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Spatial solitons interaction in liquid crystalline waveguides

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Recently, there have been shown that in planar liquid crystalline waveguides light beams can form spatial soliton due to the reorientational nonlinearity. Such self-trapped beams require only a few tens of mW of light power and their stability is controlled by state of the light polarization. In this paper, the collisions of previously observed solitons are analyzed theoretically. Obtained results show that analyzed self-trapped beams became unstable due to the interaction with other light beams.

Keywords: liquid crystalline waveguides, spatial solitons.

1. Introduction

Nematic liquid crystals are excellent medium for nonlinear optics, both in three-dimensional bulk systems [1-2] as well as in waveguide structures [3-4]. The main contribution to nonlinear optical phenomena in liquid crystals arises from thermal and reorientational processes. While the thermal effect is similar to that observed in other materials, the reorientational effect is characteristic only in the liquid crystalline phase. Reorientational nonlinearity in nematic liquid crystals can also form spatial solitons [5]. Experimental results showed that for light power of the order of only a few mW it could be achieved self-trapped beams at distances of the order of a few mm. The stability of such beams can be controlled by external fields or by state of the light polarization. The experiments showed the existence of the self-focused light beams inside liquid crystals in capillaries [6-9], in planar cells [10], and in planar waveguides [11].

In this paper, the collisions of the optical solitons in planar waveguides are analyzed theoretically. Such solitons were previously observed experimentally and analyzed theoretically in a thin layer with homeotropically-aligned nematics [11]. By controlling the state of polarization of the incident light the stable self-trapped beams were obtained. They were named spatial soliton, but in fact only their stability were proved. In the exact definition, solitons need to be stable and need to maintain their properties after the collision with another solitons. Obtained results in this paper show that analyzed self-trapped beams are rather solitary waves than solitons. They are stable during the propagation but the interaction between two such beams can destroy them.

2. Beam propagation

The analyzed configuration is presented in Figure 1. Nematic liquid crystal (NLC) with homeotropic orientation creates the film of the optical planar waveguide. Assuming the arbitrary polarization in the light beam it could be introduced that the electric field components of the electromagnetic monochromatic wave have the form:

$$E_x = A(y,z)\psi(x)\exp(i\omega t - ik_0 N_x z), \quad (1)$$

$$E_y = B(y,z)\varphi(x)\exp(i\omega t - ik_0 N_y z), \quad (2)$$

where $\psi(x)\exp(i\omega t - ik_0 N_x z)$ and $\varphi(x)\exp(i\omega t - ik_0 N_y z)$ are modes of planar waveguides with homeotropic texture, N_x, N_y are effective refractive indices and A, B are amplitudes slowly varying in respect to z. The TE-like field, connected with E_y component, is assumed to be much weaker than the TM-like field, connected with E_x component, i.e. $|A|>>|B|$. This assumption is necessary to obtain the stable self-trapped light beams in the analyzed configuration [5-11]. For the waveguides thicker than the wavelength and the

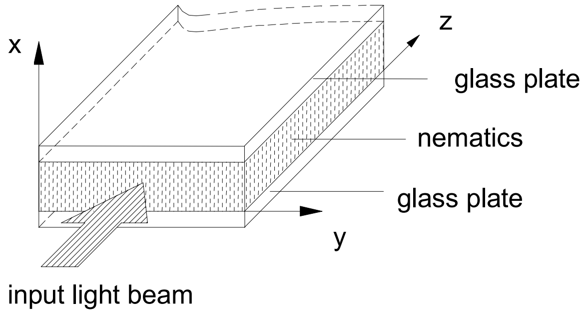

Fig. 1. Schematic of the analyzed liquid crystalline waveguides

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light beam wider than the wavelength the pair of equations for amplitudes A and B are obtained in the form:

$$\left[\kappa_1 + \frac{1}{2k_0 N_y}\frac{\partial^2}{\partial y^2} - i\frac{\partial}{\partial z}\right]B = -\kappa_{12} A \exp\left(-ik_0\left(N_x - N_y\right)z\right), \quad (3)$$

$$\left[-\kappa_2 + \frac{1}{2k_0 N_x}\frac{\partial^2}{\partial y^2} - i\frac{\partial}{\partial z}\right]A = -\kappa_{12} B \exp\left(ik_0\left(N_x - N_y\right)z\right), \quad (4)$$

where κ_1, κ_2 and κ_{12} are coefficients dependent on the NLC reorientation. The coefficient κ_{12} is responsible for a coupling between TE and TM waves, while the coefficients κ_1 and κ_2 effectively modify the propagation constants of both fields. For pure homeotropic alignment all coefficients are equal to zero and both fields propagate independently with different phase velocities.

In the nonlinear regime the electromagnetic field induces reorientation of liquid crystalline molecules. The orientation angle θ, defined as an angle between the director and x-axis, is calculated from the Euler-Lagrange equation:

$$\frac{d^2\theta}{dx^2} + \frac{\varepsilon_0 \Delta\varepsilon}{4K}\left[2|AB|\varphi\psi\cos\Delta\alpha\cos 2\theta + \left(|B\varphi|^2 - |A\psi|^2\right)\sin 2\theta\right] = 0, \quad (5)$$

where $\Delta\alpha$ is a phase difference between E_x and E_y field components and K is an elastic constant in the one-elastic constant approximation. The solution of equation (5) allows calculate the nonlinear coefficients κ_1, κ_2 and κ_{12}, which are proportional to:

$$\kappa_1 \sim \int \sin^2\theta\,\varphi^2 dx,\ \kappa_2 \sim \int \sin^2\theta\,\psi^2 dx,$$

$$\kappa_{12} \sim \int \sin\theta\cos\theta\,\varphi\psi dx$$

Their dependence on fields in both polarizations can be approximated with a very high accuracy by functions:

$$\kappa_{12} \sim \frac{|AB|\cos\Delta\alpha}{1 + |A/A_S|^2}, \quad (6)$$

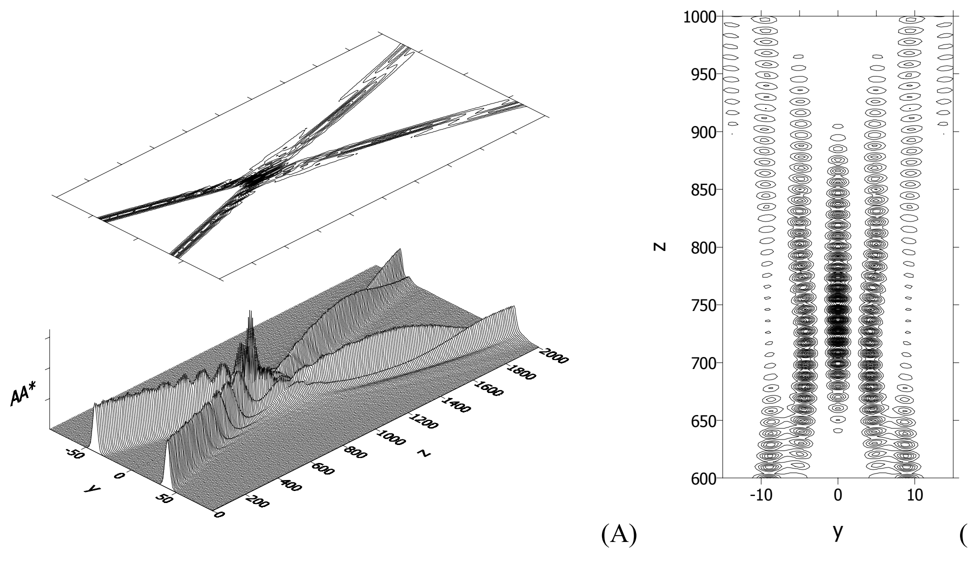
$$\kappa_{1,2} \sim \left(\frac{|AB|\cos\Delta\alpha}{1 + |A/A_S|^2}\right)^2, \quad (7)$$

where A_S is a saturation amplitude.

For $\Delta\alpha=0$ the reorientation is the largest while for $\Delta\alpha=\pi/2$ it disappears (because $|A|>>|B|$). The phase difference $\Delta\alpha$ in linear case is equal to $k_0(N_x-N_y)z$ and this causes changes of the light polarisation with a period equal to the birefringence length $L_B=\lambda/(N_x-N_y)$. The beat length L_B in NLC could be as low as a few wavelengths. Consequently, the nonlinear changes of NLC orientation should be periodic with a spatial period roughly equal to L_B.

3. Numerical results

The light propagating in the analyzing configuration can form a stable self-trapped beam. Such stable solitary waves were used at the input to observe their collisions. The numerical simulations were done for the 10μm thick NLC layer filled with homeotropically aligned 6CHBT (*4-trans-4'-n-hexyl-cyclohexyl-isothiocyanatobenzene*) nematics. Refractive indices for the wavelength λ=842 in analyzed configuration are taken as: extraordinary $n_{e|}$=1.69, ordinary n_o=1.52 and for glass n=1.45. The parameters values correspond to the configuration, which were previously analyzed theoretically and used in experimental observation. Results of solitary waves collisions are presented in figures 2-4, where the light intensity in the TE-field and the nonlinear coefficient κ_1 are plotted.



(A) (B)

Fig. 2. Collisions of the solitary waves at an angle 3°: (A) the distribution of the light intensity $|A|^2$ for the TE-field and (B) distribution of the coefficient κ_1 in the region of interaction. Distances are plotted in micrometers and the light intensity in a dimensionless unit

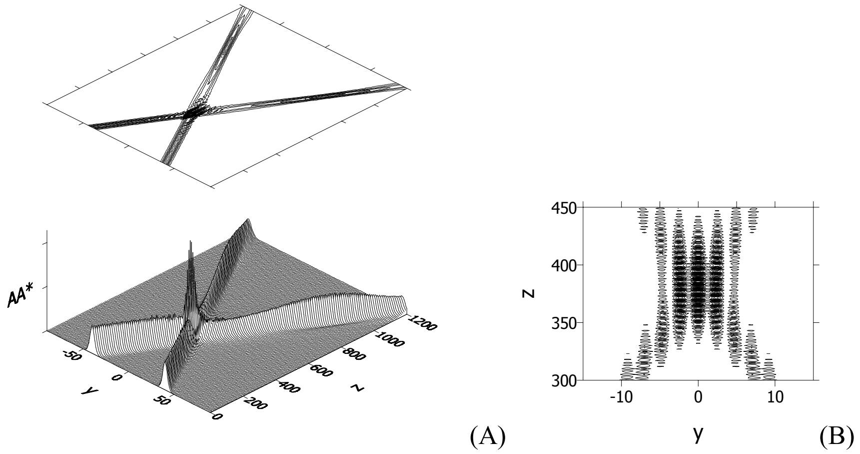


Figure 3. The same as in figure 2 for collisions of solitary waves at an angle 6^{o}

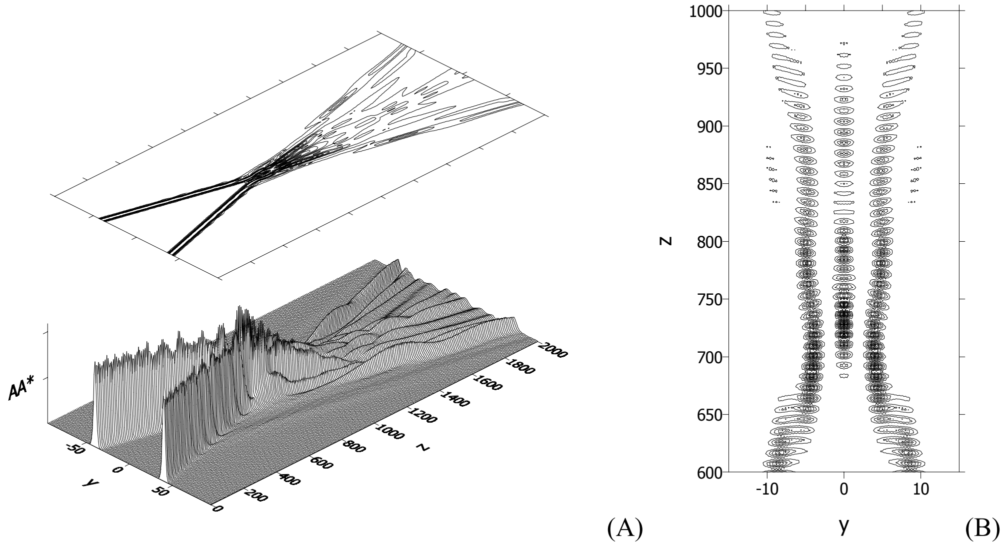


Figure 4. The same as in figure 2 for collisions of solitary waves with higher power (and smaller width)

4. Conclusions

Presented numerical results show that collision of two solitary waves makes them unstable and can lead to their break-up. The collision induces smaller disturbance when the angle between both beams are larger (compare fig. 2 and fig. 3). This is caused by the fact, that for larger angle the region of interaction is shorter and consequently the influence one beam on another beam is smaller. The effect of beam destroying is enhanced when solitary waves have higher intensities (compare fig. 2 and fig. 4). Then the reorientation of NLC is larger and larger is mutual interaction between both beams.

The mechanism of beams breaking-up due to their collision is connected with the mechanism of the self-focusing in analyzed configuration. The solitary waves are focused by periodic reorientation in the NLC film. When two solitary waves interfere the resulting reorientation has also the periodic form. This is shown in figures 2B, 3B, and 4B, where the value of the nonlinear coefficient κ_1 represents the effective reorientation of the NLC film. The periodic structures in the region of collision act as the diffraction gratings, which scatter the light beams. It should be pointed out, that in another configurations of liquid crystalline layers we could expect existence of the stable solitons, which are stable also after the collision with another beams.

5. References

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