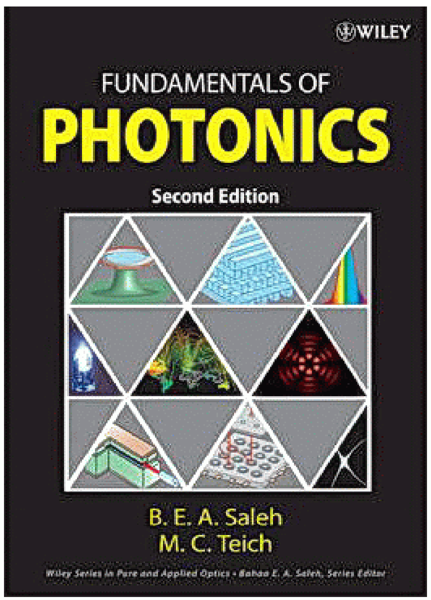
WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

Quantum Electronics
Lecture 4



Introduction to nonlinear optics

Lecturer:

Bozena Jaskorzynska

Royal Institute of Technology (KTH)
Sweden, bj@kth.se

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Saleh Ch.21

Motto

"Physics would be dull and uninteresting and life most unfulfilling if all physical phenomena around us were linear. Fortunately we are living in a non-linear world. While linearization beautifies physics, non-linearity provides excitement"

Y.R. Shen in "Non-Linear Optics", Wiley

Where from the nonlinear effects come?

In contrast to electrons photons can only interact through polarization of the medium

Incident photons make an electron claud or a molecule oscillate

- Those oscillations in turn result in photon emission

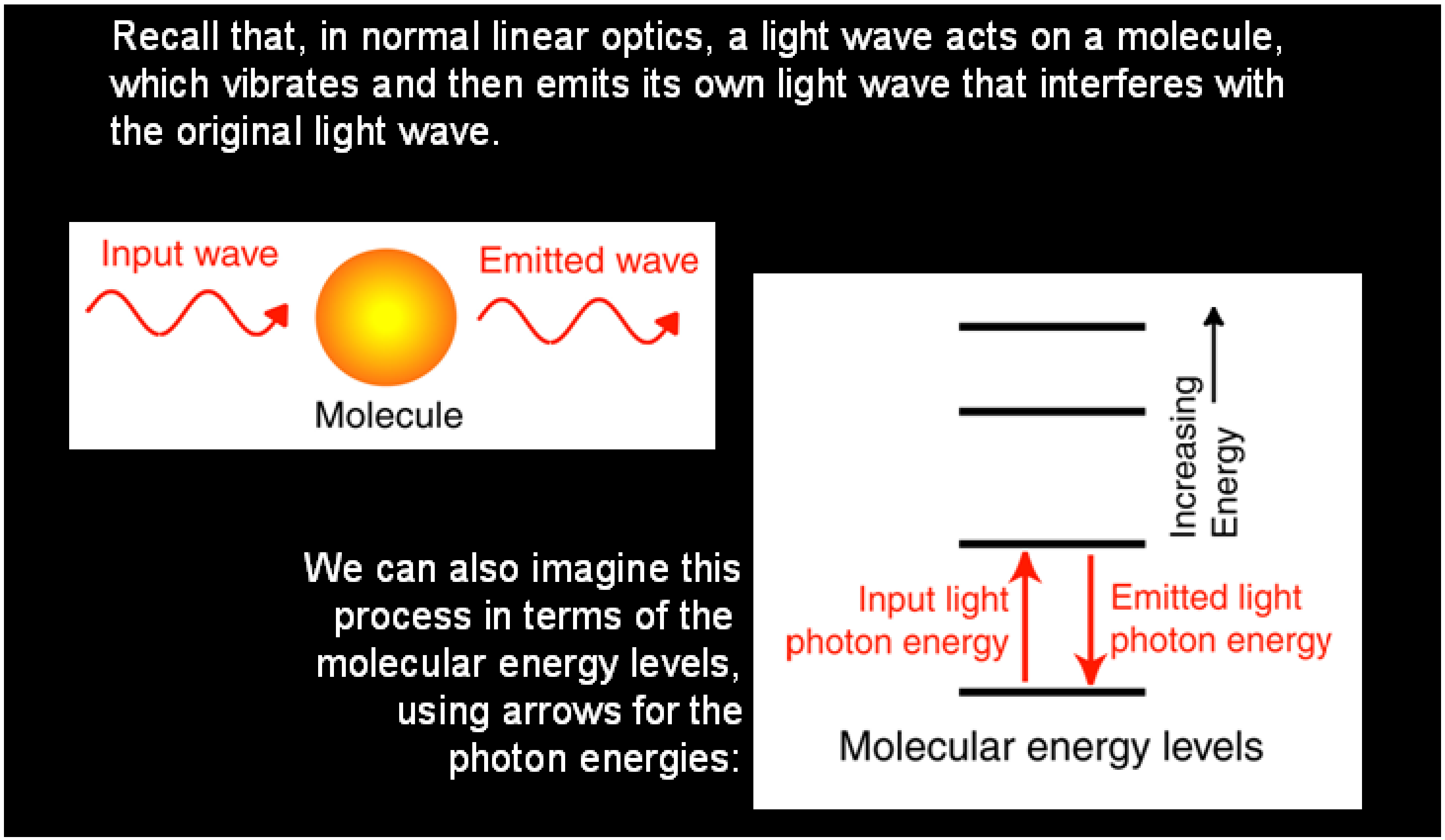
Linear regime: (small deviations from equilibrium) the emitted photons have the same frequency as the incident ones – there is no effective interaction between photons

Nonlinear regime: (more photons, large oscillation amplitude) different frequency (higher harmonics) are generated – photons effectively ”interact”

All media are nonlinear

However, for low light intensities the nonlinear response of the medium is typically negligible in ananlogy with a harmonic oscillator

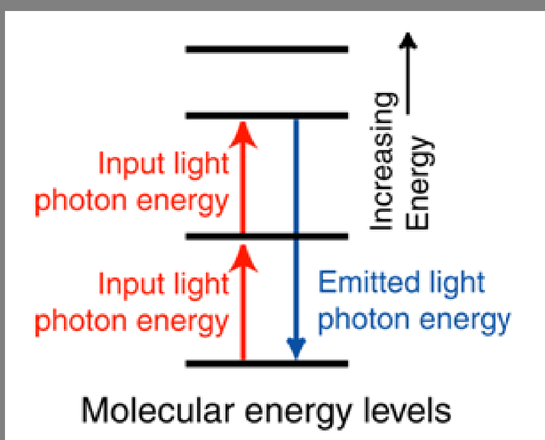
Linear response of the medium

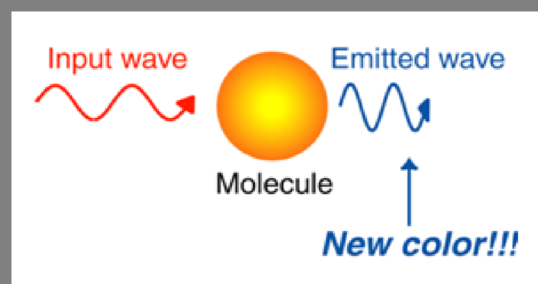


Gatech

Origin of nonlinear effects

Now, suppose the irradiance is high enough that many molecules are excited to the higher-energy state. This state can then act as the lower level for additional excitation. This yields vibrations at all frequencies corresponding to all energy differences between populated states.





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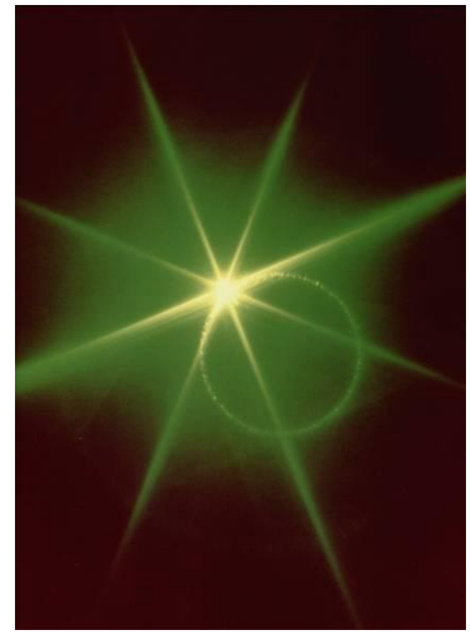
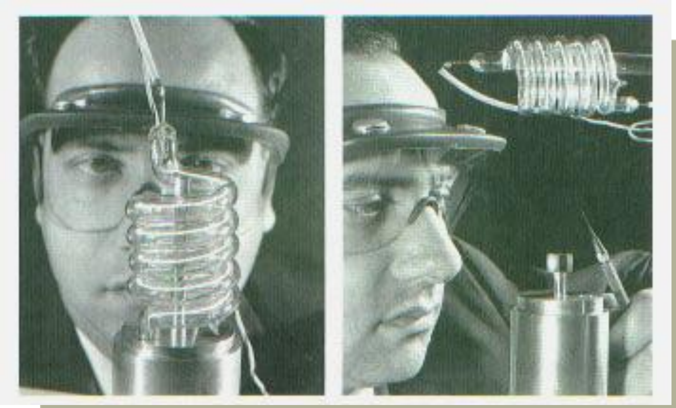
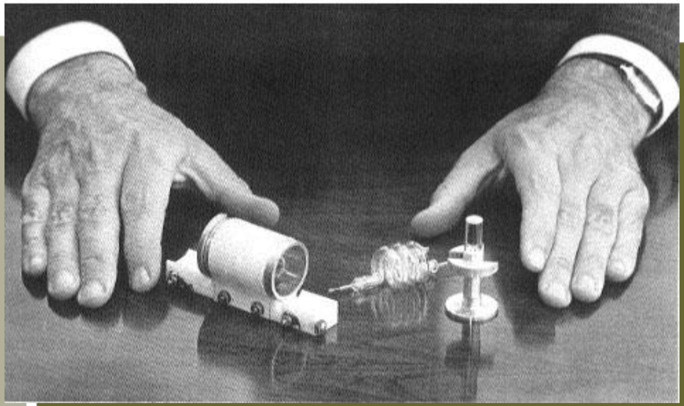


Green light obtained by illuminating the crystal with infrared light

Invention of the first lasers

(1960) **Theodore Maiman** Invention of the first Ruby Laser





(1960) **Ali Javan**
The first He-Ne Laser

Birth of nonlinear optics

The advent of the laser as an intense, coherent light source gave **birth to nonlinear optics**

Optical Second-Harmonic Generation – the first nonlinear effect observed with coherent input generating coherent output

P.A. Franken, et al, Physical Review Letters 7, p. 118 (1961)

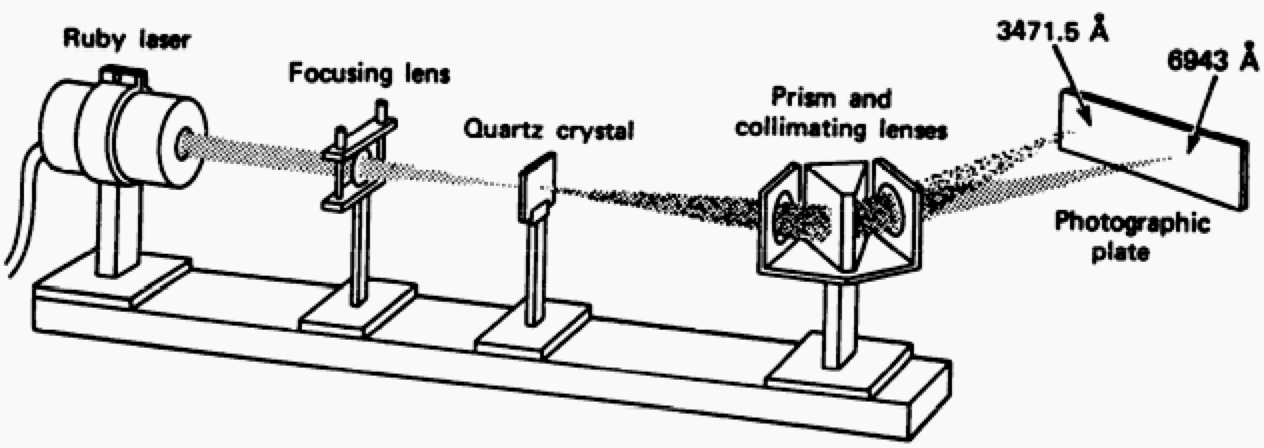


Figure 12.1. Arrangement used in the first experimental demonstration of second-harmonic generation [1]. A ruby-laser beam at $\lambda = 0.694\ \mu m$ is focused on a quartz crystal, causing the generation of a (weak) beam at $\frac{1}{2}\lambda = 0.347\ \mu m$. The two beams are then separated by a prism and detected on a photographic plate.

Peter Franken

First theoretical description of nonlinear phenomena

N. Bloembergen, Harvard, Cambridge (a book "Nonlinear Optics", 1964)

R.V. Khokhlov and S.A. Akhmanov, Moscow University (a book "Problems of Nonlinear Optics", 1964)

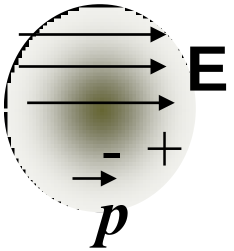
Nicolas Bloembergen

Rem Khokhlov

Sergey Akhmanov

Induced polarization – medium response

The polarization is induced by E-field:



$$\mathbf{P} = f(\mathbf{E})$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

D is the electric flux density - caused by:

- E-field in the absence of the medium: ε_0 **E**,
- Plus the field created by the response of the medium: **P**
  - **P** $= 0$ in free space
  - **P** $\neq 0$ in a dielectric

Polarization is a driving term for the wave equation:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{dt^2}$$

Nonlinear polarization

For high intensity fields oscillatory motion of bound electrons becomes anharmonic in analogy to a simple pendulum motion: sufficiently small oscillations are harmonic - the larger ones include higher harmonics

Higher order (nonlinear) terms in the induced polarization P:

$$P = \varepsilon_0 \left[\chi^{(1)} E + \chi^{(2)} EE + \chi^{(3)} EEE + \ldots \right]$$

P*L* (linear term: $\chi^{(1)}E$) ***P****NL* (nonlinear terms: $\chi^{(2)}EE + \chi^{(3)}EEE + \ldots$)

$\chi^{(n)}$ - **n***th* order susceptibility **tensor**

Generally ($\chi^{(1)}$ **E** >> $\chi^{(2)}$ **E E** >> $\chi^{(3)}$ **E E E**)

In centrosymmetric media: $P(-E) = -P(E) \Rightarrow \chi^{(2n)} = 0$

$\chi^{(2)}$ - only in media lacking a centre of symmetry (i.e. crystals)
$\chi^{(3)}$ -in all dielectric media

Note alternative definition in some books (e.g. Saleh) : $\varepsilon_0 \chi^{(2)} \to 2d \quad \varepsilon_0 \chi^{(3)} \to 4\chi^{(3)}$

Wave mixing by 2nd order nonlinearity

Let us look at **polarization** frequecies generated by a sum of two waves:

$$E = E_1\cos(\omega_1 t - k_1 x) + E_2\cos(\omega_2 t - k_2 x) = \frac{1}{2}\left[E_1 e^{i\omega_1 t}e^{-ik_1 x} + E_2 e^{i\omega_2 t}e^{-ik_2 x} + E_1^{*} e^{-i\omega_1 t}e^{ik_1 x} + E_2^{*} e^{-i\omega_2 t}e^{ik_2 x}\right]$$

$$P \propto \chi^{(2)} EE$$

$$P \propto \chi^{(2)}\Big[E_1^{\,2} e^{i2\omega_1 t}e^{-i2k_1 x} + E_2^{\,2} e^{i2\omega_2 t}e^{-i2k_2 x}$$ **2nd-harmonic generation**

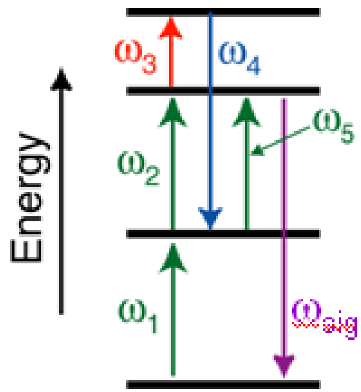
$$+2E_1E_2 e^{i(\omega_1+\omega_2)t}e^{-i(k_1+k_2)x}$$ **Sum-frequency generation**

$$+2E_1E_2^{\,*} e^{i(\omega_1-\omega_2)t}e^{-i(k_1-k_2)x}$$ **Difference-frequency generation**

$$+\left|E_1\right|^2 + \left|E_2\right|^2\Big]$$ **dc rectification**

Can light at those frequencies be efficiently generated?
Answer: only if the conservation laws are fullfiled – next slide

Conservation laws for elastic nonlinear processes



N-wave-mixing (or N-photon) mixing

N - number of photons involved (including the emitted one)

$\chi^{(2)}$ can mix 3 waves, $\chi^{(3)}$ - 4 waves, etc

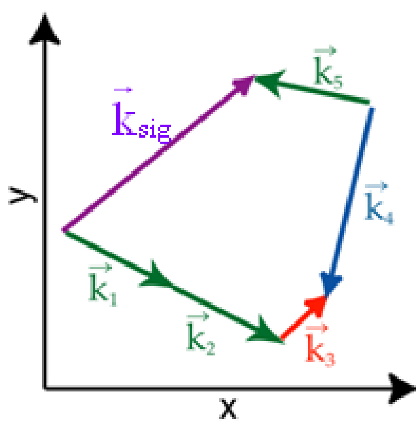
Absorbed photons ↑

Emitted photons ↓ - contribute to **complex conjugate** of their field

$$P \propto E_1 E_2 E_3 E_4^* E_5$$

Energy must be conserved: $\omega_1 + \omega_2 + \omega_3 - \omega_4 + \omega_5 = \omega_{sig}$

Automatic ! ($\hbar$'s are cancelled)



Momentum must also be conserved: $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{k}_4 + \vec{k}_5 = \vec{k}_{sig}$

Typically, $\mathbf{k_{sig}}$ DOES NOT correspond to a light wave at frequency ω_{sig} **!**

Satisfying these two relations simultaneously is called "phase-matching"

Phase-matching problem for SHG

k-vector of the polarization: $k_{sig} = 2k_{\omega} = 2\frac{\omega}{c}n(\omega)$

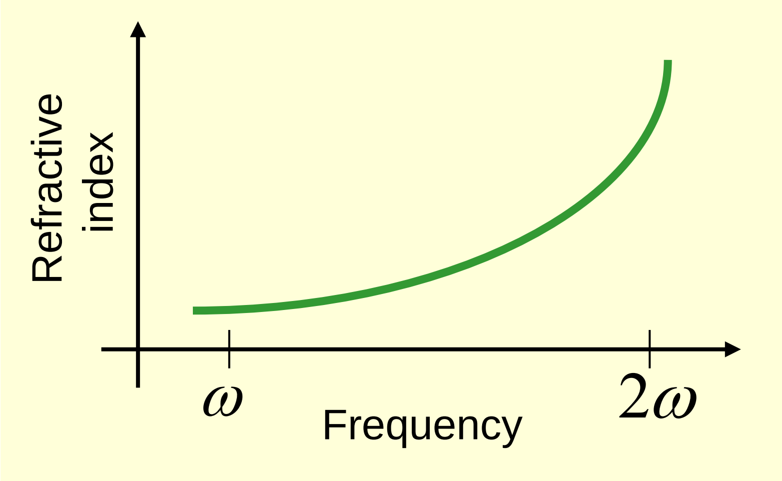
k-vector of the second harmonic: $k_{2\omega} = \frac{2\omega}{c}n(2\omega)$

For phase-matching they should be equal: $k_{sig} = k_{2\omega}$

which requires: $n(2\omega) = n(\omega)$

Unfortunately, dispersion prevents this from ever happening!

Dispersion couples energy and momentum conservation



"Tricks" to inforce phase matching will be discussed later

2nd order NL tensor – contracted notation

Due to intrinsic permutation symmetries one can reduce the NL polarization tensor to its contracted form. For the 2nd order polarization it is:

$$\begin{vmatrix} P_x \\ P_y \\ P_z \end{vmatrix} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{vmatrix} \begin{vmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_zE_y \\ 2E_zE_x \\ 2E_xE_y \end{vmatrix}$$

$$P_i = \chi_{ijk}{}^{(2)} E_j E_k \equiv 2d_{ijk} E_j E_k \equiv \sum_{jK} E_j E_K = 2d_{iK} \left(EE\right)_K$$

Units of d (F/m or m/V)

- There are different conventions in the use of d
- in some texts you will find that the polarisation is written as

$$P=\varepsilon_0 d E^2$$

- while in other texts you will find polarisation written as

$$P=\mathit{d}E^2$$

(we use *d* in italics her simply to distinguish it from the d above)

- recall that the polarisation P is the dipole moment (C.m) per unit volume (m^3). Units of P are $C.m^{-2}$
- units of E are $V.m^{-1}$
- units of ε_0 are $F.m^{-1}$ or $C.V^{-1}.m^{-1}$
- units of d are $d=P/(\varepsilon_0 E^2)=C.m^{-2}/(C.V^{-1}.m^{-1}.V^2.m^{-2})=m.V^{-1}$
- units of *d* are $\mathit{d}=P/E^2=C.m^{-2}/(V.m^{-1})=F.m^{-1}$

http://phys.strath.ac.uk/12-370/sld018.htm

Three-Wave Mixing - mathematical description

Maxwell equations:

$$\left.\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \vec{E} + \vec{P}\right) \\ \vec{P} &= \varepsilon_0 \chi^{(1)} \vec{E} + \vec{P}_{nl} \end{aligned}\right\} \Rightarrow$$

Wave equations:

$$\nabla^2 \mathbf{E} = \mu_0 \frac{\partial^2}{\partial t^2}(\varepsilon_0 \mathbf{E} + \mathbf{P}) = \mu_0 \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial^2}{\partial t^2} \mathbf{P}_{NL}$$

Consider interaction of three harmonic fields via $\chi^{(2)} \equiv 2d_{ijk}$ with $\omega_3 = \omega_1 + \omega_2$

Crystal axes

Express each of the fields $E_i^{\omega_1}(t) = \frac{1}{2}(E_{0i}^{\omega_1} e^{i(\omega_1 t - k_1 z)} + \text{c.c.}) = \frac{1}{2}(a_{1i} E_1 e^{i(\omega_1 t - k_1 z)} + \text{c.c.}) \qquad (i = x', y', z')$

Unit polarization vectors

Mix the total field: $E_i(t) = E_i^{\omega_1}(t) + E_i^{\omega_2}(t) + E_i^{\omega_3}(t)$

to obtain nonlinear polarization: $Pi_{NL} = P_i = 2d_{ijk} E_j E_k$

$$P_i(t) = 2d_{ijk} \tfrac{1}{2}(E_{0j}^{\omega_1} e^{i\omega_1 t} + E_{0j}^{\omega_2} e^{i\omega_2 t} + \text{c.c.}) \times \tfrac{1}{2}(E_{0k}^{\omega_1} e^{i\omega_1 t} + E_{0k}^{\omega_2} e^{i\omega_2 t} + \text{c.c.})$$

Group terms at different frequencies:

$$[P_{NL}^{\omega_3 - \omega_2}(z,t)]_i = d_{ijk} a_{3j} a_{2k} E_3 E_2^* e^{i[(\omega_3 - \omega_2)t - (k_3 - k_2)z]} + \text{c.c.}$$

$$[P_{NL}^{\omega_3 - \omega_1}(z,t)]_i = d_{ijk} a_{3j} a_{1k} E_3 E_1^* e^{i[(\omega_3 - \omega_1)t - (k_3 - k_1)z]} + \text{c.c.}$$

$$[P_{NL}^{\omega_1 + \omega_2}(z,t)]_i = d_{ijk} a_{1j} a_{2k} E_1 E_2 e^{i[(\omega_1 + \omega_2)t - (k_1 + k_2)z]} + \text{c.c.}$$

Coupled equations for Three Wave Mixing (1)

Substituting nonlinear polarizations to the wave equation, and applying Slowly Varying Amplitude Approximation:

$$\frac{d^2}{dz^2}E_s \ll k_s \frac{d}{dz}E_s \quad (s = 1, 2, 3)$$

we obtain, after a few steps of algebra,

$$\frac{d}{dz}E_1 = -i\omega_1\sqrt{\frac{\mu_0}{\varepsilon_1}}\, dE_3E_2^* e^{-i(k_3-k_2-k_1)z}$$

$$\frac{d}{dz}E_2^* = +i\omega_2\sqrt{\frac{\mu_0}{\varepsilon_2}}\, dE_1E_3^* e^{+i(k_3-k_2-k_1)z}$$

$$\frac{d}{dz}E_3 = -i\omega_3\sqrt{\frac{\mu_0}{\varepsilon_3}}\, dE_1E_2 e^{+i(k_3-k_2-k_1)z}$$

where d is the effective second-order nonlinear coefficient

$$d = \sum_{ijk} d_{ijk}a_{1i}a_{2j}a_{3k}$$

Energy is conserved:

$$\frac{d}{dz}\left(\sqrt{\varepsilon_1}\,|E_1|^2 + \sqrt{\varepsilon_2}\,|E_2|^2 + \sqrt{\varepsilon_3}\,|E_3|^2\right) = 0$$

Coupled equations for Three Wave Mixing (2)

Introducing variables: $A_m = \sqrt{\frac{n_m}{\omega_m}}E_m, \quad m = 1, 2, 3$ ***n-index*** **of refractio** $|A_m|^2 \propto$ **photon flux**

one obtains:

$$\frac{d}{dz}A_1 = -i\kappa A_3 A_2^* e^{-i\Delta kz}$$

$$\frac{d}{dz}A_2^* = +i\kappa A_1 A_3^* e^{+i\Delta kz}$$

$$\frac{d}{dz}A_3 = -i\kappa A_1 A_2 e^{+i\Delta kz}$$

$\Delta k = k_3 - (k_1 + k_2)$ **Momentum mismatch**

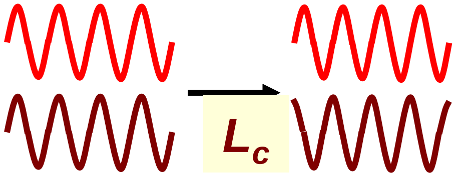
Coupling coefficient:

$$\kappa = d\sqrt{\frac{\mu_0\omega_1\omega_2\omega_3}{\varepsilon_0 n_1 n_2 n_3}} = \left(\sum_{ijk} d_{ijk}a_{1i}a_{2j}a_{3k}\right)\sqrt{\frac{\mu_0\omega_1\omega_2\omega_3}{\varepsilon_0 n_1 n_2 n_3}}$$

Interaction most efficient for: Δk=0 (phase matching)

Collinear propagation - large field overlaps and large interaction lengths

Constructive interaction only possible over coherence length Lc:



$$\frac{1}{L_c} \propto \Delta k = \left| \ \vec{k}_p - \left(\vec{k}_s + \vec{k}_i\right) \right|$$

Conservation laws

⇨ $$\frac{\partial}{\partial z}I_1 + \frac{\partial}{\partial z}I_2 + \frac{\partial}{\partial z}I_3 = 0$$ energy conservation

$$I_i = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}n_i|E_i|^2 = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\omega_i|A_i|^2$$

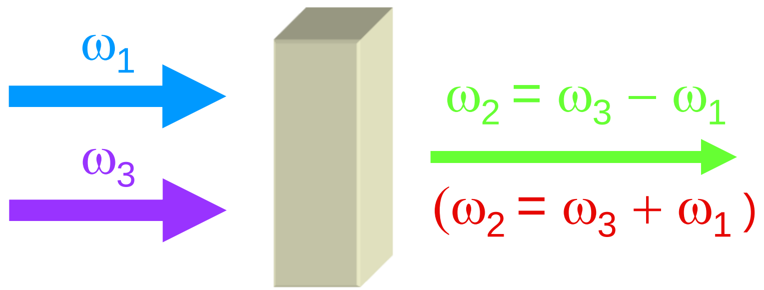
and : $$\frac{\partial}{\partial z}\left(\frac{I_1}{\omega_1}\right) = \frac{\partial}{\partial z}\left(\frac{I_2}{\omega_2}\right) = -\frac{\partial}{\partial z}\left(\frac{I_3}{\omega_3}\right)$$ Manley-Rowe relations

note : the ratio of intensity I and freq. ω is proportional to the number of photons : $$I = \frac{N\hbar\omega}{\Delta t\,\Delta A} \propto N\omega$$

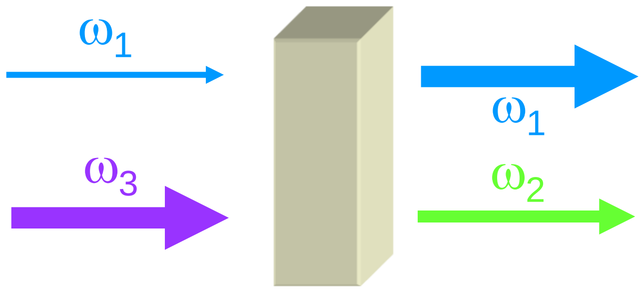
⇨ interpretation of the Manley-Rowe relation : the generation of one photon at freq. ω_3 requires the annihilation of one photon at ω_1 and one photon at ω_2

$$\Phi = \frac{I}{h\omega}$$ Photon flux density

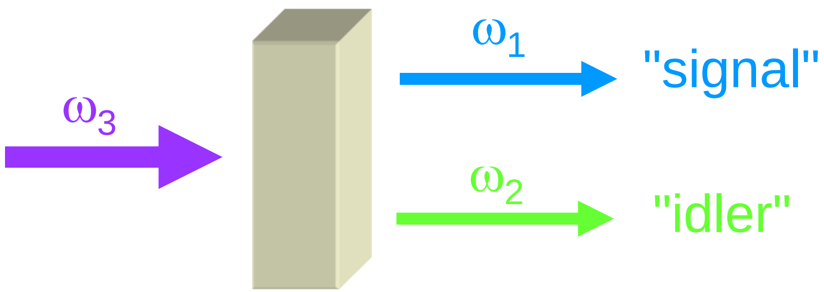
Examples of three waves mixing effects



Parametric Down (Up)-Conversion
[Difference (Sum)-frequency generation]

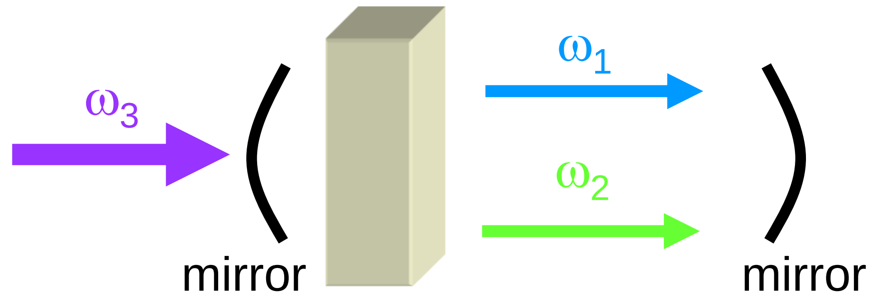


Optical Parametric
Amplification (OPA)



Optical Parametric
Generation (OPG)

$\omega_2 = \omega_3 - \omega_1$



Optical Parametric
Oscillation (OPO)

Strong pump laser at ω_3 amplifies a weak, phase matched, signal at ω_1

Second Harmonic Generation

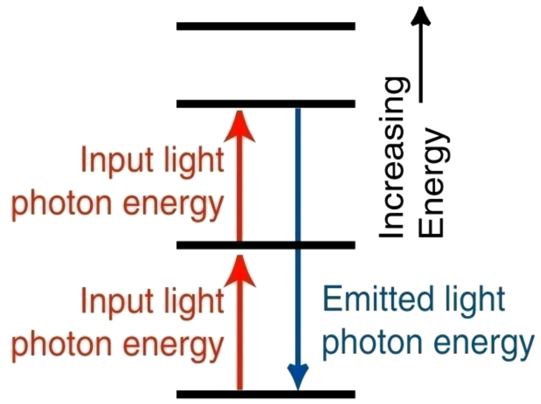
Input intense beam at frequency ω to 2nd order nonlinear crystal

Since $E(t) \propto E_0 \exp(i\omega t) + E_0^* \exp(-i\omega t)$,

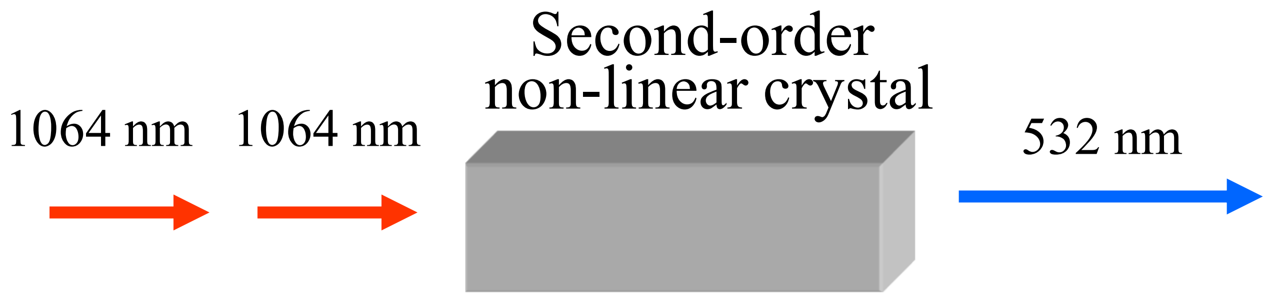
$$E(t)^2 \propto E_0^2 \exp(2i\omega t) + 2|E_0|^2 + E_0^{*2} \exp(-2i\omega t)$$

↑

2ω = 2nd harmonic!



Molecular energy levels



Second Harmonic Generation - SHG

Undepleted pump solution

$$\omega_1 = \omega_2 \equiv \omega \qquad \omega_3 = 2\omega \qquad A_1 = A_2 \equiv A \approx const$$

$$\frac{dA_3}{dz} \approx -i\frac{1}{2}\kappa A^2 e^{i\Delta kz}$$

$$A_3(L) = -i\frac{1}{2}\kappa A^2 \frac{e^{i\Delta kL}-1}{i\Delta k} = -i\frac{1}{2}\kappa A^2 L e^{i(\Delta k/2)L}\left[\frac{\sin[(\Delta k/2)L]}{(\Delta k/2)L}\right]$$

Conversion efficiency for SHG

$$\eta_{SHG} = \frac{I^{(2\omega)}}{I^{(\omega)}} = \frac{2\omega^2 d^2 L^2}{n^3}\left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2}\left[\frac{\sin[(\Delta k/2)L]}{(\Delta k/2)L}\right]^2 I^{(\omega)}$$

Note interaction length and intensity dependence

Coherence length and Beat length

$$A_3(L) = -iA^2 \frac{\kappa}{\Delta k} e^{i(\Delta k/2)L} \sin[(\Delta k/2)L] = \begin{cases} A^2 \dfrac{\kappa}{\Delta k} & for\ (\Delta k/2)L = \pi/2 \\ 0 & for\ (\Delta k/2)L = \pi \end{cases}$$

$$\Rightarrow \quad L_C = \frac{\pi}{\Delta k}$$

Coherence length

$$L_B = \frac{2\pi}{\Delta k}$$

Beat length

$$\rightarrow \infty \quad for\ \Delta k \rightarrow 0$$

Depleted Pump SHG

In the treatment of SHG so far was assumed that the input intensity at ω was not affected by the interaction, i. e. that the pump remained undepleted. This limits the validity of the result to situations where the fraction of the power converted from ω to $2\ \omega$ is small.

Generally the conversion efficiency for **phase matched** SHG is:

$$\eta \equiv \frac{I^{(2\omega)}}{I^{(\omega)}} = \tanh^2\left[\frac{A_1(z=0)}{\sqrt{2}}\kappa z\right],$$

where $A_1 = \sqrt{\frac{n_1}{\omega}} E_1^{\omega}$,

with: $\omega_1 = \omega_2 = \omega$, $\omega_3 = 2\omega$

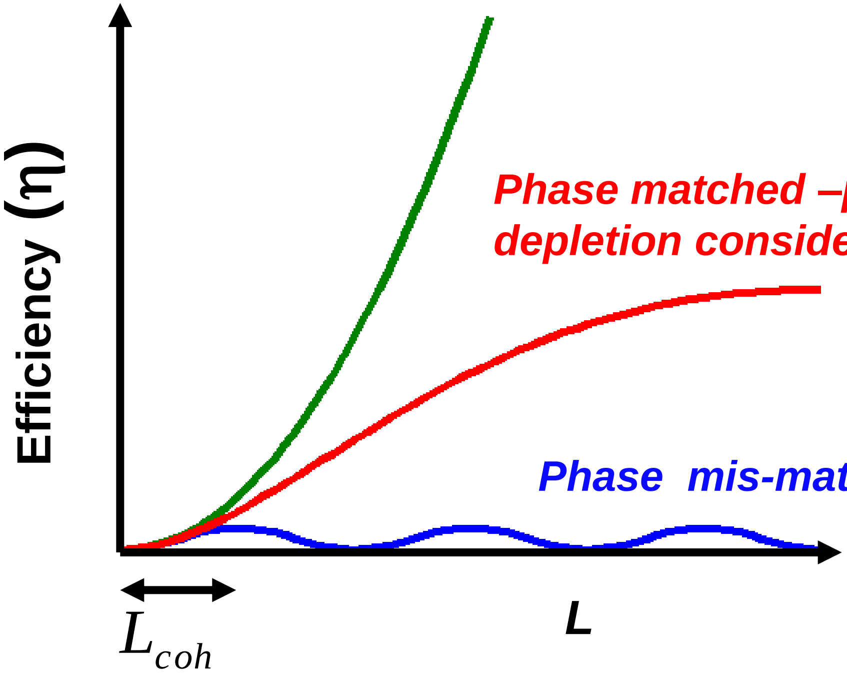
and $\kappa \equiv \mathrm{d}\sqrt{\frac{\mu_0}{\varepsilon_0}\frac{\omega_1\omega_2\omega_3}{n_1 n_2 n_3}}$.

SH conversion efficiency vs phase mismatch

Phase matched – undepleted pump approximation $\eta \propto L^2$

Phase matched –pump depletion considered $\eta = \tanh^2\left[\sqrt{\frac{n_1}{2\omega}} E^{\omega}_{L=0} \kappa L\right]$

Phase mis-matched $\eta \propto \sin^2(\frac{\pi}{2} L / L_{coh})$

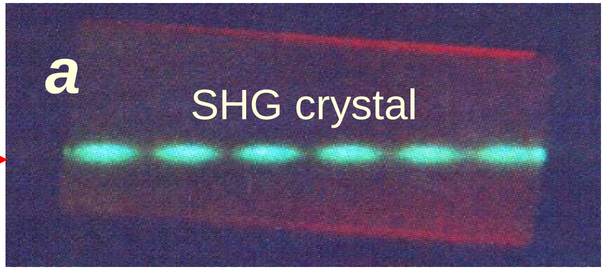


SH evolution in nonlinear crystals

$$L_B = \frac{2\pi}{\Delta k}$$

Far from phase-matching:

ω

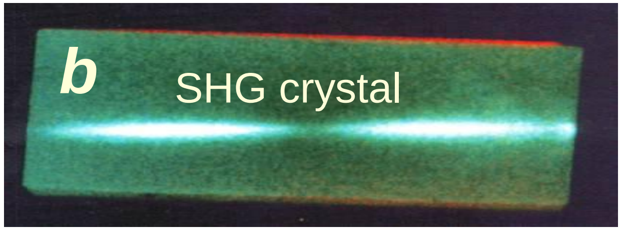


ω

2ω

Closer to phase-matching:

ω



ω

2ω

Oscillation period (beat length) increases and the SH beam becomes brighter - conversion efficiency increases

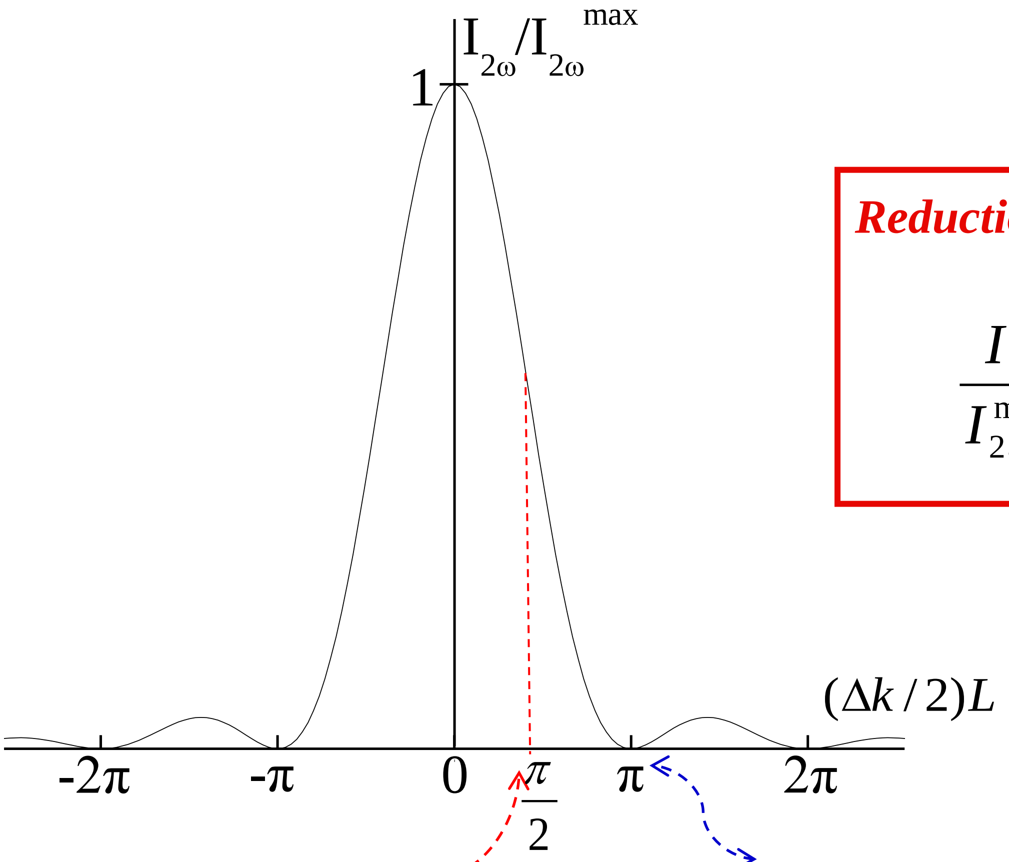
$$n_a(\omega) = n_b(\omega), \quad n_a(2\omega) \neq n_b(2\omega)$$

$$\Delta_a n \equiv n_a(2\omega) - n_a(\omega), \quad \Delta_b n \equiv n_b(2\omega) - n_b(\omega)$$

$$\Delta_b k \equiv k_0 \Delta_b n \;<\; \Delta_a k \equiv \Delta_a n$$

Phase mismatch penalty for SHG

for fixed crystal length



***Reduction** of SH output by the factor:*

$$\frac{I_{2\omega}}{I_{2\omega}^{max}} = \left[\frac{\sin[(\Delta k/2)L]}{(\Delta k/2)L}\right]^2$$

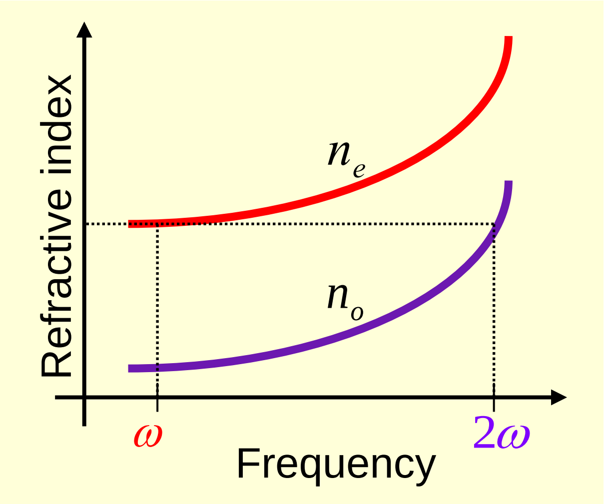
$\Delta k = 2\pi / L_B$ *Mismatch at which the crystal length = beat length*

$\Delta k = \pi / L_C$ *Mismatch at which the crystal length = coherence length*

SHG – birefringence phase-matching

One can utilize this for satisfying phase-matching condition

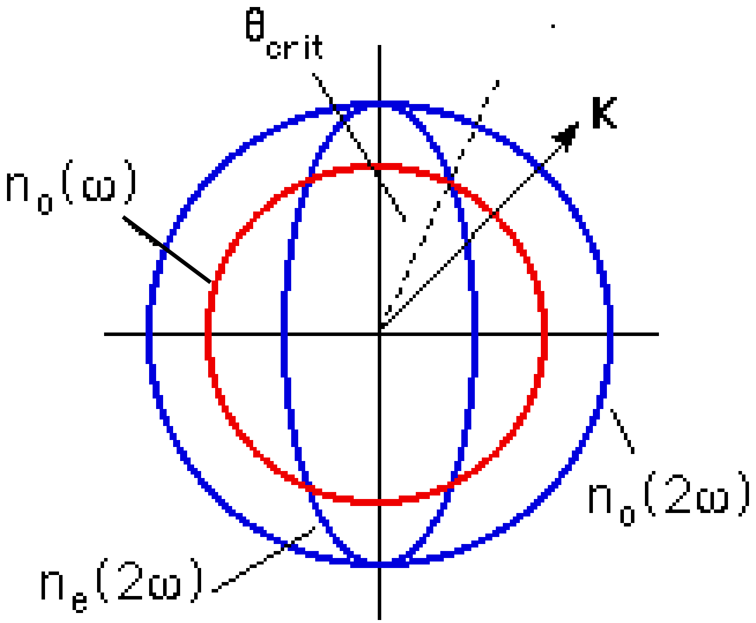
In positive uniaxial crystals $n_e > n_o$ chose the extraordinary polarization for ω and the ordinary for 2ω: →



n_e depends on propagation angle, so we can tune for a given ω.

$$n_o(2\omega) = n_e(\omega)$$

Birefringence Phase-Matching



$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

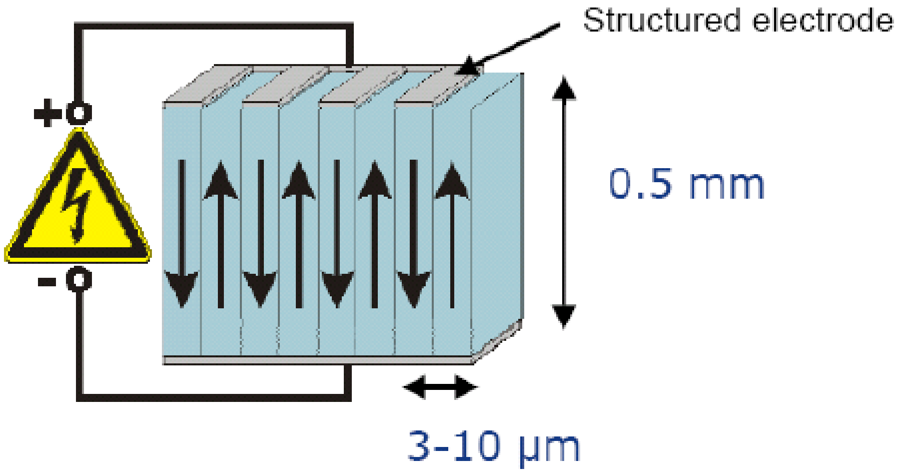
If $n_e < n_o$, (negative uniaxial crystal) there exists an angle θ_{crit} at which $n_e^{2\omega}(\theta_{crit}) = n_o^{\omega}$. So if the fundamental beam (at ω) is launched along θ_{crit} as an ordinary ray, the second-harmonic beam will be generated along the same direction as an extraordinary ray

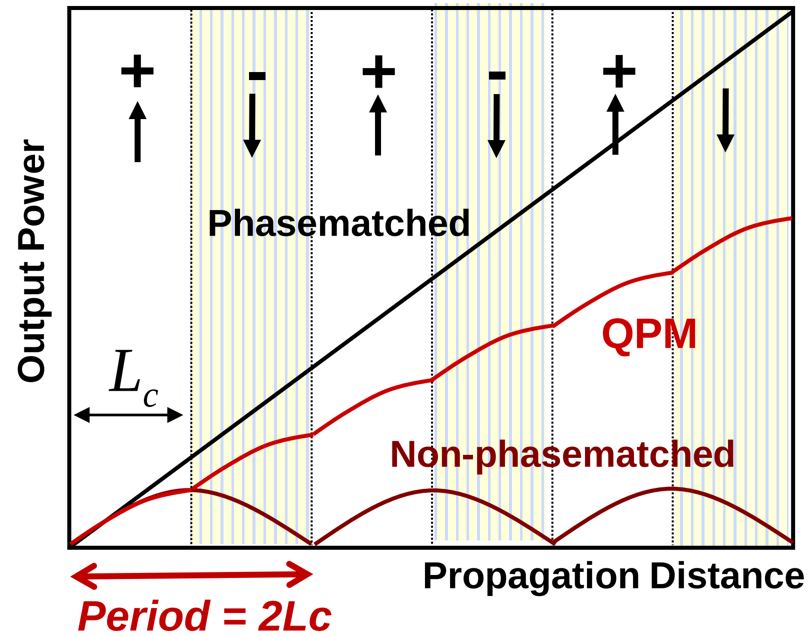
Quasi-Phase Matching (QPM)

Reverse sign of nonlinear coefficient every coherence length: rephase waves

$$\vec{k}_p = \vec{k}_s + \vec{k}_i + \vec{K}_g$$

$$\vec{K}_g = 2\pi / 2L_c = \Delta k$$





In ferroelectrics (*e.g.*, $LiNbO_3$ or KTP), reversed permanent electric dipole moment is obtained by applying high voltage of electric field – so called **”periodic poling”**

Third order nonlinear effects

$$E_n = A_n e^{i\omega_n t} e^{-i\beta_n z}$$

$$P^{NL}_{\omega_n} = \chi^{(3)} A_k A_l A_m^* \, e^{i(\omega_k+\omega_l-\omega_m)t} e^{-i(\beta_k+\beta_l-\beta_m)z} \Longrightarrow A_n e^{i\omega_n t} e^{-i\beta_n z}$$

ENERGY CONSERVATION: $\omega_k + \omega_l - \omega_m = \omega_n$

MOMENTUM CONSERVATION: $\beta_k + \beta_l - \beta_m \cong \beta_n$ — PHASE MATCHING

Self-phase matched:

- $\beta_k + \beta_k - \beta_k = \beta_k$ — SPM – self-phase modulation
- $\beta_k + \beta_l - \beta_l = \beta_k$ —
  - XPM – cross-phase modulation
  - SRS – stimulated Raman scattering
  - SBS – stimulated Brillouin scattering

Requires phasematching:

- $\beta_k + \beta_l - \beta_m \cong \beta_n$ — FWM – four-wave mixing (with small frequency shift)

Phase-matched effects build up

Self-Phase Modulation - SPM

The incident optical field: $\tilde{E}(t) = E(\omega)e^{-i\omega t} + \text{c.c.}$

Third order nonlinear polarization:

$$P^{(3)}(\omega) = 3\chi^{(3)}(\omega = \omega + \omega - \omega)\,|E(\omega)|^2\,E(\omega)$$

The total polarization can be written as:

$$P^{\text{TOT}}(\omega) = \chi^{(1)}E(\omega) + 3\chi^{(3)}(\omega = \omega + \omega - \omega)\,|E(\omega)|^2\,E(\omega)$$

One can define an effective susceptibility: $\chi_{\text{eff}} = \chi^{(1)} + 4\pi\,|E(\omega)|^2\,\chi^{(3)}$

The refractive index can be defined as usual: $n^2 = 1 + 4\pi\chi_{\text{eff}}$

Define: $\boxed{n = n_0 + n_2 I}$ with: $n_2 = \dfrac{12\pi^2}{n_0^2\chi}\chi^3$ $\quad I = \dfrac{n_0 c}{2\pi}\left|E(\omega)\right|^2$

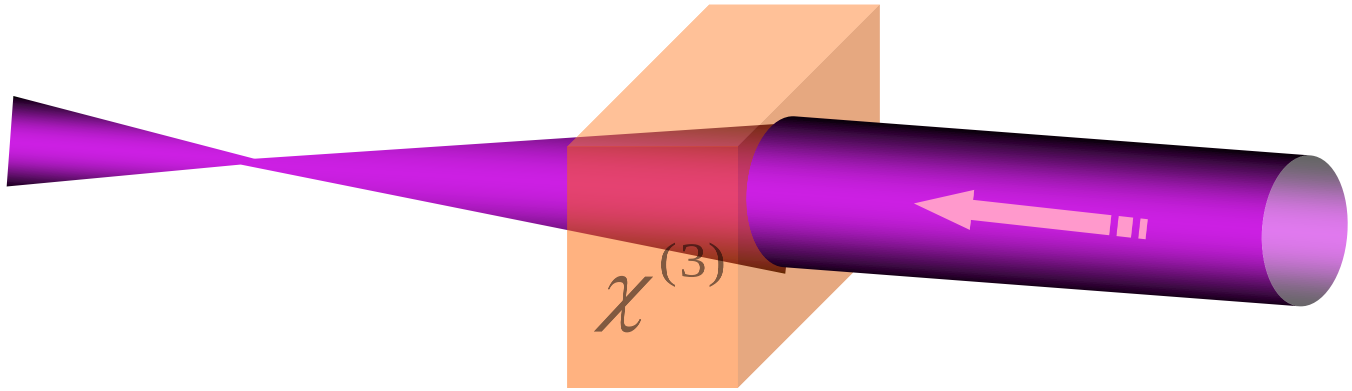
Intensity dependent refractive index

Optical Kerr effect

Self focusing due to SPM

Optical Kerr effect: $n = n_0 + n_2 I$

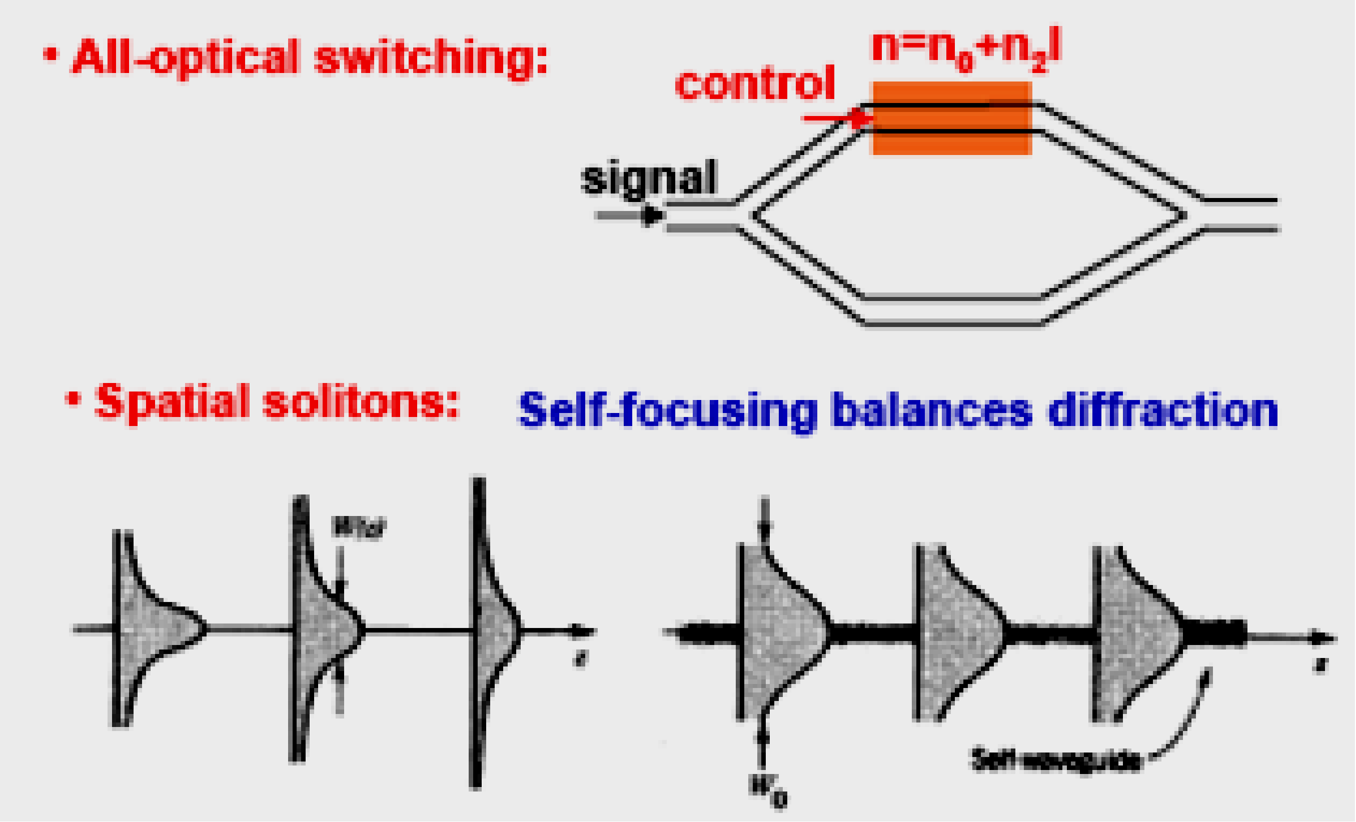
The laser beam has Gaussian intensity profile. It can induce a Gaussian refractive index profile inside the NLO sample – **a "Kerr lens" !**



Self-focusing of Gaussian beam in **Kerr medium**

Utilized e.g. for passive mode-locking

Kerr effect - examples of other applications



Spectral pulse broadening due to SPM

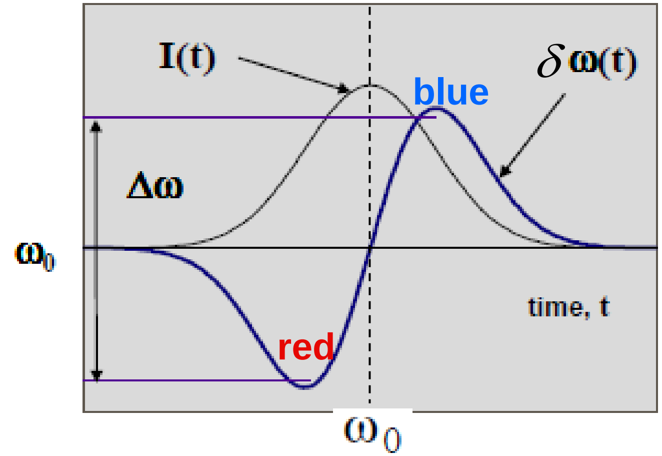
$$E_{sig}(z,t) = E_{sig}(0,t)\exp\left[i n k z\right] = E_{sig}(0,t)\exp\left\{i[n_0 + n_2 I(t)]kz\right\}$$

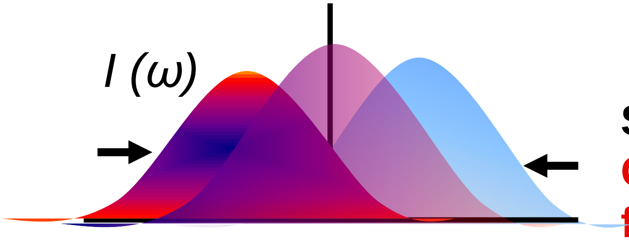
$$\phi(z,t) = n_2 k z I(t)$$

The phase is modulated according to the pulse **time envelope *I(t)*,** and increases with the **propagation distance *z***

Instaneneous frequency shift:

$$\delta\omega = \omega(t) - \omega_0 = \frac{d\phi}{dt} = -n_2 kz\frac{dI}{dt}$$





Spectral broadening
Generation of new frequencies

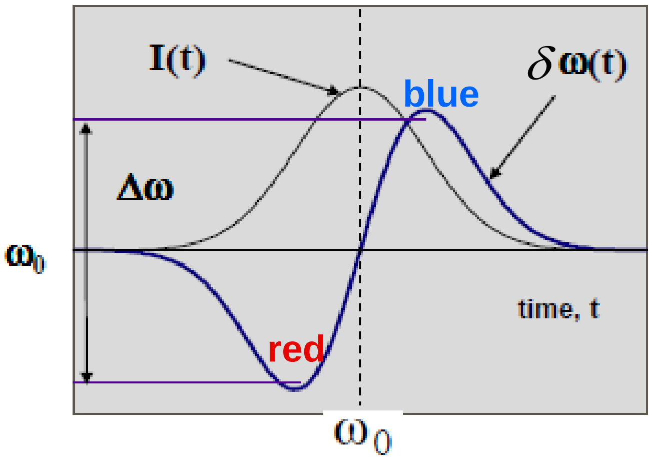
$$\Delta\omega = 2n_2 kz\left[\frac{dI}{dt}\right]_{\max}$$

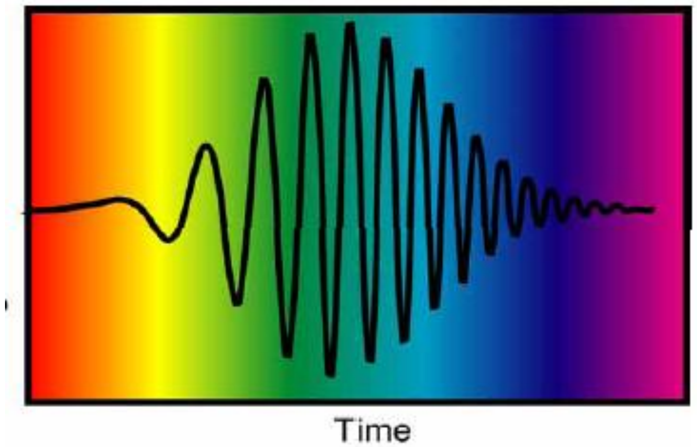
In the absence of GVD only spectrum broadens - temporal shape is preserved !

SPM – frequency chirp

Pulse frequency varies in time - the leading edge is shifted to lower frequencies (red shift), the trailing edge to the higher ones (blue shift)

In analogy to bird sounds the pulse is called "chirped"

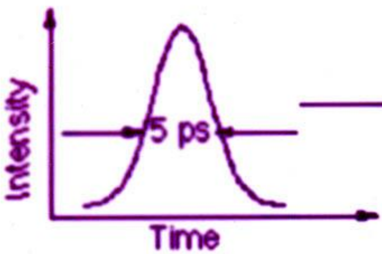


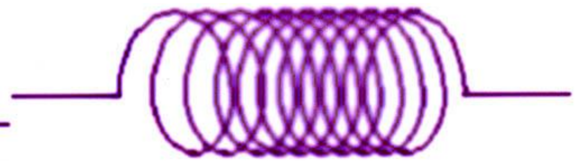


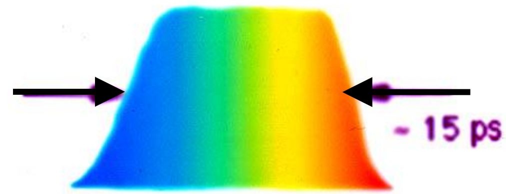
Frequency chirp:

$$C = \frac{d\omega}{dt} = \frac{d^2\phi}{dt^2} = -n_2 kz \frac{d^2 I}{dt^2}$$

SPM and GVD induced frequency chirp

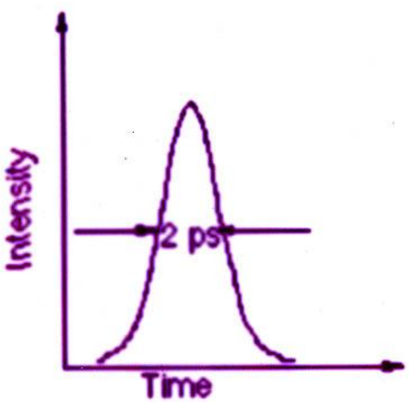


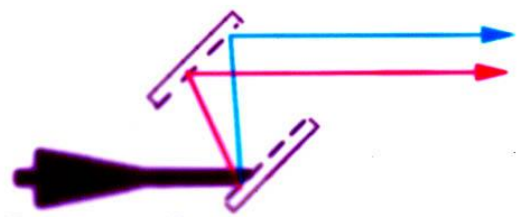




Transform limited laser pulse

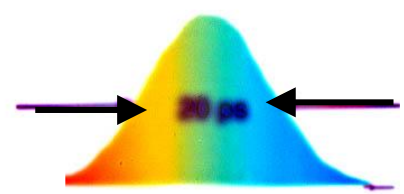
Frequency chirp generated by SPM in optical fiber
With normal GVD - enhanced pulse broadening





Temporal broadening
but ***no spectral broadening***

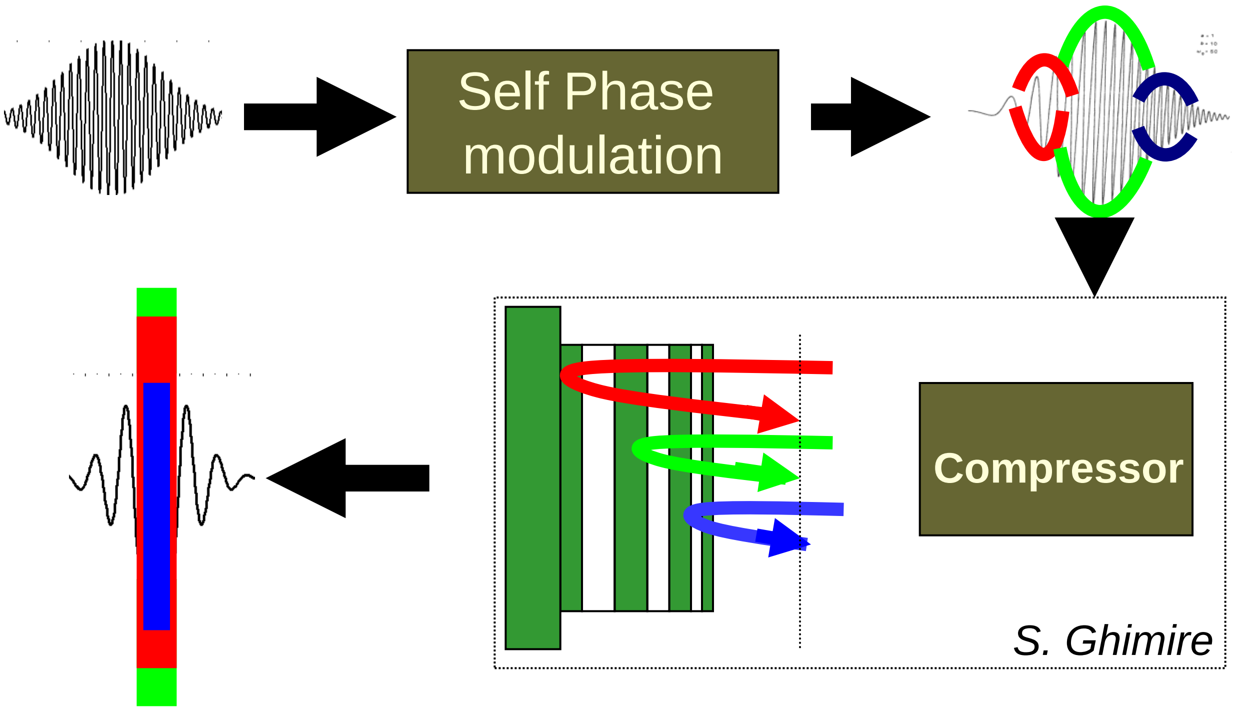
Frequency chirp generated by a grating or prism pair



20 ps

Pulse compression by SPM+GVD

With normal GVD – enhanced pulse broadening
With anomalous GVD – pulse compression!

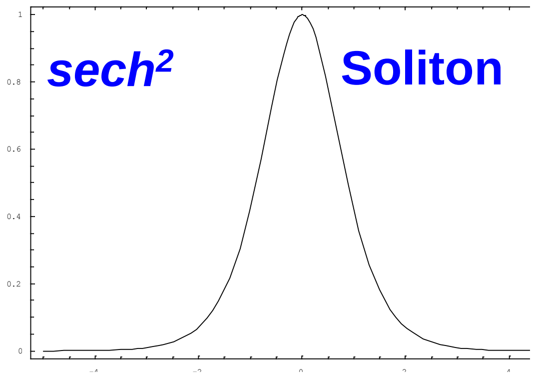


Note: at a given spectral width the shortest possible pulse is unchirped (transform limited)

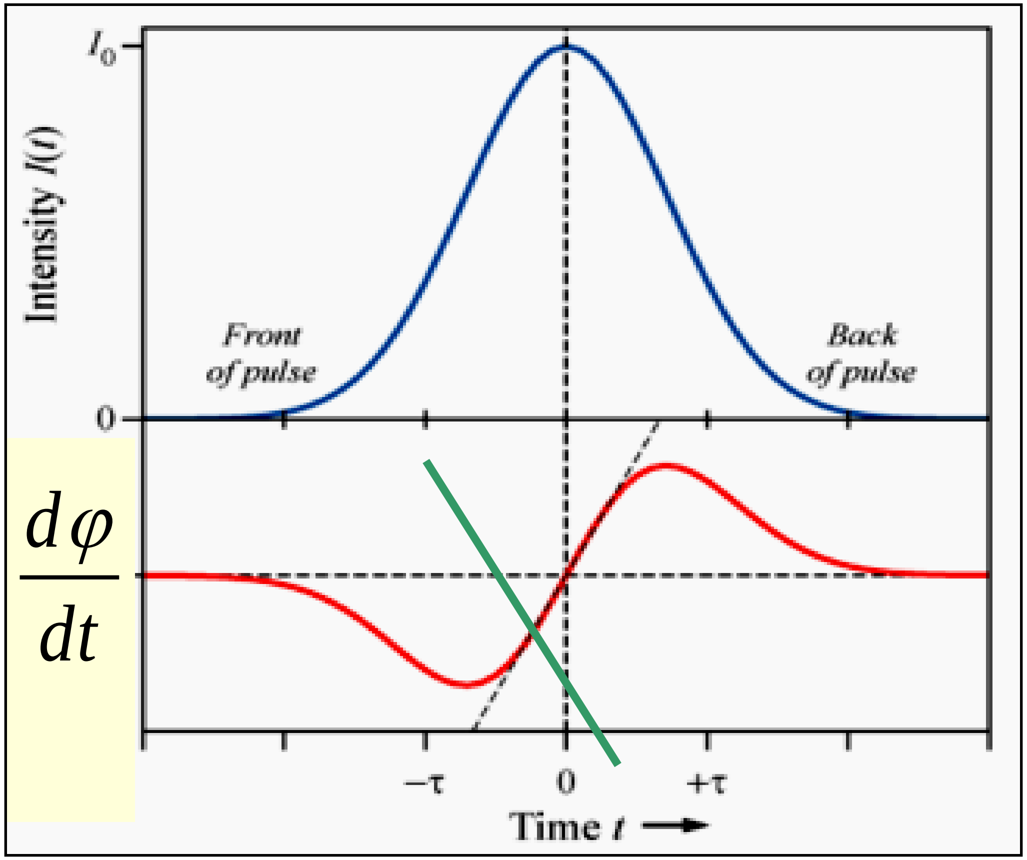
→ **Chirped pulses have a ”compression potential”**

Optical soliton

Perfect balance between GVD and SPM



Transform-limited (no chirp)
In lossless media preserves its shape !



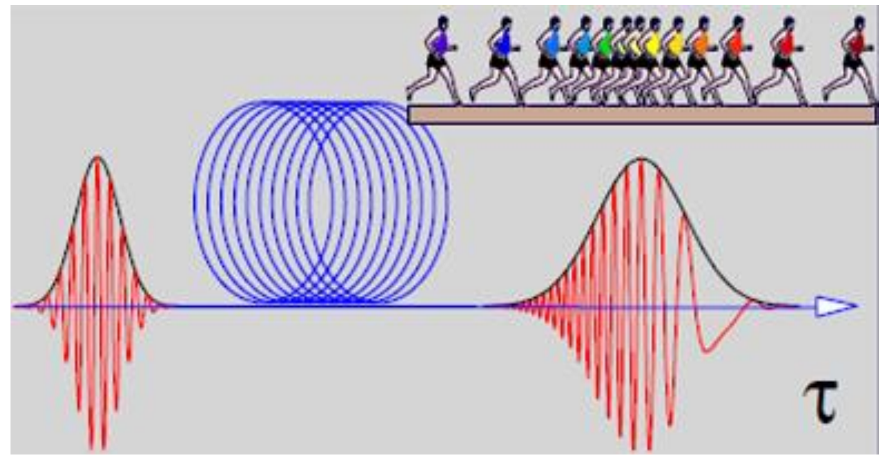
Negative (anomalous) dispersion needed to form solitons

Soliton – NL Schrödinger Equation

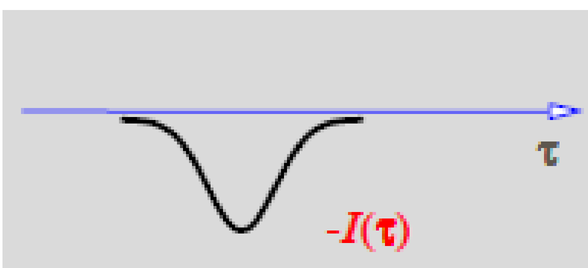
GVD

$$i\frac{\partial \tilde{E}}{\partial z'} = -\frac{\beta_2}{2}\frac{\partial^2 \tilde{E}}{\partial \tau^2} + \Gamma |\tilde{E}|^2\,\tilde{E}$$

SPM



In anomalous dispersion regime pulse forms a bounding potential

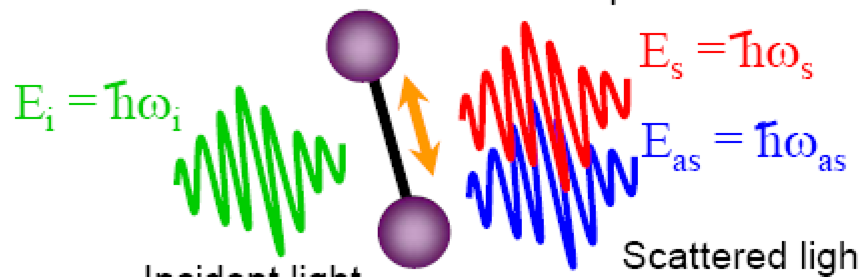


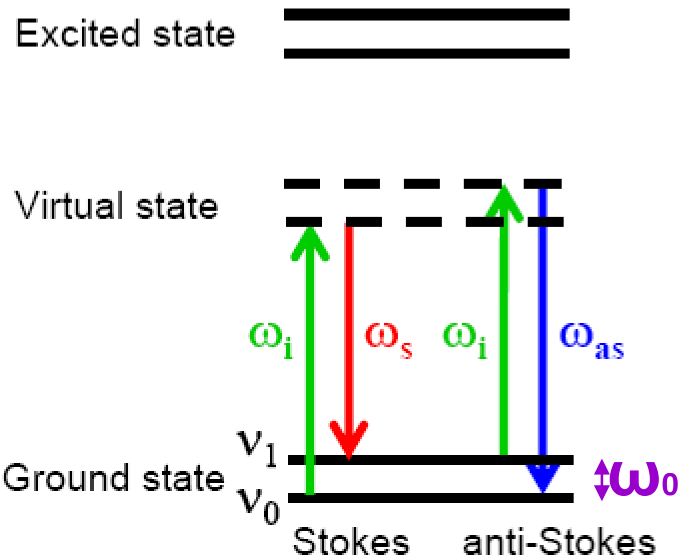
Pulse is trapped in time (shape does not change)

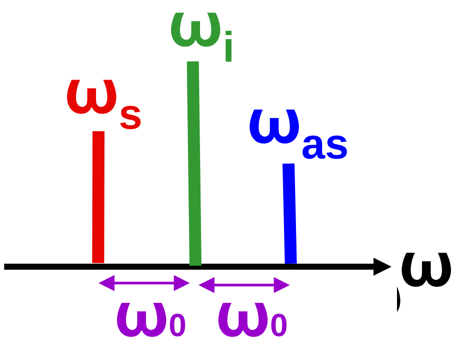
Raman scattering

Interaction of photons and molecular vibrations (optical phonons)

C.V. Raman
1930 Nobel Prize







+ Boltzmann distribution **broadening**

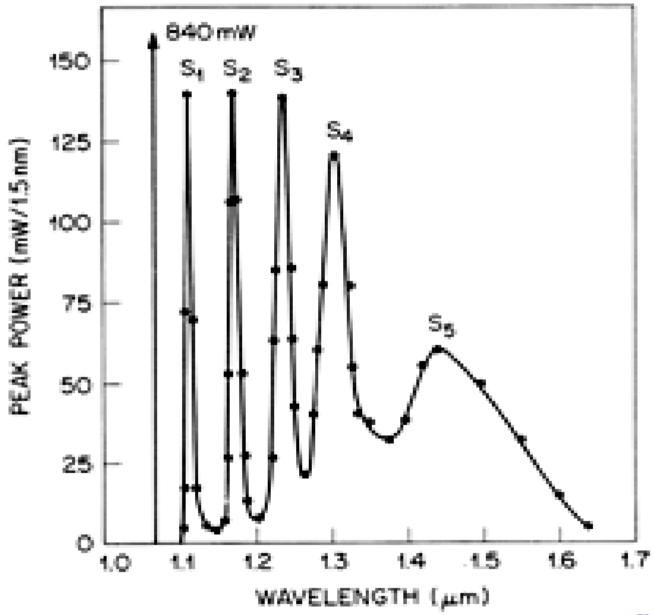
$$\omega_{as} - \omega_i = \omega_i - \omega_s = \omega_0$$

Stimulated Raman scattering

New phenomenon observed in 1962 by Gisela Eckhardt et al:
for intense pump the Stokes wave rapidly grows

Spontaneous Raman Scattering provides a weak signal that is amplified by the pump ↔ **Stimulated Raman Scattering SRS**

When the Stokes power becomes large enough it can act as a pump to the next order Stokes →

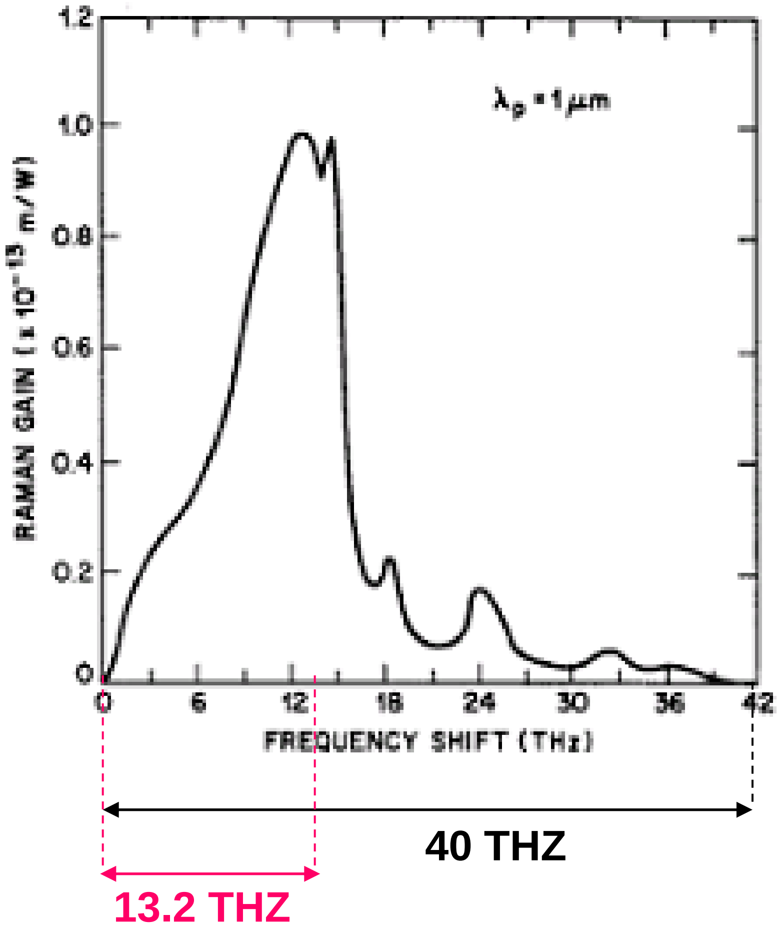


SRS is utilized for single pass or cascaded amplifiers and lasers

Raman gain in optical fibers

Raman amplification in an optical fiber was first observed and measured in 1973 by ***Stolen and Ippen***

- Raman gain peak is shifted from the pump by resonance frequency of molecular vibrations
- Gain spectrum in optical fibers is broad and smooth due to amorphous nature of glass
- For a given fiber the normalized gain profile does not depend on pump wavelength (in contrast to EDFA)



Single-pass Raman amplifier