

## **Light Beam Propagation in Twisted Nematics Nonlinear Waveguides**

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In the paper, the light beam propagation in twisted nematic liquid crystalline waveguides is analyzed theoretically. It is shown that optical reorientation nonlinearity in analyzed waveguides is large enough to observe spatial solitons formation with milliwatts of light power. The reorientation nonlinearity induces self-focusing and changes the direction of propagation of the light beam.

Keywords: liquid crystalline waveguides; spatial solitons;

### **INTRODUCTION**

Nematic liquid crystals are unique materials for nonlinear optics. The use of nematics leads to the numerous nonlinear optical phenomena arising from molecular reorientation or/and thermal effects<sup>[1,2]</sup>. Nematic liquid crystals are also very interesting medium for creation of optical spatial solitons. Experimental results showed that for light power of the order of only a few mW there could be achieved self-trapped beam at distances of the order of a few mm<sup>[3-9]</sup>. The self-focusing in liquid crystals were reported in capillaries<sup>[4-7]</sup>, in planar cells<sup>[8]</sup>, and in planar waveguides<sup>[9]</sup>.

In the previous works usually the planar or homeotropic texture in layer structures and axial in capillaries were used. In this paper the twisted nematics configuration is analyzed. The thickness of the liquid crystalline layer is assumed to be comparable with the wavelength, that allows treating the nematics layer as an optical planar waveguide. Light beam propagating in such waveguide diffracts and due to the structural anisotropy propagates at some angle to the direction of the input light.

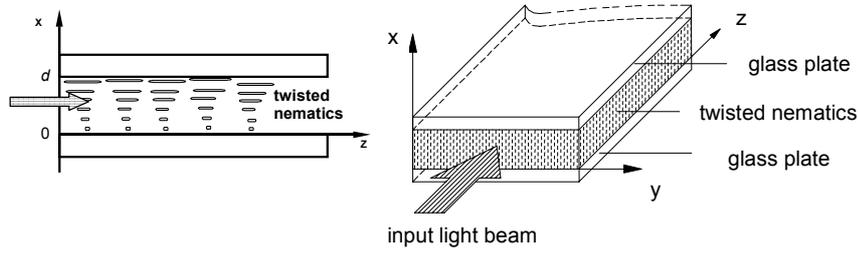


FIGURE 1 Schematic drawing of planar waveguides with twisted nematic liquid crystal

Optical nonlinearity due to the reorientation process causes that for higher light intensities the light beam is self-focused and changes the direction of propagation. The proposed configuration of liquid crystalline waveguide can be applied to switching of the light beam in the low-power all-optical systems.

### THEORY OF LIGHT BEAM PROPAGATION

The light beam propagating in a planar waveguide filled with twisted nematic liquid crystal (see Figure 1) is taken into consideration. The electromagnetic field with dominating  $E_y$  component of the electric field is assumed. In nonlinear regime the liquid crystal molecules are forced to reorient in the  $yz$  plane and consequently the electric permittivity tensor has the form:

$$\epsilon = \begin{pmatrix} \epsilon_{\perp} & 0 & 0 \\ 0 & \epsilon_{\perp} + \Delta\epsilon \cos^2 \theta & \Delta\epsilon \sin \theta \cos \theta \\ 0 & \Delta\epsilon \sin \theta \cos \theta & \epsilon_{\perp} + \Delta\epsilon \sin^2 \theta \end{pmatrix}, \quad (1)$$

where  $\Delta\epsilon = \epsilon_{\parallel} - \epsilon_{\perp}$  is an optical anisotropy,  $\epsilon_{\perp} = n_o^2$  is an ordinary and  $\epsilon_{\parallel} = n_e^2$  is an extraordinary electric permittivity, and  $\theta$  is an orientation angle measured as an angle between the director and the  $y$ -axis. In this medium the Maxwell's equations for monochromatic electromagnetic waves have the form:

$$\begin{aligned} & \left[ k_0^2 \epsilon_{yy} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \frac{\epsilon_{yy}}{\epsilon_{xx}} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y \partial z} \frac{\epsilon_{yz}}{\epsilon_{xx}} \right] E_y \\ & = \left[ -k_0^2 \epsilon_{yz} - \frac{\partial^2}{\partial y^2} \frac{\epsilon_{yz}}{\epsilon_{xx}} - \frac{\partial^2}{\partial y \partial z} \left( \frac{\epsilon_{zz}}{\epsilon_{xx}} - 1 \right) \right] E_z, \end{aligned} \quad (2)$$

$$\begin{aligned}
 & \left[ k_0^2 \varepsilon_{zz} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \frac{\varepsilon_{zz}}{\varepsilon_{xx}} + \frac{\partial^2}{\partial y \partial z} \frac{\varepsilon_{yz}}{\varepsilon_{xx}} \right] E_z \\
 & = \left[ -k_0^2 \varepsilon_{yz} - \frac{\partial^2}{\partial z^2} \frac{\varepsilon_{yz}}{\varepsilon_{xx}} - \frac{\partial^2}{\partial y \partial z} \left( \frac{\varepsilon_{yy}}{\varepsilon_{xx}} - 1 \right) \right] E_y,
 \end{aligned} \tag{3}$$

where  $\varepsilon_{ab}$  are the components of the permittivity tensor (1) and  $k_0 = \omega/c$  is the wave vector.

The  $E_z$  component of the field is assumed to be much weaker than the  $E_y$  component. This approximation allows introducing the ansatz:

$$E_y = A(y, z) \varphi(x) \exp(i\omega t - ik_0 Nz), \tag{4}$$

$$E_z = A(y, z) \phi(x) \exp(i\omega t - ik_0 Nz), \tag{5}$$

where  $\varphi \exp(i\omega t - ik_0 Nz)$  and  $\phi \exp(i\omega t - ik_0 Nz)$  are components of the planar waveguide mode,  $N$  is an effective refractive index and  $A$  is a complex amplitude slowly varying in respect to  $z$ . The mode envelopes  $\varphi(x)$  and  $\phi(x)$  fulfill the equations:

$$\left[ \frac{\partial^2}{\partial x^2} + k_0^2 (\varepsilon_{yy}^{(0)} - N^2) \right] \varphi = -k_0^2 \varepsilon_{yz}^{(0)} \phi, \tag{6}$$

$$\left[ \frac{\partial^2}{\partial x^2} + k_0^2 \varepsilon_{zz}^{(0)} \left( \frac{N^2}{\varepsilon_{xx}} - 1 \right) \right] \phi = k_0^2 \varepsilon_{yz}^{(0)} \left( \frac{N^2}{\varepsilon_{xx}} - 1 \right) \varphi, \tag{7}$$

where  $\varepsilon^{(0)}$  is an electric permittivity corresponding to the initial orientation of the nematics.

In this paper the calculations are done for asymmetrically twisted nematics i.e. for the orientation of liquid crystalline molecules at boundaries equal to  $\theta(0)=0$  and  $\theta(d)=\pi/2$  which corresponds to the initial orientation:  $\theta(x)=\pi x/2d$ . The distribution of mode envelopes for both components of electric field in such waveguide is presented in Figure 2. Note, that the value of  $E_z$  is much smaller than  $E_y$ . The numerical results were calculated for waveguides made by liquid crystal film of thickness  $d=2\mu\text{m}$  with refractive indices  $n_o=1.52$  and  $n_e=1.69$  surrounded by glass plates with refractive index  $n_c=1.45$  and for the wavelength  $\lambda=842$  nm. The values of refractive indices correspond to *4-trans-4'-n-hexyl-cyclohexyl-isothiocyanatobenzene* (6CHBT) nematic liquid crystal.

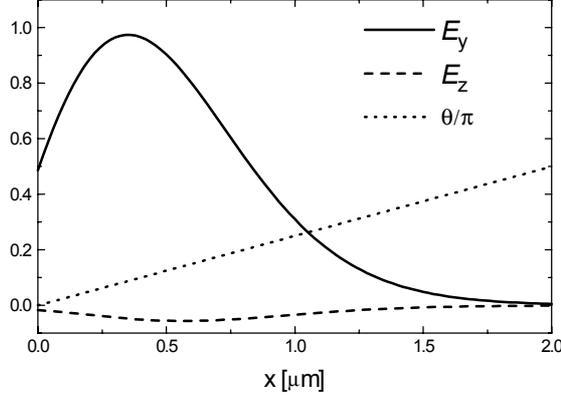


FIGURE 2 Electric field components  $E_y$  and  $E_z$  distribution in guided mode and orientation angle  $\theta$  in analyzed twisted nematics waveguide

Assuming that the field component  $E_z \ll E_y$ , the slowly varying complex amplitude  $A$  fulfills the equation obtained from integration of equation (2) over the cross section:

$$2ik_0N \frac{\partial}{\partial z} A = \frac{\partial^2}{\partial y^2} (1 + \kappa_1) A - ik_0N \frac{\partial}{\partial y} \kappa_2 A + k_0^2 N^2 \kappa_3 A, \quad (8)$$

where nonlinear coefficients are defined as follows:

$$\kappa_1 = \frac{\Delta\epsilon \int \cos^2 \theta \varphi^2 dx}{\epsilon_{\perp} \int \varphi^2 dx}, \quad (9)$$

$$\kappa_2 = \frac{\Delta\epsilon \int \sin \theta \cos \theta \varphi^2 dx}{\epsilon_{\perp} \int \varphi^2 dx}, \quad (10)$$

$$\kappa_3 = \frac{\Delta\epsilon \int (\cos^2 \theta - \cos^2 \theta^{(0)}) \varphi^2 dx}{N^2 \int \varphi^2 dx}. \quad (11)$$

The distribution of the orientation angle  $\theta$  is calculated from the Euler-Lagrange equation for the nematic liquid crystals in the form<sup>[2]</sup>:

$$\frac{d^2\theta}{dx^2} + |A|^2 [2\varphi\phi \cos 2\theta - (\varphi^2 - \phi^2) \sin 2\theta] = 0, \quad (12)$$

where  $K_{22}$  is an elastic constant corresponding to the twist deformation. Additionally, the normalization of amplitude  $A$  were assumed in such a form that the power in the light beam is equal to:

$$P = \frac{4K_{22}N}{\varepsilon_0 \Delta \varepsilon Z} \int |A|^2 dy, \quad (13)$$

where  $Z$  is the impedance of the vacuum. For 6CHBT liquid crystal the elastic constant  $K_{22}=3.5 \cdot 10^{-12}$  N and the amplitude value  $|A|^2=1/\mu\text{m}$  corresponds to the light power density  $\approx 14\text{mW}/\mu\text{m}$ .

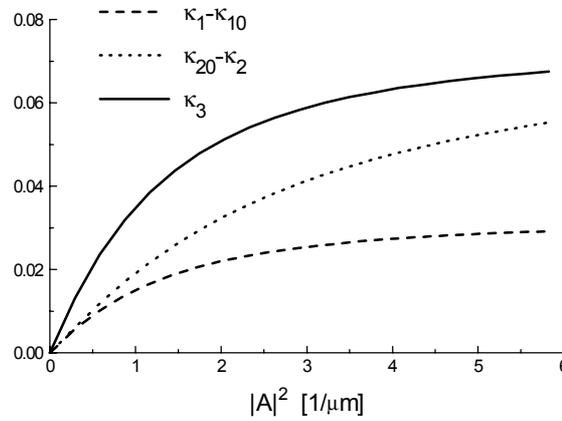


FIGURE 3 Dependence of the nonlinear coefficients on light intensity ( $\kappa_{10}$  and  $\kappa_{20}$  are the value of the coefficients for  $A \rightarrow 0$ )

In Figure 3 the dependence of nonlinear parameters  $\kappa_1$ ,  $\kappa_2$  and  $\kappa_3$  on power density are presented. They were obtained by solving the equation (12) for strong anchoring conditions, i.e. for boundary conditions  $\theta(0)=0$  and  $\theta(d)=\pi/2$ . Following the discussion presented in Ref. [10] the nonlinear coefficients could be approximated by analytical formulas:

$$\kappa_1 = \kappa_{10} + \alpha_{10} \frac{|A|^2}{1 + |A/A_{1S}|^2}, \quad (14)$$

$$\kappa_2 = \kappa_{20} - \alpha_{20} \frac{|A|^2}{1 + |A/A_{2S}|^2}, \quad (15)$$

$$\kappa_3 = \alpha_{30} \frac{|A|^2}{1 + |A/A_{3S}|^2}, \quad (16)$$

where saturation amplitudes  $A_S$  as well as values of  $\kappa_{p0}$  and  $\alpha_{p0}$  ( $p=1,2,3$ ) could be fitted from exact calculations.

All coefficients depend on light intensity in a saturated-like form and all are much lower than 1 ( $\kappa \ll 1$ ). Therefore the coefficient  $\kappa_1$  could be neglected in the equation (8). The coefficient  $\kappa_2$  is responsible for walking off the light beam. For low intensities (in the linear case:  $A \rightarrow 0$ )  $\kappa_2$  is the largest (equal to  $\kappa_{20}$ ) and increasing intensities causes decreasing their values ( $\kappa_2 \rightarrow 0$  for  $A \rightarrow \infty$ ). It means that for higher intensities the light beam walk-off is lower, i.e. the light beam direction is changing with changing the light power. The last term in equation (8), connected with the coefficient  $\kappa_3$ , is purely nonlinear and it is responsible for self-focusing of light beam and creation of the spatial soliton. However, this nonlinear component could also modify the direction of light beam propagation. If we neglect the coefficient  $\kappa_1$ , and assume that  $\kappa_2$  do not depend on light intensity (i.e.  $\kappa_2 = \kappa_{20}$ ), and simplify the nonlinearity of coefficient  $\kappa_3$  to the Kerr-like (i.e.  $\kappa_3 = \alpha_{30}|A|^2$ ) then the equation (8) will have the form:

$$2ik_0N \frac{\partial}{\partial z} A = \frac{\partial^2}{\partial y^2} A - ik_0N\kappa_{20} \frac{\partial}{\partial y} A + k_0^2N^2\alpha_{30}|A|^2 A. \quad (17)$$

This equation (17) has analytical solutions in a form of a beam preserving their shape along propagation distance:

$$A(y, z) = A_0 \frac{\exp(-i\beta z + i\gamma y)}{\cosh(\sigma(y - \rho z))}, \quad (18)$$

where  $\sigma^2 = k_0^2 N^2 \alpha_{30} A_0^2 / 2 = \gamma^2 + kN(2\beta - \gamma\kappa_{20})$ , and  $2\gamma = k_0 N(\kappa_{20} - 2\rho)$ .

The solution (18) is similar to the bright solitons governed by the nonlinear Schrödinger equation. However, the light beam defined by equation (18) propagates into the direction  $y - \rho z = 0$  and has a phase parallel to the plane  $\beta z - \gamma y = \text{const}$ . The beam for  $\gamma = 0$  propagates at a direction the same as a linear beam (for  $\alpha_{30} = 0$ ). In this case  $\rho = \kappa_{20}/2$  and the phase plane is perpendicular to the  $z$ -axis. This solution should appear for lower light intensities. On the other hand, the soliton with  $\rho = 0$  propagates along the  $z$ -axis but their phase is not perpendicular to the  $z$ -axis.

## NUMERICAL RESULTS AND CONCLUSIONS

Propagation of light beam in twisted nematics waveguide and possibility of creation solitons in a form (18) were tested by using the

numerical Beam Propagation Method. At the input the Gaussian beam were used in the form:

$$A(z = 0) = A_0 \exp(-y^2/w^2). \quad (19)$$

The numerical results are presented in the Figure 4. The value of parameters approximating the nonlinear coefficients  $\kappa$  were fitted from the dependence showed in the Figure 3 as follows:  $\kappa_{20}=0.07$ ,  $\alpha_{20}=0.025$ ,  $(A_{2s})^2=3.6$ ,  $\alpha_{30}=0.06$ ,  $(A_{3s})^2=1.4$ , and  $\kappa_1=0$  were taken. The input light beam with  $w=3\mu\text{m}$  were used, which means that  $A_0^2=1/\mu\text{m}$  corresponds to the total power  $\approx 50\text{mW}$  in 6CHBT nematics.

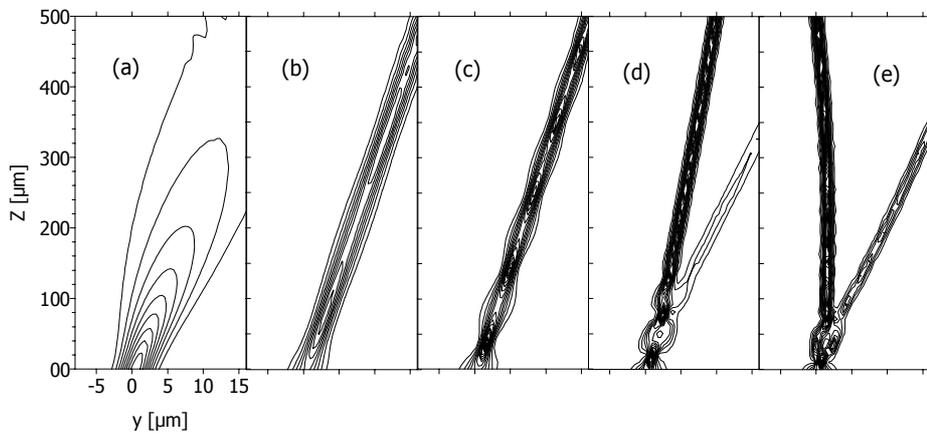


FIGURE 4 Light beam propagation in twisted nematics waveguide for different input light power:  $A_0^2 = 0.0$  (a);  $0.1$  (b);  $0.2$  (c);  $0.5$  (d);  $1.0$  (e) [ $1/\mu\text{m}$ ]. Distances are measured in micrometers

In the linear case (for low light power) the typical diffracted light beam propagating at some angle to the  $z$ -axis exists (Figure 4(a)). For larger light power the diffraction is compensated by the nonlinearity and soliton-like beam propagating at the direction not perpendicular to the  $z$ -axis is obtained (Figure 4(b)). This beam corresponds to the soliton described by the equation (18) for  $\gamma=0$ . For higher values of the light power, i.e. larger nonlinear terms light beam width oscillates (Figures 4(c)) like in breathing solitons. Next, for large enough value of the light power the beam forms spatial soliton propagating at another angle (Figure 4(d)) and finally, propagating parallel to the  $z$ -axis (Figure 4(e)). The last beam corresponds to the soliton solution described by equation (18) with  $\rho=0$ .

Presented numerical results show that reorientational nonlinearity can support self-trapped beams propagating in different directions while

the direction of propagation is dependent on the light intensity. The light power necessary to observe such behavior is of order of milliwatts. It should be noted, that the main source of such behavior is the nonlinear self-focusing term connected with the coefficient  $\kappa_3$ . The nonlinear changes of the coefficient  $\kappa_2$  (which is responsible for the beam walk-off) are not very large: even in the center of the output beam in Figure 4(e) the coefficient  $\kappa_2 > 0.03$ . This means, that without the self-focusing effect the changes of the direction of the light beam propagation will require much larger values of the light power.

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