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LIGHT INTERFERENCE
NEWTON’S RINGS, MICHELSON’S INTERFEROMETER

1. Fundamentals

Interference is one of the most typical phenomena of the wave motion. Generally speaking, it is an effect of waves’ superposition which can lead to strengthening of the output wave (the overlapping waves are in phase) or its weakening (the overlapping waves are in opposite phases). The argument of the periodic function describing the propagating wave is called **phase**. The interference phenomenon is observable when superposing waves have to feature a constant phase difference i.e. they have to be coherent. If this condition is not fulfilled, there are some moments in time when the waves are in phase – at the chosen point of the space, leading to the strengthening, and in the other moments, on the contrary, they lead to weakening. The rapidly changing strengthening and weakening leads to lack of a constant image in time and the interference image cannot be observed.

Most of light sources are not coherent. This is the consequence of the fact that each atom, making a transition from the higher to lower energy level, emits a short wave pulse which is independent of the other atoms in the excited states. Even the light emitted by a monochromatic source (i.e. of a single wavelength) is a superposition of short wave pulses sent in a random way (their phases are uncorrelated). Thus the source as a whole is incoherent.

We can observe the interference even with the application of an incoherent light source if we can assure a mutual coherence of interfering rays (ray is a light flux of a very narrow cross section). An appropriate method of this is splitting a ray sent from one light source into two when each of them travels a different path and then causing them to superpose once again. One can assume that those two rays are sent from two mutually coherent sources. The mutual coherence of these rays will be however maintained only if the difference of paths they travel will not be too significant. If this condition is not fulfilled, then the ray that travels a longer distance “may not be able” to meet with its original wave and the mutual coherence will not be maintained.

1.1 Wave interference

In this chapter, we are going to discuss conditions for obtaining a stable interference image. We consider an optical system consisting of a lens and a glass plate – the created image will be called “Newton’s rings”. Generally, one can say that a stable interference image can be obtained only when the phase difference of waves of the same frequency will be constant at each moment of the observation of the phenomenon.

Let’s assume that two plain, harmonic electromagnetic waves 1 and 2 (having identical frequency ω and the same direction of linear polarization) propagate in the positive direction of the x axis. Those waves are described by their electric field values E_1 and E_2.

Let the wave 2 travels an additional distance Δ. Then the propagation of waves 1 and 2 can be described by the equations: $\mathbf{E_1 = E_{01}\sin(\omega t - kx)}$ **and** $\mathbf{E_2 = E_{02}\sin[\omega t - k(x+\Delta)]}$ where E_{01} i E_{02} denote wave amplitudes 1 and 2, $k = \frac{2\pi}{\lambda}$ is a **wave number**, and λ – **wavelength** (in air). When wave 2 travels the additional distance Δ in other medium the air, its wavelength in the other medium changes, and consequently the wave number k changes as well. If the refraction index at this path section is equal to *n*, the wavelength will decrease to the value $\lambda^1 = \frac{\lambda}{n}$, and the wave number $k^1 = \frac{2\pi}{\lambda^1} = \frac{2n\pi}{\lambda}$ will increase and will be equal to *nk*. The equation describing wave 2 for this case will be : $E_2 = E_{02}\sin(\omega t - kx - kn\Delta)$.

The product found in the sine function argument ***n*Δ** is called a **difference of optical paths (optical path = refraction index multiplied by geometrical path)**. Whereas the product *kn*Δ, characterizing the phase change caused by travelling of additional optical path is called the **phase shift angle *φ*** ($\varphi = kn\Delta = \frac{2\pi}{\lambda} n\Delta$).

Mathematical calculations describing superposition of waves 1 and 2 are presented in the Appendix at the end of the manual. Their main conclusion is that the intensity of the output wave is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi \quad (1)$$

The first component of the right hand side of the equation (1) (i.e. I_1) is the intensity of the wave 2, the second one is the intensity of the wave 2, whereas the third one describes result of the interference of waves 1 and 2. Depending on the **angle of phase shift** $\varphi = \frac{2\pi}{\lambda} n\Delta$, the value of this component is changing within the range:

from $-2\sqrt{I_1 I_2}$ (when cosφ = -1, and $\varphi = \frac{2\pi}{\lambda} n\Delta = (2m+1)\pi$ where m = 0,1,2,...)

to $2\sqrt{I_1 I_2}$ (when cosφ = 1, and $\varphi = \frac{2\pi}{\lambda} n\Delta = m2\pi$).

In the first case, the intensity will be weakened ($I = I_1 + I_2 - 2\sqrt{I_1 I_2}$), and in the second case - it will be strengthened ($I = I_1 + I_2 + 2\sqrt{I_1 I_2}$). **The most convenient way of constructing of the condition for weakening (or strengthening) of intensity is to do if with the reference of optical paths nΔ.** Based on the above considerations, one can conclude that **the weakening takes place when nΔ=(2m+1)λ/2, and the strengthening when nΔ=mλ.**

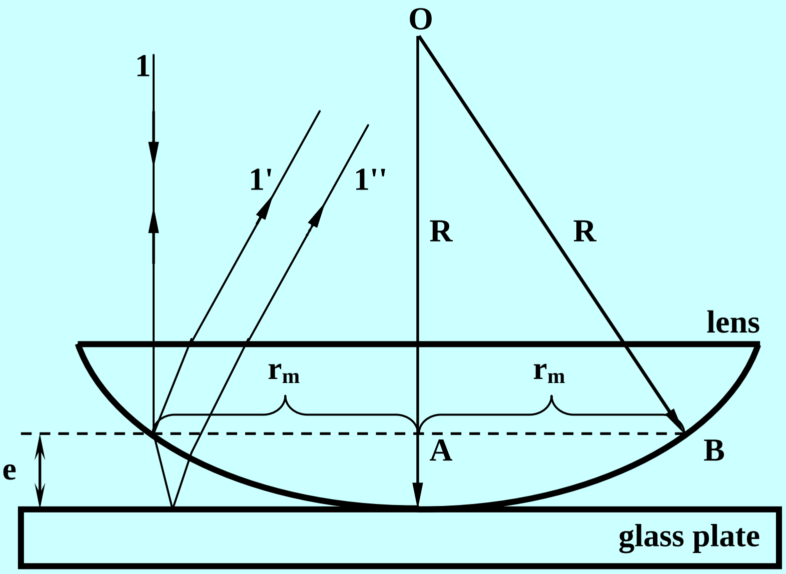
Let's analyze a particular situation when $I_1 = I_2 = I_0$. Once we plug these values to equation (1) we obtain $I = 2I_0 + 2I_0\cos\varphi$. For the strengthening (i.e. cosφ = 1) $I = 4I_0$. It means that at superposition of waves 1 and 2 the output intensity is four times higher than the intensity of the component wave than the intensity of the component wave. Does it mean that the energy conservation law is not conserved anymore? This seeming infringement of the energy conservation law can be easily explained when we take into consideration the fact that there are some points in the space where I = 0 (and the wave is extinct). So this is not an example of infringement of the energy conservation law but this is the example of energy redistribution. In the above considerations, the phase shift was caused by travelling of an additional path. However this is not the only cause of phase change. Light reflection, depending on the type of a reflection surface and an angle of incidence, can also lead to phase change (in an abrupt way). And for example, the light reflection from the optically more dense medium which is also an isolator, causes the phase shift of π.

2. Newton's rings

Let's lay a semi-convex lens with a large curvature radius on a glass plate in such way that the convex side touches the plate (fig. 1a). There will be an air gap of variable thickness between the lens and the plate. Let's illuminate this system with a monochromatic light of wavelength λ travelling perpendicularly to the plate surface.

The rays reflected from the convex side of the lens (1') can interfere with the rays reflected from the upper surface of the plate (1") as they are mutually coherent, because they originate from the split of the incident ray (1), and their optical path difference is not big (Δ <100λ). The other rays do not fulfill these conditions.

According to the previously presented considerations, the strengthening will take place when nΔ = $m\lambda$ (m=0,1,2,3...) and the weakening (attenuation) when $n\Delta = (2m+1)\frac{\lambda}{2}$. The optical path difference nΔ in this case (rys.1a) is equal 2e (as n=1 and the light travels e section twice).



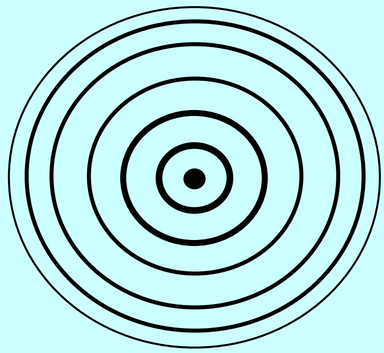


Fig.1b Newton's rings image seen in microscope

Fig. 1a Light ray paths creating the Newton's ring image: 1 - incident ray, 1' - a ray reflected on the convex side of the lens, 1'' - ray reflected on the top surface of the plate; R - the radius of curvature of the lens; r_m - radius of the Newton's ring of the m order

Due to the phase change into the opposite one at reflection from a more optically dense medium, one has to add $\frac{\lambda}{2}$ to 2e. The experimental confirmation of the mentioned above phase step is a creation of dark circle at the point where the lens touches the plate (it's a zero-order fringe). Taking into consideration the above remarks, the condition for extinction **(Newton's rings are dark!)** will be $2e + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$, and after the transformation:

2e = $m\lambda$. **(2)**

Now, let's link e with the other parameters that can be relatively easily measured. Looking at the AOB triangle (Fig. 1a) we see the relation $R^2 = r_m^2 + (R-e)^2$. After rising R-e to the power of 2, we obtain: $R^2 = r_m^2 + R^2 - 2\,\mathrm{Re} + e^2$. As e<<R the component with e^2 can be ignored. After a reduction we finally obtain: $2e = \frac{r_m^2}{R}$.

After plugging this component to (2), we obtain a relation between the Newton's ring radius r_m of an order m with the lens curvature R, wavelength λ and interference order m.

$r_m^2 = R\lambda m$. **(3)**

We have to underline ones again that the relation (3) is valid for **dark fringes**, in case of observation of rays reflected from the system of lens and plate.

3. Michelson's Interferometer

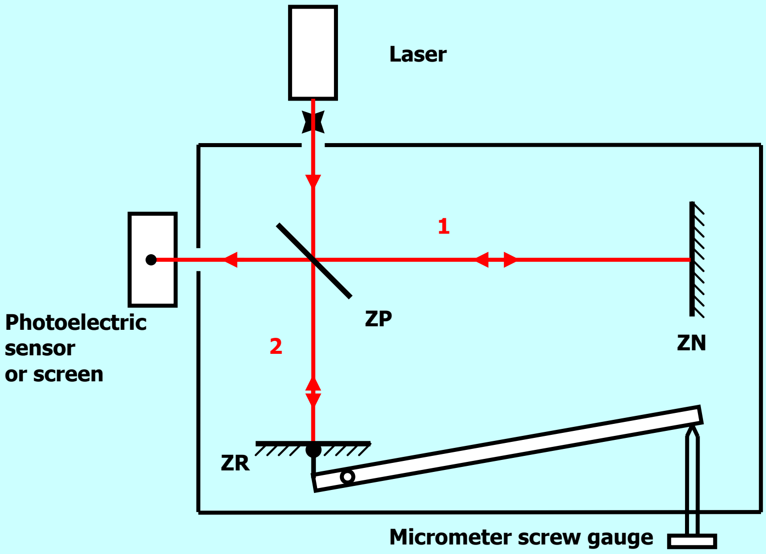


Fig.2. Michelson's interferometer scheme; ZP - half-silvered mirror; ZN - fixed reflecting mirror; ZR - adjustable reflecting mirror

The laser beam falls on a half-silvered mirror (a so-called beam splitter) that splits the beam into two beams: (1) one of them falls on the mirror **ZN** and after reflection falls on a screen or a sensor; the second beam (2) falls on the mirror **ZR** and after a sequence of reflections reaches the screen as well. Both beams interfere, creating an interference image on the screen. The shape of the interference image depends on the type of the applied mirrors. With application of a parallel beam and ideally plane mirrors the screen should be homogenously illuminated (from light to dark level) and the intensity of illumination should depend of the mutual setting of both mirrors i.e. the optical paths' difference of both beams. The shift of adjustable mirror **ZR** should cause changes of intensity of illumination of the screen in the range from the maximum value to the full extinction. In the laboratory setup, a divergent beam is used (the laser is equipped with a short focal length lens) as a result a phenomenon similar to the creation of Newton's rings. The interference rings are created on the screen. The shift of the mirror **ZR** leads to "rings' shift" as a result of change of conditions of strengthening at a given point of the screen. One has to remember that the shift of the mirror at a distance of **d** causes the change of optical paths of interfering rays by**2d**. Thus the condition of creation of maxima is:

$$N\lambda=2d \qquad (4)$$

There is a lever **1:10** in the mirror shift, so the mirror shift is ten times smaller than the shift shown on the gauge.

The Michelson's interferometer is an example of application of the interference phenomenon in measurement devices. This device is usually used for measurement of the wavelength or measurement of very small displacements comparable with a wavelength used in the interference.

Michelson's interferometer is an example of application of the interference phenomenon in measurement devices. It is usually used for measurement of the wavelength or measurement of very small displacements of the same order of magnitude as the light source wavelength. The Michelson's interferometer significantly supported the progress in physics as it was used, among others, in the **Michelson-Morley experiment**. This experiment is the experimental base for the special theory of relativity. It was carried out to confirm or deny the existence of ether and dependence of the speed of light on the direction of its propagation. The decisive measurement were carried out by Albert Michelson and Edward Morley at the beginning of July 1887. They come to the following conclusion "There is no significant difference in the speed of light independent of the direction of motion of the observer". (American Journal of Science, nr 207, 1887).

On 15th January 1931 after the scientific conference there was a banquet to honor Albert Einstein, during which he said among others following words: "It was you, my honored Dr. Michelson, (…) who led the physicists into new paths, and through your marvelous experimental work paved the way for the development of the theory of relativity. You uncovered the insidious defect in the ether theory of light as it then existed. " In 1907, A. Michelson was awarded a Nobel Prize "for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid".

It is notable that Albert Abraham Michelson was born on 19th December 1852 in Strzelno, Kujawy, Poland (Prussia at the time) in a family of a Jewish merchant. The Michelsons family left Strzelno in 1855 and moved to the United States and thus in all the encyclopedias is described as an American scientist of Prussian (more often) or Polish (unfortunately less often).

4. Measurements

Newton's rings

1. Turn the monochromatic light source on (for example the sodium lamp of the wavelength: λ=589.3nm).
2. Place the convex lens with the plain plate on XY table and find the focus image of the Newton's rings.
3. Measure the **diameters** (not radiuses as it is difficult to find the centre location) of 10 Newton's rings in x and y direction, writing down their interference order m.
4. Using the light of unknown wavelength, measure 10 diameters of Newton's rings (writing down *m* order). The light of unknown wavelength can be obtained by letting the white light through the interference filter.

Michelson's Interferometer

<u>Operation of the electronic frequency meter-counter</u>

The device connected to the photodetector is a universal meter that can be used as frequency meter, timer or pulse counter. In this experiment, it is used as pulse meter.

1. Turn the device on - the display will show "**P1-F**" and the device will start in frequency measurement mode. (If the external light is on, the display should show the value of approx. 100.)

2. Press the right top button twice (labeled **UP**). The display will show "**P3-CU**" and the device will switch to pulse counting mode. (If the room light is on, the meter should count the pulses of fluorescent lamps.)

3. Turn the room lights off – the counter should stop counting.

4. The left bottom button serves as the counter reset. If the micrometer screw is at the desired position, reset the counter (the display should show zero).

CAUTIONS:

The interferometer is a very precise and sensitive optical device. All the operations should be performed with care. The measurement range is within the range of 2.5 to 4.5 mm on the micrometer screw (green laser). During the measurement the screw has to be rotated VERY SLOWLY in one direction only (do not turn the screw in the opposite direction).

The fringes should be clearly visible, and their width on the screen should exceed the diameter of the hole. During the measurement the screw step should remain within the 0.2 - 0.5 mm range (the mirror shift is 10 times smaller!).

Measurement procedure:

1. Set the screw position according to the recommended value given on the plate.
2. Reset the counter.
3. Rotate the screw SLOWLY by about 0.2 mm for example (less than 0.5 mm).
4. Repeat the measurement several times by turning the screw in both directions (please remember not to exceed the measurement range) by 0.1; 0.2; 0.3; 0.4 i 0.5 mm respectively.
5. Based on the measurement result find the laser light wavelength and estimate the uncertainty (do not surpass the screw measurement range!).

5. Results

Newton's Rings

1. Plot a graph r_m^2 vs. λ_m for the light of the known wavelength.
2. By applying the least squares method and using the equation (3) with substitutions $y = r_m^2$ and $x = \lambda_m$ find R. Figure out u(R). Based on the χ^2 test decide whether the equation (3) is true or false.
3. For the light of unknown wavelength (for both colors), plot the graph r_m^2 vs. R_m (where R - convex radius found in the step 2) to find λ, using the R value found in the step 2. Calculate the combined uncertainty $u_c(\lambda)$ and extended uncertainty $U_c(\lambda)$. Report your final results correctly.
4. Comment on the results you have found.

Michelson's Interferometer

1. Convert the equation (7) in the form that will enable you to apply the least squares method.
2. Plot a graph in Origin software and based on the linear approximation find the unknown wavelength λ. Calculate the uncertainty. What conclusions can be drawn from the χ^2 test results?

6. Questions (a full list is available on the laboratory website)

1. What conditions have to be fulfilled to make the interference phenomenon observable?
2. How to obtain mutual ray coherence?
3. How to calculate the output intensity of interfering waves?
4. What are the conditions of strengthening (weakening) of the output wave intensity in the interference phenomenon?
5. How to obtain the Newton's rings image (their mathematical description)?
6. What is the Michelson's interferometer?
7. How to figure out the laser wavelength based on the measurement of fringes' shift in the Michaleson's interferometer?

7. References

1. D. Halliday, R. Resnick, J. Walker, Fundamentals of Physics, Wiley (2011), part.4, chapter 35.

Appendix

Wave superposition - mathematical calculations

Let's assume that two plane, harmonic electromagnetic waves 1 and 2 (having identical frequency ω and the same direction of linear polarization) propagate in the positive direction of the x axis. Those waves are described by their electric field values E_1 and E_2.

Let the wave 2 travels an additional distance Δ. Then the propagation of waves 1 and 2 can be described by the equations: $\mathbf{E_1 = E_{01}\sin(\omega t - kx)}$ **and** $\mathbf{E_2 = E_{02}\sin[\omega t - k(x+\Delta)]}$ where E_{01} i E_{02} denote wave amplitudes 1 and 2.

Let's calculate the result of superposition of those two waves.

$$E = E_1 + E_2 = E_{01}\sin(\omega t - kx) + E_{02}\sin(\omega t - kx - \varphi) \quad \textbf{(D1a)}$$

Electromagnetic wave detectors (including our eyes) react on the wave intensity I, i.e. **the average amount of energy falling on the unitary area in the time unit**. The energy carried by the wave is proportional to the square of the intensity of electric field. For the case we analyze (see (D1a)) the energy will be proportional to:

$$E^2 = (E_1 + E_2)^2 =$$
$$= E_{01}^2\sin^2(\omega t\text{-}kx) + E_{02}^2\sin^2(\omega t\text{-}kx\text{-}\varphi) + 2E_{01}E_{02}\sin(\omega t\text{-}kx)\sin(\omega t\text{-}kx\text{-}\varphi) \quad \textbf{(D1b)}$$

According to the trigonometric equation $\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$, the last component of the equation for E^2 can be converted into the following form:

$$2E_{01}E_{02}\underbrace{\sin(\omega t\text{-}kx)}_{\frac{\alpha+\beta}{2}}\underbrace{\sin(\omega t\text{-}kx\text{-}\varphi)}_{\frac{\beta-\alpha}{2}} = E_{01}E_{02}\{\cos(\underbrace{\varphi}_{\alpha}) - \cos[\underbrace{2(\omega t\text{-}kx)\text{-}\varphi}_{\beta}]\}$$

Taking into consideration the last result, E^2 can be expressed as:

$$E^2=E_{01}^2\sin^2(\omega t\text{-}kx)+E_{02}^2\sin^2(\omega t\text{-}kx\text{-}\varphi)+E_{01}E_{02}\{\cos\varphi\text{-}\cos[2(\omega t\text{-}kx)\text{-}\varphi]\} \quad \textbf{(D2)}$$

According to the above equation the energy carried by the wave depends on time. However, the detector registers not an instantaneous value of the wave intensity, but the value of the energy flux average in time. For the analyzed electromagnetic waves, one can calculate this average based on the equation:

$$\left\langle E^2\right\rangle = \frac{1}{T}\int_0^T E^2 dt \quad \textbf{(D3)}$$

Based on (D2) and (D3), finding $\langle E^2\rangle$ can be limited to calculation of average values of the functions of type $\sin^2(\omega t + \delta)$ i $\cos(2\omega t + \gamma)$ within their period.

Averaging of the first mentioned above function gives $\frac{1}{2}$ and the second one 0. Thus, we obtain:

$$\left\langle E^2\right\rangle = \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01}E_{02}\cos\varphi\,. \quad \textbf{(D4)}$$

Finally the intensity of the output wave will be equal to:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi. \quad \textbf{(D5)}$$