Warsaw University of Technology
Faculty of Physics
Physics Laboratory I „P”

12

RELAXATION PROCESSES IN ELECTRICAL CIRCUITS

1. Fundamentals

Relaxation processes are a very common class of phenomena taking place in nature. They can be generally described as a transition of a macroscopic system to an equilibrium state. (We understand here the equilibrium state as the state of a minimum thermodynamic potential). These processes are irreversible, as they are accompanied by energy dissipation – i.e. conversion of part of energy into heat.

Relaxation processes cover both thermodynamic phenomena – heating up and cooling down of bodies, gas expansion and radioactive decay, as well as charging and discharging of capacitors in electric RC circuits.

All the mentioned here phenomena have one feature in common: the process speed $\left(-\frac{dy}{dt}\right)$ is proportional to $y(t)$ deviation from equilibrium state at a given moment in time. This means that at the beginning of the relaxation process the change of describing it parameter is dramatic and it decreases to zero as time passes to infinity.

These phenomena are mathematically described with:

- decreasing exponential functions (energy dissipation processes – the system is undisturbed at equilibrium state and $y_e = 0$) or $y(t) = y_0 e^{-\lambda t}$ (fig.1) or
- complementary exponential functions (local energy accumulation, at the initial moment the system energy equals to 0) or $y(t) = y_e(1 - e^{-\lambda t})$ (fig. 2).

In the above formulas *y(t)* is a momentaneous value of a physical parameter describing a given process, y_0 -its initial value, y_e - final value, *e* - base of a natural logarithm (e = 2,72), λ - coefficient of proportionality of a physical unit that is reciprocal of time. (Derivation of the above relations can be found in **Appendix 1** at the end of this manual.)

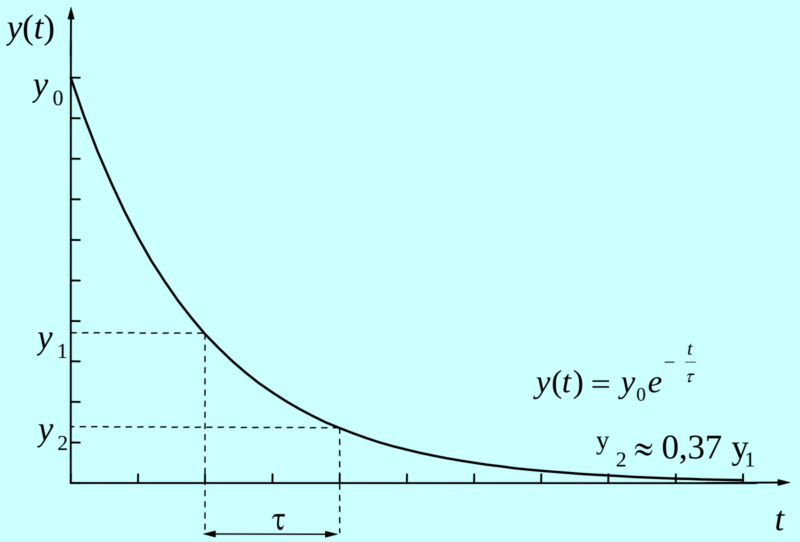


Fig.1. Reaching the equilibrium state through relaxation with decrease of y value. τ - relaxation time, y_0 - initial value of a changing parameter.

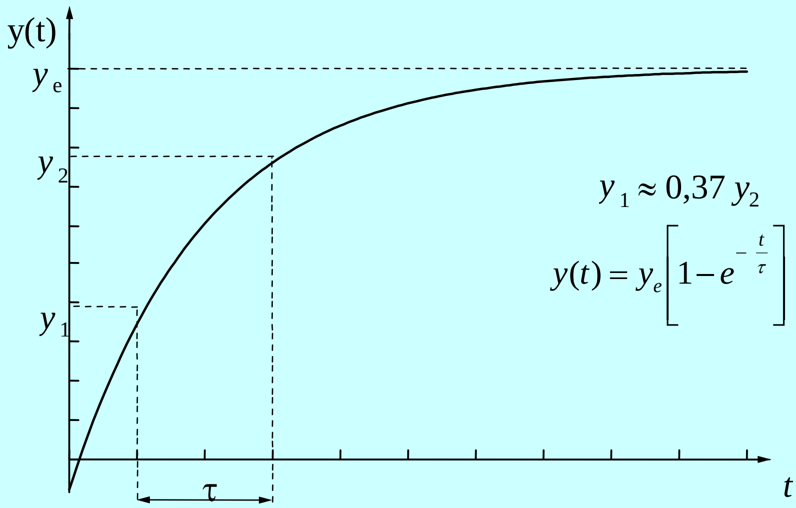


Fig. 2. Reaching the equilibrium state through relaxation with increase of y value. τ - relaxation time, y_e - end value of a changing parameter.

Instead of λ coefficient, one uses $\tau = \frac{1}{\lambda}$ parameter which is represented in time domain. This quantity is called **relaxation time**. The above equations can be then re-written as:

$$y(t) = y_0 e^{-\frac{t}{\tau}} \quad (1)$$

$$y(t) = y_e(1 - e^{-\frac{t}{\tau}}) \quad (2)$$

The interpretation of τ value is very simple**: relaxation time is the time Δ*t*=τ after which observed parameter will decrease *e* times.** The advantage of using such parameter as a process descriptor is independence of τ value of the choice of the observation moment. The τ itself characterizes the speed of the progress of relaxation process.

Both processes of forced energy increase and its dissipation can occur alternately if during the energy transfer from the environment to the system, the system reaches the unstable equilibrium state. At this moment the system can spontaneously dissipate the aggregated energy **many times faster** than it is transferred. In case of a **continuous energy supply**, its changes will take place periodically as each time the unstable equilibrium is reached the energy obtained will be dissipated. The oscillations called **relaxation oscillations** will be created in the system. These oscillations proceed in a completely different way than harmonic oscillations.

2. Experiment

In order to observe relaxation process of energy increase in a system, dissipation of energy and creation of relaxation oscillations, we are going to analyze electric circuit RC (consisting of resistors and capacitors). The circuits will be additionally extended with a neon lamp that can open or close a circuit depending on the voltage applied to its terminals.

A direct current (DC) cannot flow through a circuit containing a resistor *R* and a capacitor *C* in series. In such circuits, equipped additionally with an ammeter (fig. 3 and 4), once they are closed we can only notice a briefly lasting capacitor charge/discharge current. *K* switches are used for closing circuits and discharging capacitors before the next measurement.

We use **Kirchhoff's voltage law (KVL or Kirchhoff's second law)** telling that the directed sum of the electrical potential differences (voltage) around any closed network is zero.

2.1. Charging a capacitor in the RC circuit

Closing the switch *K* in the circuit presented in fig. 3 causes flow of a direct current of a maximum value $I_o=\varepsilon/R$ where **ε is the electromotive force of the power supply.** After the *K* switch is open, the capacitor *C* is being charged - the charge *q* is being collected on the capacitor's plates. This leads to the voltage increase (U_C) on the capacitor and to the simultaneous decrease of the charging current.

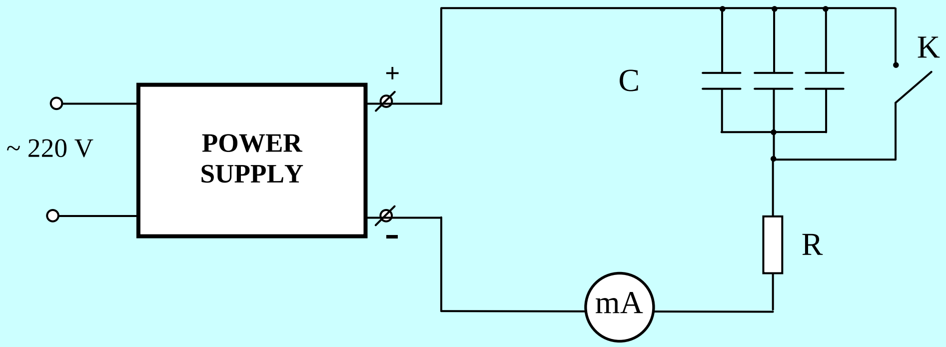


Fig.3. Capacitor charging circuit.

Kirchhoff's voltage law for the capacitor charging circuit can be written as:

$$\mathcal{E} = IR + \frac{q}{C}\ , \tag{3a}$$

where *IR* denotes a momentaneous potential drop (voltage) on the *R* resistor, and q/C - the momentaneous potential difference on the capacitor's plates. Taking into consideration that current $I=dq/dt$, we obtain a differential equation of one variable *q* which can be written as:

$$\mathcal{E} = \frac{dq}{dt}R + \frac{q}{C}\ . \tag{3b}$$

Through some elementary transformations leading to separation of variables *q* and *t* (see Appendix 1), we obtain the last equation in the form:

$$\frac{dq}{q-\mathcal{E}C} = -\frac{1}{RC}dt\ . \tag{3c}$$

After integrating both sides and taking into consideration initial conditions $q(t=0)=0$, giving the value of integrating constant $A=-\varepsilon C$, we obtain solution for a dependence of the charge *q* with respect to time during the process of charging the capacitor as a complementary exponential curve (compare with equation (2)):

$$q(t) = \mathcal{E}C\left(1-e^{-\frac{t}{RC}}\right) = q_e\left(1-e^{-\frac{t}{RC}}\right)\ . \tag{4}$$

According to the above formula, the charge increases exponentially from the value $q = 0$ to the value $q_e = \varepsilon C$. The voltage changes in analogical way. This conclusion results directly from the relation between charge and voltage on the capacitor plates U_c.

$$U_C(t) = \frac{q(t)}{C} = \mathcal{E}\left(1-e^{-\frac{t}{RC}}\right), \tag{4a}$$

The charging current flowing after the circuit is closed, as a derivative of charge with respect to time is described with the exponential formula:

$$I(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} \quad . \tag{5}$$

2.2. Discharging a capacitor in the *RC circuit*

A circuit in which we are going to analyze discharging characteristics is shown in fig. 4.

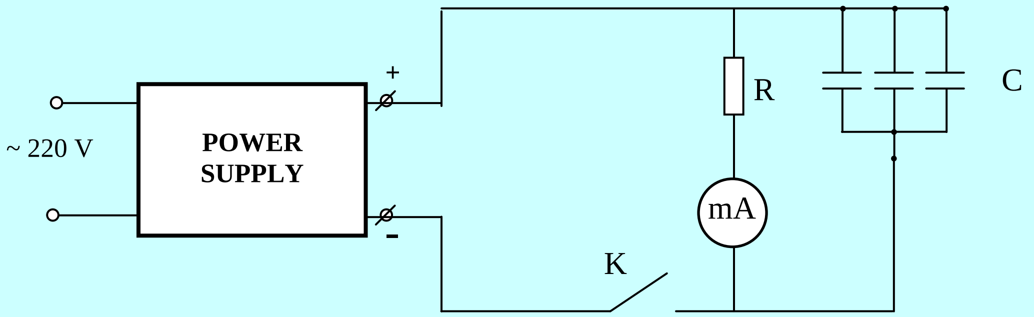


Fig.4 Capacitor discharge circuit.

Capacitor *C* that has already been charged to the voltage ε is being discharged through the resistor *R*. When any electromotive force is absent in discharge circuit, the Kirchhoff's voltage law can be written as:

$$IR + \frac{q}{C} = 0 \ , \tag{6a}$$

where, like previously, *IR* expresses drop of potential on the resistor, and *q*/*C* is a momentaneous potential difference on the capacitor's plates. Taking into consideration the relation between current and charge, we obtain a differential equation of one variable *q* which can be written as:

$$\frac{dq}{dt} R + \frac{q}{C} = 0 \ , \tag{6b}$$

then after separating the variables we obtain an analogical formula to (3c) when ε = 0:

$$\frac{dq}{q} = -\frac{1}{RC} dt \ , \tag{6c}$$

which is a typical formula for processes of relaxation dissipation. Its solution describes a momentaneous value of charge *q*(*t*) on the capacitor's plates (compare equations (4) and (5)):

$$q(t) = \mathcal{E} C e^{-\frac{t}{RC}} = q_0 e^{-\frac{t}{RC}} \ . \tag{7}$$

Based on the relation between the voltage and the charge on capacitor plates we obtain a time dependence of voltage changes in discharge process:

$$U_C(t) = \frac{q(t)}{C} = \varepsilon e^{-\frac{t}{RC}} \ , \tag{8}$$

additionally, after differentiating formula (7), we obtain a time dependence of discharge current:

$$I(t)=\frac{dq}{dt}=\frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}=I_0e^{-\frac{t}{RC}}\,. \tag{9}$$

Comparison of exponents in equations (7) and (9) with exponents in equations (1) and (2) indicates that for analyzed circuits λ = 1/*RC*, and relaxation time τ = *RC*, where *R* is a resistance and *C* capacitance in the circuit.

2.3. Relaxation oscillations in RC circuits

Process of obtaining energy from environment by a system can be interrupted if the reached state is the state of instable equilibrium and the obtained energy can be dissipated rapidly. If the process of energy supplying lasts continuously, the aggregation of energy will be continued until the moment of reaching the state of instable equilibrium and energy dissipation.

In the system shown in fig. 5, the process of energy aggregation and dissipation will periodically take place which is called **relaxation oscillations**. In the serial *RC* circuit "looping" capacitor charging and discharging is obtained through connection of **neon lamp** (fig. 5a) to the capacitor plates. For **voltages lower than ignition (or striking) voltage U_i the lamp practically does not conduct any current (resistance near to infinity)** and does not disturb capacitor charging process by the power supply (fig. 5b). As soon as the charge *q* is aggregated on capacitor's plates for which the potential voltage is U_i, the avalanche ionization of the gas inside the bulb takes place and the conductivity of the bulb increases by several orders of magnitude. The further increase of voltage between the capacitor plates is interrupted as a result of short circuit its plate and the capacitor discharge process takes place (fig .5c). We have to remember that **the discharge process in being continued** (the capacitor **WASN'T** disconnected from the power supply), but the discharging current is higher than the charging current. Such a situation can only take place if the neon lamp resistance in conduction state R_n is **LOWER** than the serial resistance R.

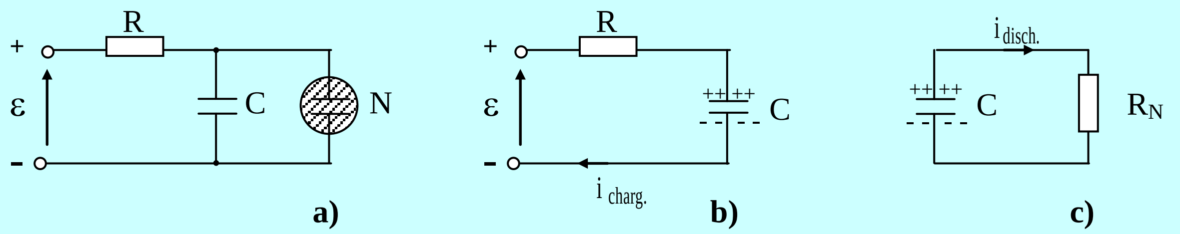


Fig. 5 Circuit for relaxation oscillation analysis.

Neon lamp resistance R_N in conduction state is many times **lower** that the resistance of charging circuit, so the speed of energy dissipation exceeds its aggregation speed. A specific feature of neon lamp is maintenance of avalanche ionization despite voltage decrease and its definitive decline only when voltage is lower by dozen or so volts which is called extinction voltage U_e.

At this moment, the neon lamp resistance R_N increases once again to the value near to infinity and the current in the branch with the neon lamp stops flowing. In this system, there are periodic capacitor charging - energy aggregation (in circuit shown in fig. 5b) and discharging - energy dissipation (in circuit shown in fig. 5c). Voltage on capacitor plates increases and decreases exponentially, oscillating between the values U_i and U_e (compare fig. 6a and 6b). The changes of voltage during the charging process are described with a complementary exponential function $U_C=\mathcal{E}(1-e^{-\frac{t}{RC}})$ (see fig. 4a).

The capacitor is being charged in t_1 time: from voltage $U_c(t) = U_e$ to $U_c(t+t_1) = U_i$. Equations for the boundary values can be written as:

$$\mathcal{E}-U_e=\mathcal{E}\cdot e^{-\frac{t}{RC}}\;;\qquad \mathcal{E}-U_i=\mathcal{E}\cdot e^{-\frac{t}{RC}}e^{-\frac{t_1}{RC}}\,. \tag{10}$$

After dividing both equations side by side and logarithmizing we obtain a formula for capacitor charging time t_1:

$$t_1 = RC\ln\left(\frac{\varepsilon - U_e}{\varepsilon - U_i}\right) = RCK \text{, where } K = \ln\left(\frac{\varepsilon - U_e}{\varepsilon - U_i}\right) \quad (11)$$

Voltage changes during the discharging process (8) are described with exponential equation $U_c(t) = U_0 e^{-\frac{t}{R_N C}}$ and they progress from voltage $U_c(t) = U_i$ to voltage $U_c(t + t_2) = U_e$, related with a formula:

$$U_e = U_i e^{-\frac{t_2}{R_N C}} \quad (12)$$

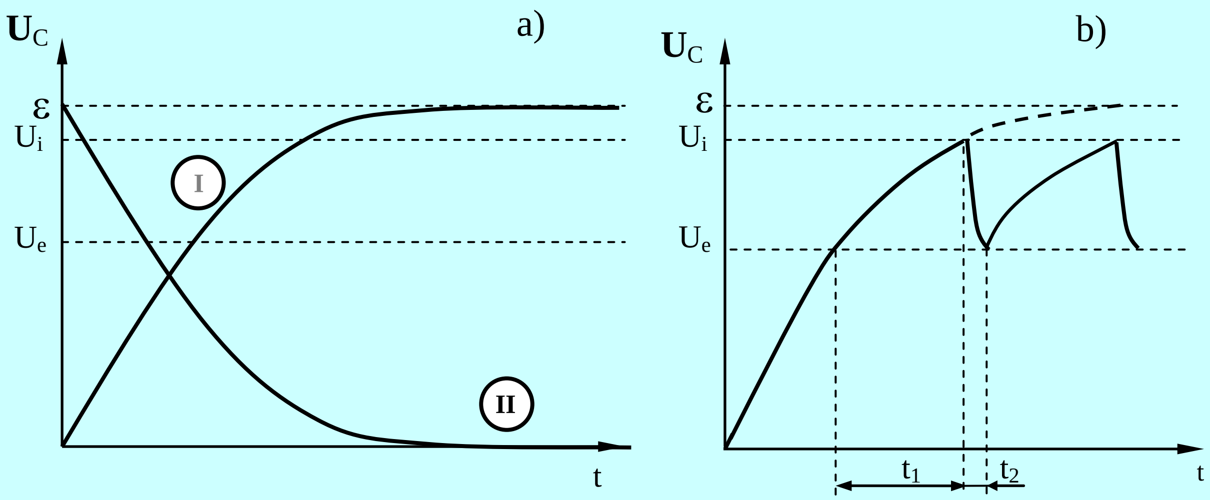
Thus the discharging time t_2 is:

$$t_2 = R_N C \ln\left(\frac{U_i}{U_e}\right). \quad (13)$$

The period of relaxation oscillations T is a sum of charging t_1 and discharging times t_2:

$$T = t_1 + t_2 \quad (14)$$

Fig. 6b shows voltage changes $U(t)$ on capacitor plates with respect to time, created graphically through tiling respective sections of capacitor charging and discharging curves $U_c(t)$ (fig. 6a). Thanks to gas glowing phenomenon, we can observe this process directly and measure the relaxation oscillation period.



Rys.6. Changes of capacitor voltage during the relaxation oscillations.
a) changes of capacitor voltage: charging curve (I), discharging curve (II) in a circuit without a neon lamp;
b) changes of capacitor voltage during the relaxation oscillations.
U_i - neon lamp ignition voltage, U_e - neon lamp extinction voltage, ε - power supply voltage, t_1 - capacitor charging time, t_2 - capacitor discharging time.

3. Measurements

The number of measurements depends on the time assigned to the experiment and on the supervisor's request. A stopwatch is needed to perform the measurements.

3.1 Capacitor charging (discharging) process analysis

The circuits used for analysis of capacitor charging or discharging process are built based on diagrams shown in fig. 3 or fig. 4. The switch K (fig. 3) connected to capacitor plates is used for its discharging before the next measurement and it should be open during the measurement. Switch K in fig. 4 has to be shortcut for a brief moment to charge the capacitor and then it has to open to begin the discharging process.

Use the table templates shown below (table 1), to report capacitor charging (discharging) currents read every 5 seconds. Stop the measurements after the time $t=3\tau$ when their value falls to about 5% of the initial value.

Table 1

C[μF]	R[kΩ]	t(s) / I_0 (μA)	0	5	10	15	20	25...		
C_1=	R_1 =									
	R_2 =									
C_2 =	R_1 =									
	R_2 =									

3.2 Finding relaxation oscillations period

As we remember, thanks to the phenomenon of neon gas glowing in a neon lamp accompanying avalanche ionization, we can observe this process directly and measure oscillation period.

Relaxation oscillation period T is a sum of capacitor charging t_1 and discharging t_2 times, $T = t_1 + t_2$. The relaxation time of discharging circuit $\tau = R_N C$ is very short due to low resistance of the neon lamp in conduction state (avalanche ionization) R_N. This is why observation of oscillations with stopwatch in hand requires, due to limited speed of human perception, extension of period through extension charging time requiring high values of resistance R in charging circuit. As a result, due to the above condition, the time ratio is

$\frac{t_1}{t_2} = \frac{R}{R_N} \frac{\ln[(\mathcal{E}-U_e)/(\mathcal{E}-U_i)]}{\ln(U_e/U_i)} >> 1$ and relaxation oscillation period $T \cong t_1$. On the other hand, observation of oscillations on the oscilloscope's screen requires quick waveform due to difficulties in synchronization of slow waveform. For this reason, a resistor R of a resistance comparable to R_N has to be used in the circuit. Thus the oscillation period in this circuit will be equal to $T = t_1 + t_2$. To calculate t_1 and t_2 times, neon lamp ignition U_i and extinction U_e voltages are required as well as power supply voltage ε (see formulas (11), (13)).

3.3. Measurement of ignition U_i and extinction U_e voltages of neon lamp

1. Build a circuit shown in fig. 7.
2. Turn the voltage knob on the power supply very slowly to increase the voltage till the moment the lamp ignites.
3. Write down the highest voltage read **BEFORE** the ignition, U_i (when the lamp ignites, the voltage falls by a dozen or so volts to the so-called maintaining voltage).
4. Reduce the voltage and write down U_e voltage at each the gas stops glowing. Repeat the measurement several times, and write down the results in table 2. Calculate their average values. Consider the uncertainties type A and B.

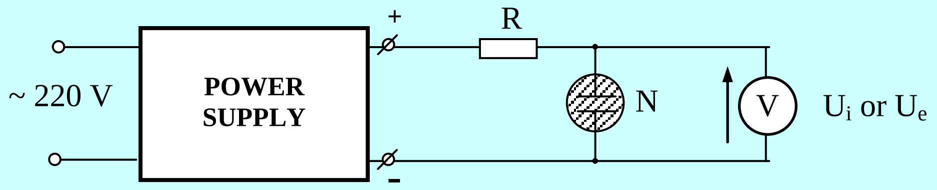


Fig. 7. Circuit for measurement of neon lamp ignition and extinction voltages.

Table 2.

n / U[V]	1	2	3	4	5	6	...	...	U_{av}
U_i									
U_e									

3.4. Analysis of oscillation period dependence with respect to resistance R and capacitance C

1. Build a circuit shown in fig. 8.

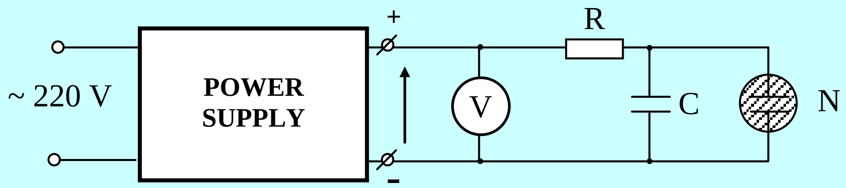


Fig. 8. RC circuit with a neon lamp for observation of relaxation oscillations.

2. Set such a voltage U, to see neon lamp flashing for each measured resistance R (the power supply voltage has to remain constant during the measurement).
3. Measure at least twice time of n=20 flashes of neon lamp for various subsequent R values. The measurement result and the calculated oscillation period have to be written in Table 3.
4. Estimate the uncertainties R, C, U_i, U_e, U.

Table 3

C [F]	R [kΩ]	t_{20} [s]	T_{exp} [s]
C_1 =	R_1 =		
	......		
	R_k =		
C_2 =	R_1 =		
	....		
	R_k =		

3.5. Observation of relaxation oscillation on the oscilloscope screen

1. Build a circuit shown in fig. 9. Make sure to connect correctly the GND terminal of the power supply and the oscilloscope, and to use in the circuit a resistor *R* of a lower resistance.
2. Basing on observations of the waveform in the screen, estimate the capacitor's charging and discharging times.

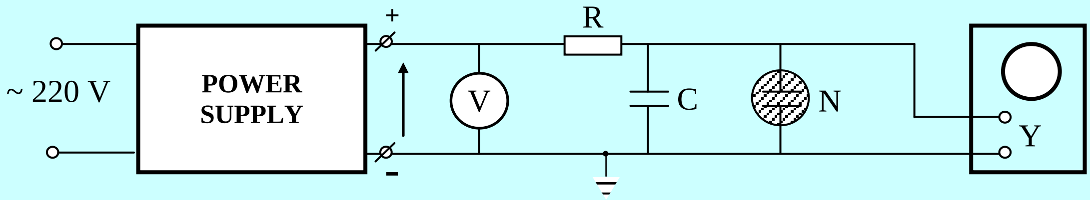


Fig. 9. Circuit for observations of relaxation oscillations in oscilloscope screen.

4. Results

4.1. Capacitor charging (discharging) process analysis

1. Using a software, plot graphs of charging (discharging) currents in the coordinate system $I = f(t)$ and $\ln I = f(t)$. In the first case, the measurement points should be spread along the exponential curve and in the second case – along the straight line.
2. Figure out the relaxation times of analyzed processes for various values of the *RC* product, with application of the method **chosen by the supervisor**.
   a) **Method based on finding the slope of the ln *I* = *f*(*t*) graph**. In this coordinate system, measurement points should be spread along the straight lines $\ln I = -t/\tau + \ln I_0$, ($y = bx + a$), so the calculation of $\tau = -1/b$ is limited to calculation of reciprocal of the slope of the line with application of the least squares method (in Origin software). Confirm or deny the exponential dependence of the current with respect to time, with application of the χ^2 test.
   b) **Method based on the relaxation time τ definition**. In the graph $I(t)$, on the current axis, mark values I_1 and I_2 with the ratio $I_2 = I_1/e$. Then the difference of their abscissa determines the time range $\Delta t = \tau$. Repeat the same procedure for several various point on the plot and calculate the average value.
   Ask the supervisor to choose the *τ* calculation method to be applied. The results have to be reported in the table together with *τ* value calculated based on the half-life time and the product $\tau = RC$.
   The combined uncertainty has to be calculated according to the chosen method.

3. Report the calculation results and measurement uncertainties in table 4.

Table 4

R [kΩ] C [μF]	RC [s]	$u(\tau_{calc})$ [s]	τ_{exp} [s]	$u(\tau_{exp})$ [s]
R_1= C_1=				
R_1= C_1=				
....				

4.2 Analysis of relaxation oscillations

1. Figure out the combined uncertainty $u_c(T)$ based on the estimated uncertainties R, C, U_i, U_e, U.
2. Compare the measured periods T_{exp} with the ones calculated according to the formula $T_{calc} = RCK$ (fig 11), and report the results in table 5.
3. Create plots $T = f(R)$ for constant C values.

Table 5

R [kΩ] C [μF]	T_{exp} [s]	T_{calc} [s]	$u(T_{exp})$ [s]	$u(T_{calc})$ [s]
R_1, C_1				
……..				
R_k, C_j				

5. Questions

1. Describe properties of the system in which relaxation oscillations can take place.
2. Compare relaxation and harmonic oscillations.
3. Why do we use *RC* circuit to analyze relaxation oscillations?
4. What changes in the capacitor voltage U_c waveform we are going to see in the oscilloscope screen after the replacement of the neon lamp with another one of parameters $U_i' = 2U_i$ i $U_e' = U_e$?
5. Describe operation principles of a neon lamp.

6. References

1. D. Halliday, R. Resnick, J. Walker, Fundamentals of Physics, Wiley (2011), part 2, Chapter 31.

APPENDIX 1

The differential equation describing the speed of changes of y value in time is:

$$\frac{dy}{dt} = -\lambda[y(t) - y_k] \quad , \qquad (A1)$$

where λ is the proportionality coefficient. We can find the solution very easily when y decreases from the initial value y_0 to the final value $y_k (y_0 > y_k)$. This is the differential equation of the first order that can be solved with the variable separation method. By multiplying both sides of the equation (1) by the expression $\frac{dt}{y - y_k}$ and integrating it side-by-side (for the simplicity sake, let's assume the equilibrium condition $y_k = 0$), we will obtain a formula:

$$\int \frac{dy}{y - y_k} = -\int \lambda dt \qquad (A2)$$

whose primitive function (antiderivative) is function $\ln(y - y_k)$

$$\ln(y - y_k) = -\lambda t + \ln A \qquad (A3)$$

and the solution after taking into consideration the initial condition $A = y(t = 0) = y_0$, and $y_k = 0$ will be:

$$\ln y = -\lambda t + \ln y_0 , \qquad (A4)$$

because

$$\ln y - \ln y_0 = \ln \frac{y}{y_0} \qquad (A5)$$

so

$$\ln \frac{y}{y_0} = -\lambda t \qquad (A6)$$

after the transformation we obtain the exponential function form:

$$\frac{y}{y_0} = e^{-\lambda t} \qquad (A7)$$

so

$$y(t) = y_0 e^{-\lambda t} \qquad (A8)$$

In the reverse process when the equilibrium state is reached through the increase of y value, its changes in time are described with a complementary exponential dependence:

$$[1 - \exp(-\lambda t)]. \qquad (A9)$$

After taking into consideration in equation (A3) initial condition y(0) = 0 leading to the constant condition $A = y_k$, we obtain an equation:

$$\ln(y - y_k) = -\lambda t + \ln y_k \qquad (A10)$$

which, after a transformation will have a form of a complementary exponential function:

$$y(t) = y_k (1 - e^{-\lambda t}) . \qquad (A11)$$

APPENDIX 2

Neon lamp properties

A neon lamp is a glass bulb with two electrodes merged and filled with a noble gas under a reduced pressure (order of magnitude of kPa). At a low gas pressure, a free path of particles is extended, making it easier to obtain a glowing discharge while a low voltage of several tens of volts is applied to the lamp electrodes. We are going to present here a simplified description of this phenomenon.

Electrical conductivity depends on the number of charge carriers. For a gas, charge carriers concentration is a dynamic value and its momentaneous value is defined by the velocity of generation and recombination of carriers.

When the external voltage is absent, gas in the neon lamp is ionized thorough a thermal dissociation and radiation into positive (cations) and negative (anions) inions what happens only at insignificant level and the carrier concentration level is near zero.

Electric field, created by the voltage applied to lamp electrodes, pushes the existing charges towards the respective electrodes, creating a current of a very low value. The lamp resistance is low at that time but its value is finite. The increase of voltage leads to increase of drift velocity which results in increase of the current until the saturation current is reached which corresponds to removal of all carriers present between the electrodes. However, due to insignificant carrier concentration, gas enclosed in the bulb can be treated as an insulator.

At higher voltages, electrons accelerated in electric field reach kinetic energy comparable with ionization energy of gas particles. Ionization energy is the energy sufficient to separate electron from a gas particle which leads to transformation of an electrically neutral molecule into a free electron and a positively charged cation. When the kinetic energy of electrons, due to increase of voltage between the electrodes, reaches ionization energy of gas particles, the probability of Ionizing collision of electrons with molecules dramatically increases, leading to increase of current carrier concentration. In strong electric fields, the kinetic energy of an electron surpasses significantly ionization energy and one electron, as a result of subsequent collisions, creates an avalanche of positive ions and electrons. The remaining electrons ionize subsequent gas molecules. The multiplication process increases rapidly towards anode and the area between the electrodes gets fully ionized.

Increase of the avalanche process leads, due to the difference in the mobility of positive ions and electrons, to the creation of inhomogeneous field distribution between the electrodes. Electrons as carriers of a high mobility reach the anode without any obstacle. By contrast, the heavy gas ions (cations) are drifted significantly slower in the opposite direction, creating a spatial charge between the electrodes. The electric field which was initially homogeneous, due to the presence of the spatial charge, is concentrated between the spatial charge and the anode. In the proximity of anode, due to the increase of the avalanche breakdown, the cation concentration increases and an extremely strong electric field is created.

The cathode surface is being struck by the cations, and the kinetic energy of those cations that went through the section from the anode is high enough to strike out the secondary electrons from the cathode (This is a so-called secondary emission.). These electrons, additionally accelerated in electric field, begin the ionization process at the cathode. The initial factors do not play a significant role anymore in the carrier generation process compared by the electron striking from the cathode and the ionizing collisions. This phenomenon is called electric breakdown of a gas, and its corresponding voltage U_i – ignition voltage.

Collisions of these electrons that have energy lower that ionization energy of gas molecule lead only to excitation of a hit molecule. This energy is radiated out as light quanta. This is why gas glowing accompanies avalanche ionization and we can confirm visually presence of this phenomenon.

The ionized gas is such a good conductor that the current flowing through the lamp is only limited by the resistance of the external circuit.

The important feature of the glowing discharge is its ability to self-sustaining even after the voltage falls below U_i value due to the secondary electron emission effect from the cathode that is stroke by the gas cations. For this reason, the discharge disappears at the voltage U_e lower than U_i by about 20 – 30V.

According to this description that induction of a glow discharge depends primarily on the gas ionization energy and work function of a cathode material. During the optimization of design of neon lamps aiming to reduce ignition voltage U_i, it was found that U_i depends on the product *pd* where *p* is the gas pressure, *d* – the distance between electrodes (see fig. 10). Increase of U_i value for high pd values (in high pressure range) is caused by reduction of free path, and in the range of low pressures and big sizes of the bulb – decrease of the probability of the ionizing collisions.

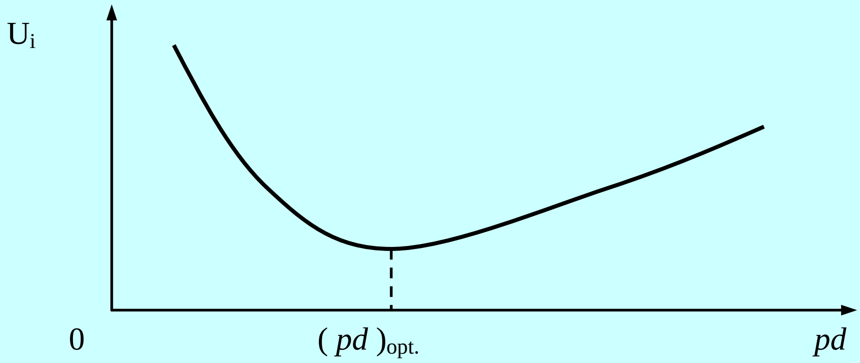


Fig.10. Dependence of neon lamp ignition voltage U_i ***on the*** pd ***product, where*** p ***is the gas pressure,*** d – ***the distance between the electrodes***