020864 12683575295
/tmp/525740.dat.378 12683575296 12717129727
/tmp/525740.dat.379 12717129728 12750684159
/tmp/525740.dat.380 12750684160 12784238591
/tmp/525740.dat.381 12784238592 12799999999
    
```

## Arbitrary Rank Objects

Canonical Mapping :: (e.g. rank-4 object)

$(i_0, i_1, i_2, i_3) \rightarrow k$

$$k = i_0 + \text{dim}[0] ( i_1 + \text{dim}[1] ( i_2 + \text{dim}[2] ( i_3 ) ) ) )$$

$i_0 \rightarrow$  inner most loop (fastest varying)

$i_3 \rightarrow$  outer most loop (slowest)

Example Use: `kfil_2dbc_rd()`, `kfil_2dbc_wr()`

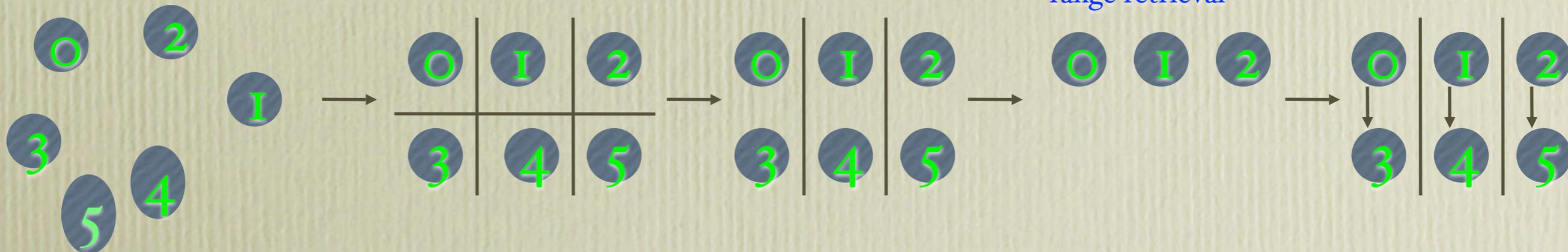
MPI\_COMM\_WORLD

Virtual rectangular  
Process grid

Subgroup  
formation

Multithreaded  
range retrieval

Subgroup  
Bcast, parse



\*replace PATH w/ struct holding MPI id and ptr

- Tasks for years 2 , 3

- finish ooc hybrid code for enhanced scalability
- parallelize dft solver , begin coupling to time dependent code ,write test code that diagonalizes for comparison
- implement Aurel's theoretical changes to homogeneous unitary fermi gas code
- start implementation of data storage of computed densities and analysis tools for these data sets

- Contributions this year

- fully parallel td-slida code with nice numerical conservation of critical observables as function of time - as well as exceptional scaling features
- full support for threaded, asynchronous io for arbitrary rank objects of arbitrary size