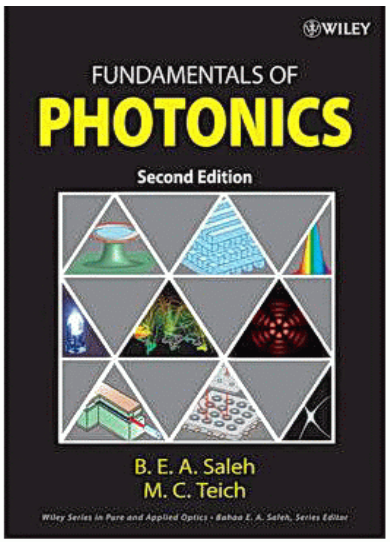
WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

Quantum Electronics
Lecture 5



Electro-optical modulation of light

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♦ **Introduction to light modulation**

♦ **Linear Electro-optic effect, phase retardation**

♦ **Electro-optic modulation of amplitude or phase**

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Optical beam modulation

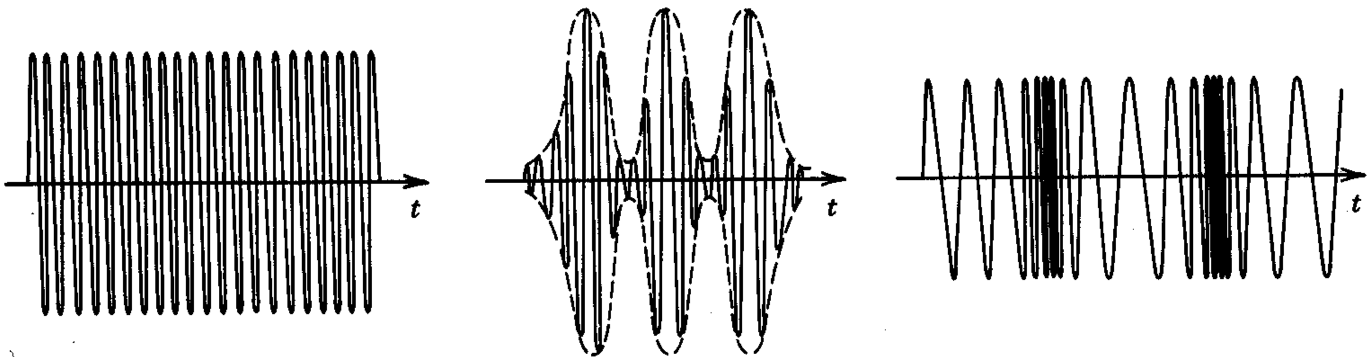
Optical field - very high frequency carrier (e.g. 200 THz for λ=1.5 μm)

→ **large modulation frequency possible**

→ **large amounts of information can be coded**

Modulation formats:

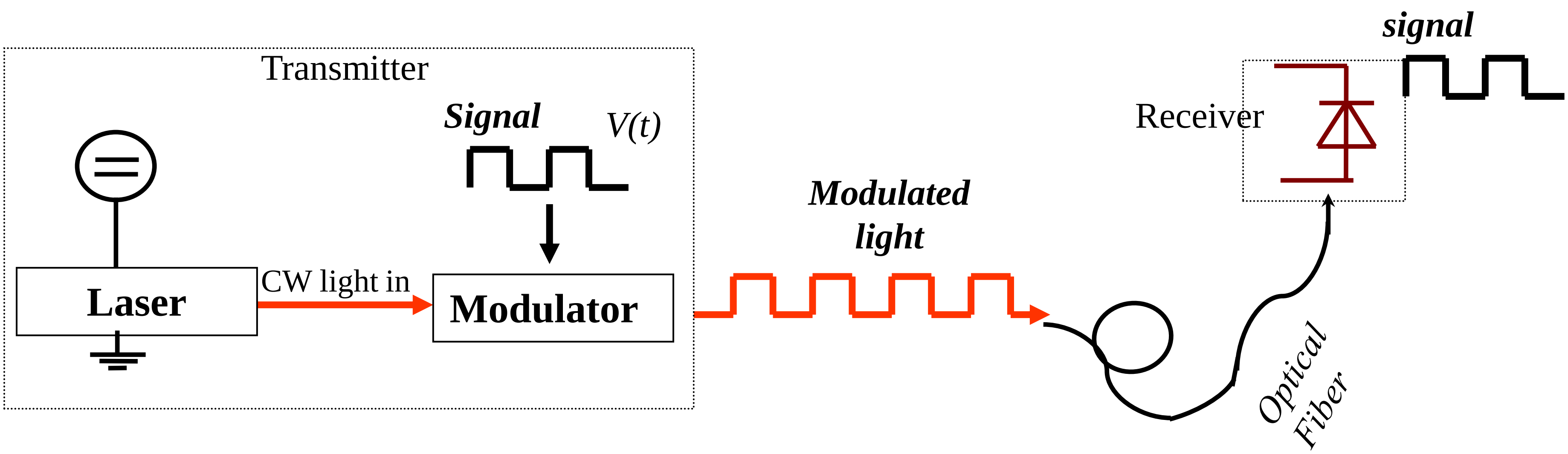
Amplitude Modulation (**AM**), **P**hase **M**odulation (**PM**), Frequency Modulation (**FM**)



Optical carrier beam   Amplitude Modulation   Frequency Modulation

Applications: ***data encoding in optical communication,***
active mode locking of lasers, short pulse generation,
beam deflectors, etc

Optical beam modulation for data encoding



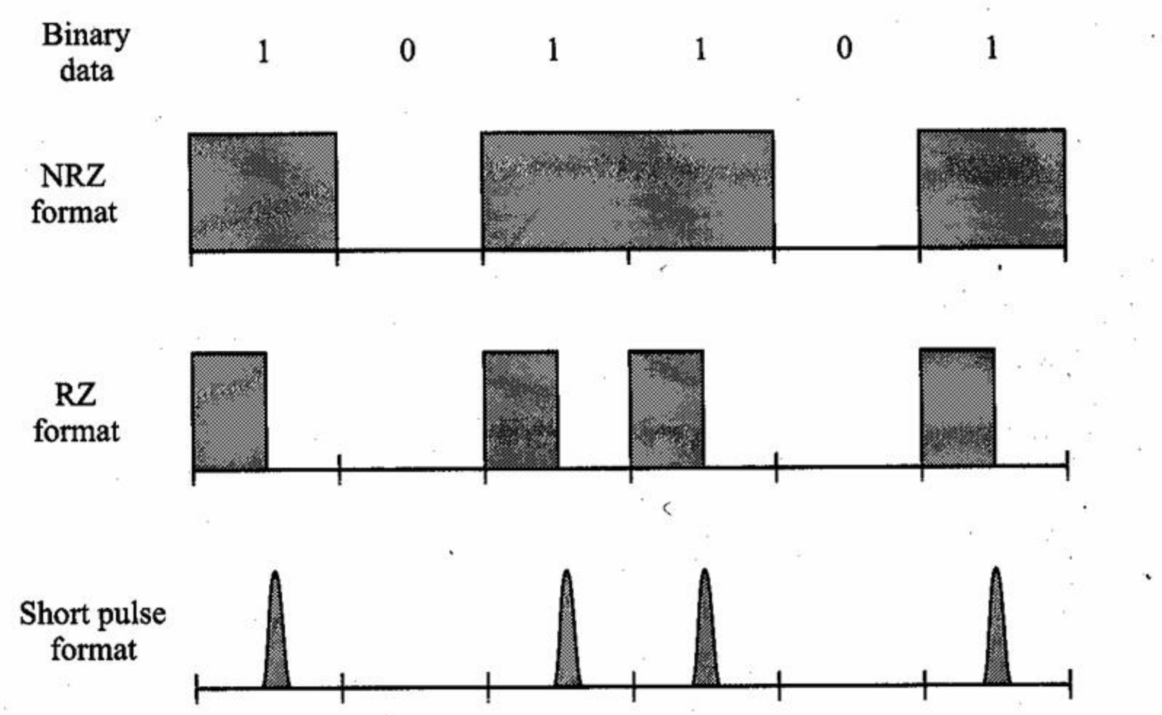
Communication system: a physical variable (light intensity, field amplitude, frequency, phase, or even polarization) is modulated at one point and detected at another point

Amplitude modulation

Most popular for optical fiber communication systems, primarily due to the simplicity of envelope photo-detection

Non-Return-to-Zero

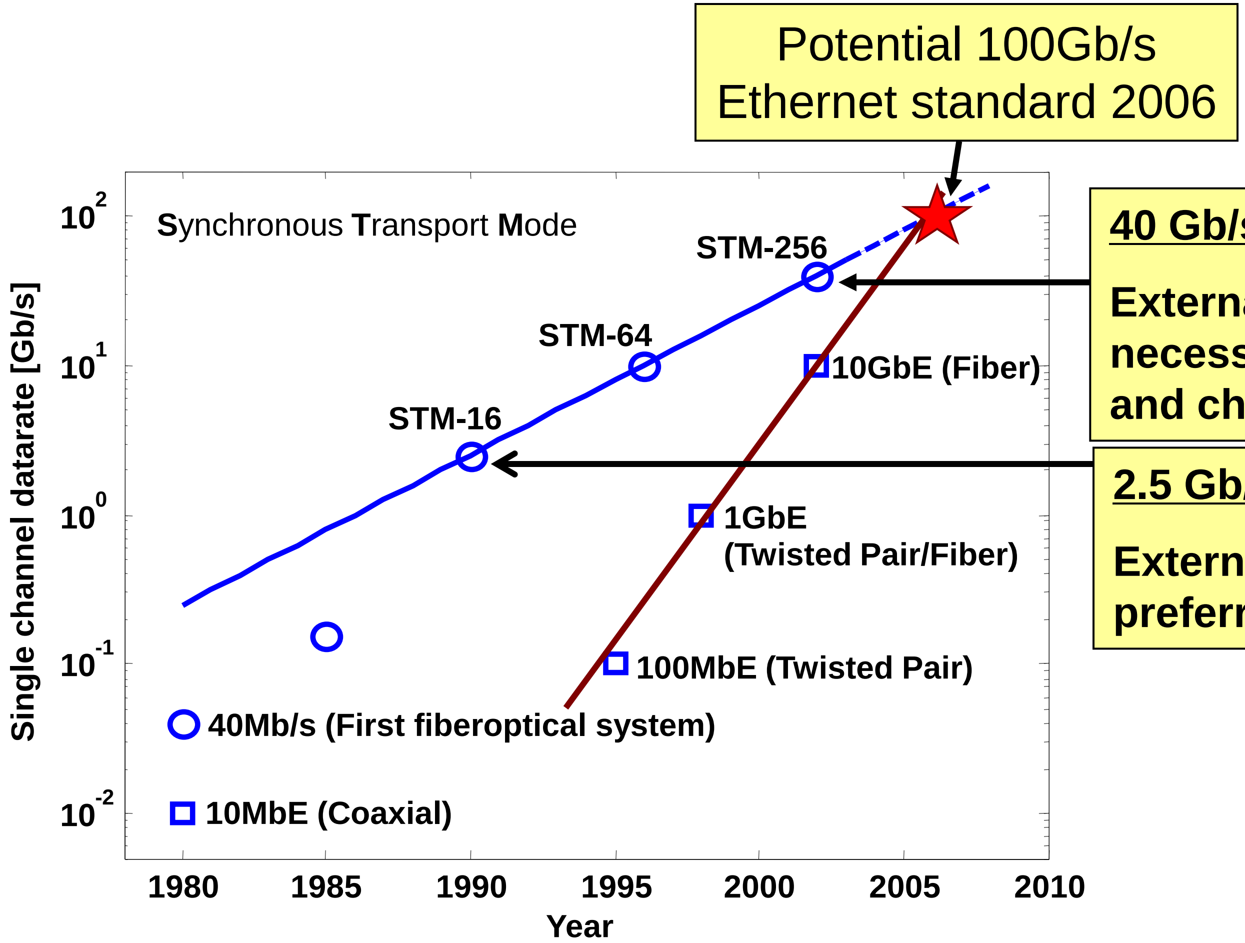
Return-to-Zero



40 Gb/s commercially available (Lithium Niobate), **Target: Terabit (1000 Gb/s) speed**

For modulation 2.5 Gb/s and above external modulators preferred to avoid chirp

High speed modulator: Beyond 40 Gb/s



40 Gb/s and above:

External modulators necessary due to speed and chirp.

2.5 Gb/s and above:

External modulators preferred due to chirp.

Motivation:

Investigate transmitter technologies suitable for 100Gb/s

Electro-Optic (EO) effects

Combine a DC (or low frequency field) $\mathbf{E_o}$ with a wave $\mathbf{E_\omega}$ **cos(ω t)** at optical frequency **ω**:

For $E_o \ll E_\omega$

$$\mathbf{P} = \varepsilon_\mathbf{o}\chi^{(1)}\left(\mathbf{E_o} + \mathbf{E}_\omega\right) + \varepsilon_\mathbf{o}\chi^{(2)}\left(\mathbf{E_o} + \mathbf{E}_\omega \cos(\omega\, t)\right)^2 + \varepsilon_\mathbf{o}\chi^{(3)}\left(\mathbf{E_o} + \mathbf{E}_\omega \cos(\omega\, t)\right)^3$$

$$P_\omega = \varepsilon_o\chi^{(1)}E_\omega + 2\varepsilon_o\chi^{(2)}E_oE_\omega + 3\varepsilon_o\chi^{(3)}E_o^2E_\omega + 3\varepsilon_o\chi^{(3)}E_\omega E_{-\omega}E_\omega + \ldots$$

Pockels effect > DC Kerr effect > AC Kerr effect

Friedrich Pockels
(1865 - 1913)

John Kerr
(1824-1907)

Linear EO (Pockels) effect

$$D = \varepsilon_0 E + P = \varepsilon\, E = \varepsilon_0 n^2 E$$

$$P_\omega = \varepsilon_o \chi^{(1)} E_\omega + 2\varepsilon_o \chi^{(2)} E_o E_\omega + \ldots$$

$$\Delta\varepsilon = \varepsilon_0 \Delta n^2 = \varepsilon_0 2\chi^{(2)} E_0$$

← Linear electro-optic effect discovered by **Pockels** in 1883

External variation of the DC field provides phase modulation of light
→ optical switching, wavelength tuning

$$\chi^{(2)}_{ijk} = -\frac{1}{2} n_{ii}^2 n_{ji}^2 r_{ijk}$$

Relation between the 2nd order susceptibility $\chi^{(2)}_{ijk}$ and the Pockels tensor r_{ijk}

Refractive indices along the principal axes

$$\Delta n_{ij}^2 = -n_{ii}^2 n_{jj}^2 \sum_k r_{ijk} E_k$$

typical values for r: 10^{-12} to 10^{-10} m/V ⟶ Δn for E=10^6 V/m : 10^{-6} to 10^{-4} (crystals)

Impermeability tensor

Convenient to describe the induced changes in terms of **impermeability tensor** η

$$\eta_{ij} = \frac{1}{n_{ij}^2}$$

$$\eta_{ij} = \eta_{ij}^{(0)} + \sum_k r_{ijk} E_k$$

$$\Delta\eta_{ij} = \Delta\left(\frac{1}{n_{ij}^2}\right) = \sum_k r_{ijk} E_k$$

$$\sum_{ij} \eta_{ij}(E) x_i x_j = 1$$ **Index ellipsoid**

Useful **scalar** relations to estimate order of magnitudes:

$$\Delta n^2 = \left(\frac{dn^2}{dn}\right) \cdot \Delta n = 2n \cdot \Delta n = -n^4 r \cdot E \longrightarrow \Delta n = -\frac{1}{2} n^3 r \cdot E$$

$$\Delta\eta = \left(\frac{d\eta}{dn}\right) \cdot \Delta n = \left(\frac{-2}{n^3}\right) \cdot \left(-\frac{1}{2} r \cdot n^3 E\right) = r \cdot E$$

Impermeability tensor – contracted form

In lossless and optically inactive media $\eta_{ij} = \dfrac{1}{n_{ij}^2}$ **is symmetric:**

$$\eta_{ij} = \eta_{ji}$$

Hence 3x3 matrix can be reduced (contracted) to a column of 6 independent elements:

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \to \begin{bmatrix} \eta_1 & \eta_6 & \eta_5 \\ \eta_6 & \eta_2 & \eta_4 \\ \eta_5 & \eta_4 & \eta_3 \end{bmatrix} \to \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \end{bmatrix}$$

$$\eta_1 \equiv \left(\frac{1}{n^2}\right)_1 = \frac{1}{n_x^2}, \quad \eta_2 \equiv \left(\frac{1}{n^2}\right)_2 = \frac{1}{n_y^2}, \quad \eta_3 \equiv \left(\frac{1}{n^2}\right)_3 = \frac{1}{n_z^2}$$

From diagonal elements

$$\eta_4 \equiv \left(\frac{1}{n^2}\right)_4 = \frac{1}{n_{yz}^2}, \quad \eta_5 \equiv \left(\frac{1}{n^2}\right)_5 = \frac{1}{n_{xz}^2}, \quad \eta_6 \equiv \left(\frac{1}{n^2}\right)_6 = \frac{1}{n_{xy}^2}$$

From off-diagonal elements

Pockels tensor – symmetries, contracted notation

In a centrosymmetric crystal $r = 0$ **:**

From the symmetry r should not change under lattice inversion: $r^{inv} = r$

From physics (linear charge displacement under DC field): $r^{inv} = -r$

$\longrightarrow r = 0$

In lossless and optically inactive media: $\boxed{r_{ijk} = r_{jik}}$ ***Permutation symmetry***

Contracted notation: $r_{ijk} = r_{lk}$

$ij:$	11	22	33	23,32	31,13	12,21
$l:$	1	2	3	4	5	6

$$
\begin{aligned}
r_{1k} &= r_{11k} \\
r_{2k} &= r_{22k} \\
r_{3k} &= r_{33k} \\
r_{4k} &= r_{23k} = r_{32k} \\
r_{5k} &= r_{13k} = r_{31k} \\
r_{6k} &= r_{12k} = r_{21k}
\end{aligned}
$$

$k = 1,2,3$

To be further reduced by spatial (group) symmetry

Impact of linear EO effect in contracted notation

The change induced by the DC electric field E=(*Ex, Ey, Ez*): $\Delta\eta_{ij} = \Delta\left(\frac{1}{n_{ij}^2}\right) = \sum_k r_{ijk} E_k$

can now be expressed in the contracted form:

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\Delta\eta_l \equiv \Delta\left(\frac{1}{n^2}\right)_l = r_{lk} E_k$$

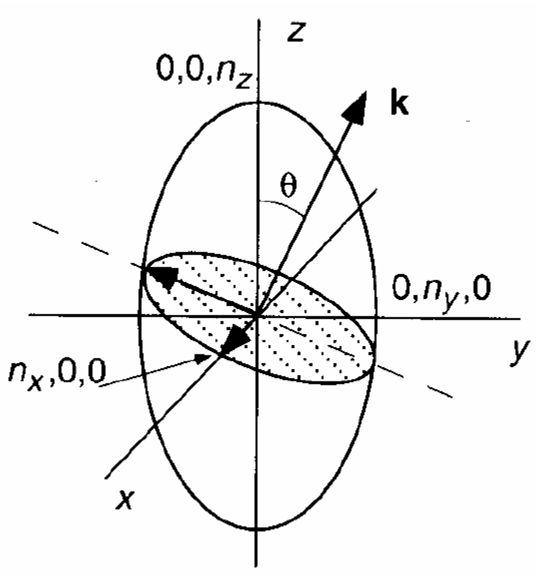
summation over repeated indices!

Linear EO effect – impact on Index ellipsoid

External electric field ***E*** distorts the Index ellipsoid. Possible impact:

change of the axes length (diagonal elements of η)

rotation ⟶ off-diagonal (mixed) terms appear in η



Unperturbed (***E=0***) Index ellipsoid

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

$$\left[\frac{1}{n_x^2} + \Delta\left(\frac{1}{n^2}\right)_1\right]x^2 + \left[\frac{1}{n_y^2} + \Delta\left(\frac{1}{n^2}\right)_2\right]y^2 + \left[\frac{1}{n_z^2} + \Delta\left(\frac{1}{n^2}\right)_3\right]z^2 +$$

$$+ 2\Delta\left(\frac{1}{n^2}\right)_4 yz + 2\Delta\left(\frac{1}{n^2}\right)_5 xz + 2\Delta\left(\frac{1}{n^2}\right)_6 xy = 1$$

$$\Delta\left(\frac{1}{n^2}\right)_l = r_{lk}E_k$$

Examples of electro-optic tensors

Isotropic | **Anisotropic**

$$\bar{4}3m \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{pmatrix}$$

GaAs
Gallium Arsenide

$$\bar{4}2m \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$

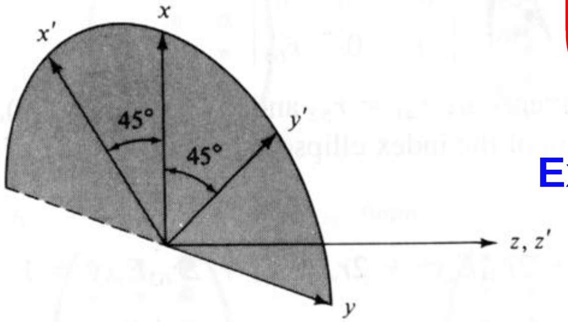
KDP: KH_2PO_4
Potassium Dihydrogen Phosphate

$$3m \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$$

$LiNbO_3$:
Lithium Niobate

Linear EO effect in KH_2PO_4 (KDP)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$



Obtain the equation for the index ellipsoid

$$\frac{x^2}{n_0^2}+\frac{y^2}{n_0^2}+\frac{z^2}{n_e^2}+2r_{41}E_x yz+2r_{41}E_y xz+2r_{63}E_z xy=1$$

Consider DC field along the optic axis z: $E=(0,0,E_z)$

Diagonalize the equation. Here, by rotating the reference system by 45°. For field polarized along z one obtains:

$$\left(\frac{1}{n_0^2}-r_{63}E_z\right)x'^2+\left(\frac{1}{n_0^2}+r_{63}E_z\right)y'^2+\frac{z^2}{n_e^2}=1$$

External field Ez induces the difference between $n_{x'}$ ***and*** $n_{y'}$

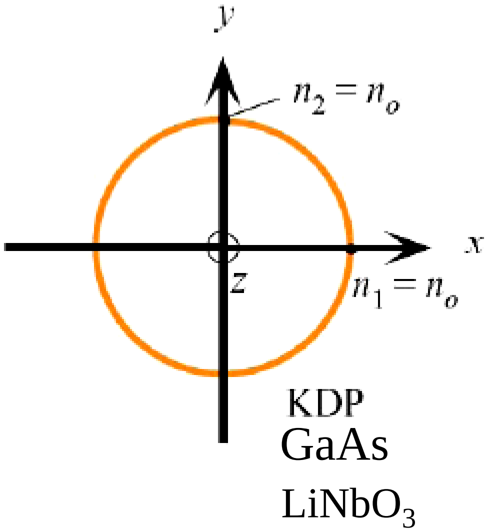
$$n_{x'}=n_0+\frac{1}{2}n_0^3 r_{63}E_z$$

$$n_{y'}=n_0-\frac{1}{2}n_0^3 r_{63}E_z$$

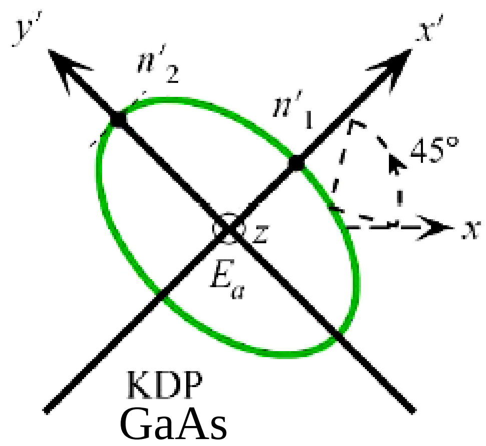
Electrically tunable birefirngence !

Electrically induced birefringence

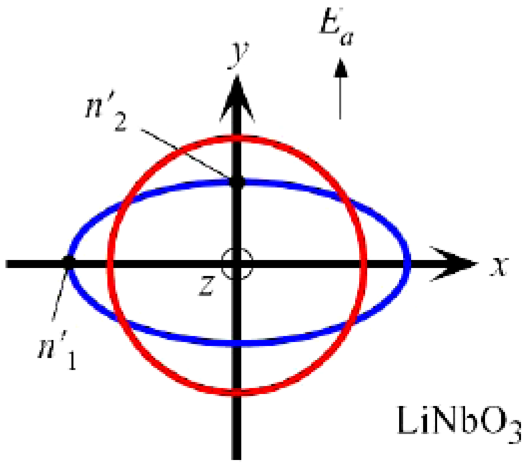
$E_a = 0$



$E_a = E_z$



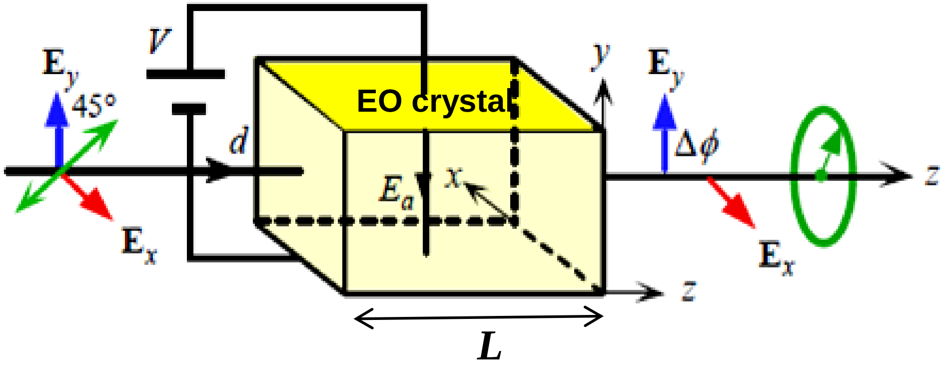
$E_a = E_y$



Isotropic GaAs became uniaxial
Uniaxial KDP and $LiNbO_3$ became biaxial

Pockels cell – phase retardation

Consider a light beam passing through a "cell" made of an electro-optic crystal, with its Index of ellipsoid modified by *Ea* field.
Assume the input light linearly polarized at some angle to the crystal axis (e.g. 45^0 as in the figure).



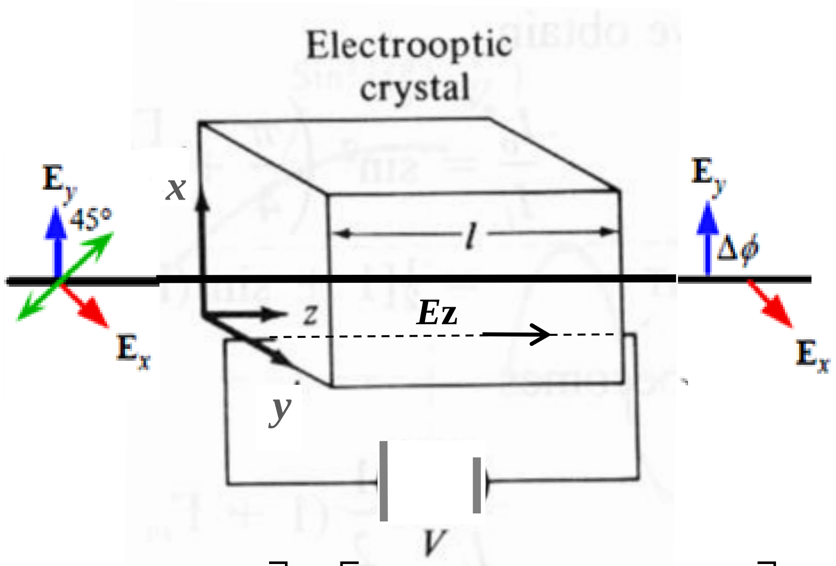
Decompose the input field into two fields *Ex* and *Ey* polarized along the crystal axes, propagate them separately, and add at the cell output.
The acquired phase retardation Γ between *Ex* and *Ey* will be determined by:

$$\Gamma \equiv \phi_x - \phi_y = k_0[n_x(E_a) - n_y(E_a)]L$$

Polarization state can thus be tuned by E_a, and if desired converted to amplitude or frequency modulation

Electro-optic retardation – longitudinal geometry

External electric field **along** the direction of light propagation



For KDP:

$$\Gamma = \phi_x - \phi_y = \left[k_0(n_0 + \frac{1}{2}n_0^3 r_{63} E_z) z\right] - \left[k_0(n_0 - \frac{1}{2}n_0^3 r_{63} E_z) z\right] = k_0 n_0^3 r_{63} E_z z = k_0 n_0^3 r_{63} V$$

Retardation depends on V but not on length

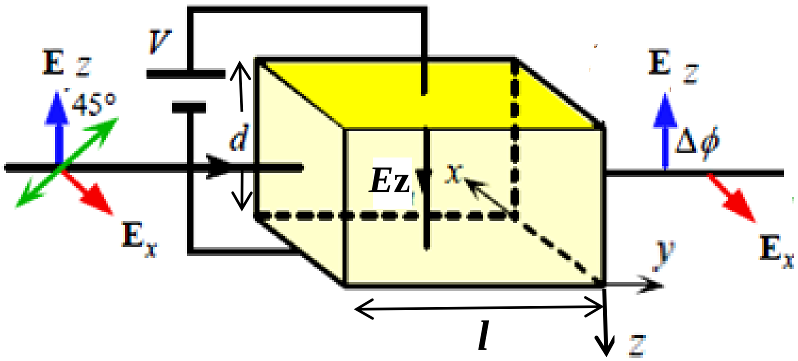
Half voltage Vπ - voltage for which the pase shift Γ= π

$$\Gamma \equiv \Delta\Phi = \pi \frac{V}{V_\pi}$$

Here: $V_\pi = \dfrac{\lambda}{2 n_0^3 r_{63}}$

Electro-optic retardation – transverse geometry

External electric field **normal** to the direction of light propagation



1. **Phase retardation depends on voltage *V*, length *l* and thickness *d***
2. **Γ has a term not depending on the applied voltage: birefringence effect**

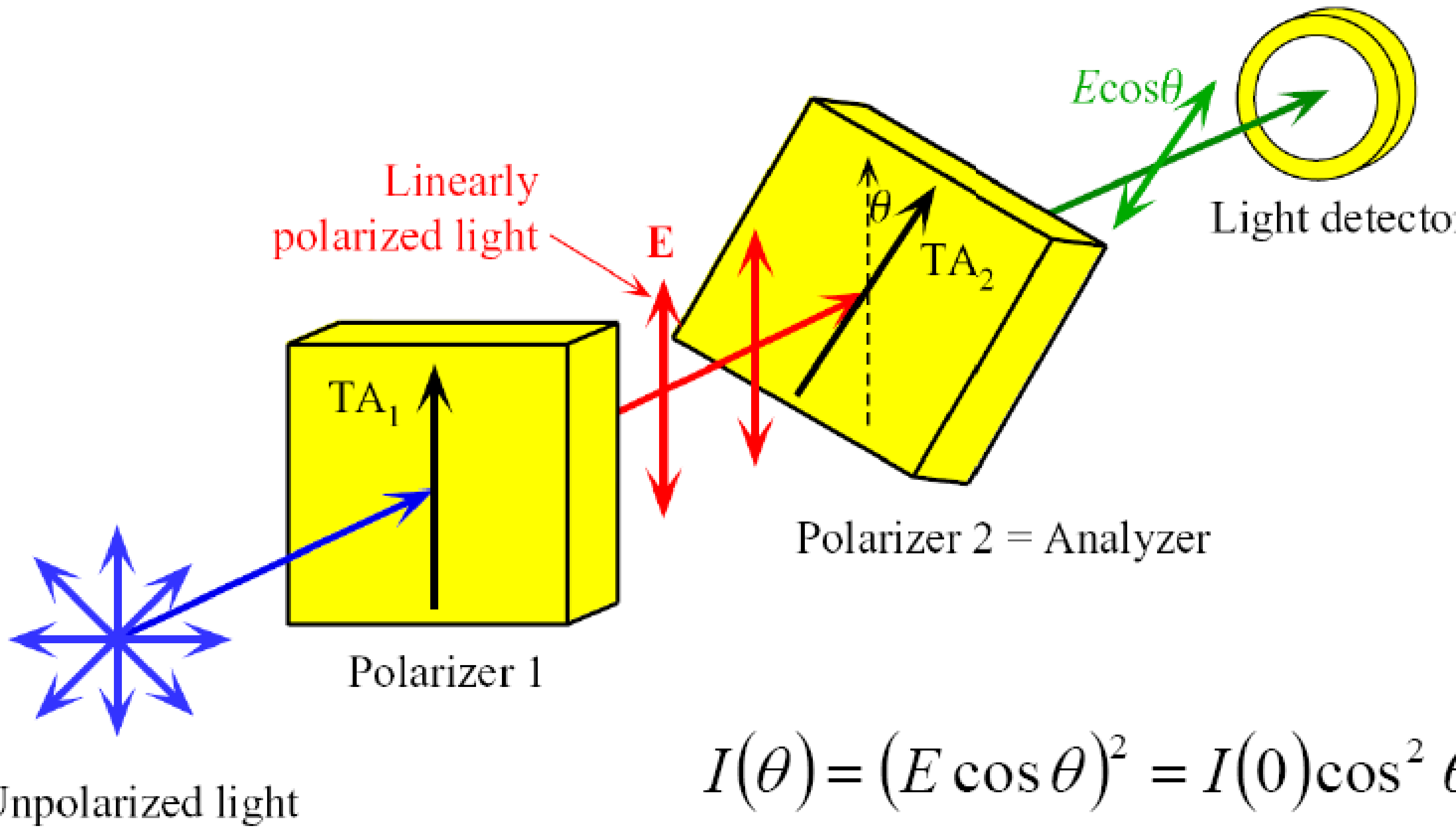
For KDP: $$\Gamma = \phi_x - \phi_z = \frac{\omega l}{c}\left[\left(n_o - n_e \right) + \frac{n_0^3 r_{63} V}{2d} \right]$$

At a given voltage *V* one can increase retardation by increasing modulator length *l* and/or decreasing its thickness *d*

$$V_\pi = \frac{\lambda d}{n_0^3 r_{63} l} - \frac{2(n_0 - n_e)d}{n_0^3 r_{63}}$$

Malus's Law

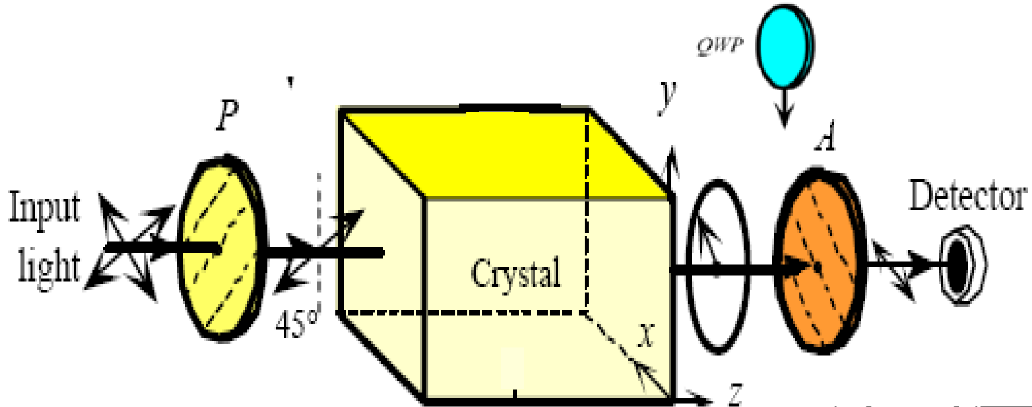
A linear polarizer will only allow electric field oscillations along some preferred directions, called the **transmission axis**, to pass through the device.



$$I(\theta) = (E\cos\theta)^2 = I(0)\cos^2\theta$$

Phase modulation converted to intensity modulation

Converting phase shift to transmitted intensity



$$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

After the polarizer P: $\vec{E}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \begin{bmatrix} e^{i\Gamma/2} & 0 \\ 0 & e^{-i\Gamma/2} \end{bmatrix}$

$$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{i\Gamma/2} & 0 \\ 0 & e^{-i\Gamma/2} \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = i\sin(\Gamma/2)\vec{E}_0$$

Relative transmitted intensity:

$$\frac{I_{out}}{I_{in}} = \sin^2(\Gamma/2)$$

With a quarter wave plate (QWP):

$$\frac{I_{out}}{I_{in}} = \sin^2(\pi/4 + \Gamma/2) = \frac{1}{2}\left[1 + 2\sin(\frac{\Gamma}{2})\cos(\frac{\Gamma}{2})\right] = \frac{1}{2}\left[1 + \sin(\Gamma)\right]$$

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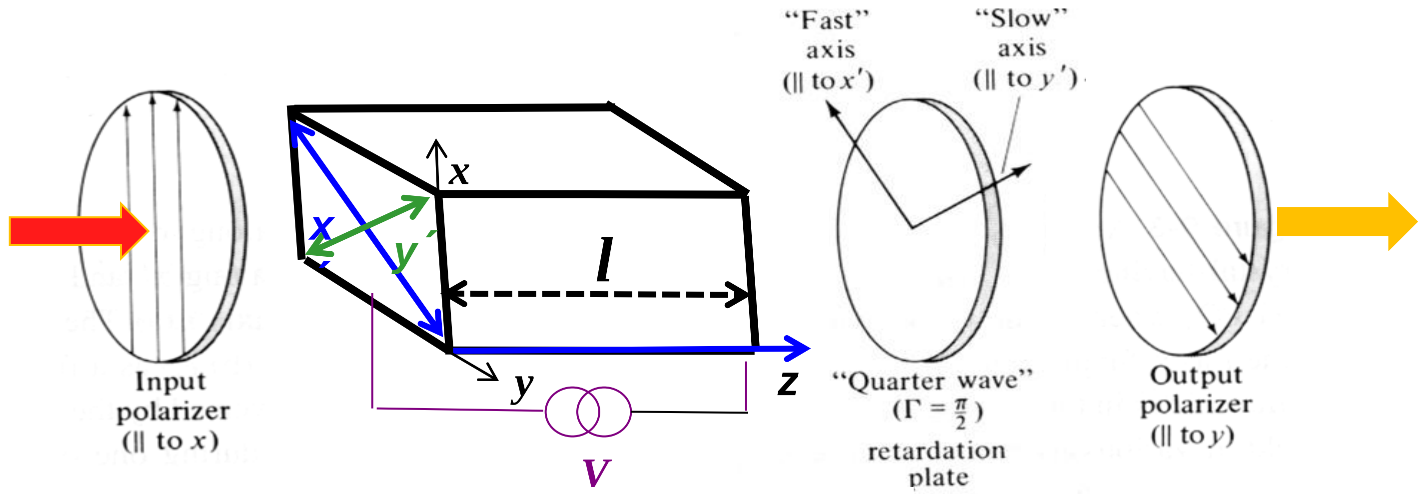
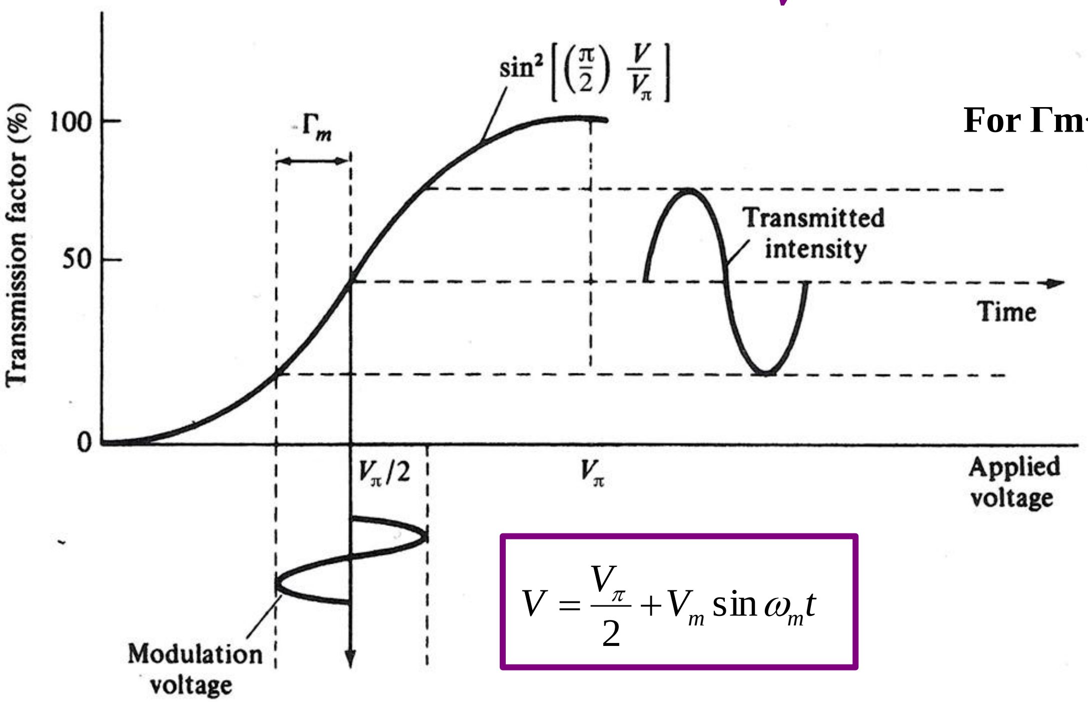
Amplitude modulation – longitudinal geometry





$$V = \frac{V_\pi}{2} + V_m \sin \omega_m t$$

For Γm<<1 - linear replica of the modulating voltage:

$$\frac{I_{out}}{I_{in}} = \frac{1}{2}\left[1 + \sin(\Gamma_m \sin \omega_m t)\right]$$

$$\cong \frac{1}{2}\left[1 + \Gamma_m \sin \omega_m t\right]$$

$$\Gamma = \pi \frac{V}{V_\pi} \qquad V_\pi = \frac{\lambda}{2 n_0^2 r_{63}} \text{ (for KDP)}$$

Phase (frequency) modulation

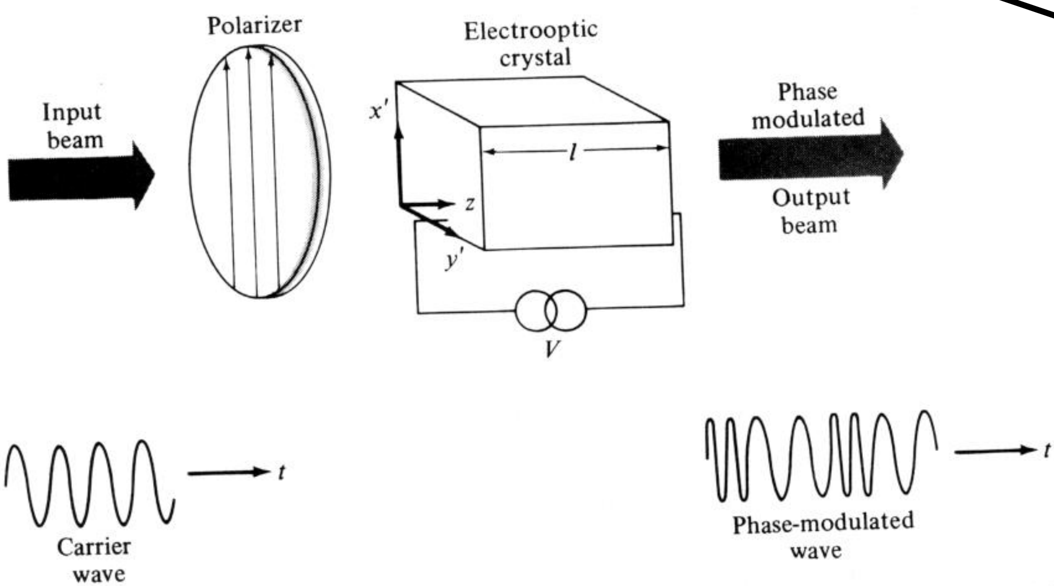
If an optical wave is incident normally on the x'-y' plane with its **E** vector along the x' direction, the electro-optic effect will simply change the output phase, without change of the polarization:

$$\Delta\phi_{x'} = -\frac{\omega n_0^3 r_{63}}{2c} E_z l$$

For an input beam $e_{in} = A\exp(i\omega t)$ and the external field $E_z = E_m \sin(i\omega_m t)$ the output becomes (disregarding the constant phase factor):

$$e_{out} = A\exp\left[i\left(\omega t + \delta\sin\omega_m t\right)\right] \qquad \delta = -\frac{\omega n_0^3 r_{63}}{2c} E_m l$$

Phase modulation index



Transit time limitation

Phase retardation for DC (or very slowly varying) field: $\Gamma = \phi_{x'} - \phi_{y'} = \frac{\omega n_0^3 r_{63} V}{c}$ $V = E_z l$

$$\Gamma_0 = \alpha E l$$

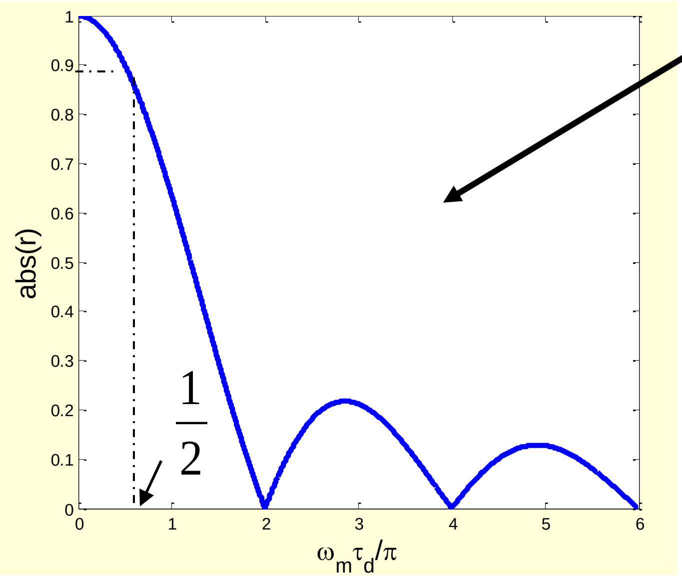
For E changing appreciably during the transit time $\tau_d = nl/c$ of the light through the crystal:

$$\Gamma = \alpha \int_0^l E(z)dz = \alpha \frac{c}{n} \int_{t-\tau_d}^{t} E(t')dt'$$

Where the wave enters the crystal at time t- τ_d, and leaves the crystal at time t

For $E = E_m \exp(i\omega_m t)$:

$$\Gamma = \Gamma_0 \left[\frac{1 - e^{-i\omega_m \tau_d}}{i\omega_m \tau_d} \right] e^{i\omega_m t}$$



r - decrease in peaking retardation resulting from the finite transit time. Modulation gets "averaged out"

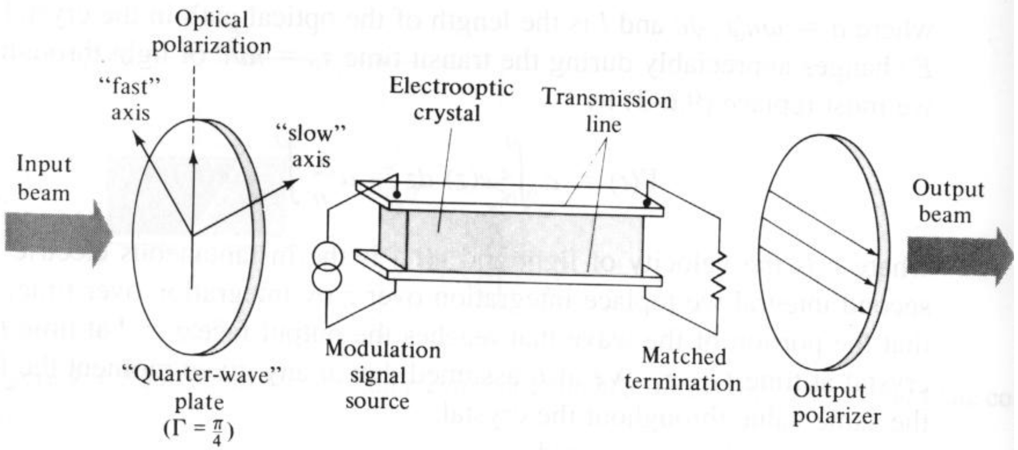
For abs(r)=0.9 as a threshold, the maximum modulation frequency:

$$(\nu_m)_{\max} = \frac{c}{4nl}$$

Traveling wave modulators

To overcome the transit –time limitation → **The modulation signal - a traveling wave phase matched with the optical wave**

Strip transmission line

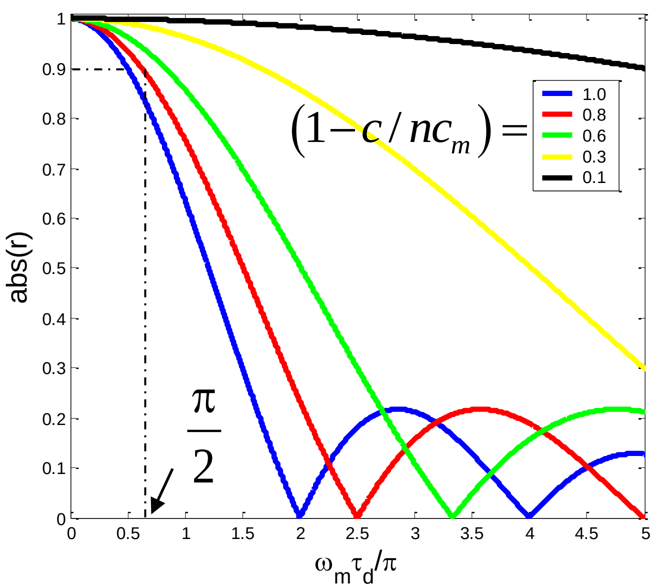


$$E = E_m e^{i\omega_m t - ik_m z}$$

c_m is the phase velocity of the modulation field

$$\Gamma = \Gamma_0 e^{i\omega_m t}\left[\frac{e^{-i\omega_m \tau_d(1-c/nc_m)}-1}{i\omega_m \tau_d\left(1-c/nc_m\right)}\right]$$

r



For perfect **phase-velocity** matching (c_m=c/n) r=1 –no frequency limitations

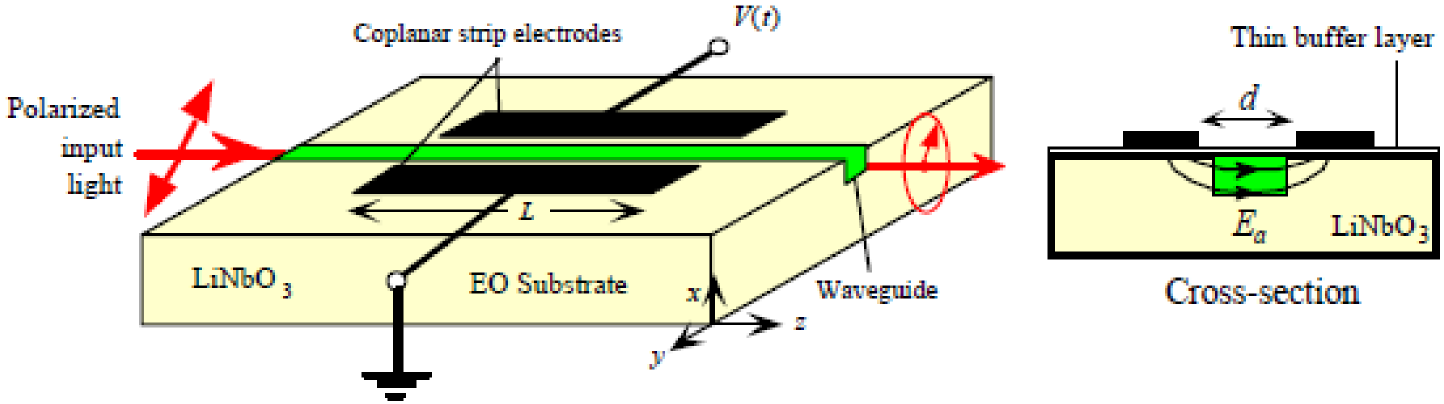
With r=0.9 the maximum operating frequency:

$$\left(\nu_m\right)_{\max} = \frac{c}{4nl\left(1-c/nc_m\right)}$$

Increased by factor:

$1/\left(1-c/nc_m\right)$!!

Integrated transverse Pockels cell

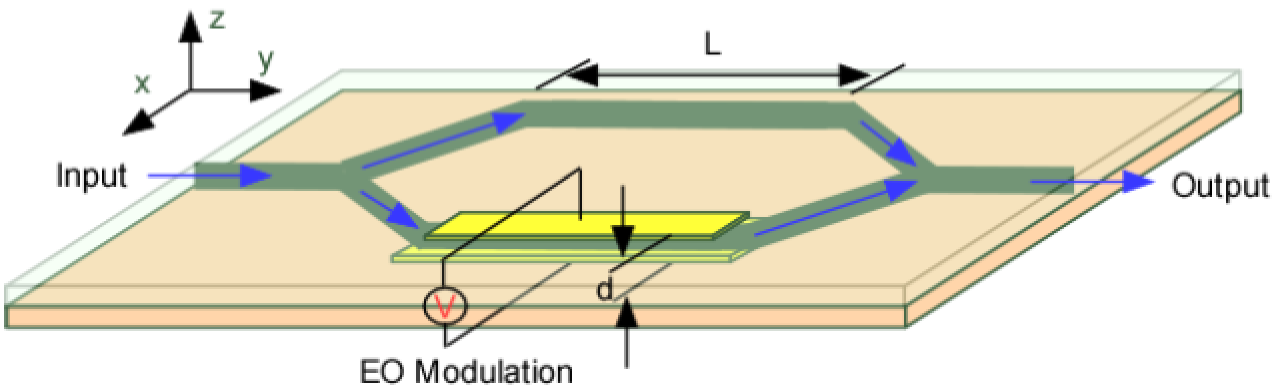


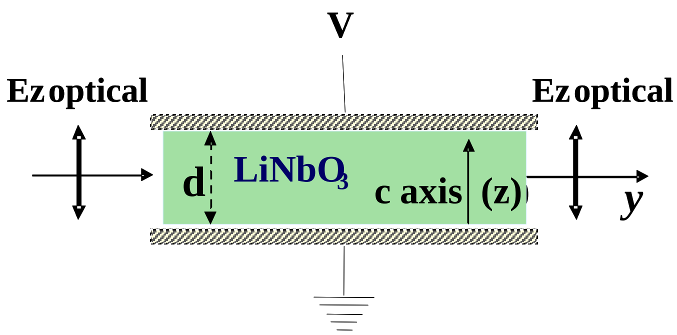
Integrated tranverse Pockels cell phase modulator in which a waveguide is diffused into an electro-optic (EO) substrate. Coplanar strip electrodes apply a transverse field E_a through the waveguide. The substrate is an *x*-cut $LiNbO_3$ and typically there is a thin dielectric buffer layer (*e.g.* ~200 nm thick SiO_2) between the surface electrodes and the substrate to separate the electrodes away from the waveguide.

$$\Delta\phi = \Gamma \frac{2\pi}{\lambda} n_o^{\ 3} \ r_{23} \frac{L}{d} V$$

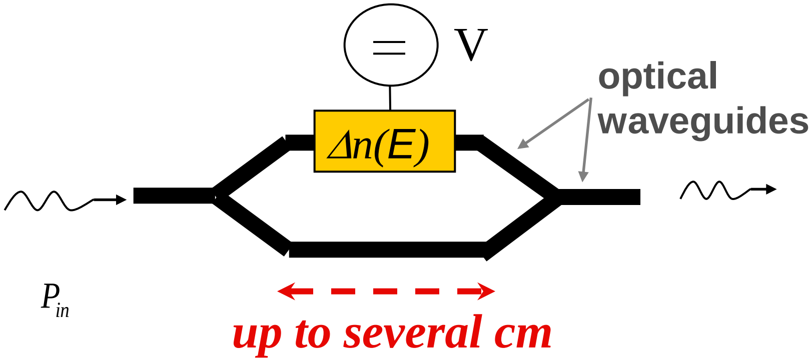
Constant Γ =0.5-0.7

EO tunable Mach-Zehnder Interferometer (MZI)





Electrooptic Mach-Zehnder modulator



Mach-Zehnder Interferometer (MZI)

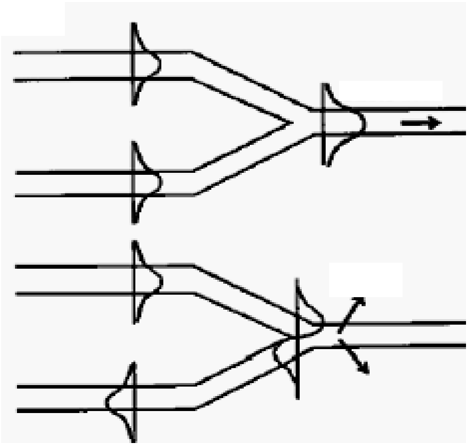
- **$LiNbO_3$**
- **GaAs-AlGaAs**
- **InP-InGaAsP**
- **(polymers)**

The input beam is split at the Y-junction into two beams that propagate in each of identical arms.

With no voltage applied they constructively interfere and recombine in the output Y-junction – MZI has no effect.

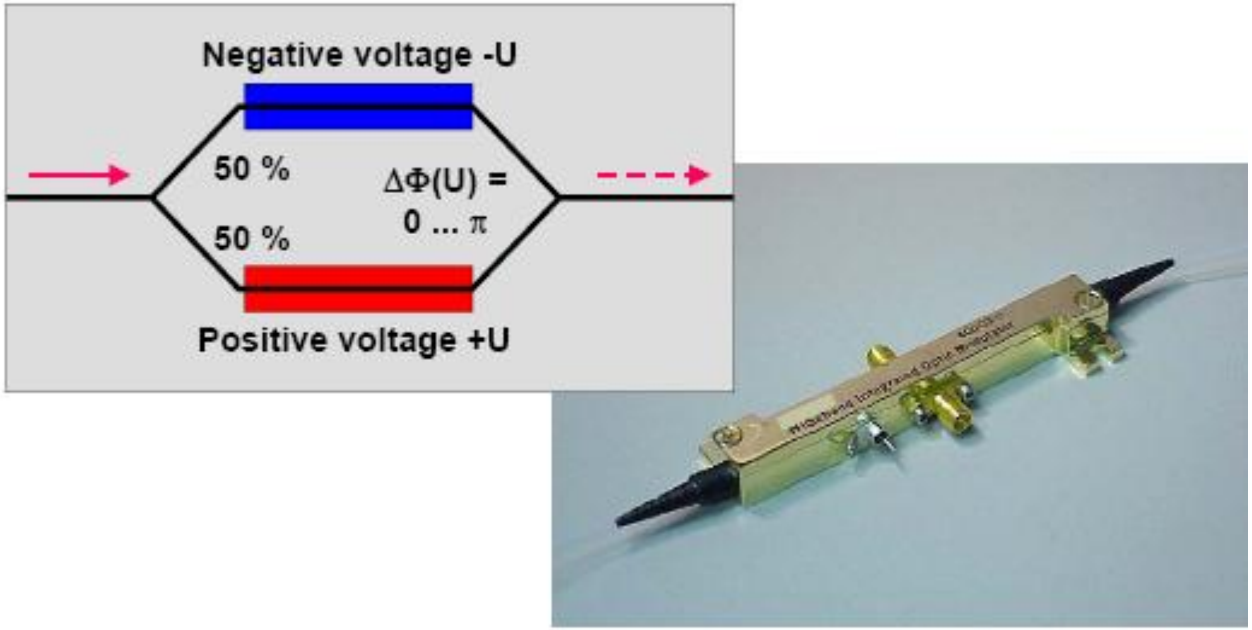
When the voltage is applied the refractive index of the arms becomes different.

The output is the sum of the two beams and the output intensity depends of their relative phase:

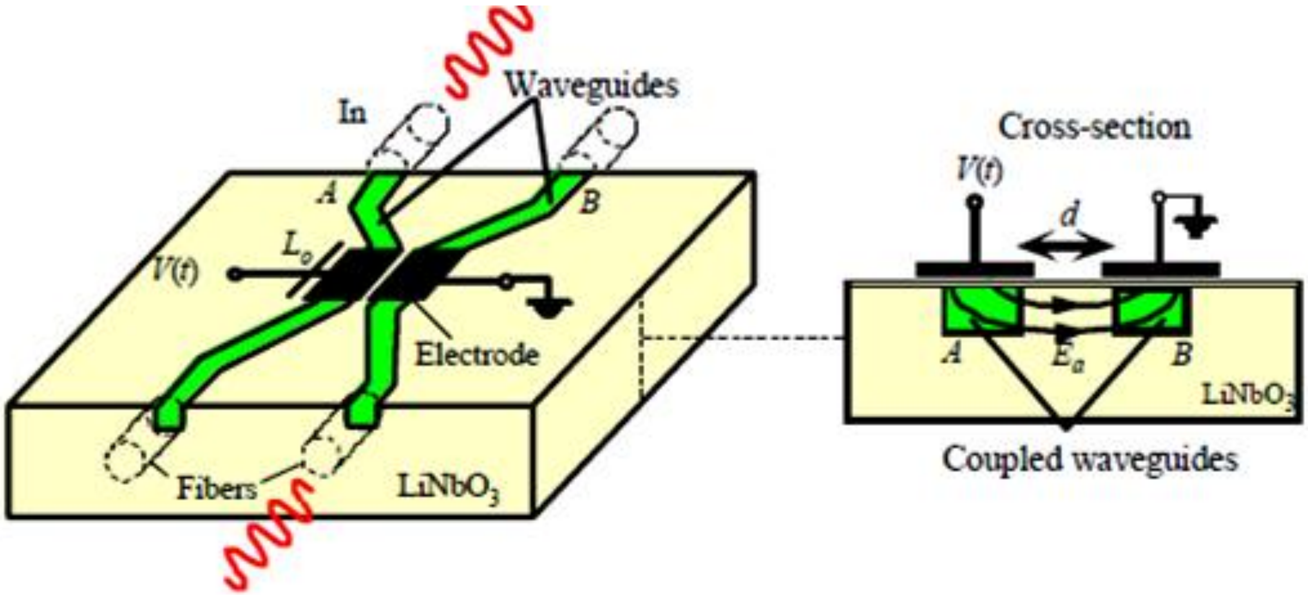


$$I_{out} \propto \left| \frac{1}{\sqrt{2}} E e^{-i\Phi_1} + \frac{1}{\sqrt{2}} E e^{-i\Phi_2} \right|^2 = \frac{1}{2}\left(1 + \cos\Delta\Phi\right) I_{inp}$$

Lithium niobate Mach-Zender modulator

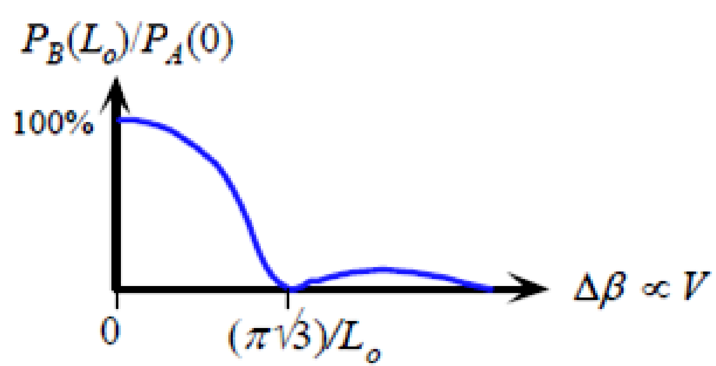


Electrooptical coupler modulator

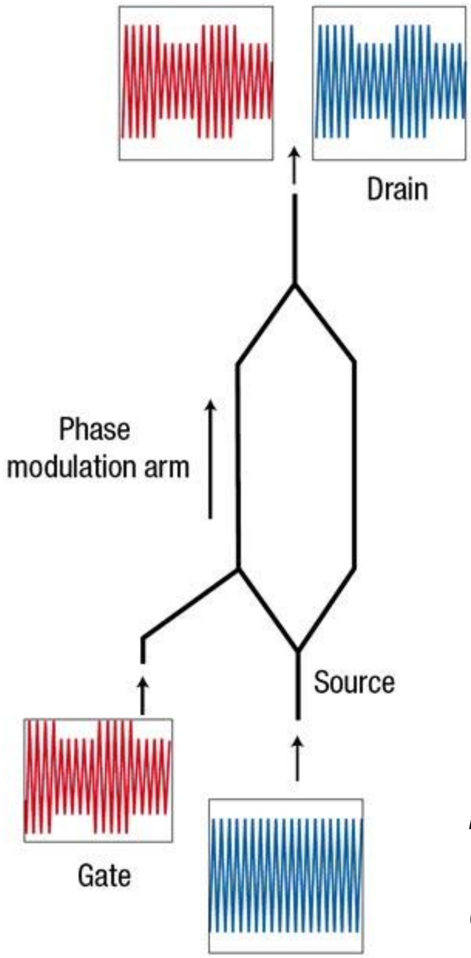


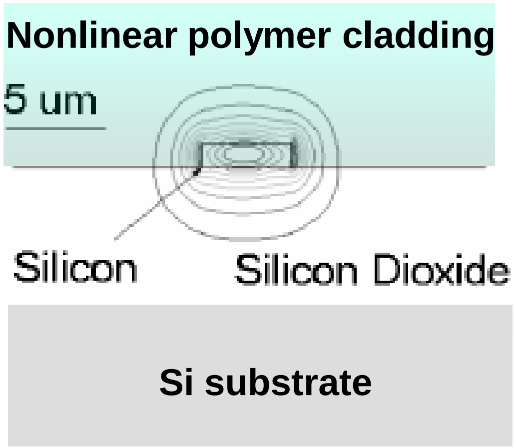
Applied voltage alters the refractive indices and the induced phase mismatch decouples the guides

Transmission power ratio from guide A to guide B over the transmission length L_o as a function of mismatch $\Delta\beta$



THz all-optical modulation in a Si–polymer hybrid system





The gate signal has its intensity modulation pattern transferred to the source via Cross Phase Modulation due to NL Kerr effect in the polymer cladding

No phase –matching needed between gate and source

Can be used to convert modulation to another wavelength

Michael Hochberg, Caltec 2006

EO modulators: pros and cons

Pros:

Very low optical loss

High power handling capability

Broad bandwidth

Zero or tunable chirp

Temperature insensitivity

Cons:

Large size

Bias-drifting issue

Polarization sensitive

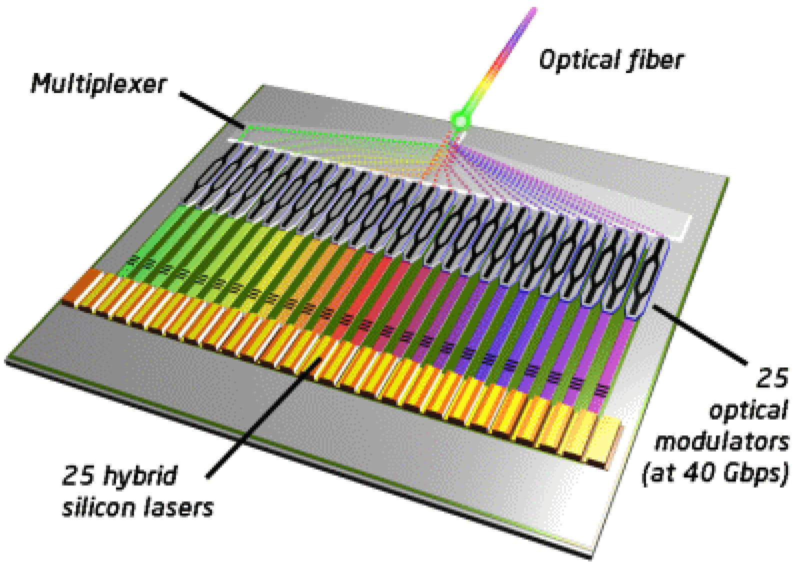
Difficult to integrated with other components

High costs for large volume production

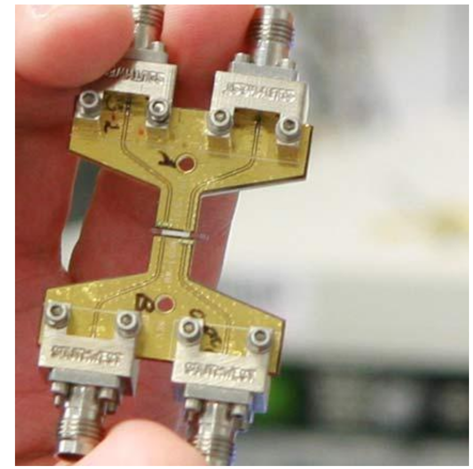
40 Gb/s Si laser modulator

Micrometre-scale 40 Gb/s laser modulator in Si developed by Intel (2007)

Based on free-carrier plasma dispersion effect – silicon's refractive index is changed when the density of free carriers (electrons/holes) is varied



Future terabit per second optical chip – Intel vision



Interesting blog: http://blogs.intel.com/research/2007/07/40g_modulator.html