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ROTATION OF THE PLANE POLARIZATION OF LIGHT

1. Fundamentals

In diffraction, interference and polarization phenomena, light acts as an electromagnetic wave - i.e. as a series of variable electric and magnetic waves propagating in space. The electromagnetic wave is described by an **electric field vector** ($\vec{E}$), **magnetic field vector** ($\vec{B}$) and **wave vector** ($\vec{k}$) that determines the direction of the wave propagation $\left|\vec{k}\right| = \frac{2\pi}{\lambda}$ where λ is the wavelength.

$\vec{E}, \vec{B}$ and $\vec{k}$ vectors are mutually perpendicular, thus an electromagnetic wave is a **transverse wave**. Electric and magnetic fields can be described as sinusoidal functions of position *x* (along the wave propagation direction) and time t:

$$E(x,t) = E_0\sin(\omega t-kx+\delta)$$
$$B(x,t) = B_0\sin(\omega t-kx+\delta) \quad (1)$$

Where **ω** denotes angular frequency, and **δ** is the initial phase.

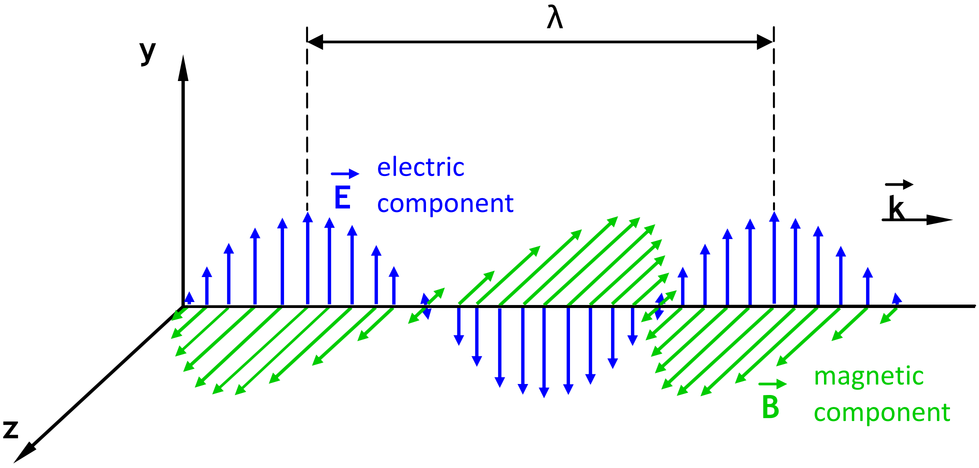

Fig. 1. An image of an electromagnetic wave propagating in x direction.

Since the electric field vector is resposible for all optical phenomena, we usually describe electromagnetic waves with electric field vector $\vec{E}$, calling it “light vector”. The electromagnetic waves emitted by an ordinary light sources like the Sun or a bulb are **unpolarized**. It means that the electric field vector is always perpendicular to the propagation direction but it can oscillate in all the possible planes located according to the propagation direction.

The light can be polarized **linearly**, **circularly** and **elliptically**. A linear polarization takes place when the electric vector vibrates in only one plane that does not change its orientation in space (Fig. 2). Speaking in other words, the vibrations’ plane of $\vec{E}$ vector remains the same along the entire ray.

If the end of the electric field $\vec{E}$ vector rotates along a circular or ellipsoidal helix, we say that the light is circularly or elliptically polarized.

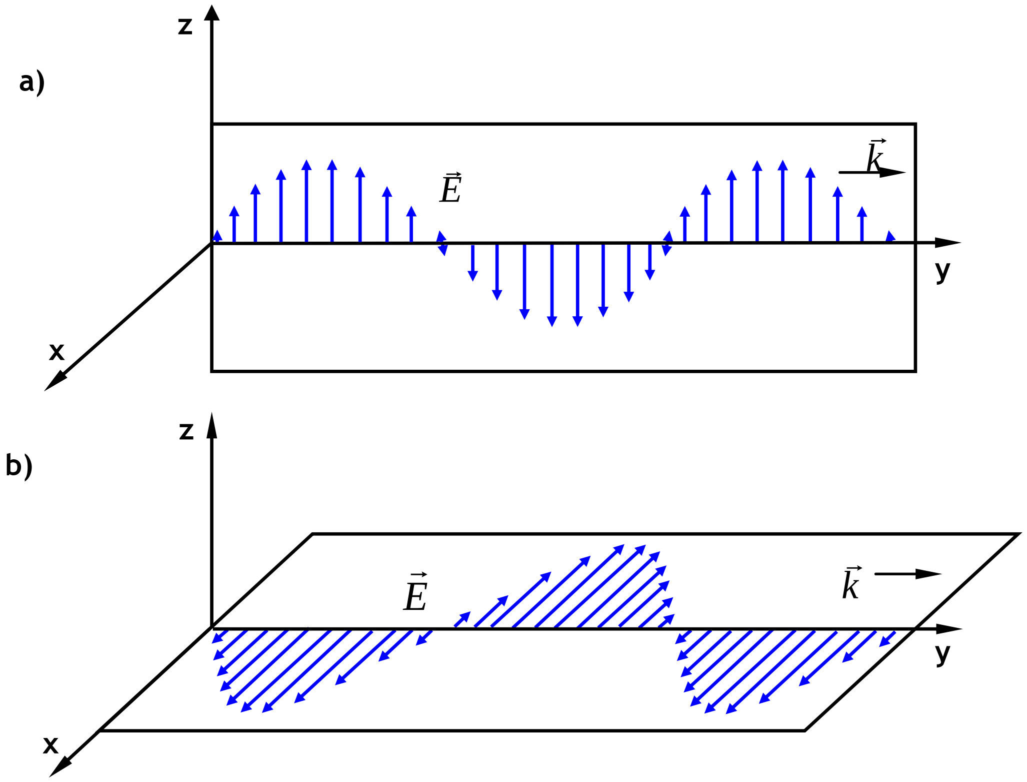

Fig. 2. A wave polarized linearly: $\vec{E}$ vector vibrates in a plane: a) yz , b) xy

1.1 Methods of making the light linearly polarized

The light linearly polarized can be obtained with application of polaroids i.e. polarizing films, reflective polarizers and birefringence polarizers.

a) Polarization with polaroids

The most commonly used H polarizer can be obtained by heating up and then stretching quickly a transparent film of polyvinyl alcohol. During the stretching process most of long particles of polymer (which is the polyvinyl alcohol) that are initially located in a random way rotate and position along almost the same direction i.e. according to the direction of the stretching force. Afterwards, the film is immersed in iodine reach solution. Iodine atoms penetrate the polyvinyl alcohol layer. Thus the iodine atoms form chains similar to the catenary particles of polymer. Quasi parallel chains saturated with iodine, thanks to a good conductivity, highly absorb electric vibrations taking place in a direction parallel to them. Vibrations in the perpendicular direction to particle chain are let through practically without any energy loss.

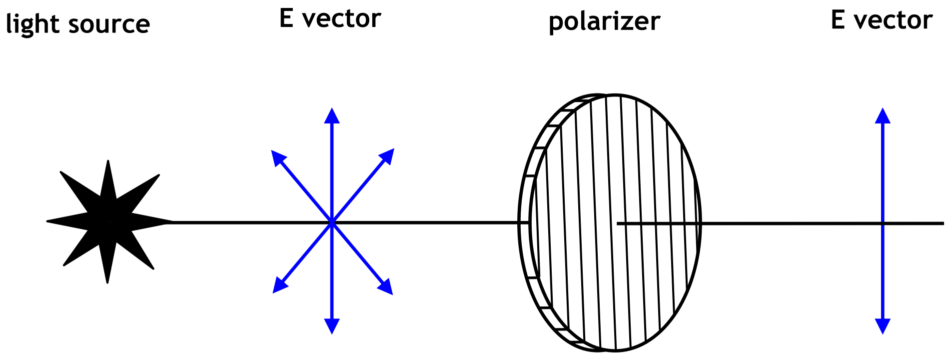

Fig. 3. Polarization of an unpolarized light beam with application of a polarizing filter.

b) Polarization by reflection

Polarized light can be also obtained by the reflection from the dielectric surface. However, the reflected beam is usually not fully polarized. A full polarization of the reflected beam can be obtained for one angle of incidence only. This angle is called a **Brewster's angle**. **For the Brewster's angle the incident ray and the reflected rays create an angle of 90^0.**

$$\alpha + \beta = 90^0 \tag{2a}$$

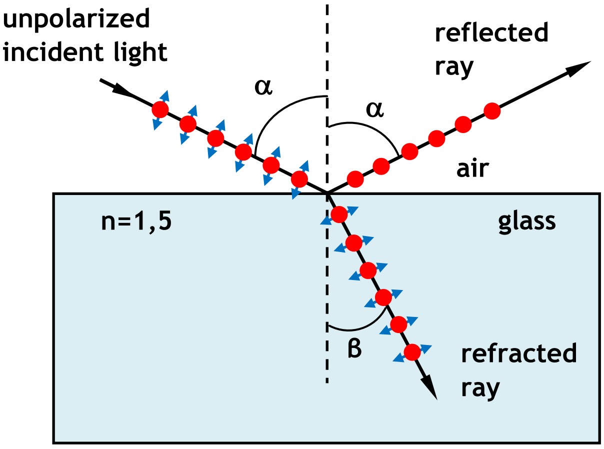

Fig.4. Light polarization by the reflection from glass plate. The electric field E component perpendicular to the plane of this paper is marked with the red dots, the parallel component is marked with the blue arrows.

Fig. 4 shows the unpolarized ray propagating in air that falls on the glass surface and **the Brewster's angle**. The electric field of the wave along the ray has been decomposed into two components: the perpendicular to this paper surface (the plane of incidence, reflection and refraction) and the parallel to the paper surface. The reflected light contains only the perpendicular component and thus it is polarized in this direction. The refracted light contains the initial components parallel to the paper surface and also the perpendicular components of a lower magnitude – this light is partially polarized.

According to the refraction law and considering equation (2a), we have:

$$n = \frac{\sin\alpha}{\sin\beta} = \frac{\sin\alpha}{\sin(90° - \alpha)} = \frac{\sin\alpha}{\cos\alpha} \tag{2b}$$

where **n is called the absolute refraction index of glass.**

Then, after the transformation:

$$\alpha = \operatorname{arctg} n \tag{2c}$$

This formula is called Brewster's law which was discovered in 1812 by David Brewster.

This phenomenon is applied to polarize the light by reflection and to reflectionless transmission of polarized light in optical instruments called **Brewster windows**, applied very often in laser equipment. Polarization by reflection can be explained as follows. Light stimulates to vibration atoms of material at which it falls. The stimulated atoms send the reflected electromagnetic wave, but only these vibrations that are perpendicular to the incidence plane can bring their contribution. Those that lay in the incidence plane do not have components transverse to the observation direction and this is why they cannot be emitted in this direction (the electromagnetic wave is a transverse wave).

c) Polarization with application of birefringence polarizers

Polarized light can be also obtained with application of **birefringence phenomenon that takes place in a certain group of crystals that are called birefringent crystals**. Birefringent crystals have an ability of separation of an incident ray into two refracted rays, as it is shown in figure 5.

The refracted rays have following properties:

1. Both refracted rays can propagate in different directions.
2. The propagation speeds are different.
3. Each ray is fully linearly polarized.
4. The vibration directions of electric field vector in both rays are mutually perpendicular.

One of the waves has always a constant speed independent of its propagation direction in a crystal. This beam has therefore a constant refraction index and fulfills the **law of refraction (also known as Snell's law)**. This is a so-called **ordinary ray**. For the second ray, **called extraordinary**, the wave speed is different and depends on the direction of propagation of this wave in crystal. The sinα/sinβ ratio for various angles of incidence α has no constant value for the extraordinary wave and it loses a physical sense for this beam. This is evidenced by the example shown in Fig. 5b when the light falls perpendicularly on the crystal surface.

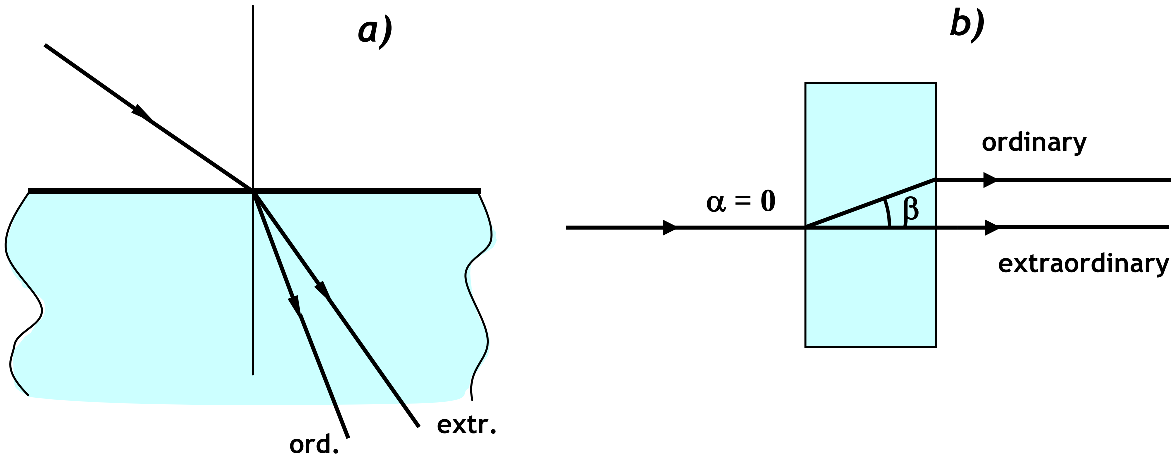

Fig.5. Birefringence of a ray in a birefringent crystal.

The problem of obtaining of the linearly polarized light with application of brefringent bodies is to find the method of removal of one of the refracted rays.

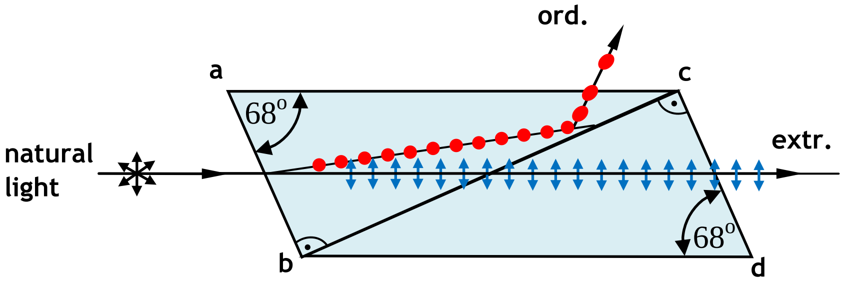

Fig.6. Light path in Nicol prism (labels the same as in Fig. 4)

One of the most commonly known birefringent polarizers is a polarizer constructed in 1828 by a Scottish physicist William Nicol (Fig. 6) In this prism, the **total internal reflection phenomenon is being applied**. When the light propagating in the medium of a refraction index n_1 strikes a boundary of a medium with the refraction index n_2 (where $n_2 < n_1$), it means that it goes from the more dense to the less dense medium, according to the refraction law we have $\frac{\sin\alpha}{\sin\beta}=\frac{n_2}{n_1}$ and the refraction β angle is larger than the angle of incidence α. At a certain angle α_{cr} called "critical angle", the refraction angle β is equal to 90° and the refracted ray is tangent to the media boundary at the point of incidence. For the angles grater α_{cr} the entire ray reflects from the boundary. None passes through. This is called **total internal reflection. (It is abbreviated as TIR.)**

Nicol prism (**called also a nicol**) is a crystal of natural calcite ($CaCO_3$) which is ground at an angle of 68^0, cut along b-c diameter (Fig. 6) and afterwards both halves are glued with Canada balsam of the refraction index 1,54. The ray of light striking the nicol in parallel to the "b-d" edge splits into ordinary ray (ord.) of a refraction index in calcite 1.66 and into extraordinary ray (extr.) with the refraction index 1.49. The extraordinary ray strikes the calcite – balsam surface at **an angle greater than critical**, and reaching the media optically less dense (of a lower refraction index – Canada balsam) undergoes **total internal reflection** and is absorbed by the blackened walls of the nicol. **The extraordinary ray** goes through the balsam (optically denser) and exits from the nicol slightly weakened and **linearly polarized.**

1.2. Optical activity phenomenon

Two polarizers are needed for observation of direction of polarization. In such a system unpolarized light travels through the first **polarizer** polarizing the light linearly and afterwards it goes through the second polarizer called an **analyzer**. If the polarization directions of the polarizer and analyzer are mutually perpendicular, the light is not transmitted by such a system.

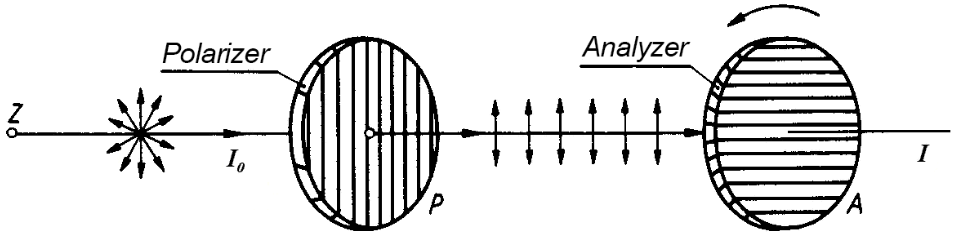

Fig. 7. Polarizer-analyzer system of perpendicular polarization directions.

a) Natural optical activity

If we place a piece of quartz or a sugar solution between polarizer and analyzer of perpendicular polarization directions, we will notice that the formerly dark view field will get brighter, but it can be fully darkened with a turn of the analyzer at a certain angle. Based on this we can conclude that this substance turns the polarization plane. The angle of rotation of the vibration plane is equal to the angle by which the analyzer has to be rotated to obtain the attenuation of the ray after placing the substance between the polarizers. The bodies of such a property are called "**optically active substances**", and the phenomenon itself is called "**optical activity**".

Optical activity is exhibited by crystals and liquids – for example turpentine and nicotine. Optical activity is also exhibited by the solid state solutions in the optically inactive liquids – for example sugar solution in water. The rotation angle of the vibration plane (**α**) is proportional to the solution concentration (**c**) and the distance travelled by light in the solution (**h**). Thus:

$$\alpha = \gamma \cdot c \cdot h \tag{3}$$

γ coefficient is called **specific rotation**. The specific rotation depends on the wavelength of the incident light. The phenomenon of dependence of specific rotation on the wavelength is called optical rotatory dispersion. The specific rotation γ usually decreases with the increase of the wavelength.

b) Optical activity caused by magnetic field. Faraday effect

The phenomenon of optical activity can be forced by some physical factors – for example magnetic field. Bodies that are optically inactive in absence of magnetic field rotate the plane of polarization when they are placed in magnetic field. This effect has been discovered in **1845 by Michael Faraday** who determined that the rotation angle is proportional to the magnetic field B and to the distance **h** travelled by the light in the analyzed medium.

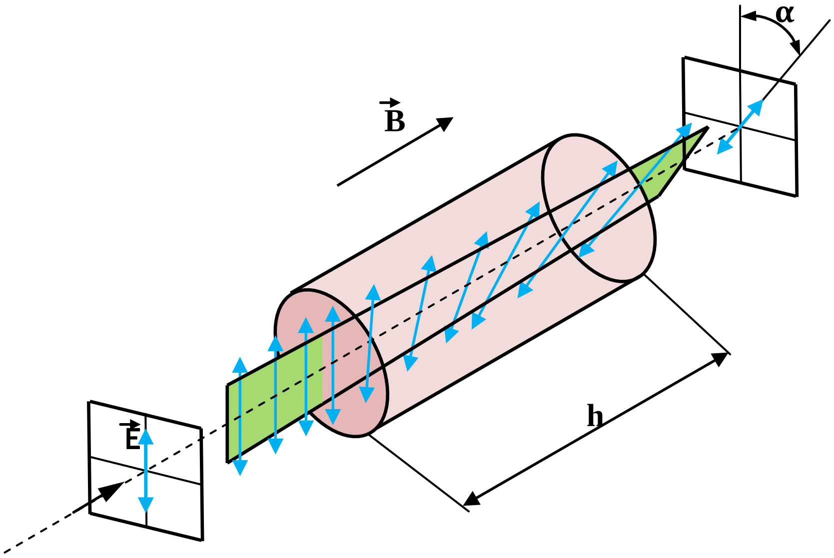

Fig. 8. Polarization rotation due to magnetic field.

Polarization rotation depends on the angle between the direction of light propagation and the direction of magnetic field vector. It is the highest when the light travels in parallel to the direction of the magnetic field vector. Rotation angle in the Faraday effect in case of the **parallel direction** of propagation to the direction of magnetic field direction B can be denoted with a phenomenological formula:

$$\alpha = V \cdot h \cdot B \tag{4a}$$

where: α - rotation angle (in radians), B – magnetic field (in Teslas), h – sample length (in meters), V – **Verdet constant.**

Verdet constant value depends strongly on the wavelength. It depends also on the medium density and temperature. The strong dependence of the Verdet constant on the wavelength causes the necessity of application of monochromatic light. The Faraday effect is applied in laser technology and for light modulation in so-called magneto-optic snapshots and unidirectional light propagation equipment.

The exact presentation of Faraday effect theory requires analysis of the electron motion in substance through which light propagates and which is additional affected by the Lorentz force originating from the external magnetic field B. This forced by the B field motion of electrons in presence of electromagnetic wave changes electrical properties of a substance and leads to the change of the refraction index, because $n = \sqrt{\varepsilon}$, where n – refraction index and ε - relative permittivity. According to the theory of Faraday effect, Verdet constant is given with the formula:

$$V = \frac{e}{2m_e} \cdot \frac{\lambda}{c} \cdot \left| \frac{dn}{d\lambda} \right| \tag{4b}$$

where: m_e - mass of electron, c – speed of light, $\left| \frac{dn}{d\lambda} \right|$ - change of the refraction index with respect to the wavelength. If the dependence of refraction index of the medium on the wavelength is known, the formula above can be used to figure out the e/m_e ratio for the electron.

Polarization rotation angle measurement principle

The system of two nicols (polarizer and analyzer) is the base of polarimeter – an instrument to measure the angle of rotation. If the polarizer and analyzer of perpendicular polarization directions is illuminated with monochromatic light, the light is totally dimmed. If we place an optically active body between the nicols, the image will get brightened. The total dimming of image will be obtained when we rotate the analyzer at an angle equal to the rotation angle caused by the optically active body.

This system, despite its simplicity, has the main drawback as the precision of stating the position of the total image dimming is very small. At present, **half-shade polarimeters** are uniquely used for the measurement of the rotation angle. Since a human eye can differentiate contrasts of neighboring areas that ware weakly but homogenously illuminated. That's the reason for the instrument's name: **half-shade polarimeter**. A **half-shade polarimeters** scheme is shown in Figure 9.

A sodium lamp light (2) passes through the polarizer (4), and then through the medium rotating the angle (5 – cuvette with sugar solution or glass rod located in the magnetic field) and second polarizer-analyzer (7). Observations are done through the viewfinder (9).

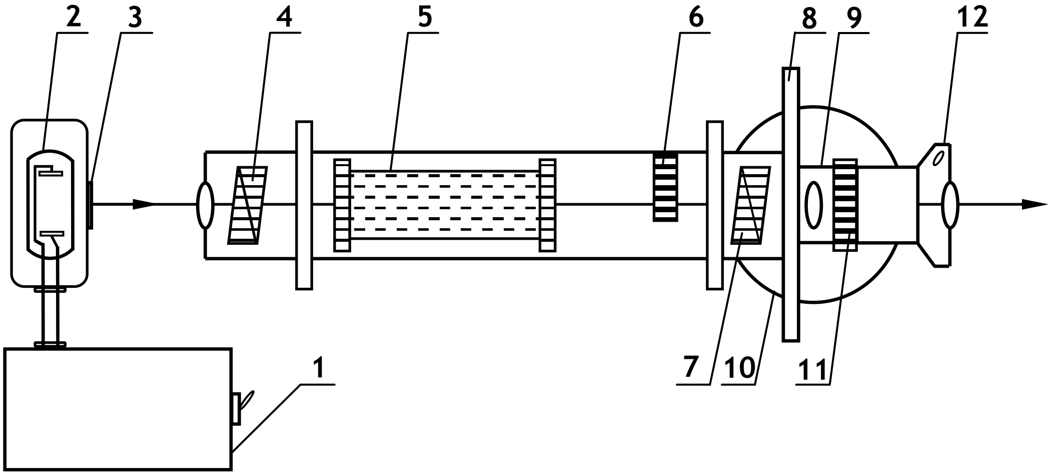

Fig. 9 Polarimeter scheme: 1- lamp power supply, 2- sodium lamp, 3- diffuser, 4- polarizer, 5- cuvette with sugar solution (glass rod), 6- half-shade plate, 7- analyzer, 8 - rotating protractor with nonius for the rotation angle read-out, 9- viewfinder, 10- protractor control knob, 11- focus control knob, 12- lens for protractor read-out.

In the half-shade polarimeter applied in this experiment, there is a half-shade plate located between polarizer and analyzer covering the middle stripe of the view field. The analyzer has to be set in this way that the middle stripe and the remaining part of the view field were homogenously illuminated (Fig. 10b). ***The analyzer is set correctly when even small angle change causes shade appearance outside or inside the view field.***

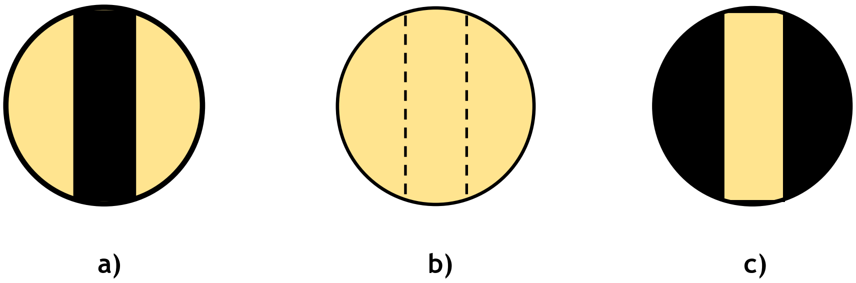

Fig.10. Images in half-shade polarimeter.

By measuring dependence of rotation angle with respect to the solution concentration or magnetic field one can determine a specific rotation of sugar or Verdet constant basing on the equation (3) or (4a).

Polarimeters in technology are often applied for determining concentration of optically active substances in solutions. Polarimeters for measurement of sugar concentration are called saccharimeters. Substances in liquid state or solutions rotating polarization plane owe this property to the structure of their particles. This is why, based on the optical activity measurement, one can obtain information on the structure of new complicated particles.

2. Measurements

I. Natural optical activity analysis

1. Turn the sodium lamp on. Wait about 5 minutes until the full light efficiency is reached. Set the viewfinder of the polarimeter in focus.
2. Put the tubular cuvette with distilled water inside the polarimeter. Make sure that the bottom of the container is directed downwards. Make sure there are no air remnants in water, and both glass windows have to be dry and clean.
3. Find such a setting of the analyzer so that all three sections (middle stripe and two side sections) of the view field were homogenously backlighted (half-shade setting).
4. Read the angle on the scale. The scale is marked with degree units and their decimal parts. Find the analyzer zero position α_0 for water. Prepare subsequently six aqueous solutions of sugar of a various concentrations.
5. Use the digital scale to prepare 1g, 2g, 4g, 6g, 8g, 10g portions of sugar.
6. Each portion of the sugar has to be dissolved in 50 ml of water. Calculate the concentration c.
7. Fill the cuvette with each of solutions. Before the measurement, purge the cuvette twice with a small quantity of the respective solution. Read the analyzer rotation angle α_p by finding the half-shade and calculate the rotation angle $\alpha = \alpha_p - \alpha_0$. Repeat each measurement several times and calculate the average value.
8. Measure the cuvette length h.
9. Fill the cuvette with the solution of unknown concentration and analyze it.

II. Faraday effect analysis

1. Turn the monochromatic light source on. (LED diode or sodium lamp; wait about 5 minutes for sodium lamp until the full light intensity is reached) Set the viewfinder in this way so that the view field image was sharp.
2. There is a glass rod in the polarimeter inside the solenoid. Set the half-shade position when the magnetic field is off. Note down the analyzer angle α_0. Repeat each measurement three times and calculate the average value.
3. Turn the solenoid power supply and, for at least 6 different current I values, measure the analyzer angles α_p corresponding to the new half-shade positions. Calculate the rotation of the polarization angles $\alpha = \alpha_p - \alpha_0$.

3. Results

Part I.

1. Use the equation (3) and assign: $y = \alpha$, $x = ch$, slope $b = \gamma$, basing on the least squares method results (ORIGIN software) figure out the specific rotation γ and combined uncertainty $u_c(\gamma)$ (consider standard uncertainties type A and B). **(The angle has to be converted to radians!)**
2. Using the χ^2 test, check if the hypothesis on the linear dependence of the rotation angle on the solution concentration is true.

3. Basing on the graph, find the concentration of the „unknown" solution and its uncertainty. Unknown concentration can be determined based on the equation (3) with application of the formerly found specific rotation γ. Find the extended uncertainty of the calculated concentration.

Part II.

1. Basing on the formula $B=\frac{\mu_0 IN}{L}$ calculate the magnetic field in Tesla units, corresponding to the given current I. ($\mu_0 = 4\pi\cdot 10^{-7}\frac{N}{A^2} = 4\pi\cdot 10^{-7}\frac{T\cdot m}{A}$; N - number of windings; L - solenoid length).
2. Using the least squares method, find the Verdet constant, using the formula (4a) and assigning: y = α, x = Bh, (use a computer and Origin software). **(The angle has to be converted into radians!)** Calculate the uncertainty of the Verdet constant.
3. Using the χ^2 test, check if the hypothesis on the linear dependence of the rotation angle on the magnetic field is true.
4. Basing on the formula (4b), using the calculated Verdet constant, calculate the $\frac{e}{m_e}$ relation value and its extended uncertainty. The dispersion value $\frac{dn}{d\lambda}$ used in the above formula can be found based on the formula $\frac{n_1-n_2}{\lambda_1-\lambda_2}$, where n_1 and n_2 are refraction indexes of waves λ_1 and λ_2, between which is the wavelength of the applied light source λ. The n(λ) dependence for glass is given on the plate near to the experimental setup. In this chart, one can read the refraction indexes for two wavelengths located symmetrically with the reference to the sodium wavelength (λ = 589,3 nm): for example for the wavelength symmetrically higher and lower by 100 nm. Compare with the chart value.

4. Questions

1. What physical parameters and equations are used to describe an electromagnetic wave?
2. What is a linearly polarized light?
3. What are the methods of obtaining a linearly polarized light?
4. What is a natural and forced optical activity?
5. Explain the principle of a half-shade polarimeter.
6. Explain the principle of Faraday effect.

5. References

1. D. Halliday, R. Resnick, J. Walker, Fundamentals of Physics, Wiley (2011), part 4, Chapter 33-7.