Fizyka statystyczna w wysokoenergetycznych zderzeniach jądrowych III

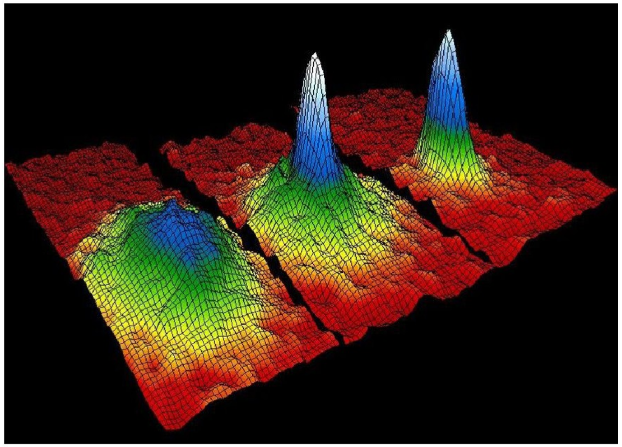
Viktor Begun

UJK, Kielce, Polska (8)
Enrico Fermi Ctr., Włochy (2)
FIAS, Frankfurt, Niemcy (8)
BITP, Kijów, Ukraina (37)

Kondensacja Bosego-Einsteina

Kondensat Bosego-Einsteina

- 1924 - Bose statistics discovered
- 1925 - Bose-Einstein condensation predicted
- 1995 - two experimental groups created BEC
- 2001 - leaders of the groups, Cornell, Wieman, and Ketterle, won the Nobel Prize



Velocity profile of the first BEC (left to right): $T > T_C$, $T < T_C$, $T \ll T_C$

The density of bosons is calculated from

$$\rho = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp[(\sqrt{p^2+m^2}-\mu)/T]-1}$$

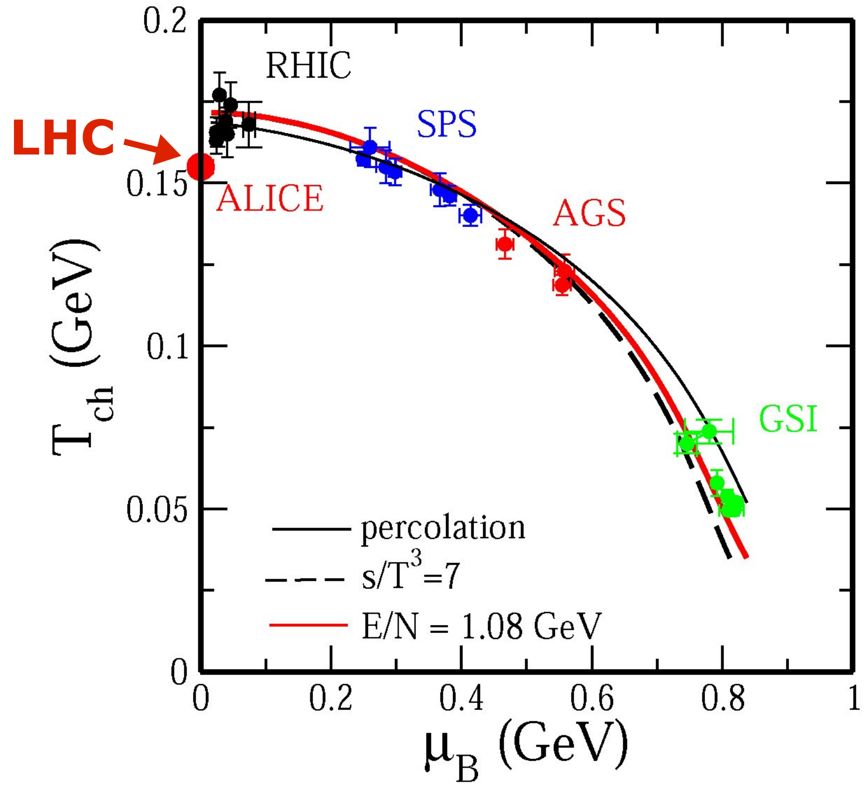
The BEC temperature for $T/m \ll 1$ is $T_C \sim m^{-1}\rho^{2/3}$. The density depends on the proper particle radius as $\rho \sim r^{-3}$. Then the ratio of the BEC temperature in the **atomic gases**, $T_C(A)$, to that in the **pion gas**, $T_C(\pi)$, in non-relativistic approximation

$$\frac{T_C(\pi)}{T_C(A)} \simeq \frac{m_A}{m_\pi}\left(\frac{r_A}{r_\pi}\right)^2 \simeq \frac{m_A}{m_\pi}10^{10}$$

- The **BEC** of **atoms** is achieved at temperatures $T_C(A) \sim 10^{-8} K$
- The **BEC** of **pions** would have 10^{12} higher temperature and very **different properties** due to different interaction forces, much smaller volumes, and higher densities

Zagadka Wielkiego Zderzacza Hadronów

- Thermal model gives the freeze-out curve
- The prediction was too high for ratios to **pions**, especially proton to pion ratio
- The best fit of the **LHC** data still gives three standard deviations for protons
- The low-transverse-momentum **pion spectra** show up to **50%** enhancement compared to hydrodynamic models
- The fit of the **LHC** data gives the parameters that **fall out** to the "wrong" side



Cleymans, Redlich, et. al., PRC (2006); EPJ (2015)

Possible explanations:

- hadronization and freeze-out in chemical **non-equilibrium** (Rafelski et al., PRC (2013))
- hadronic rescattering in the final stage (Becattini, Stock et al., PRL (2013))
- incomplete list of hadrons (Noronha-Hostler, Greiner, 1405.7298; NPA (2014))

Reasons for the non-equilibrium:

- super(over)cooling of the QGP (Shuryak, 1412.8393; Csorgo, Csernai, PLB (1994))
- gluon condensation in CGC (Blaizot, Gelis, Liao, McLerran, Venugopalan, NPA (2012); Gelis, NPA (2014))

Efekty skończonego rozmiaru

In the thermodynamic limit, $V \to \infty$, the sum over momentum levels is transformed into the integral over momentum $\sum_{\mathbf{p}} \cdots \simeq (V/(2\pi)^3) \int d^3\mathbf{p}$:

$$N = \sum_n \frac{g_n}{\exp\left(\frac{\sqrt{p_n^2+m^2}-\mu}{T}\right) - 1}$$

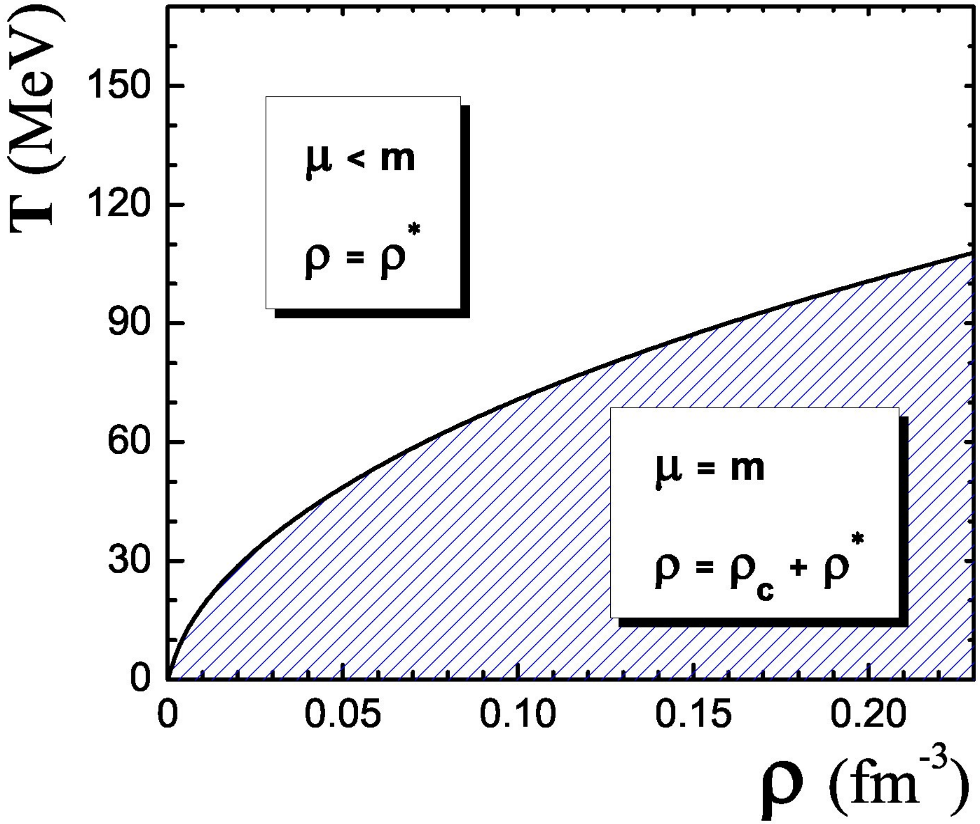
$$= \frac{g_0}{\exp\left(\frac{m-\mu}{T}\right) - 1} + \frac{g_1}{\exp\left(\frac{\sqrt{p_1^2+m^2}-\mu}{T}\right) - 1} + \ldots + V\int_{p_{\min}}^{\infty} \frac{d^3p}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right) - 1}$$

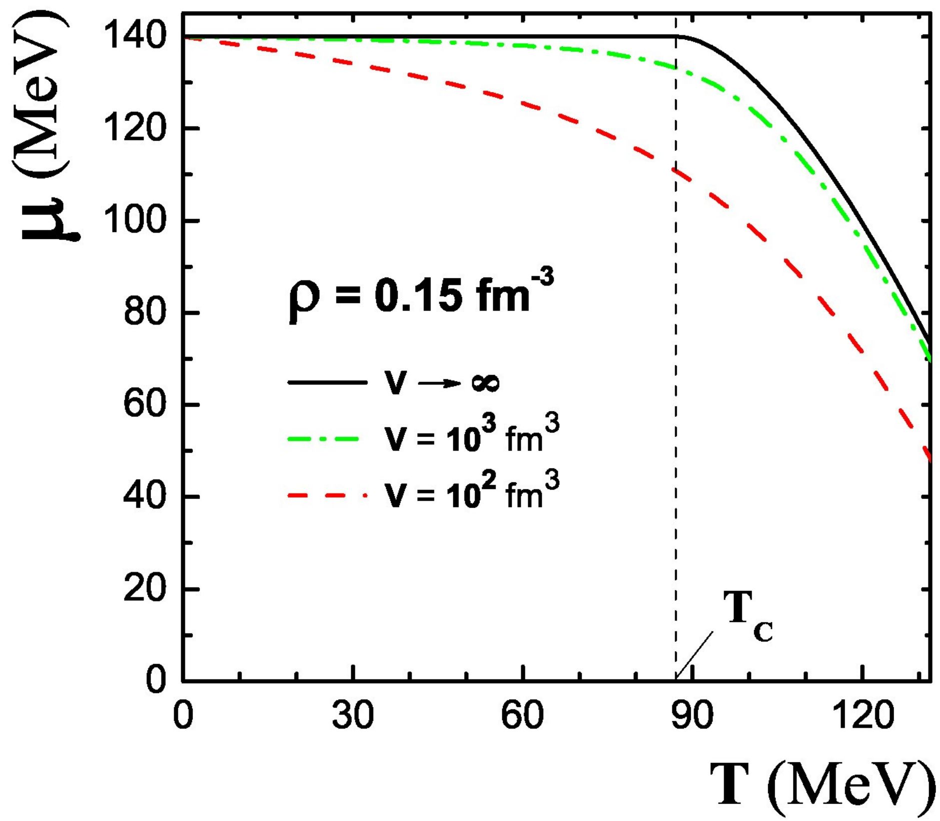
The momentum at the first excited level $p_1 \sim 1/V^{1/3}$ and the corresponding number of particles $N_1 \sim V^{2/3}$ vanish in the thermodynamic limit. While the zero momentum level grows as fast as the volume and survives (V.B., Gorenstein, PRC (2008), V.B. EPJ (2015))

$$N \simeq \frac{g}{\exp\left(\frac{m-\mu}{T}\right) - 1} + V\int_0^{\infty} \frac{d^3p}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right) - 1} = N_{\mathbf{cond}} + N_{\mathbf{norm}} \,,$$

where $N_{\mathbf{cond}}$ is the number of particles in Bose condensate and $N_{\mathbf{norm}}$ is the number of particles in normal state. For small volumes one should take into account at least the **ground state**.

Diagram fazowy, potencjał chemiczny



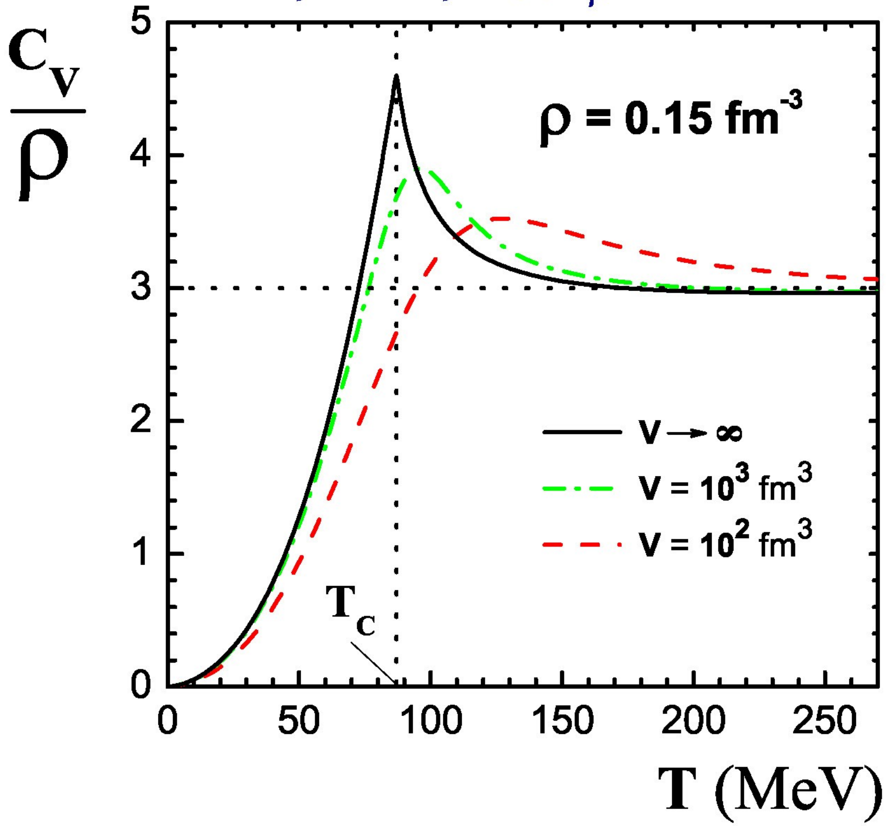


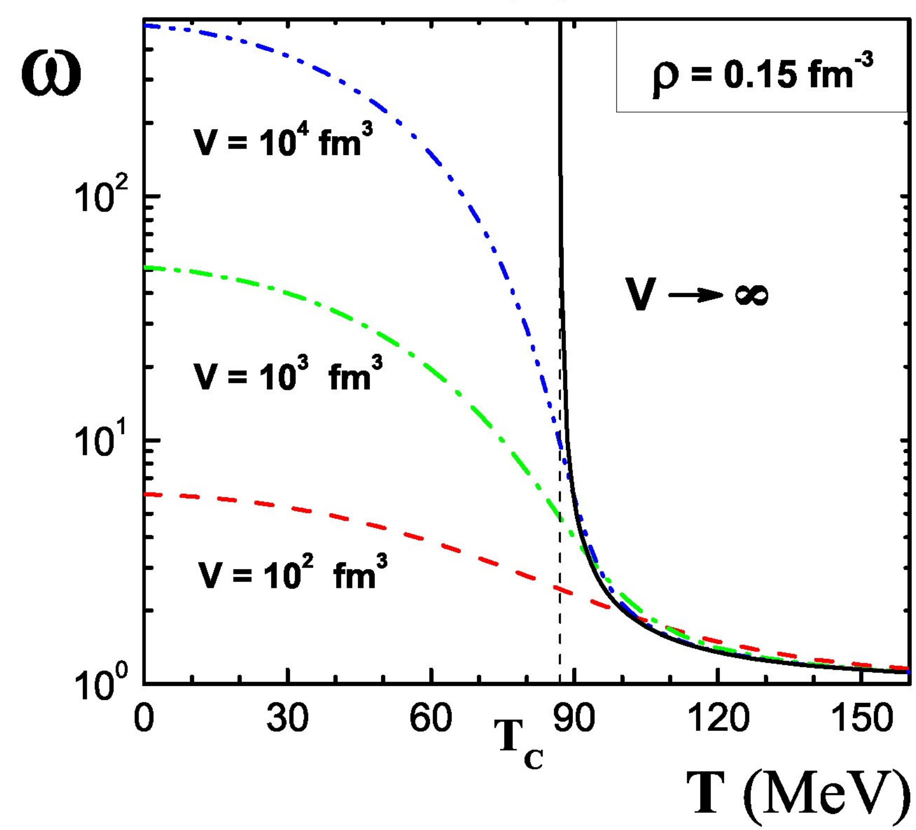
The **solid black** line is for the case $T = T_C$ in the limit $V \to \infty$ for pion gas with zero electric charge density.

- Bose-Einstein condensation is possible at any temperature, if the density is enough
- The chemical potential is always smaller than the mass in the system with a finite volume (V.B., Gorenstein, PLB (2007), PRC (2008))

Pojemność cieplna, fluktuacje krotności

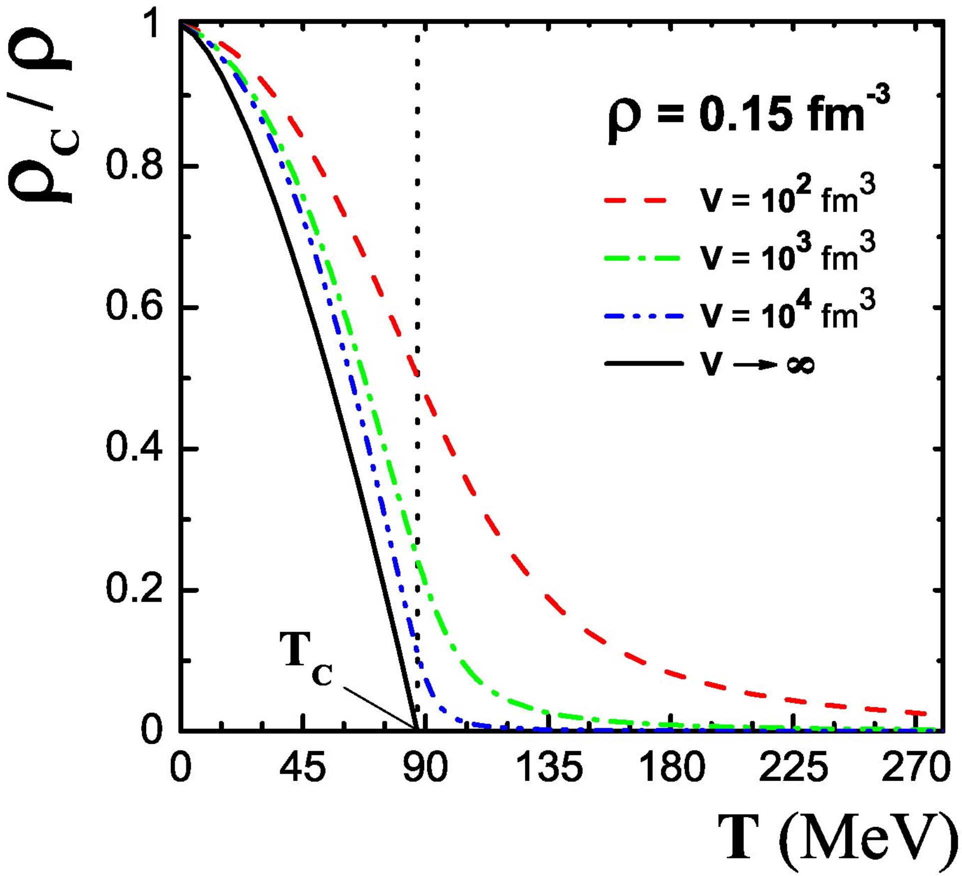
$$\frac{c_V}{\rho} \equiv \frac{1}{\rho}\left(\frac{\partial \varepsilon}{\partial T}\right)_\rho \qquad\qquad \omega \equiv \frac{\langle(\Delta N)^2\rangle}{\langle N\rangle}$$

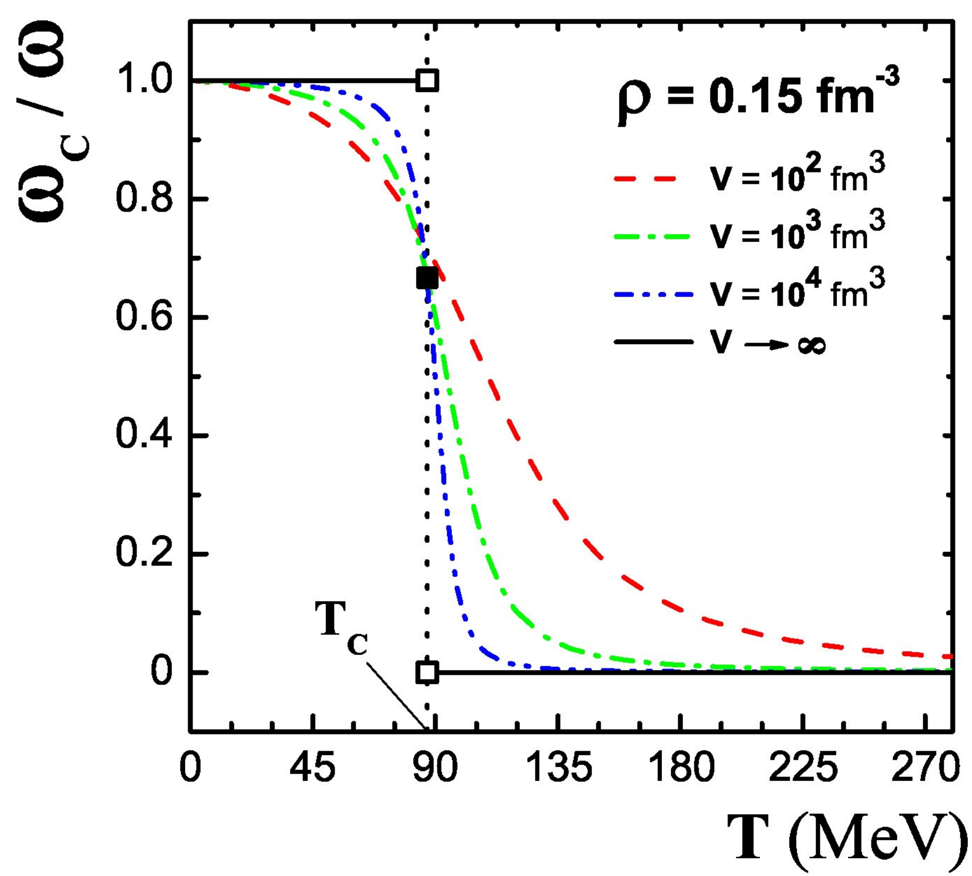




- The discontinuity in the derivative of the specific heat can be classified as a third-order phase transition
- In relativistic gas $m \ll T$ the specific heat is much larger at the maximum (**10.8** instead of **1.9**) and has the different limit at $T \to \infty$ (**3** instead of **3/2**)
- Fluctuations rapidly increase and the specific heat becomes sharper with increasing volume of the system (V.B., Gorenstein, PRC (2008))

Frakcja kondensatu





- A part of the system is always in the condensate even above T_C
- Counterintuitively, the fraction of pions in the condensate is larger for smaller systems at $T > T_C$ (V.B., Gorenstein, PRC (2008))

Model HRG + powierzchnia wymrożenia = widma

Model Krakowski z wymrożeniem

The phase-space distribution of the primordial particles has the form:

$$f_i = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\gamma_i^{-1} \exp(\sqrt{p^2 + m_i^2}/T) \pm 1}, \quad \text{where} \quad \gamma_i = \gamma_q^{N_q^i + N_{\bar{q}}^i} \gamma_s^{N_s^i + N_{\bar{s}}^i} \exp\left(\frac{\mu_B B_i + \mu_S S_i}{T}\right),$$

and N_q^i, N_s^i are the numbers of light (u, d) and strange (s) quarks in the ith hadron. It includes all well established resonances from PDG. Resonance decay according to their branching ratios.

Single-freeze out model (Broniowski, Florkowski, PRL (2001))
Monte-Carlo implementations, THERMINATOR (Chojnacki, Kisiel, Florkowski, Broniowski, Comput. Phys. Commun. (2012))

The spectra are calculated from the Cooper-Frye formula at the freeze-out **hyper surface**

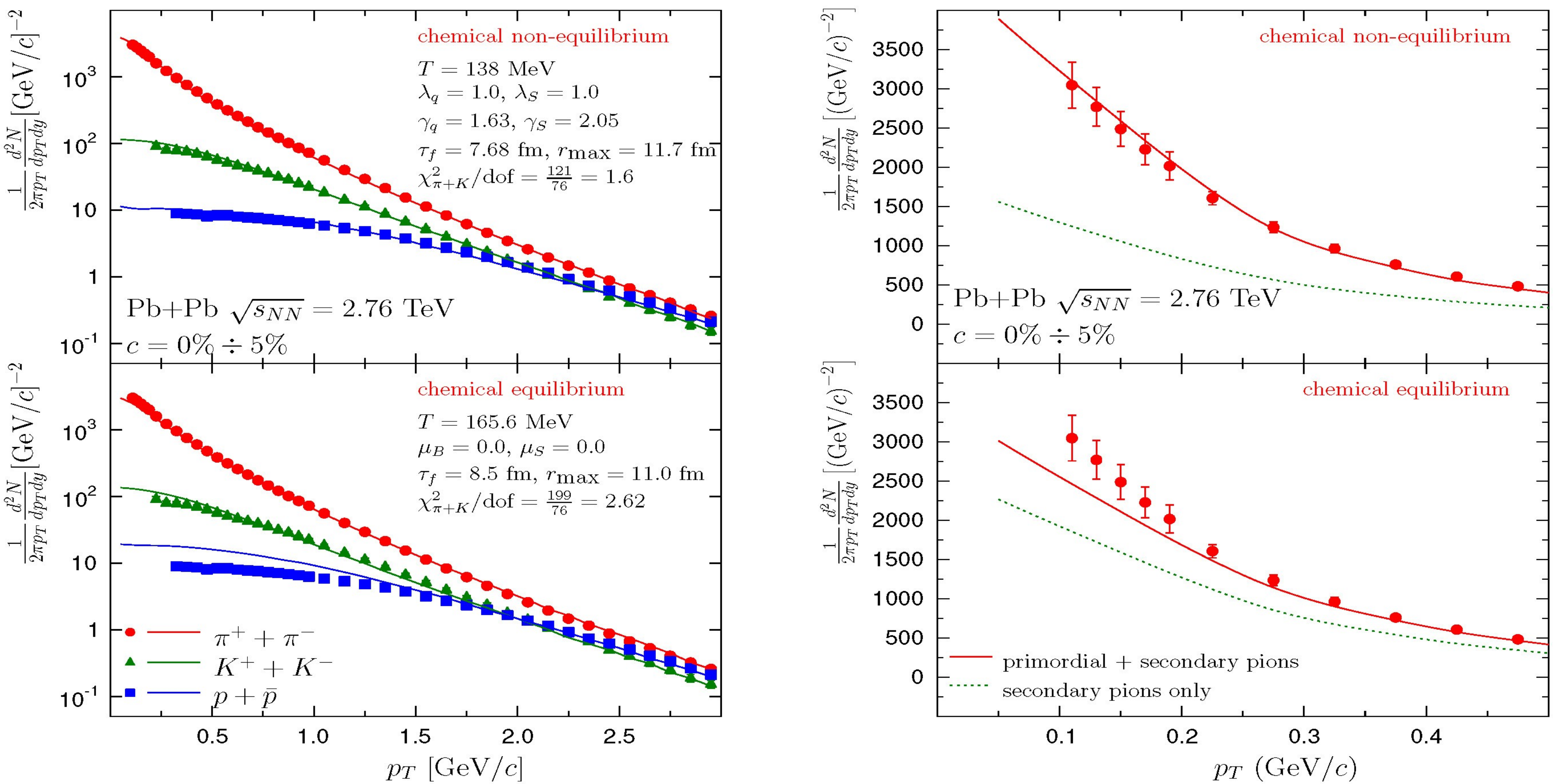
$$\frac{dN}{dy d^2 p_T} = \int d\Sigma_\mu p^\mu f(p \cdot u), \qquad t^2 = \tau_f^2 + x^2 + y^2 + z^2, \qquad x^2 + y^2 \le r_{\mathrm{max}}^2,$$

assuming the Hubble-like **flow**: $u^\mu = x^\mu / \tau_f$.

There is **only** one additional **parameter** in the model, because the product $\pi \tau_f r_{\mathrm{max}}^2$ is equal to the volume (per unit rapidity), while the ratio r_{max}/τ_f determines the **slope** of the spectra.

Mezony pi, K i protony na LHC

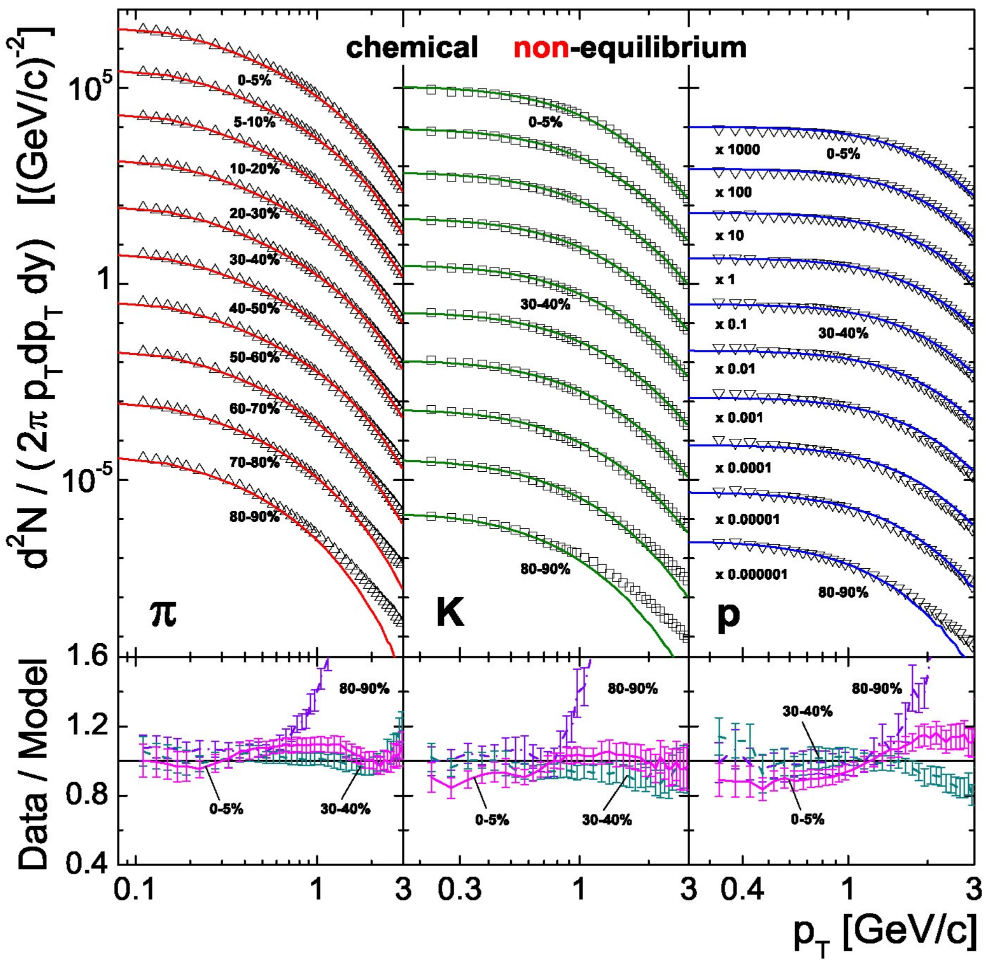
The fits to the ratios of hadron abundances (Rafelski et al., PRC (2013)) yield γ_q which is close to the critical pion chemical potential: $\mu_\pi = 2T \ln \gamma_q \simeq 134$ MeV $\simeq m_{\pi^0} \simeq 134.98$ MeV



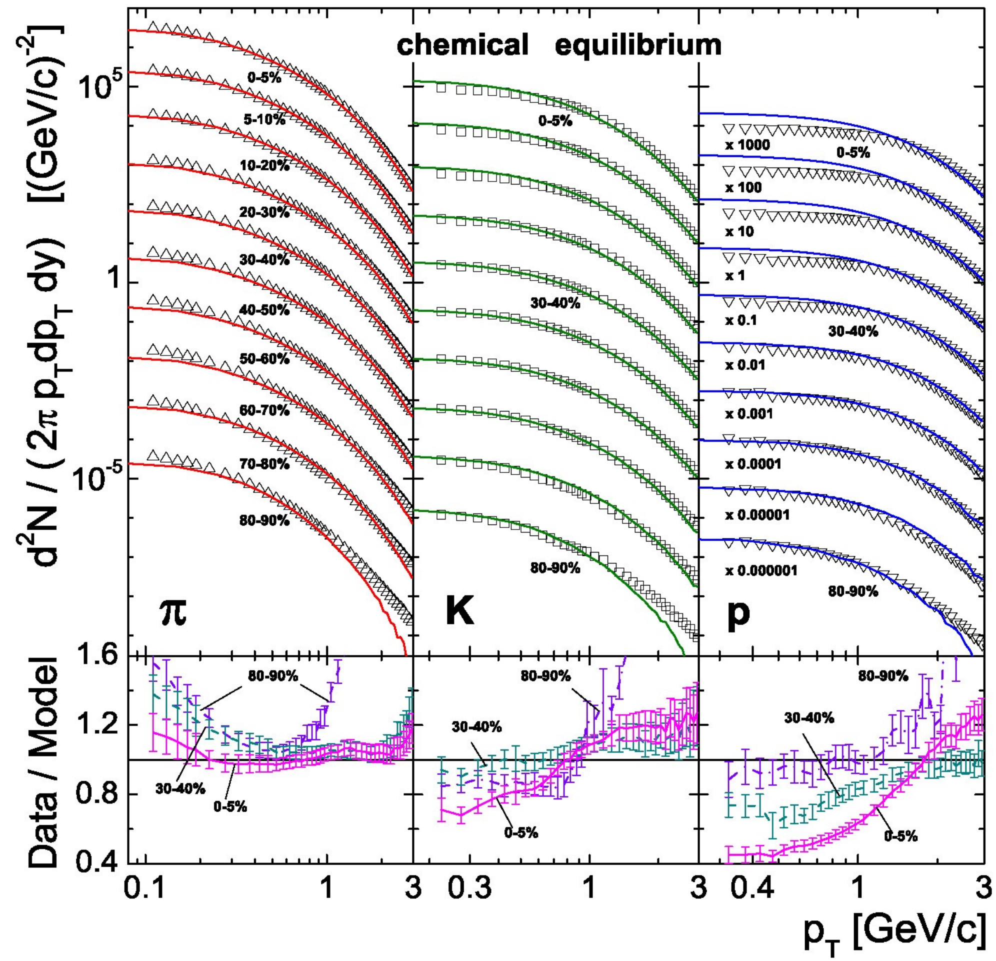
The **spectra** favor the **non-equilibrium** model. It may suggest that a substantial part of π^0 **mesons form** the **condensate** (V.B., Florkowski, Rybczynski, PRC (2014))

Mezony pi, K i protony na LHC

Chemical non-equilibrium:
V, T, γ_q, γ_s, r_{max}/τ_f

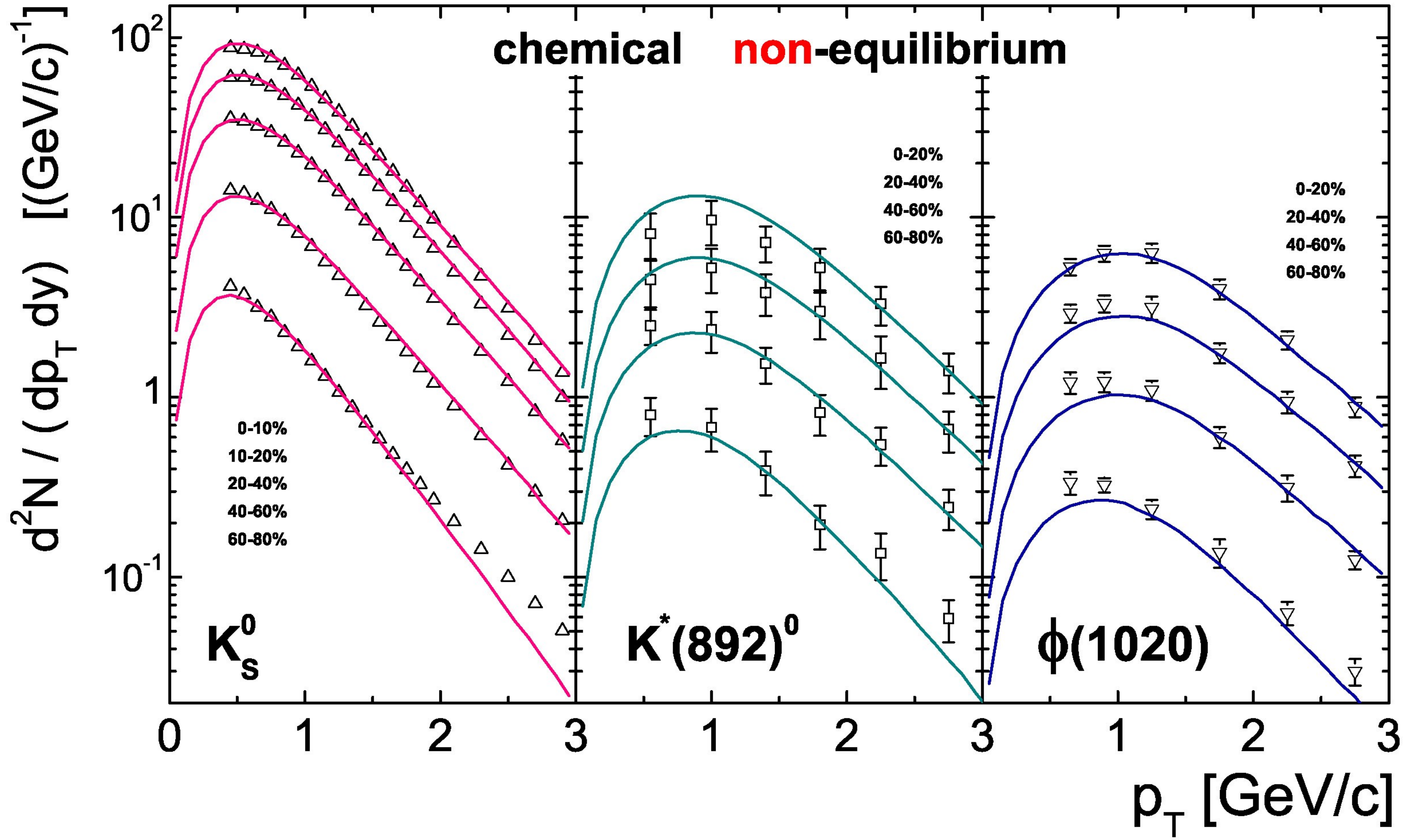


Chemical equilibrium:
V, T, r_{max}/τ_f



One can observe a good agreement for pions and kaons, however, protons in central collisions are described only in non-equilibrium (V.B., Florkowski, Rybczyński, PRC (2014) 054912).

Dziwne cząstki na LHC

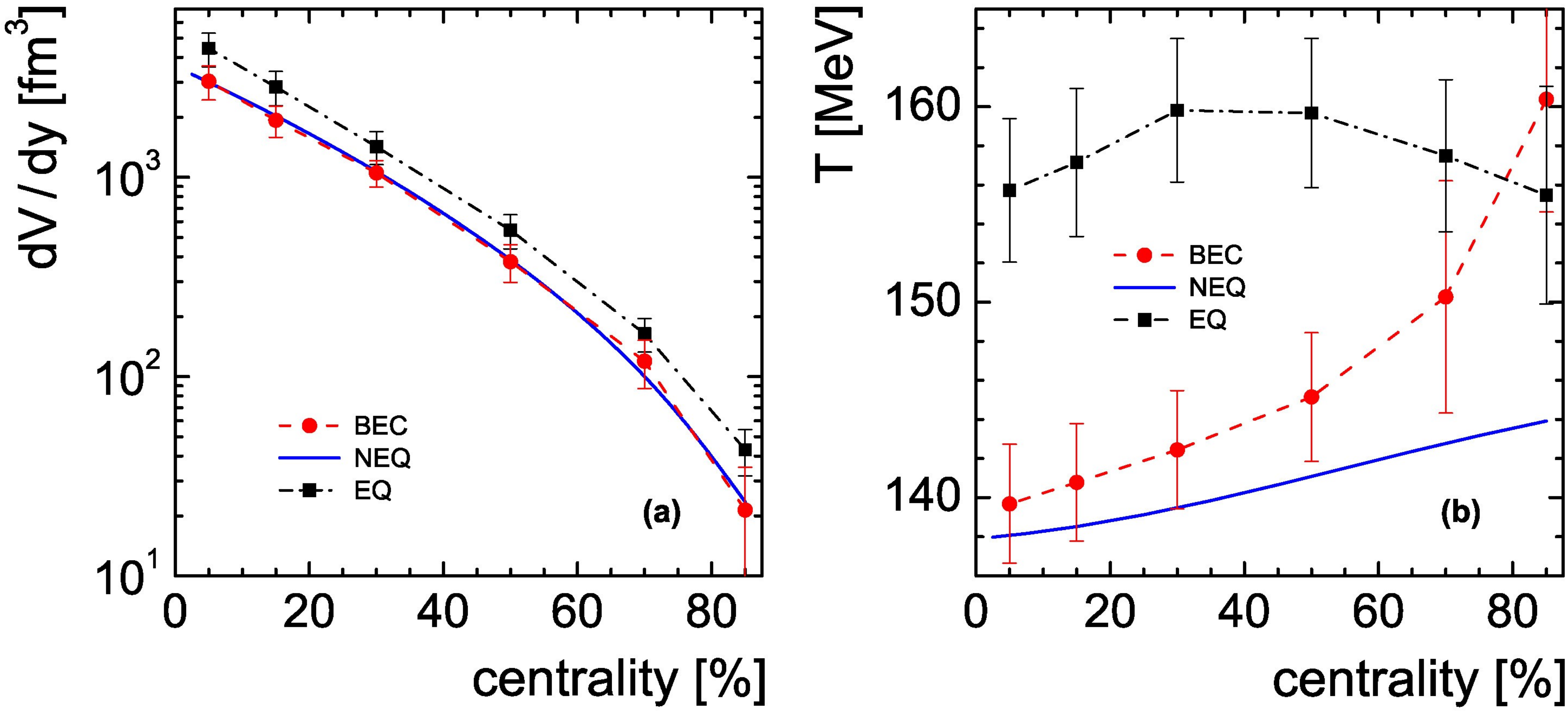


The **fit done** initially **for** $\pi^+ + \pi^-$ **and** $K^+ + K^-$ only appears also very **good for** $p + \bar{p}$, K^0_S, $K^*(892)^0$ **and** $\phi(1020)$**!** (V.B., Florkowski, Rybczyński, PRC (2014) 054912)

Uzyskane parametry hadronizacji na LHC

Objętość i temperatura

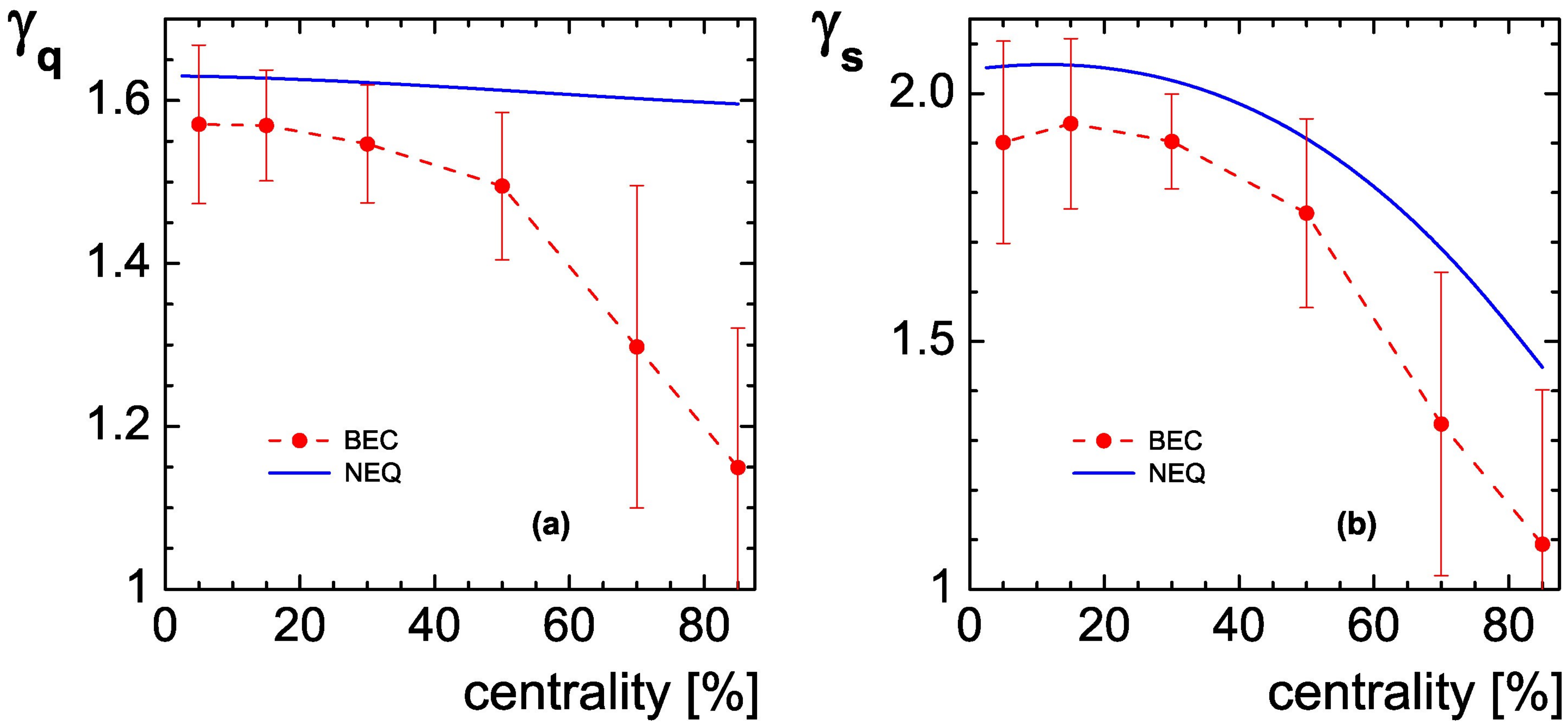
The fit of the data in SHARE (Petran, Rafelski, et. al., Comput. Phys. Commun. (2014)) that was modified to include the explicit treatment of the ground state (V.B., EPJ (2015)):



The equilibrium, $\gamma_q = \gamma_s = 1$, (EQ), non-equilibrium (NEQ), $\gamma_q \neq \gamma_s \neq 1$, and non-equilibrium with ground state (BEC).

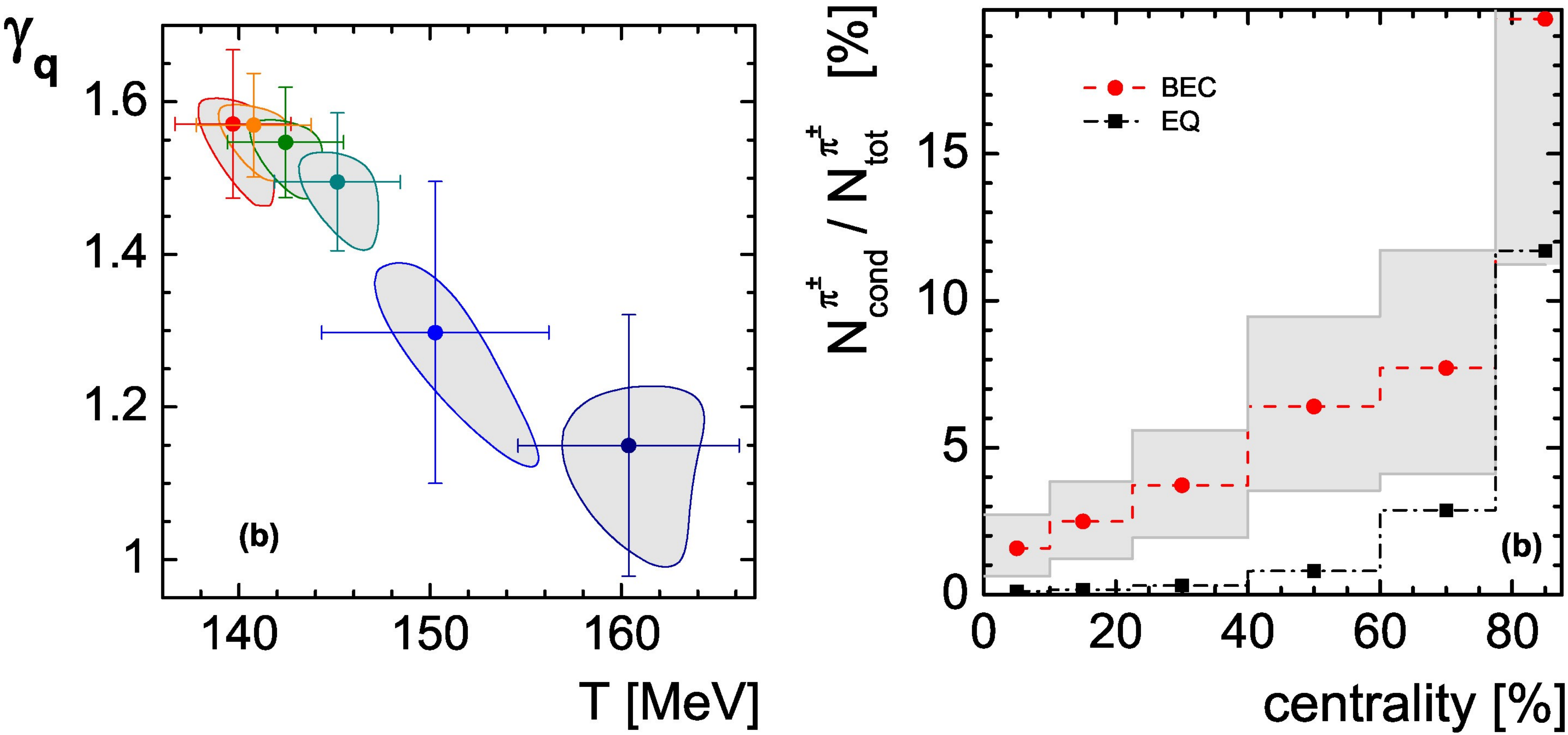
- The introduction of the ground state does not influence the volume
- The temperature in BEC increases compared to NEQ

Parametry nierównowagi



- The non-equilibrium parameters in BEC are smaller than in NEQ
- The BEC and EQ are equivalent for the very peripheral collisions (V.B., EPJ (2015))

10% odchylenie od $[\chi^2/N_{dof}]_{min}$

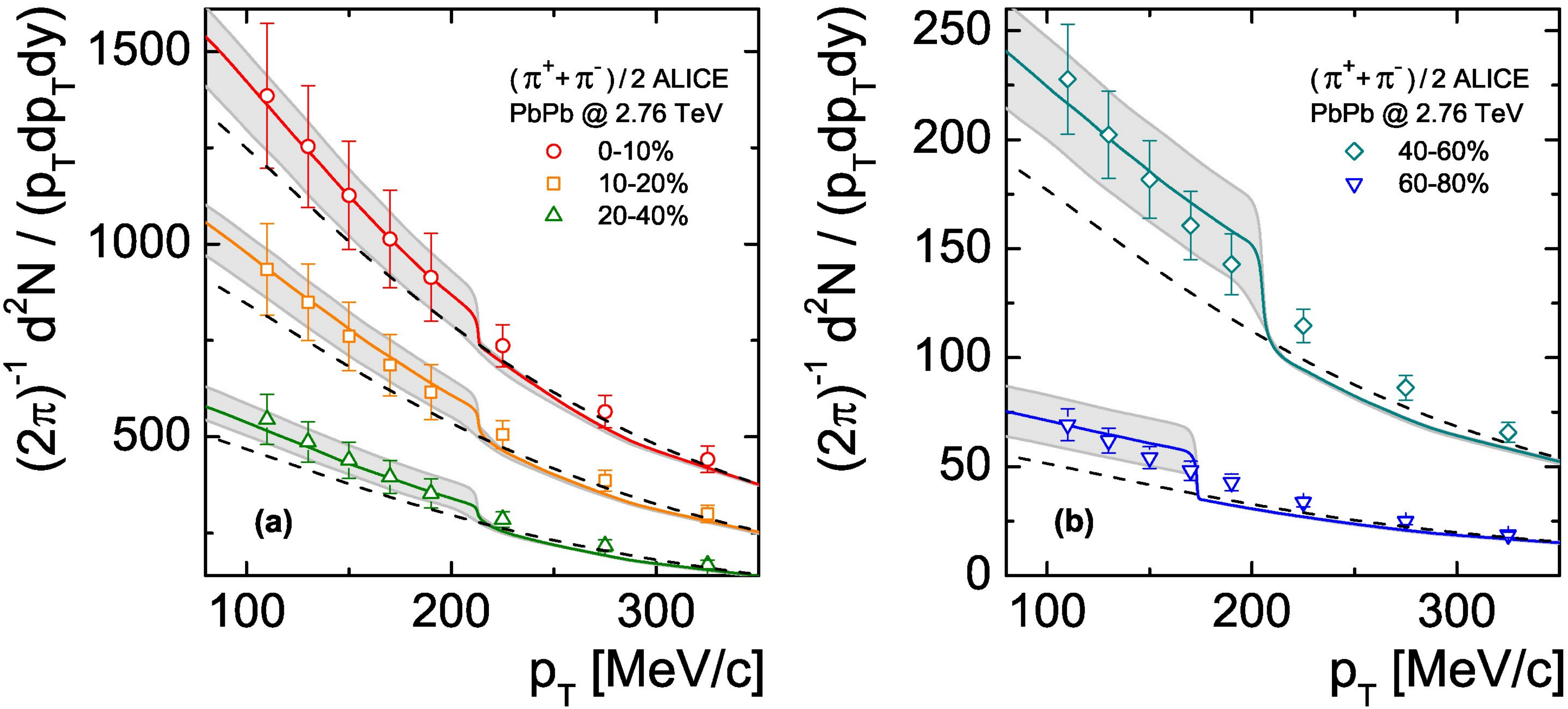


The non-equilibrium parameter as the function of temperature and the corresponding condensate rate.

- The thermodynamic parameters have a shallow minimum in BEC
- It results in a large uncertainty in the determination of the exact number of pions in the condensate. (V.B., Florkowski, PRC (2015))

Kondensacja mezonów pi na LHC ?

The p_T distribution is $\frac{dN}{dy d\phi_p\, p_T dp_T} = \frac{N_{\mathbf{cond}}}{V} \frac{\tau_f^3}{m^2} \theta\,(r_{\max} - p_T \tau_f / m)$ (V.B., Florkowski, PRC (2015)):

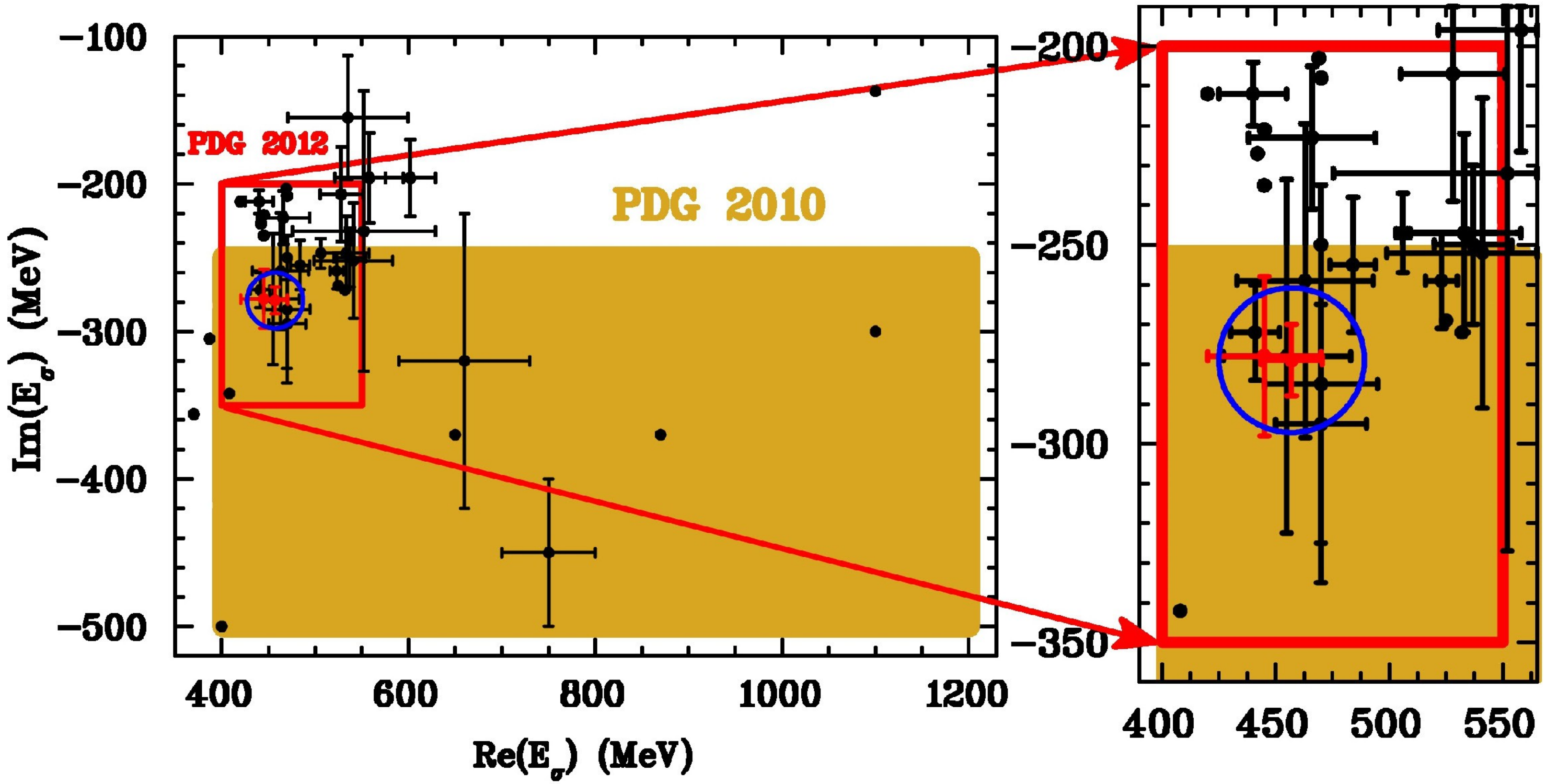


Condensate rate and p_T spectrum for charged pions. The grey area show the **10%** deviation from the best fit.

- The inclusion of several more levels would lead to finer steps
- The data on multiplicities and spectra are compatible with **5%** of the **condensate**

Czy mezon sigma może dać ten efekt?

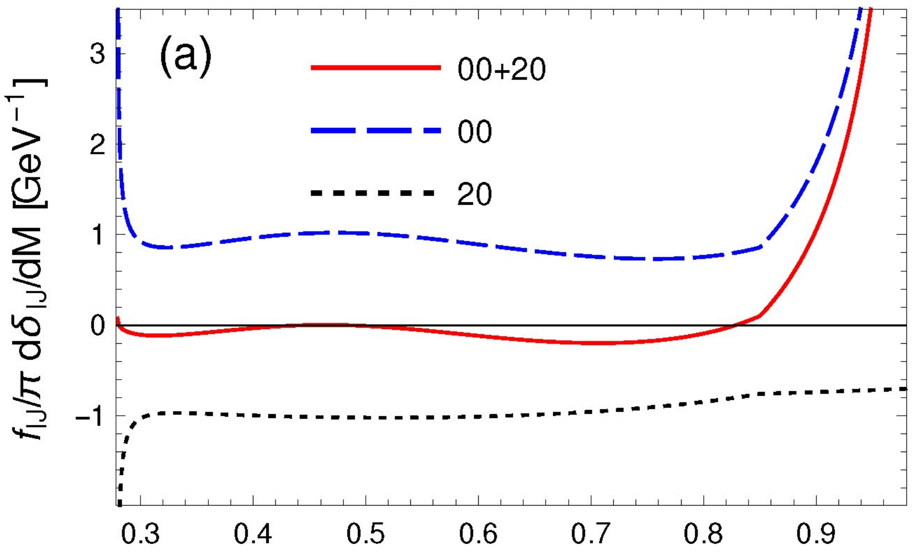
- The recent PDG reviews report much **lower mass** and width of the $f_0(500)$ or the **sigma** meson
- The lower mass of the σ would result in it's **higher multiplicity**. It decays into pions, therefore it **could add** some of the **missing pions**



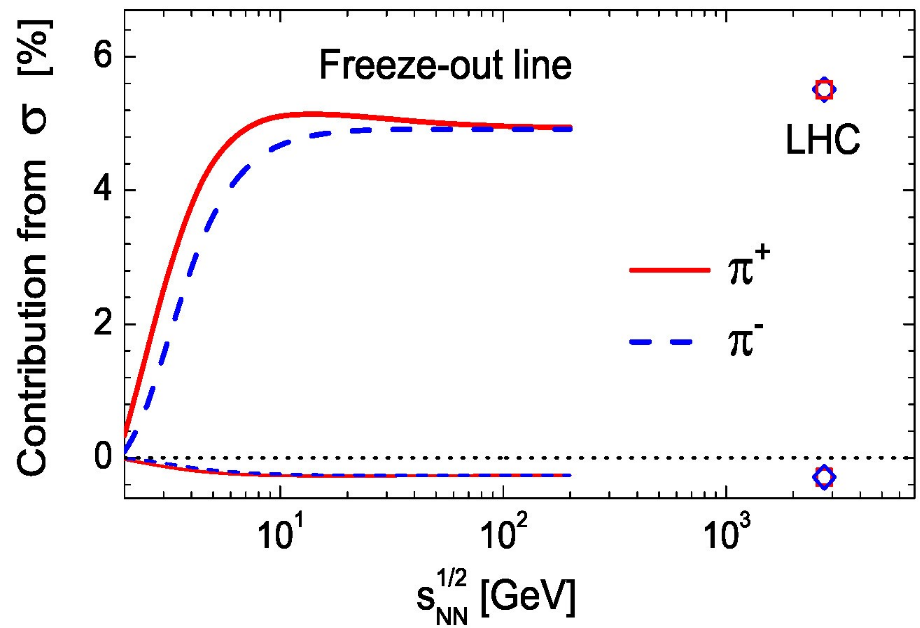
Kaminski, Acta Phys. Polon. Supp. (2015); Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira, Phys. Rev. Lett. (2011)

Nie, ponieważ jest w ogóle kasowany

- The derivative of the experimental $\pi\pi$ phase shift has **attractive** isospin-spin **channel** $(0,0)$ that is responsible for the emergence of the $f_0(500)$.
- However, the channel $(2,0)$ is **repulsive** and **cancels** $f_0(500)$ until $f_0(980)$ takes over above $M \sim 0.85$ GeV.



The derivative of the phase shift as the functions of $\boldsymbol{M_{\pi\pi}}$.



- The **cancellation** occurs at the level of the distribution functions
- Therefore it persists in **all** isospin-averaged **observables**
- The σ implemented as a Breit-Winger pole with $M_\sigma = 484$ and $\Gamma_\sigma/2 = 255$ MeV produces up to **5%** of pions, while the truth contribution is **–0.3%**
- The famous $\boldsymbol{K/\pi}$ **horn** is **affected**, as well as **all ratios** to **pions**

(V.B., Broniowski, Giacosa, PRC (2015))

Fluktuacje pomogą sprawdzić nie równowagę na LHC?

Fluktuacje krotności

Multiplicity fluctuations of any order can be calculated using the definition of susceptibility:

$$\chi_n = \left.\frac{\partial^n(\mathcal{P}/T^4)}{\partial(\mu/T)^n}\right|_T$$

where $\mathcal{P}$ is pressure, μ-chemical potential, T-temperature.

The characteristic observables are the

$\omega = \sigma^2/\langle N\rangle$ - **scaled variance**: variance over the mean,

$S\cdot\sigma$ - normalized **skewness**

$\kappa\cdot\sigma^2$ - normalized **kurtosis**

They are related to the susceptibilities and central moments:

$$\omega = \frac{\chi_2}{\chi_1} = \frac{\mu_2}{\langle N\rangle} \qquad S\cdot\sigma = \frac{\chi_3}{\chi_2} = \frac{\mu_3}{\mu_2} \qquad \kappa\cdot\sigma^2 = \frac{\chi_4}{\chi_2} = \frac{\mu_4}{\mu_2} - 3\,\mu_2$$

where $\mu_n = \langle(N-\langle N\rangle)^n\rangle = \sum_N (N-\langle N\rangle)^n \cdot P(N)$ are central moments of the $P(N)$ multiplicity distribution.

Fluktuacje pierwotnych cząstek

In the pion gas one can make **analytical calculations**

$$\langle N\rangle = \sum_p \langle n_p\rangle \qquad \langle n_p\rangle = \frac{1}{\exp\left[\left(\sqrt{p^2+m_i^2} - \mu_\pi\right)/T\right]-1}$$

$$\omega = \frac{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}{\sum_p\langle n_p\rangle}$$

$$S\cdot\sigma = \frac{\sum_p\left(\langle n_p\rangle + 3\langle n_p\rangle^2 + 2\langle n_p\rangle^3\right)}{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}$$

$$\kappa\cdot\sigma^2 = \frac{\sum_p\left(\langle n_p\rangle + 7\langle n_p\rangle^2 + 12\langle n_p\rangle^3 + 6\langle n_p\rangle^4\right)}{\sum_p\left(\langle n_p\rangle + \langle n_p\rangle^2\right)}$$

The standard replacement of the sum by the integral over the momentum levels $\sum_p \to V/(2\pi)^3\int d^3p$ leads to the divergence of all $\int d^3p\langle n_p\rangle^m$ with $m \geq 2$ and infinite variance, skewness and kurtosis at BEC.

There are no divergences in **finite volume**, that can be taken into account by keeping the **zero momentum level** of the sum (V.B., Gorenstein, PRC (2008), V.B. EPJ (2015)):

$$\langle N\rangle = \langle n_0\rangle + \frac{V}{(2\pi)^3}\int_0^\infty \langle n_p\rangle\, d^3p = N_{cond} + N_{norm}$$

Oszacowanie wkładu od rezonansów

For two uncorrelated multiplicity distributions $P_1(N_1)$ and $P_2(N_2)$:

$$\langle N \rangle = \langle N_1 \rangle + \langle N_2 \rangle = \sum_{N_1} N_1 \, P_1(N_1) + \sum_{N_2} N_2 \, P_2(N_2)$$

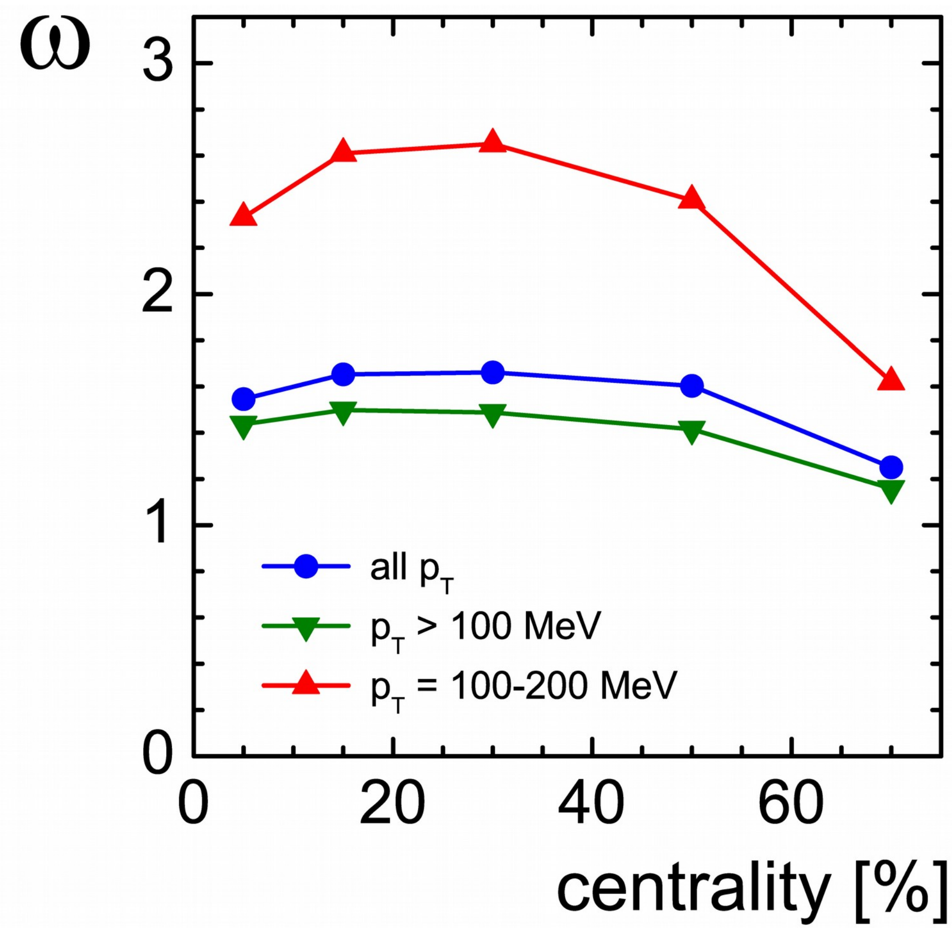
$$\omega = \omega_1 \frac{\langle N_1 \rangle}{\langle N \rangle} + \omega_2 \frac{\langle N_2 \rangle}{\langle N \rangle}$$

Similarly for kurtosis and skewness:

$$S \cdot \sigma = S_1 \cdot \sigma_1 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + S_2 \cdot \sigma_2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle}$$

$$\kappa \cdot \sigma^2 = \kappa_1 \cdot \sigma_1^2 \frac{\omega_1}{\omega} \frac{\langle N_1 \rangle}{\langle N \rangle} + \kappa_2 \cdot \sigma_2^2 \frac{\omega_2}{\omega} \frac{\langle N_2 \rangle}{\langle N \rangle}$$

For Poisson distribution $\omega_i = 1$. Experimentally seen fluctuations are $\omega_i \lesssim 2$.

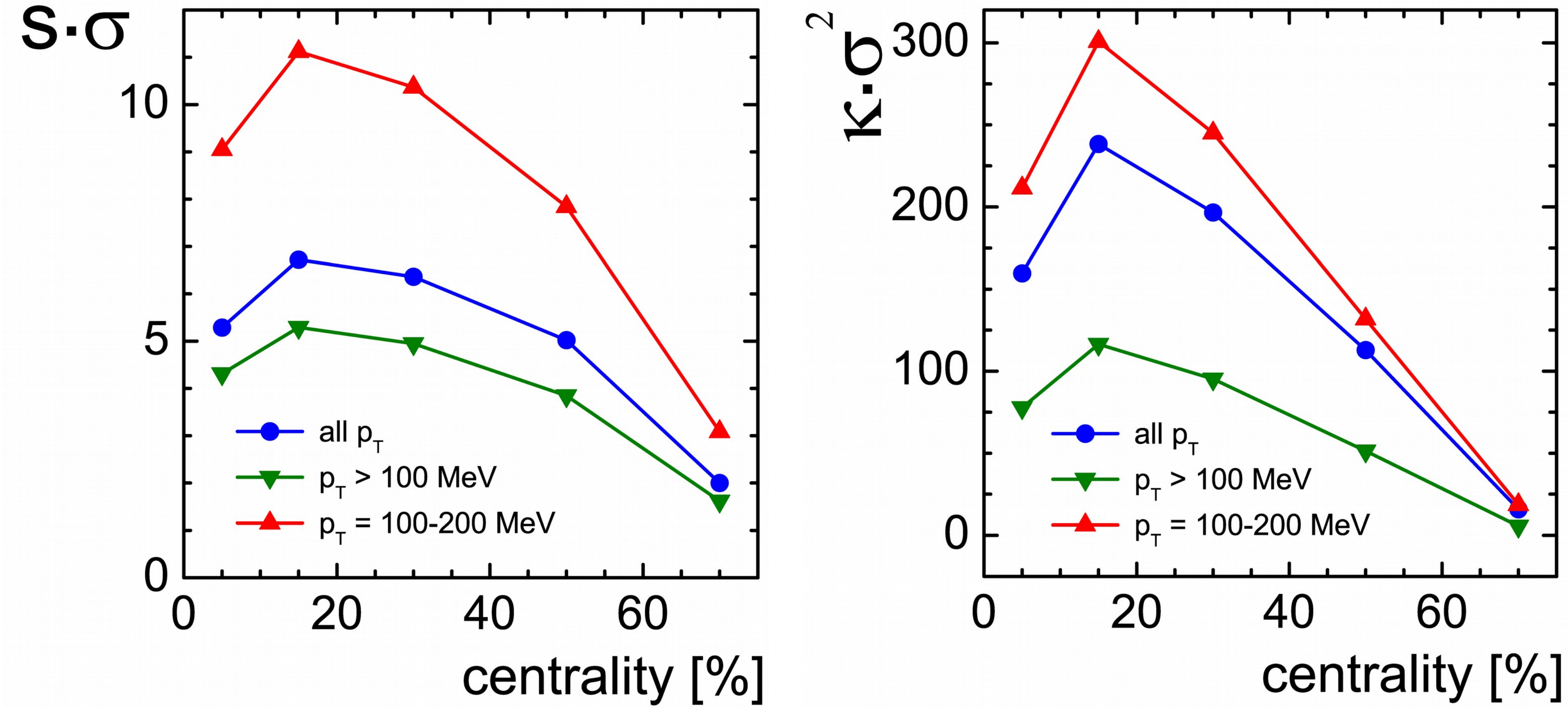


V. Begun, arXiv: 1603.02254

- One can increase the condensate rate in the most central collisions parameters up to **10%** making the cut $p_T < p_T^{BEC}$.
- **ALICE** collaboration claims that coherent fraction in charged pion emission may reach **23%** for $p_T < 300$ MeV (PRC 89, 024911 (2014)).

Skośność i kurtoza na LHC

For Poisson distribution $S \cdot \sigma = \kappa \cdot \sigma^2 = 1$, while for Gauss distribution $S \cdot \sigma = \kappa \cdot \sigma^2 = 0$.



V. Begun, arXiv: 1603.02254

- Skewness is small and positive at the LHC
- **Kurtosis** is **large** even for the current data
- The increase of the condensate rate to **10%** could lead to a **significant signal**

Podsumowanie

- Na przykładzie kondensacji Bosego-Einsteina pokazano, że **fluktuacje są o wiele bardziej czułe do zmian** w systemie przy **przejściach fazowych**

- Wyniki **na LHC** są takie, że **istnieje możliwość eksperymentalnej obserwacji kondensacji Bosego-Einsteina** za pomocą **wyższych momentów fluktuacji krotności**

Dziękuję za uwagę!