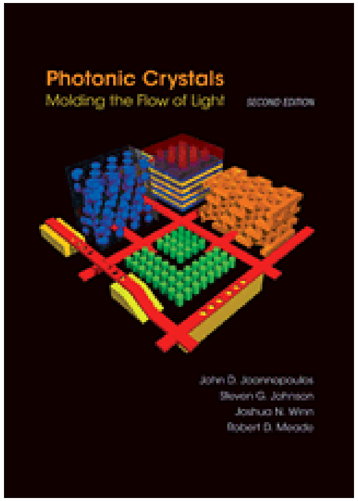
WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

Quantum Electronics
Lecture 3



Light propagation in periodic media
Photonic crystals

Lecturer:

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The book online:
http://ab-initio.mit.edu/book/

HUMAN CAPITAL
NATIONAL COHESION STRATEGY

EUROPEAN UNION
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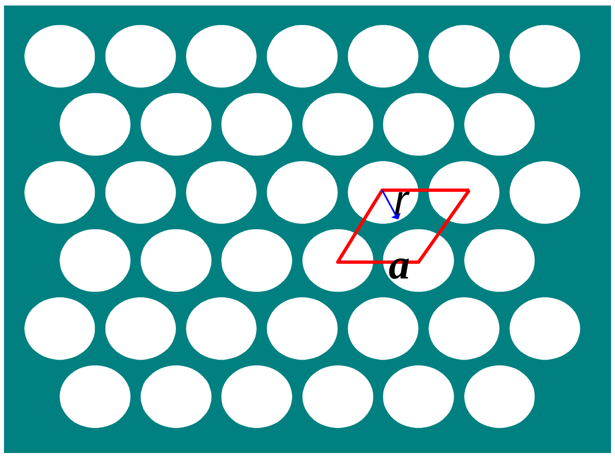
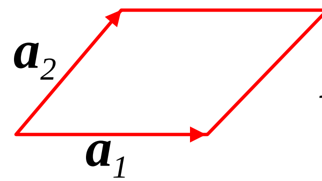
Contents

Periodic media



Translation symmetry

$$\varepsilon\ (\mathbf{r}) = \varepsilon\ (\mathbf{r} + \mathbf{R}_{mn})$$



$$\mathbf{R}_{mn} = m\boldsymbol{a}_1 + n\boldsymbol{a}_2$$

Periodically reapeted properties of the medium in analogy to a crystal lattice

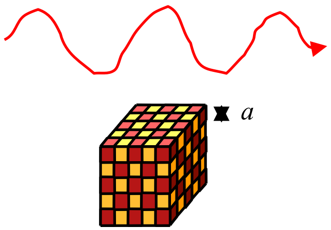
a_i – lattice constant

Periodic media - scattering regimes

$a >> \lambda$ incoherent scattering

Constructive / destructive interference effects

$a << \lambda$ averaging

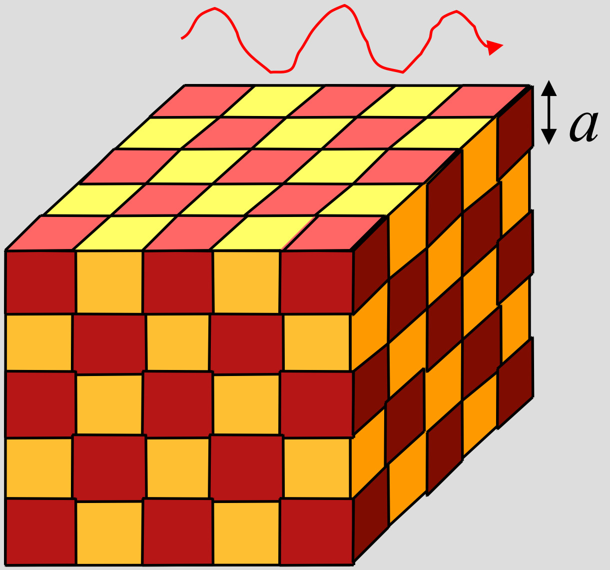


Metamaterials

Wavelength scale regime

$a \sim \lambda$ coherent scattering

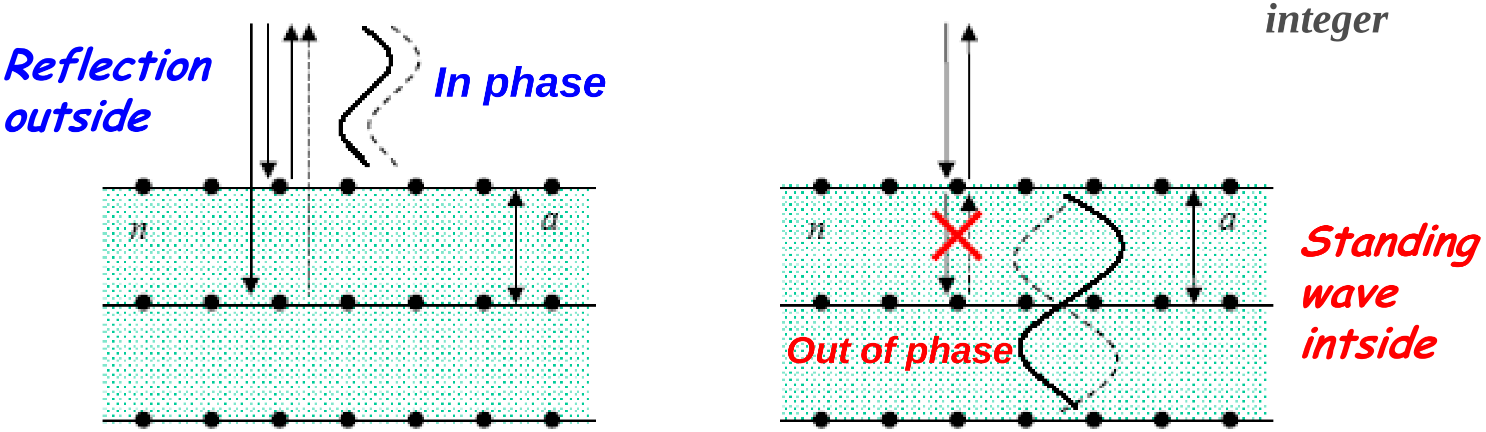
Bragg diffraction



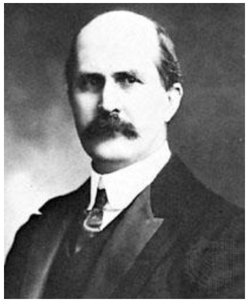
Bragg's law

Bragg Condition: $\frac{\lambda}{n} = 2a \Rightarrow a = \frac{1}{2}\frac{\lambda}{n}$ *(x m)*

integer



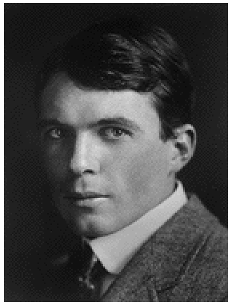
Exactly half a wavelength fits in one period of the lattice



W.H. Bragg (1862-1942)

*Formula for X-rays reflection from crystals (**Bragg's law**):*

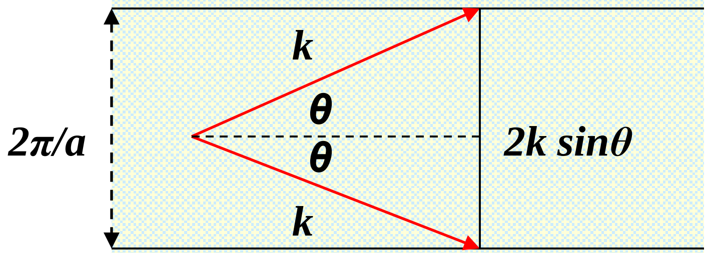
Sir William Henry Bragg
and his son William Lawrence
Nobel Prize 1915



W.L. Bragg (1890-1971)

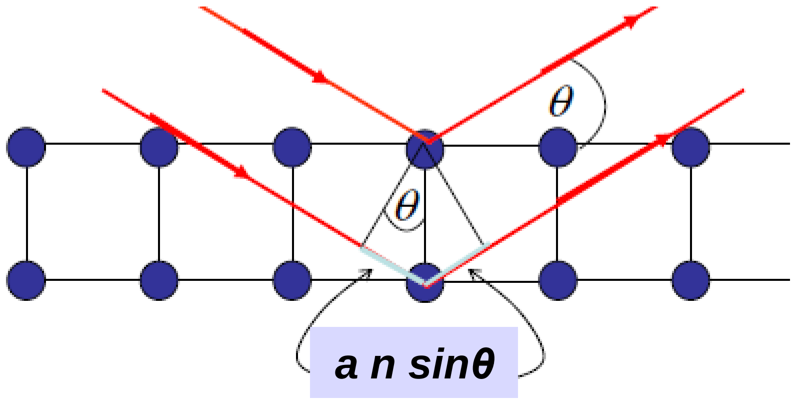
Bragg law for skew incidence

k-vector space	*real space*



Transverse resonance for projection of k-vector:

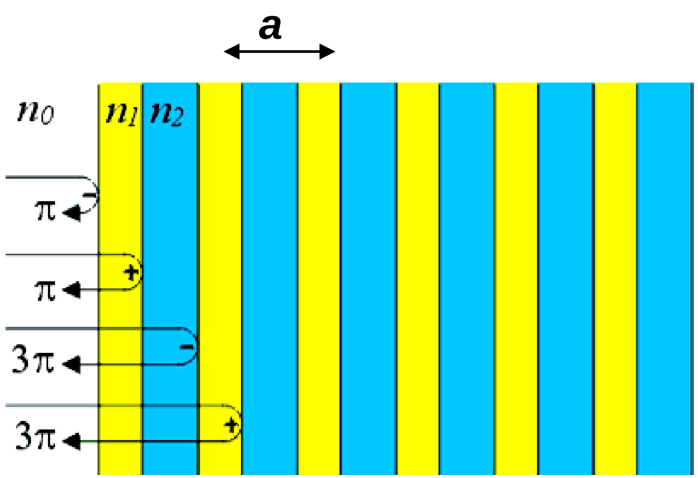
$2k \sin\theta = m\, 2\pi/a$



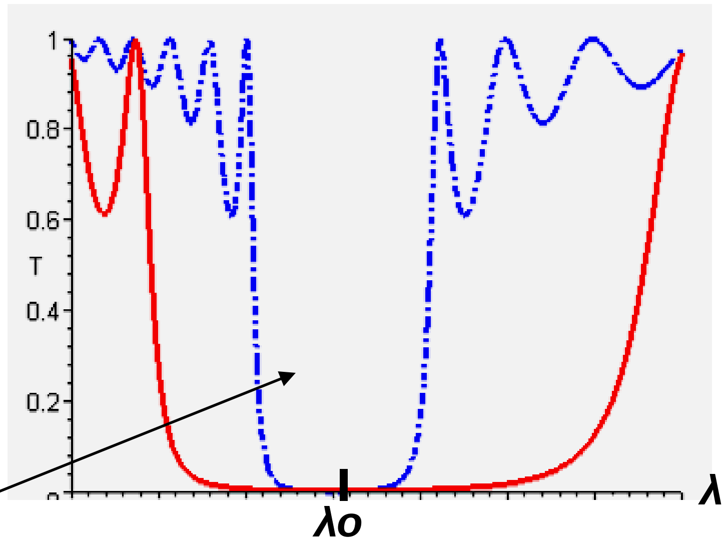
Optical path difference: $2a\, n \sin\theta = m\, \lambda$

$$a \sin\theta = m\, \lambda/2n$$

Periodic layered medium - Bragg reflector



Transmission spectra:
*solid line: index difference = **1.0**; 5 layer pairs*
*dashed line: index difference = **0.3**; 14 layer pairs*



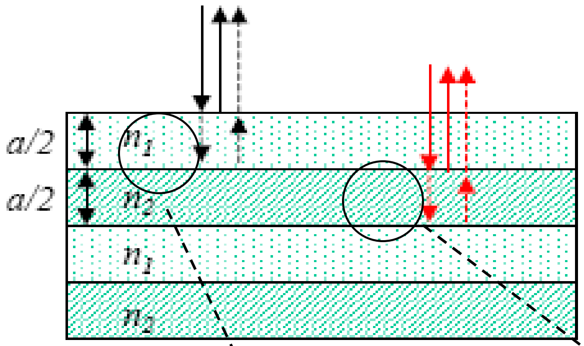
Transmission forbidden around Bragg resonance

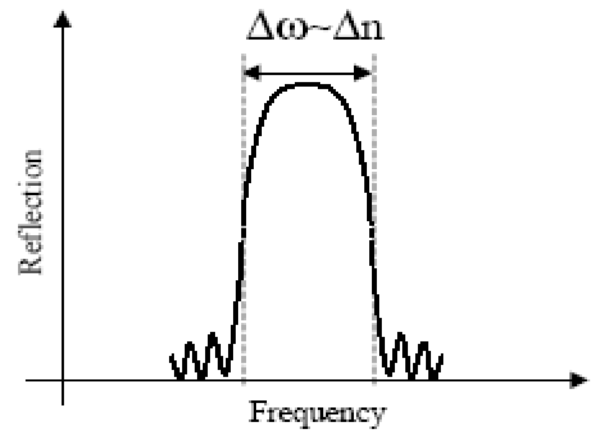
Photonic band gap !

Bragg resonance: λo =2na

Band gap width in layered media

Band gap edge - *frequency at which the incoming and reflected waves in one of the layers are in phase*



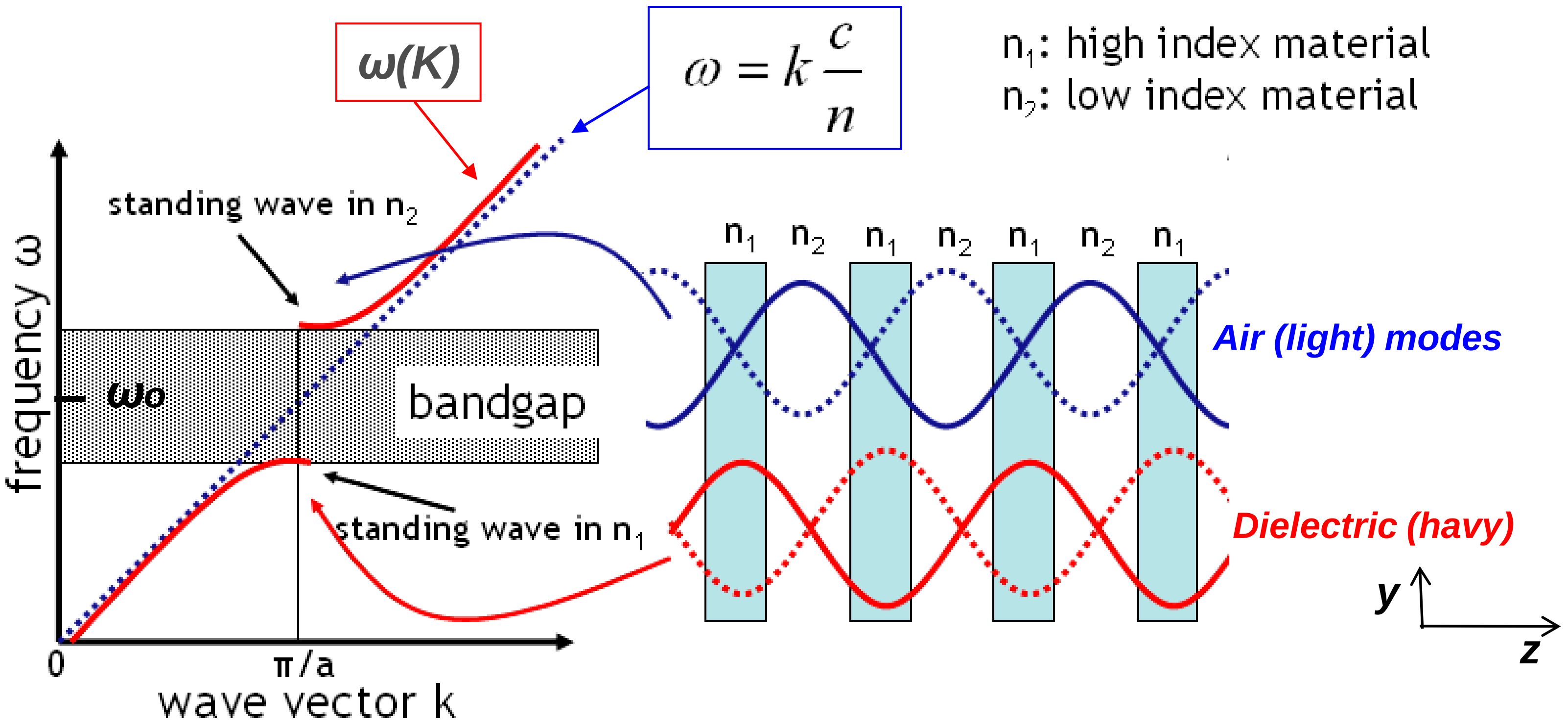


$$\lambda_1 = 2\frac{a}{2}n_1 = an_1 \qquad \lambda_2 = 2\frac{a}{2}n_2 = an_2$$

$$n_1 < n_2 \;\Rightarrow\; \lambda_1 < \lambda_2$$

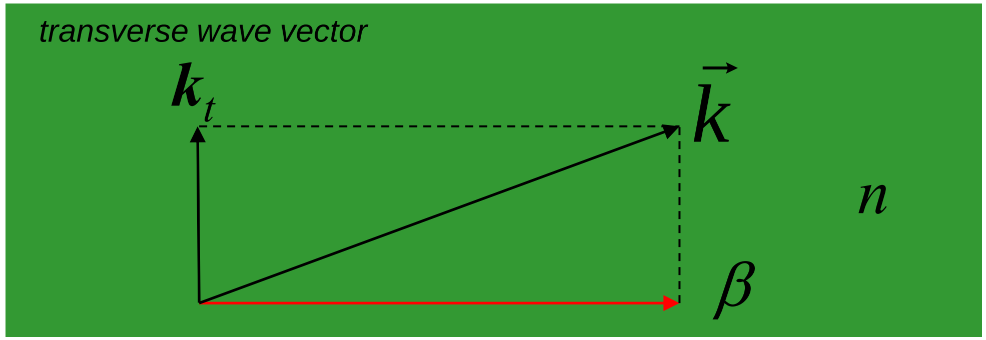
$$\omega_1 - \omega_2 = \frac{2\pi c}{a}\left(\frac{1}{n_1} - \frac{1}{n_2}\right) \sim (n_2 - n_1) = \Delta n$$

Dispersion in layered medium for k_y=0



Deviation from the straight-line dispersion curve of a homogeneous medium is to ensure Bragg reflection for $kz = \pm N(\pi/a)$ - the curve becomes horizontal

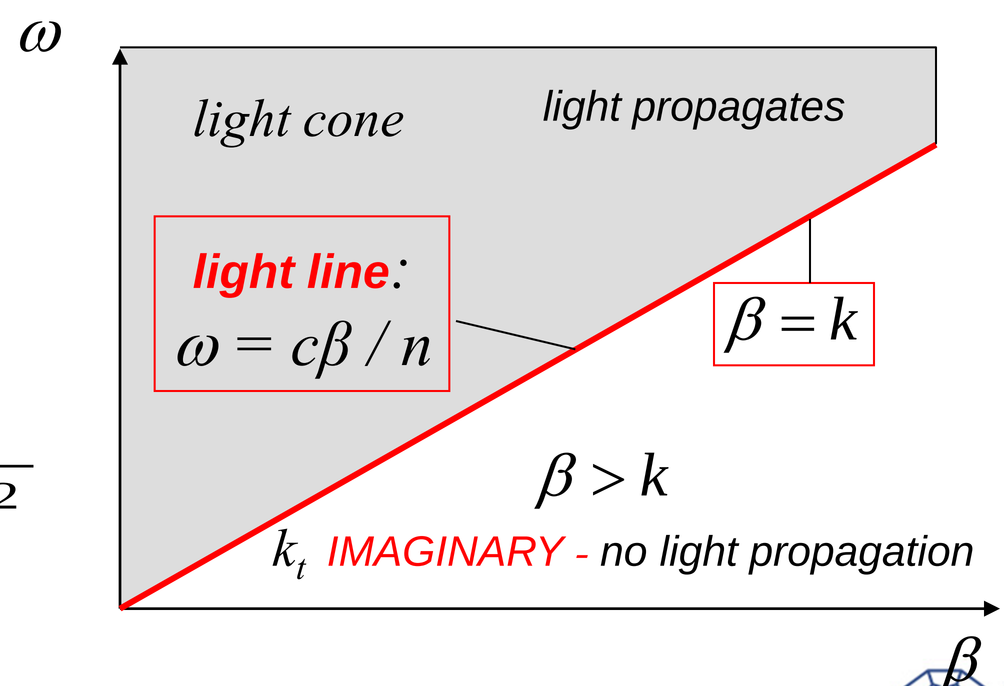
Projected dispersion diagram – uniform dielectric



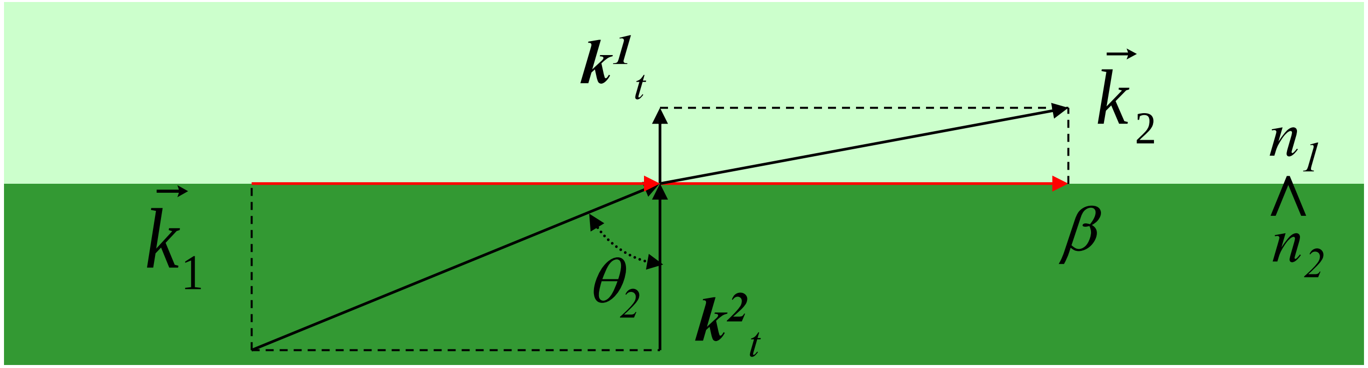
$$\left|\vec{k}\right| \equiv k \;\; = \omega n / c$$

$$\mathbf{k}_t = \sqrt{\left(\frac{\omega n}{c}\right)^2 - \beta^2}$$

$$\omega = \frac{c}{n}\sqrt{\beta^2 + \left|\mathbf{k}_t\right|^2}$$

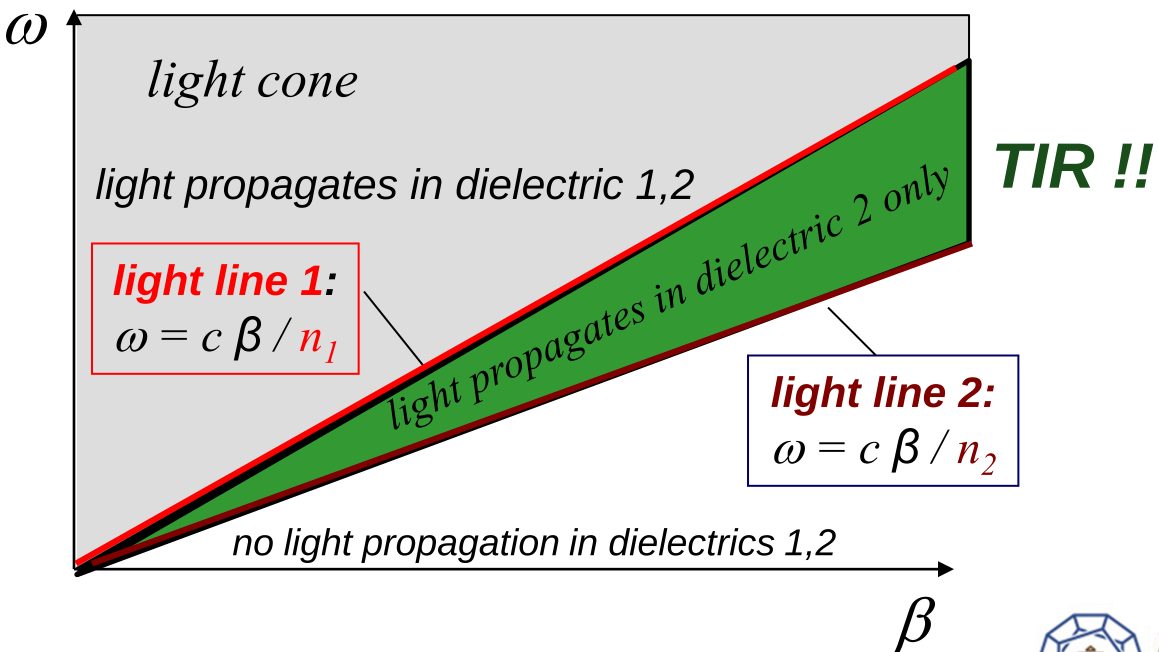


Dispersion diagram – two uniform dielectrics

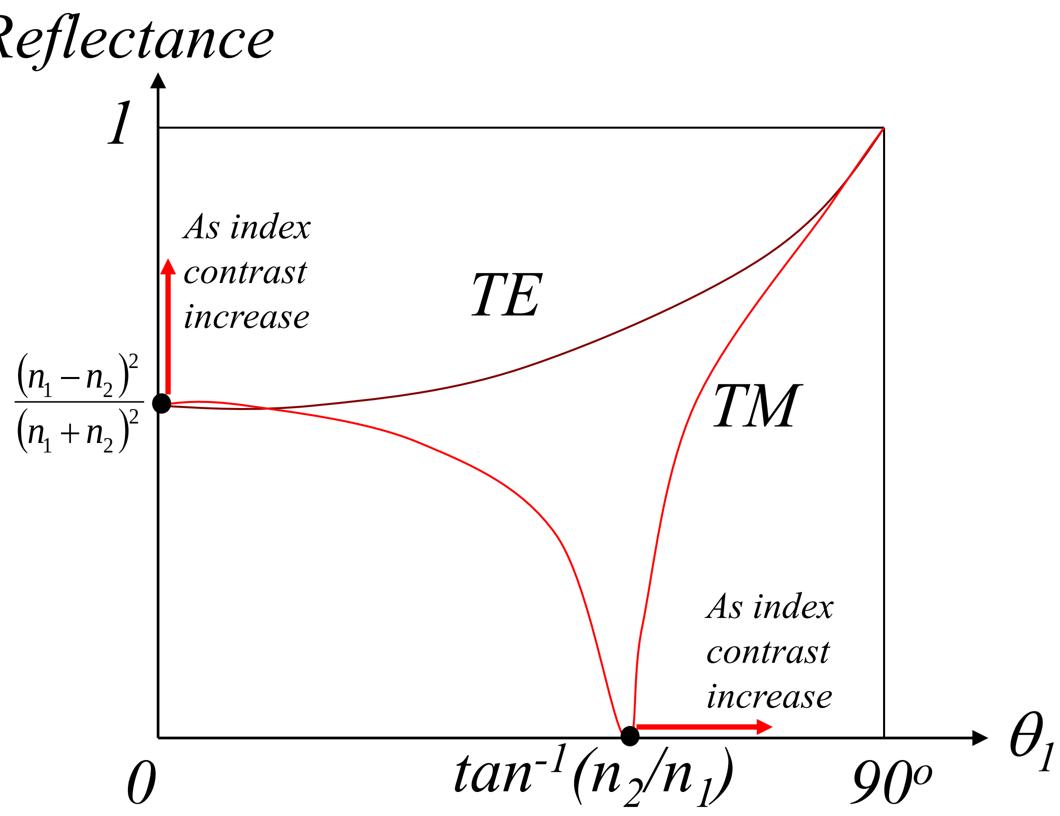


$$\mathbf{k}_t^i = \sqrt{\left(\frac{\omega n_i}{c}\right)^2 - \beta^2}$$

$$\sin\theta_i = \frac{c\beta}{\omega n_i}$$

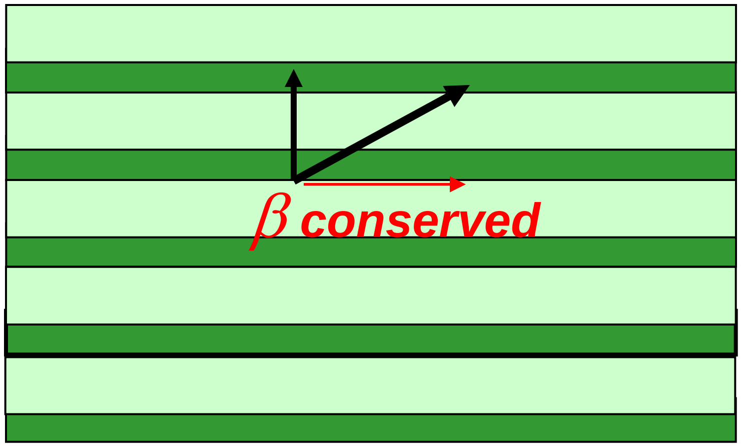


Reflectance form dielectric interface

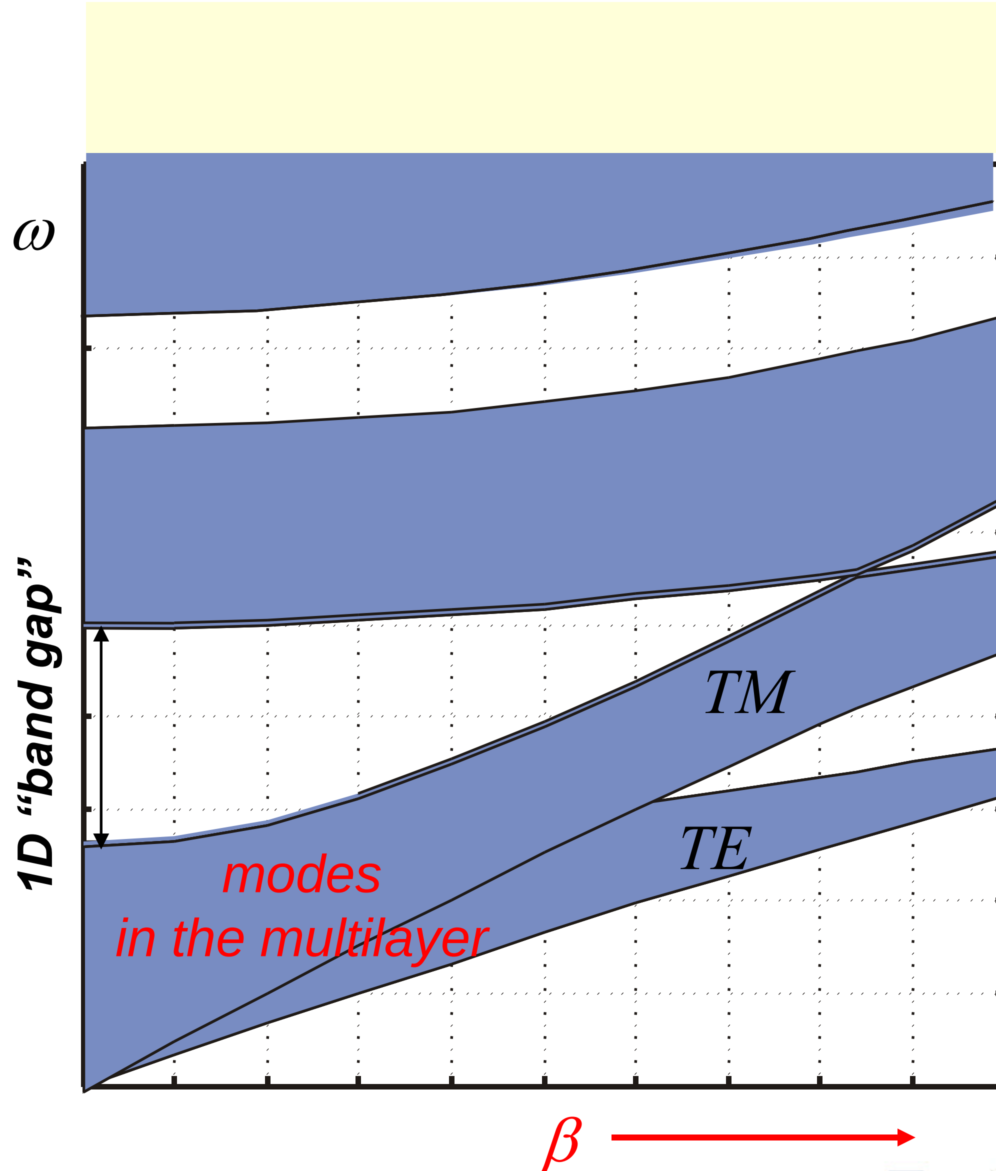


(Metallic mirror has low angular and polarization dependence, but very high loss for optical frequencies)

Dispersion diagram for 1D PhC (Bragg mirror)

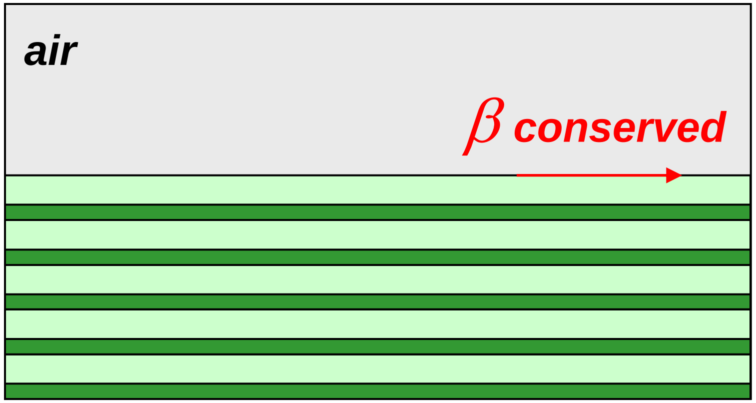


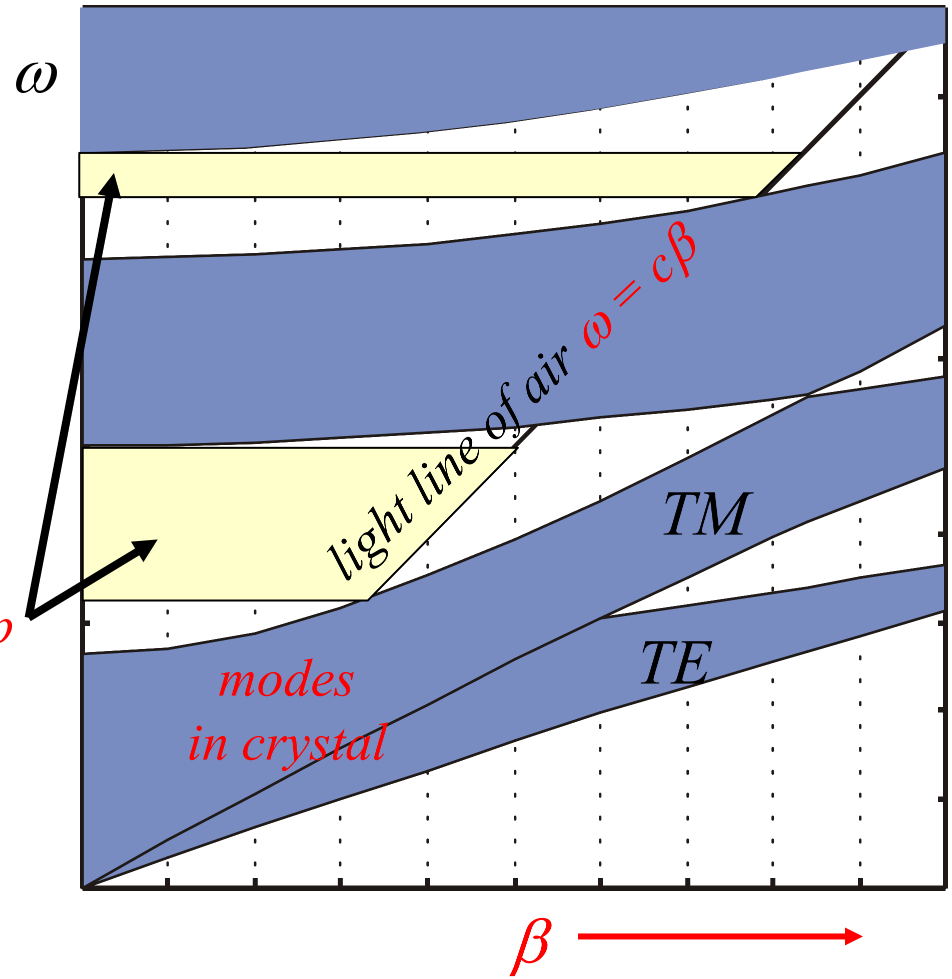
$\beta=0$ – propagation perpendicular to the layers



Omnidirectional Reflection

[J. N. Winn et al, Opt. Lett. ***23****, 1573 (1998)]*





in these ω ranges, there is no overlap between modes of air & crystal

All incident light
(any angle and polarization)
is reflected
from flat surface !!

needs *sufficient index contrast &* $n_{high} > n_{low} > 1$

Weak coupling – Coupled Mode Theory

Co-directional

$$\frac{dA_m}{dz} = -i\kappa\, A_l\, e^{i\Delta\beta z}$$

$$\frac{dA_l}{dz} = -i\kappa^* A_m e^{-i\Delta\beta z}$$

Contra-directional

$$\frac{dA_m}{dz} = -i\kappa\, B_l\, e^{i\Delta\beta z}$$

$$\frac{dB_l}{dz} = i\kappa^* A_m e^{-i\Delta\beta z}$$

Coupling coefficient: $\kappa = \dfrac{\omega\varepsilon_0}{4}\iint E_m^* \Delta n^2 E_l\, dxdy$

Phase mismatch:

$$\Delta\beta = \beta_m - \beta_l$$

$$\Delta\beta = \beta_m + \beta_l$$

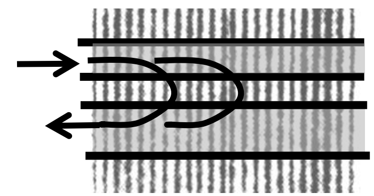
Two phase mismatched modes can be coupled when z-variation of Δn^2 ***compensates the mismatch.***

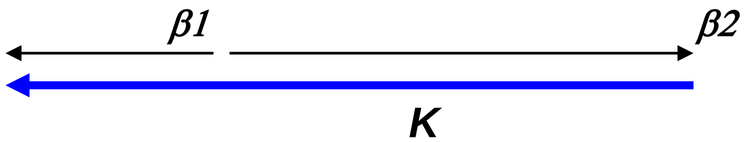
Grating assisted coupling

Periodic index modulation for compensation of pase mismatch: $K = \Delta\beta$

Grating period: $$\Lambda \equiv \frac{2\pi}{K} = \frac{2\pi}{\Delta\beta} = \frac{2\pi}{(2\pi/\lambda)\Delta n} = \frac{\lambda}{\Delta n}$$

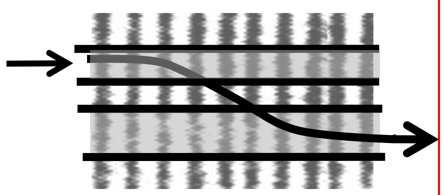
- *Short period Bragg gratings (contra-directional coupling)*
  - *Grating period shorter than λ:*

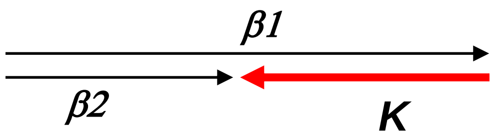




$$\Lambda = \frac{2\pi}{(\beta_2 + \beta_1)} = \frac{\lambda}{(n_2 + n_1)}$$

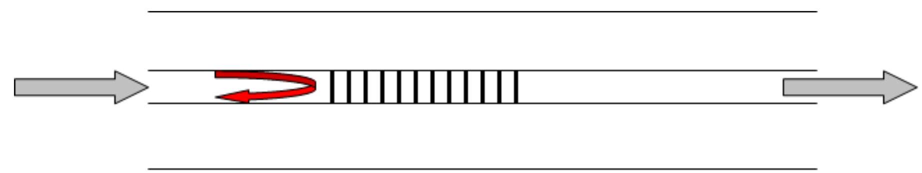
- *Long period Bragg gratings (co-directional coupling)*
  - *Grating period longer than λ*

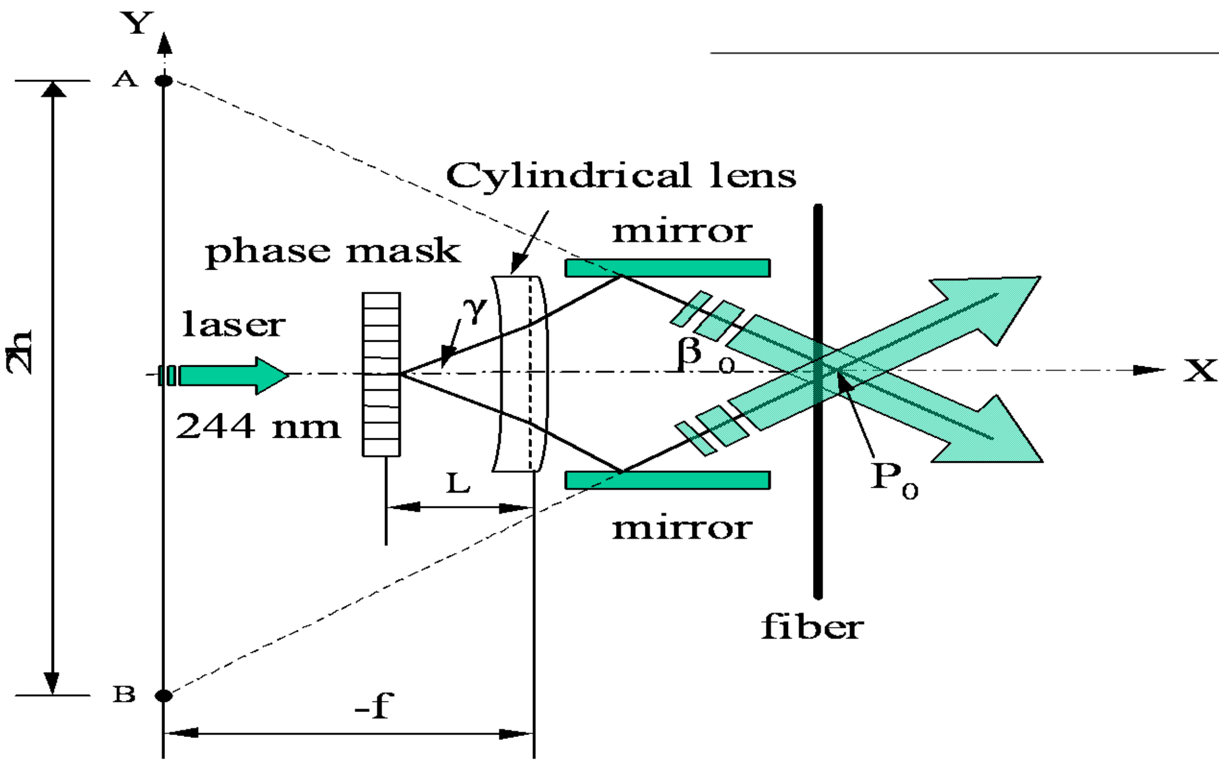




$$\Lambda = \frac{2\pi}{(\beta_2 - \beta_1)} = \frac{\lambda}{(n_2 - n_1)}$$

Fiber UV induced Bragg-Grating

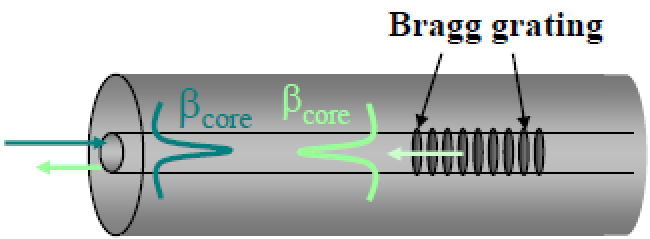




Holographic UV Lithography with 244 nm UV light

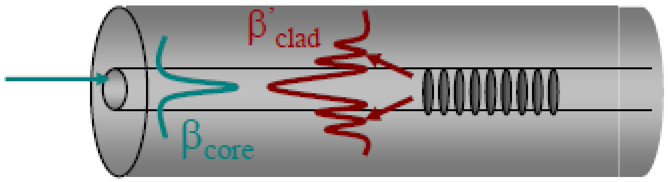
Mode coupling in fiber grating

Core-core coupling

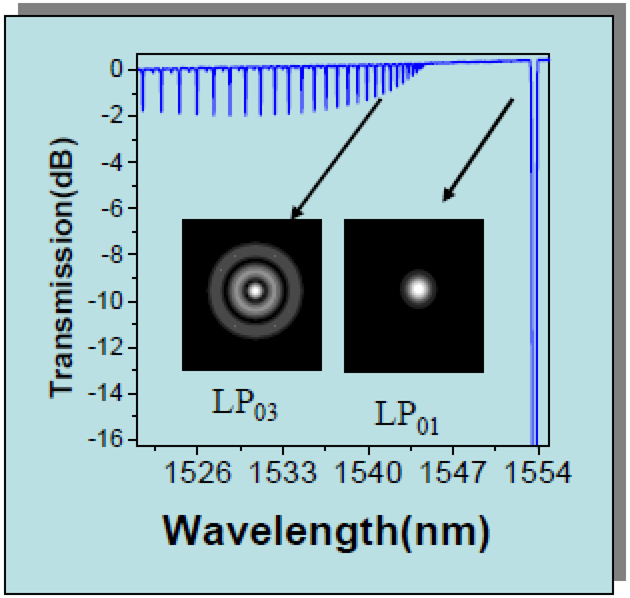


$(\lambda_B = 2 n_{core} \Lambda)$

Core-cladding coupling



$\lambda_{clad,i} = (n_{core} + n_{clad,i})\Lambda$



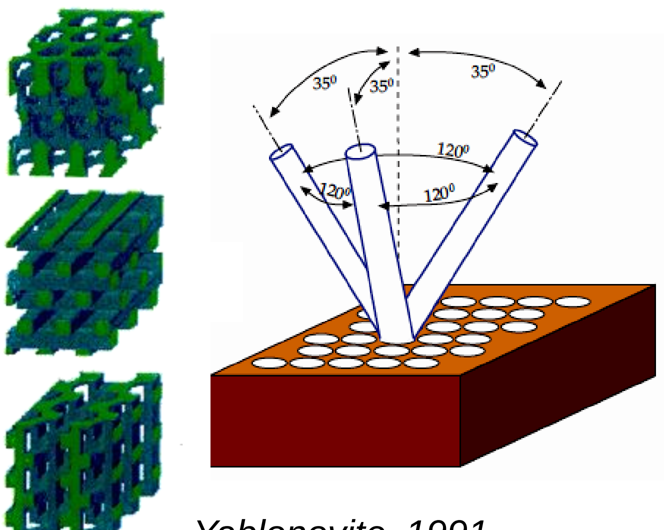
Bandgap in all 3 dimensions

Extension to 3D, full band gap 1987 :
E. Yablonovitch, S. John

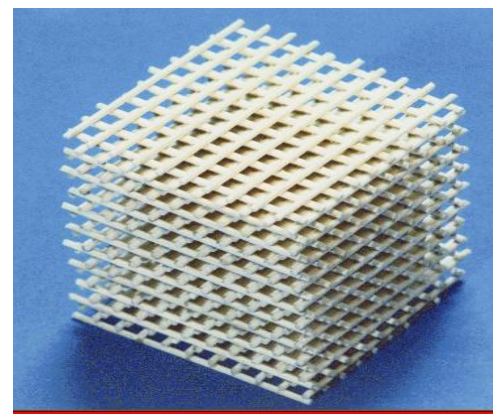
The first 3D photonic crystal made by E. Yablonovitch 1991 (microwaves)


E. Yablonovitch


S. John

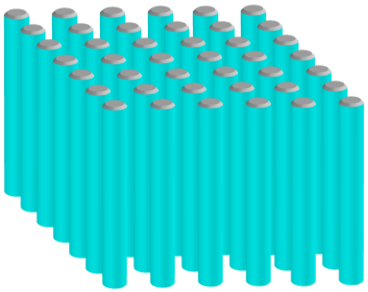


Yablonovite, 1991

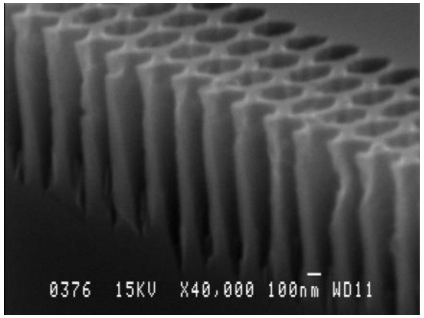

"Woodpile" stack of Alumina rods

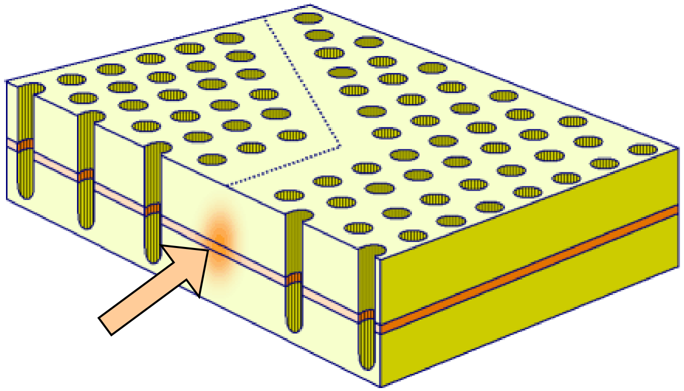
The first PhC for optical λs (Proposed by K. M. Ho et al 1994, made by Lin et al 1998)

2D Photonic Crystals



Simpler fabrication
Suitable for integrated optics
(slab waveguide confinement in vertical direction)





Theoretical description

Starting point: Maxwell Equations

$$\nabla \cdot H = 0 \qquad \nabla \cdot \varepsilon E = 0$$

$$\nabla \times H - \frac{\varepsilon}{c}\frac{\partial E}{\partial t} = 0 \qquad \nabla \times E - \frac{1}{c}\frac{\partial H}{\partial t} = 0$$

No sources

- *Look for time-harmonic states:* $\mathbf{H}(\mathbf{r},t)=\mathbf{H}(\mathbf{r})\, e^{i\omega t}$, $\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})\, e^{i\omega t}$
- *Eliminate the* ***E*** *fields* ⇨ *Hermitian eigenproblem:*

$$\nabla \times \left(\frac{1}{\varepsilon(\vec{r})} \nabla \times H(\vec{r}) \right) = \left(\frac{\omega}{c} \right)^2 H(\vec{r})$$

- *Real eigenvalues* ω
- *Orthogonal eigenfunctions (modes)* $\mathbf{H}$

In general: The eigenproblem in infinite domain → *Continuous* ω *spectrum*

But ε **(r)** *is periodic !* → *Discrete set of* ω

Maxwell meets Bloch (& Floquet in 1D)

Bloch theorem:

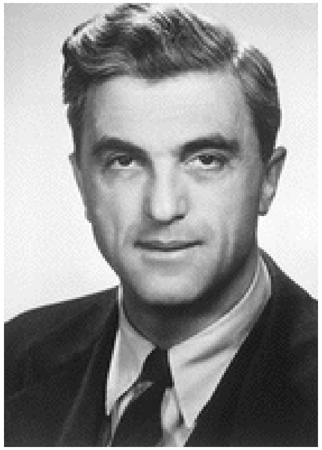
Eigen-operator periodic → *Solutions: e^{ikr} x (periodic function)*

$$H_k(r) = e^{ikr}\, u_k(r) = e^{ikr}\, u_k(r+R)$$

plane wave *spatially periodic amplitude ("envelope")*

k is conserved, i.e. no scattering of Bloch waves on periodic index-modulation !!

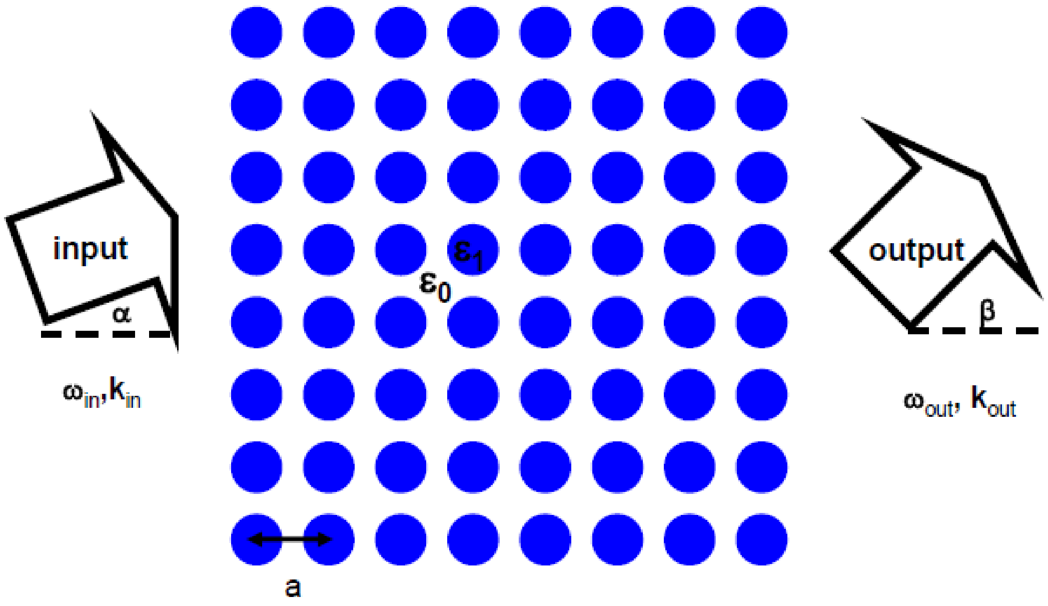
Bloch waves are nonuniform plane waves *with the* **envelope period** = **lattice period**



F. Bloch 1905-1983

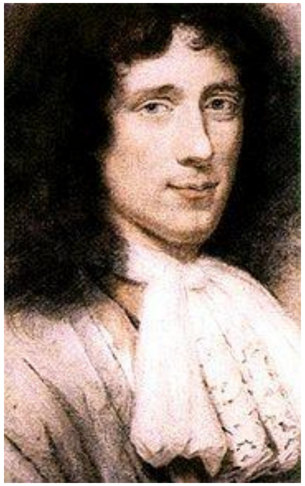
Bloch waves – ”inteference optics”

*Light **is scattered** from each of holes (rodes) in periodic media*



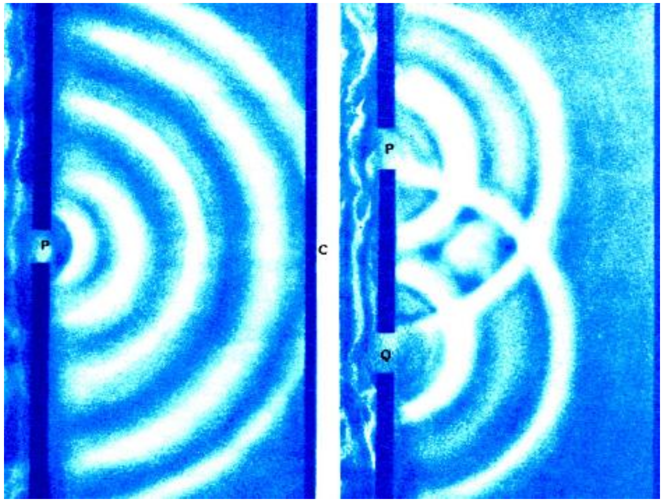
*But due to **interference (Bragg diffraction)** plane Bloch waves are formed*

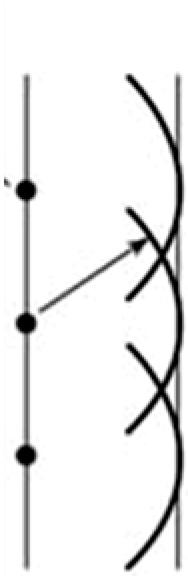
Huyghens interference



1629 – 1695, Dutch

Constructive interference between two wavefronts in certain directions, + the destructive interference in other directions



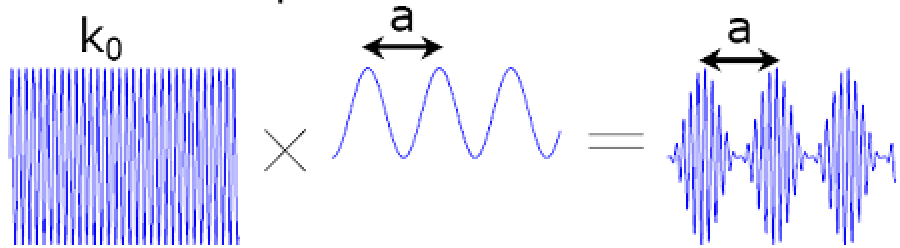


Wave front

Two ways of viewing Bloch waves

A Bloch wave is a plane wave with a modulation:

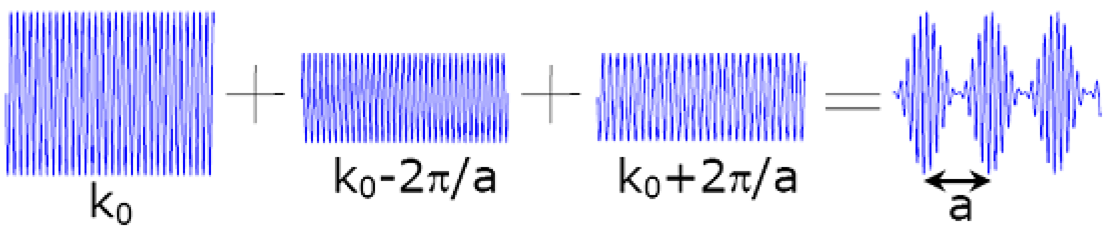
A



$$\psi_{n\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n\mathbf{k}}(\mathbf{r})$$

A Bloch wave consists of multiple wavevectors:

B

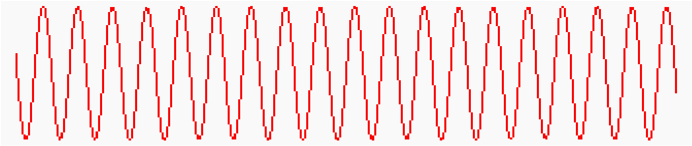


Plane Wave Expansion **:** $\psi(\mathrm{r}) = \Sigma_{\mathrm{m}}\, \mathrm{c}_{\mathrm{m}} \mathrm{e}^{\mathrm{i}(\mathrm{k}+\mathrm{m}2\pi/\mathrm{a})\mathrm{r}}$

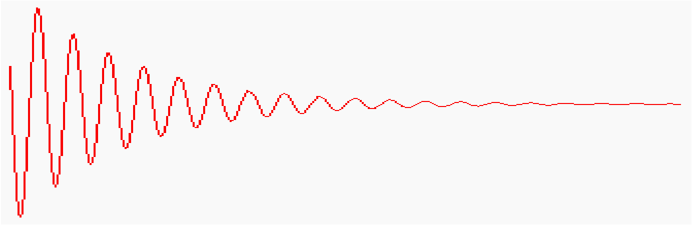
Both pictures are correct and lead to the same results

modified slide from Rob Engelen

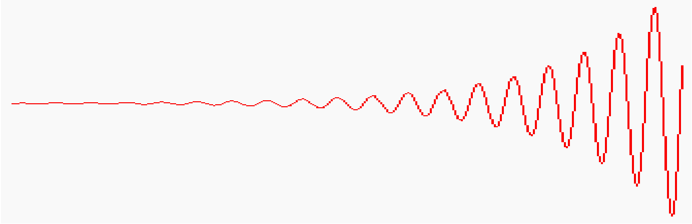
Bloch eigenmodes



Propagating mode: Im k = 0

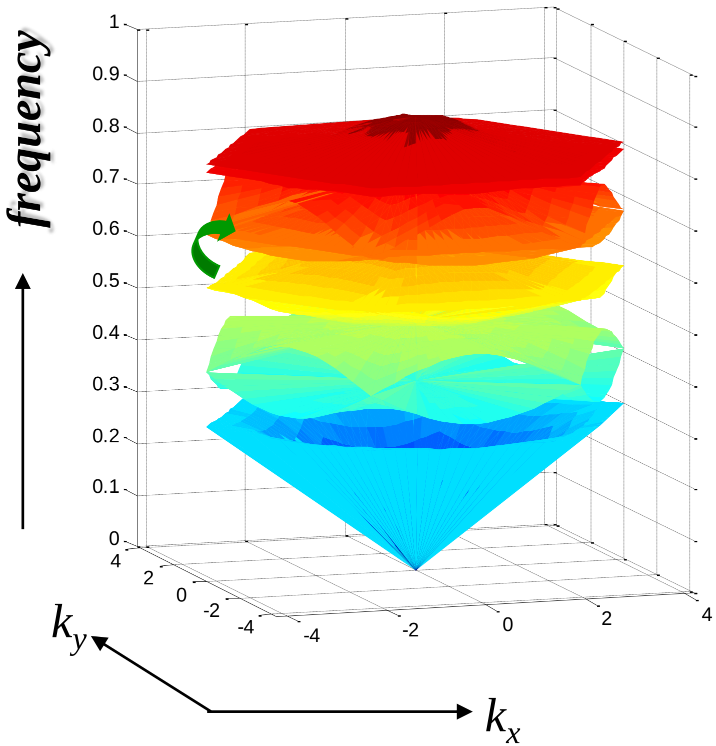


Evanescent mode: Im k > 0

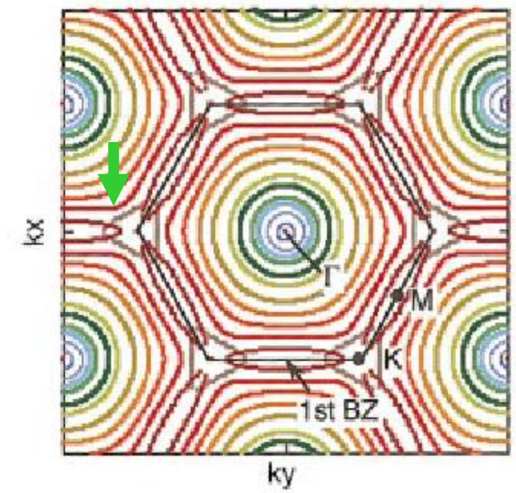


Evanescent mode: Im k < 0

Unusual dispersion relations in PhC



Equal frequency surface (EFS) plot of the band structure for frequency ***w=0.56-0.635***



For frequency ***0.60-0.64*** *EFSs are circles, but their radius shrinks for increasing* ***ω***

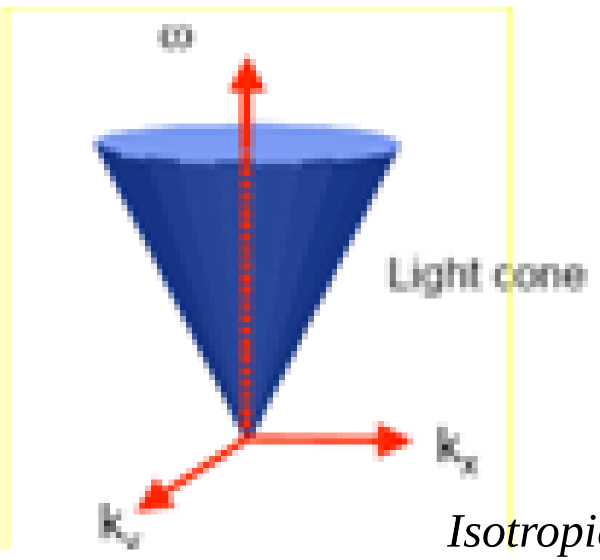
$V_g = \nabla_k \omega$ *inward* → ***All-angle Negative refraction!***

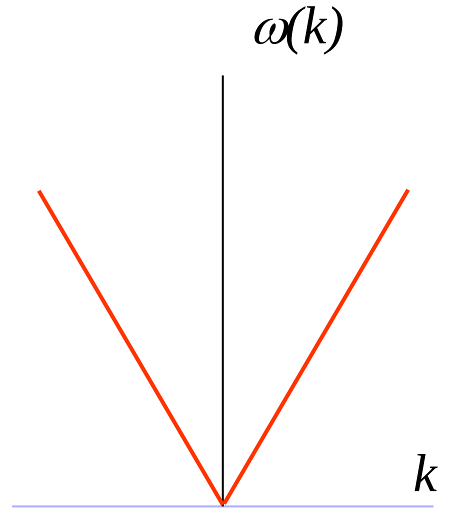
Close to ***K*** *propagation direction* V_g
ultra-sensitive to ***incidence angle*** *and* ***wavelength***
→ ***Beam steering, Superprism effect***

M. Notomi, Phys. Rev. B 62, 10697, 2000

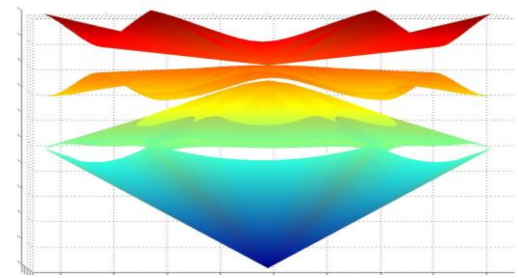
Dispersion in uniform medium and PhC

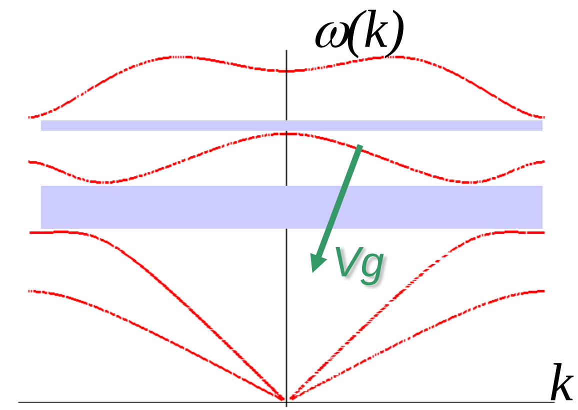
$$\vec{v}_g = \partial\omega / \partial\vec{k}$$





Isotropic non-dispersive medium:
k=nω/c

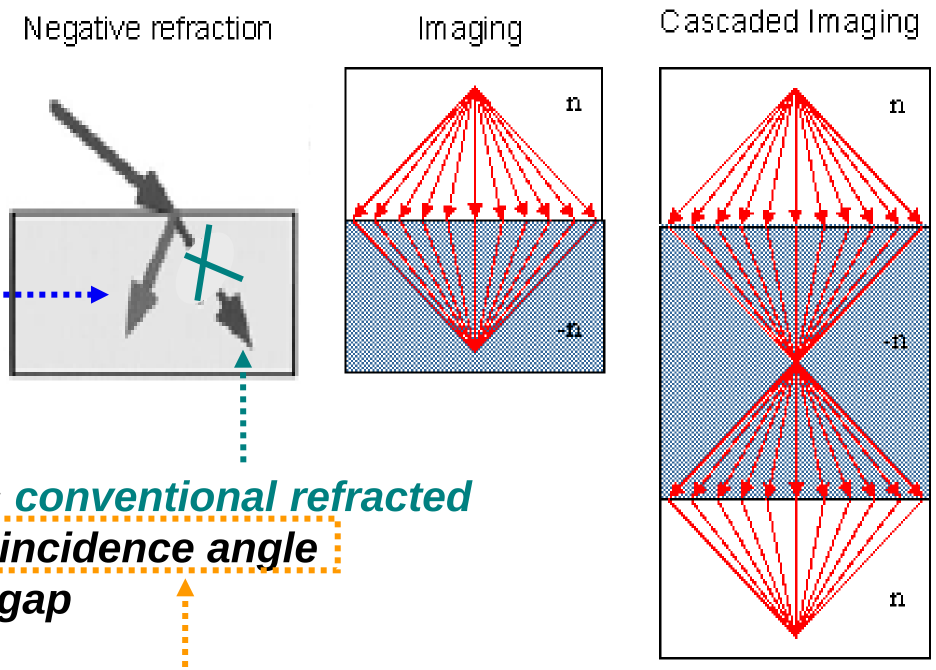




Photonic crystal for a given direction of ***k*****.**

Group velocity in photonic crystals can be inward directed due to negative slope of the dispersion !

"Negative Refraction" –> Self Focusing



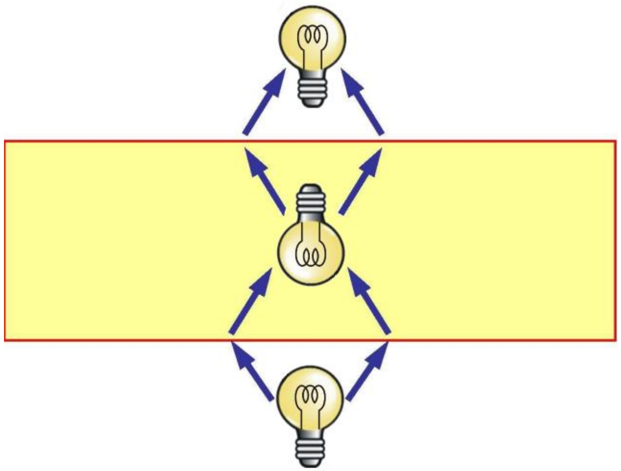
Strong mixture of diffracted waves

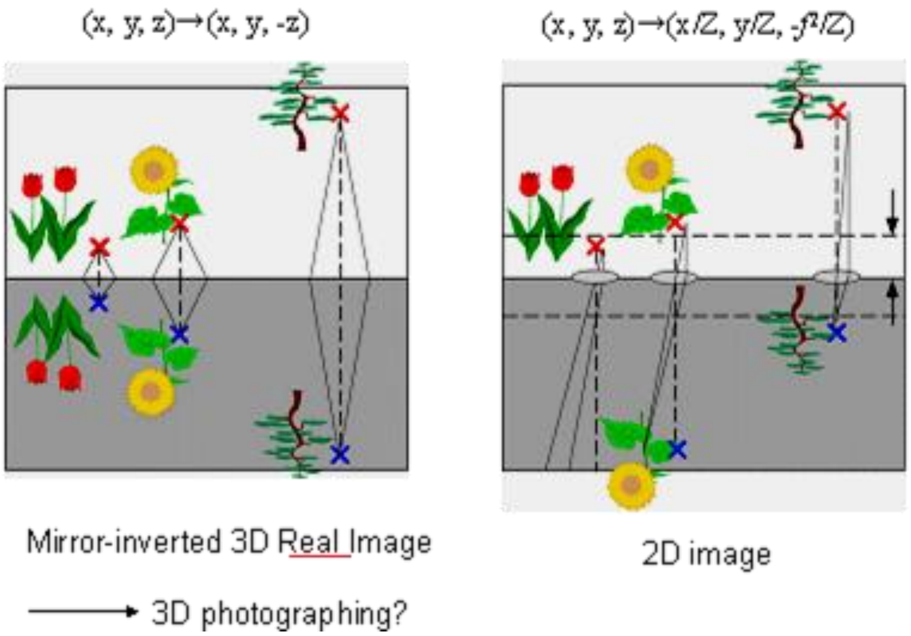
For some frequency regions conventional refracted wave does not exist for any incidence angle – it falls into photonic band gap

Not true for weak index modulation

M. Notomi, Phys. Rev. B 62, 10697, 2000

3D imaging





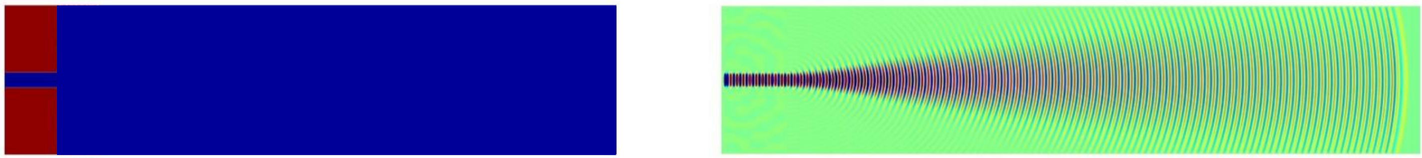
3D pictures?

Sub wavelength resolution imaging !!

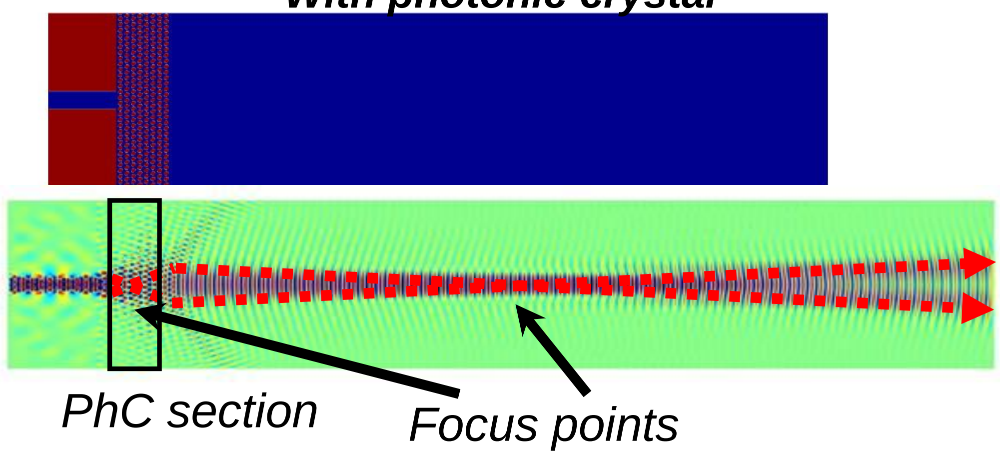
(similar to that in Negative Index Materials – NIM)

Simulation of Self-focusing due to Negative Refraction

Without photonic crystal

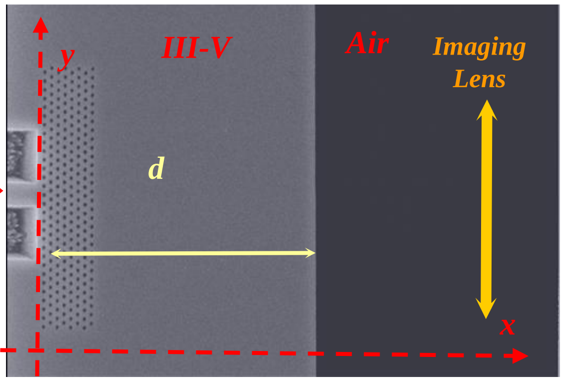


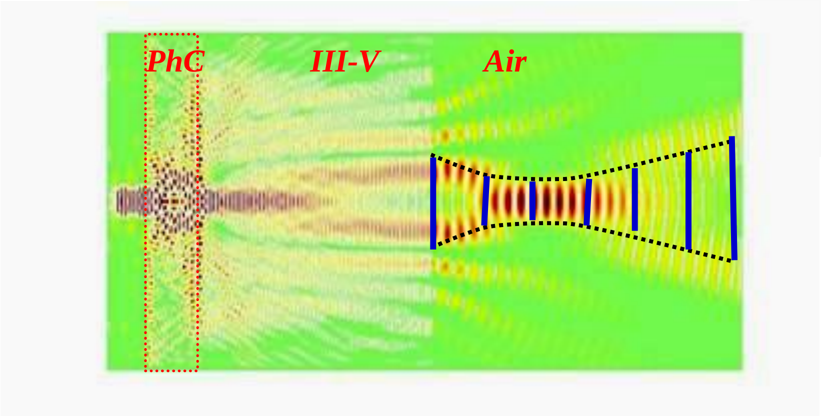
With photonic crystal

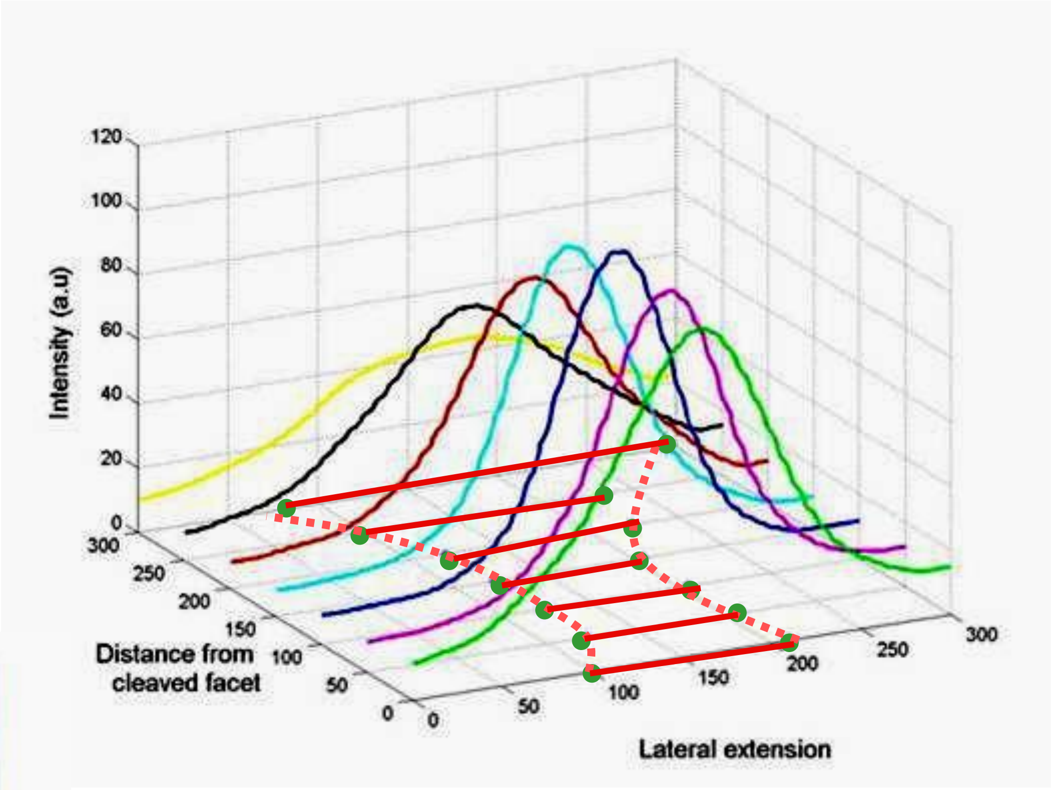


M. Qiu, L. Thylen, M. Swillo, and B. Jaskorzynska, IEEE J of Selected Topics in Quantum Electronics 9, (2003)

Demonstration of Negative Refraction at optical wavelengths - KTH





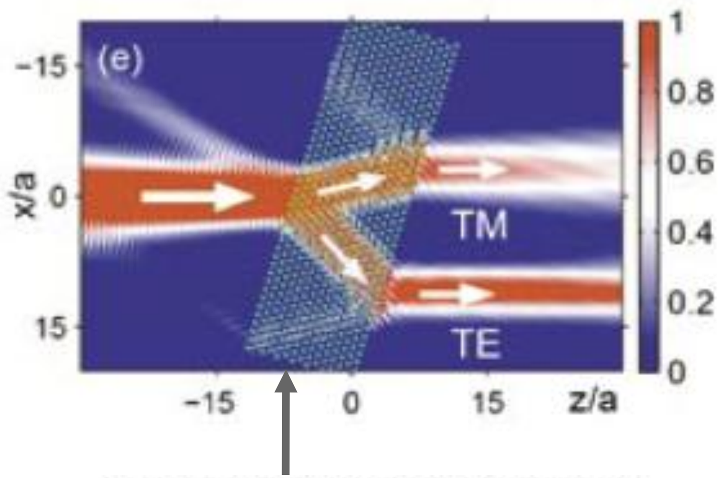


*Measured profile of the imaged output beam for different lens positions. **F**ull-**W**idth at **H**alf- **M**aximum intensity shown with green points.*

Wavelength = 1480 nm

M. Qiu et al, IEEE JSTQE, 9, 106, (2003)
A. Berrier et al, Phys. Rev. Lett. 93, 073902, (2004)

Negative Refraction for polarization splitting - KTH

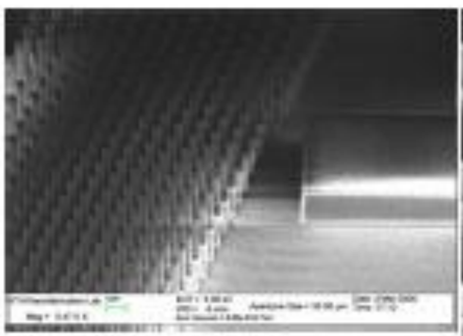


Amorphous silicon pilars on silica

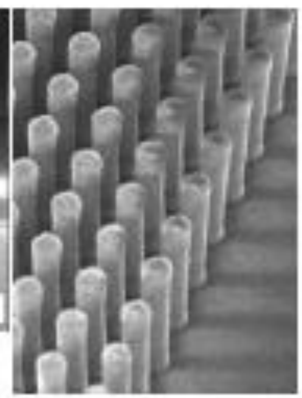
Operation range: 1530 – 1610 nm

TM polarization – positive refraction

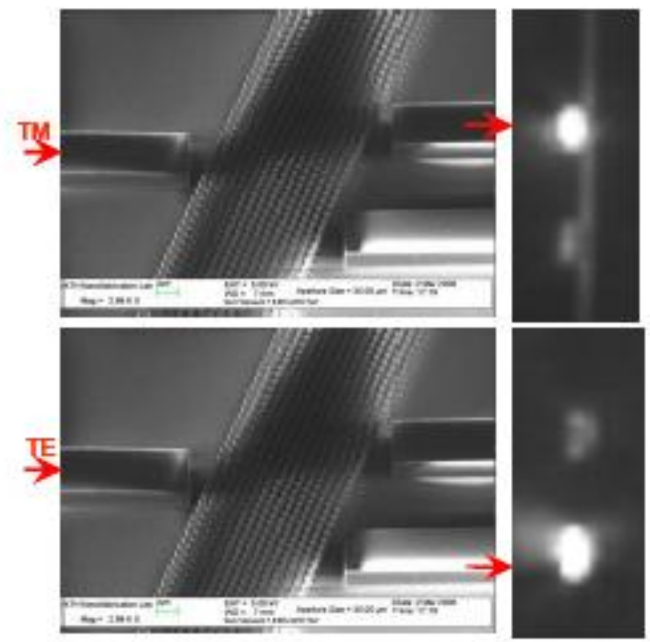
*TE polarization – **negative refraction***



Silicon access waveguide 2.2 x 6 μm



Pilars diam 450 nm x 2.2 μm
Matrix pitch 1.1 μm

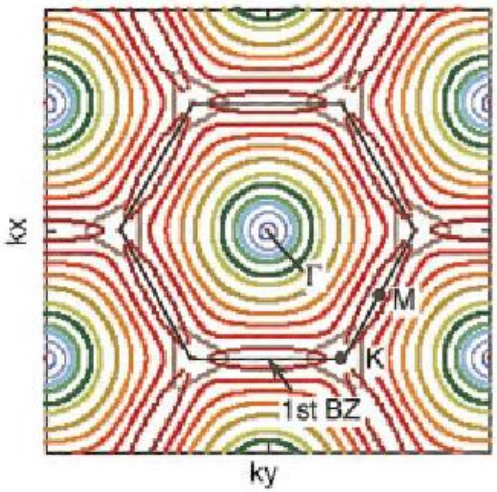


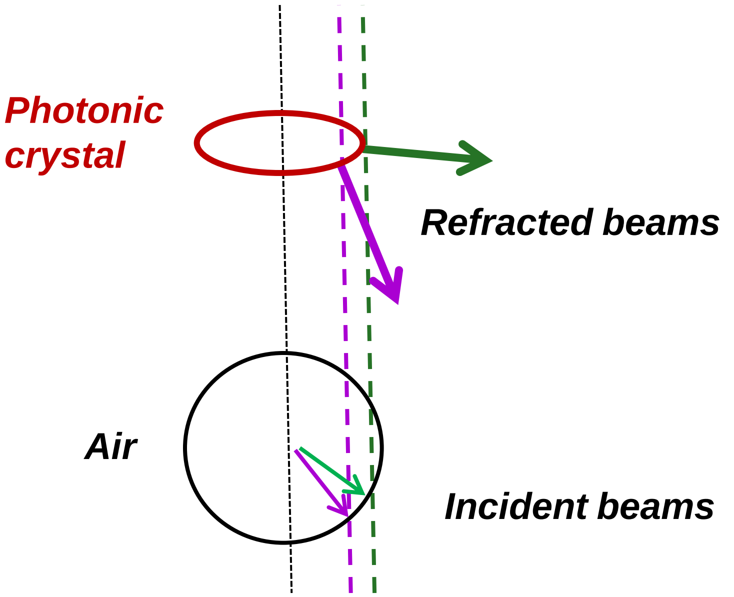
L Liu, X. Ao, L. Wosinski, S. He

Beam steering

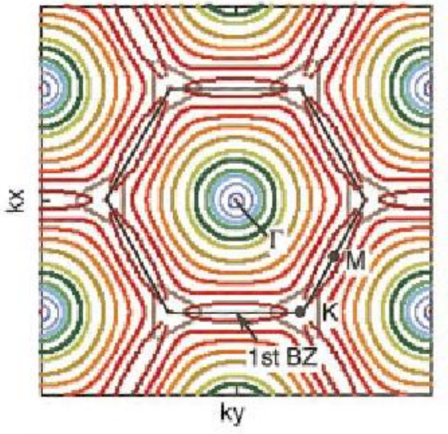
Utilizing strong band-edge anisotropy in PhC

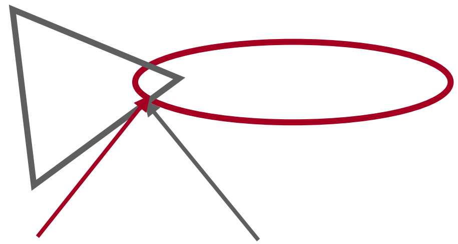
Equal-frequency surfaces in (k_x,k_y) space :



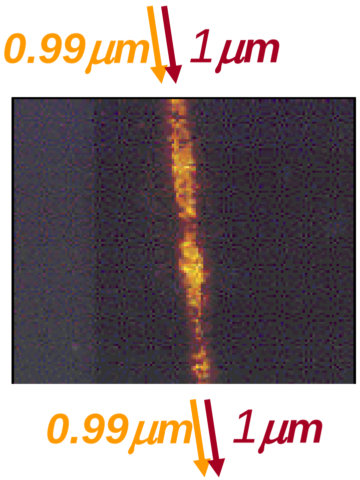


Superprism effect

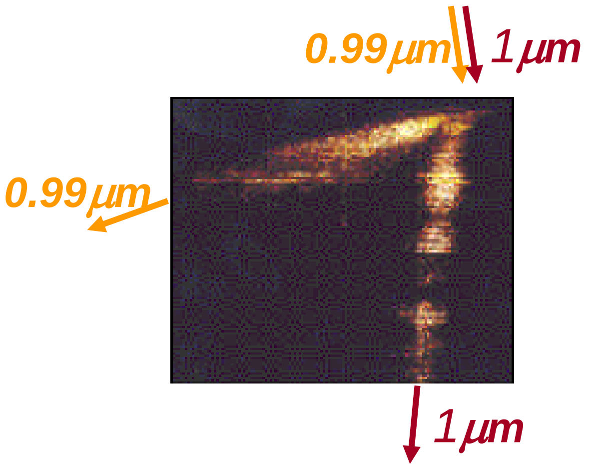




$k(\omega)$ $k(\omega+\delta\omega)$



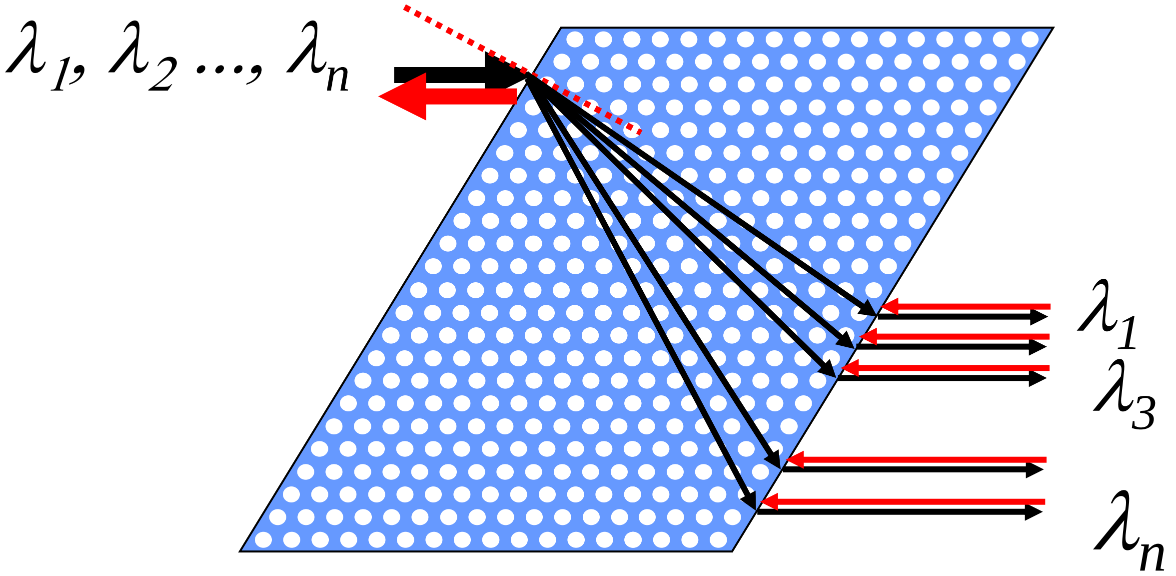
Conventional glass



3D photonic crystal

NEC Corp., To-hoku University, NTT

Superprism effect for de/multiplexing



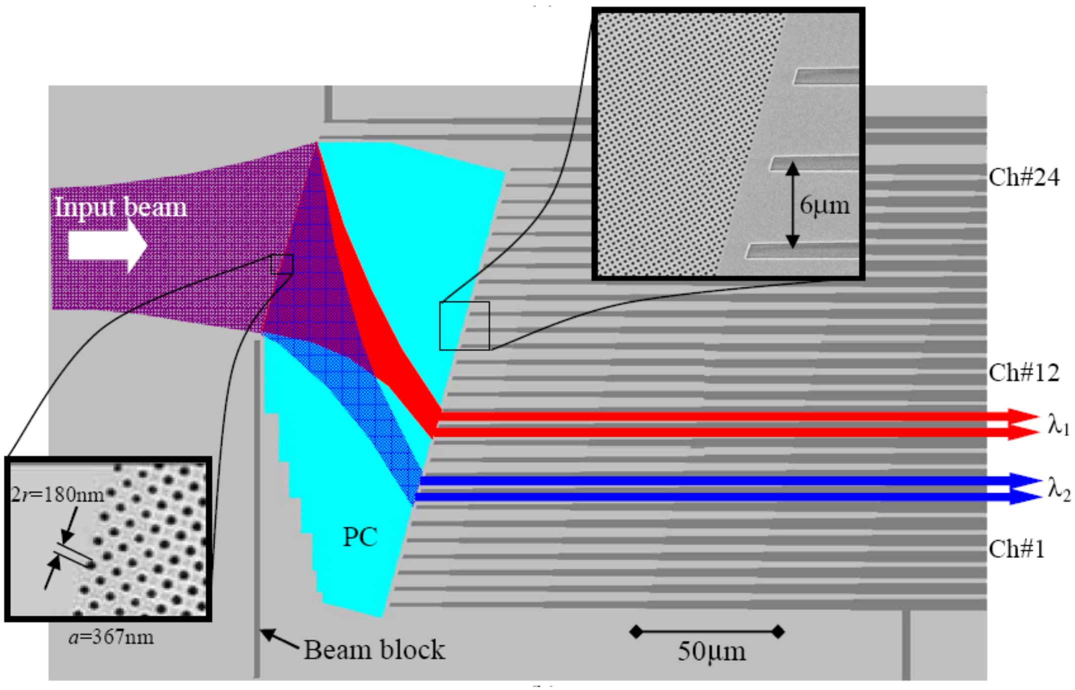
Lin, Sandia (1996) - concept, Kosaka, NEC (1998) – demonstration at optical wavelengths

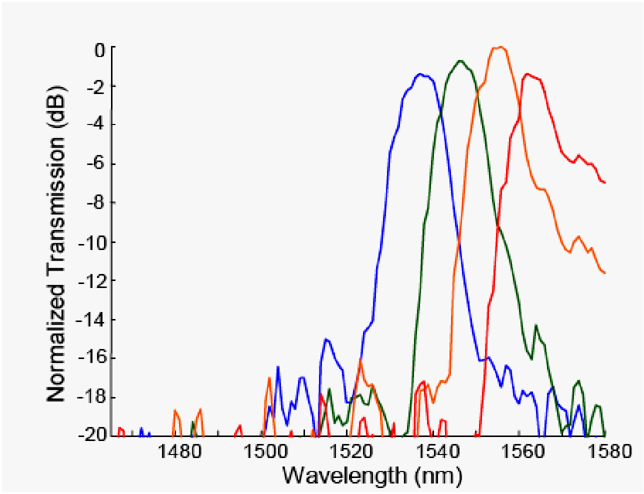
Potential applications:

De/Multiplexers for WDM optical communication systems

High-resolution spectral analyses, e.g. for biophotonics

Demonstration of PhC superprism demultiplexer





Gatech, Momeni et al, OPTICS EXPRESS, 14, p. 2413 (2006)

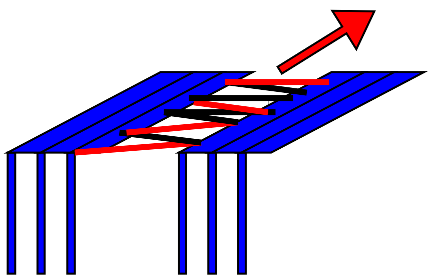
Defect structures in photonic crystals

Defects in PhC lattice trap light at frequencies within photonic band gap

Line defects → ***Waveguides, linear cavities***

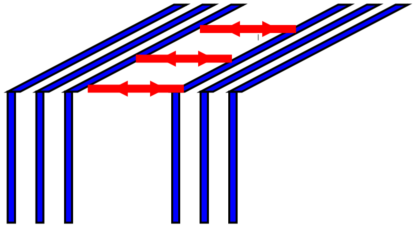
Point defects → ***Point cavities***

Waveguide in 1D PhC



Bragg Reflection Waveguide

Cavity in 1D PhC



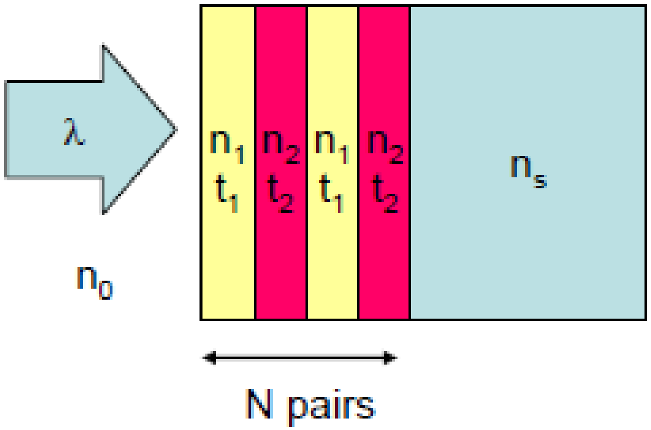
Distributed Bragg Reflectors

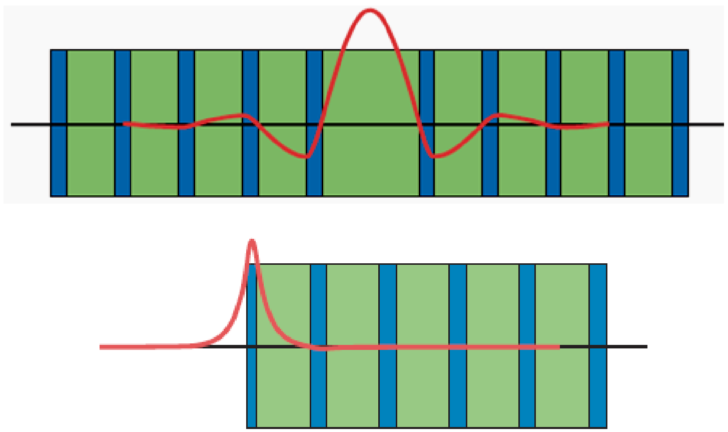
Confinement possible in a lower-index core – in contrast to index-guided waveguides

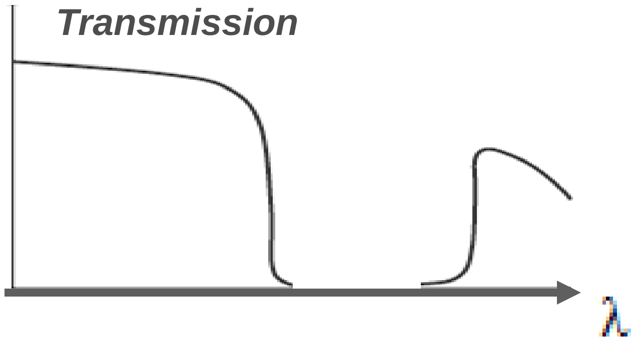
(But in analogy to metal-mirror waveguides used for microwaves)

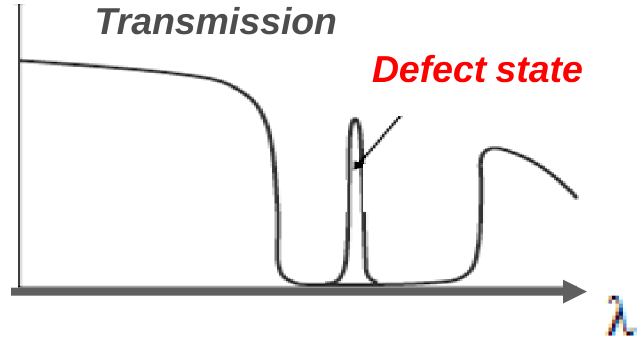
P. Yeh, A. Yariv, 1977

Defect states in 1D PhC - transmission spectrum





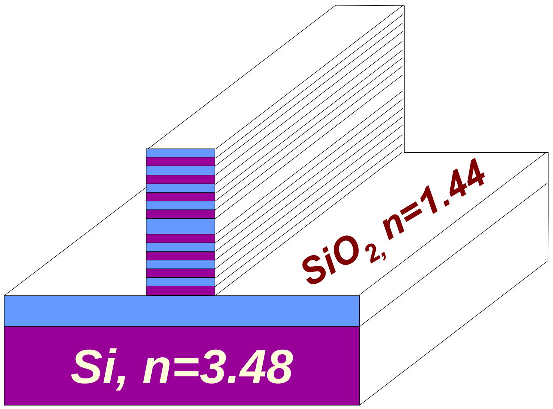


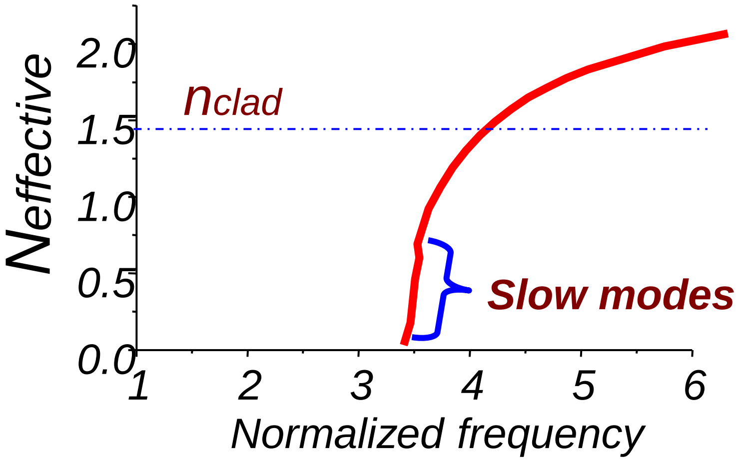


http://ab-initio.mit.edu/book/

Unusual dispersion in PC waveguides

1D PC channel waveguide *(KTH)*

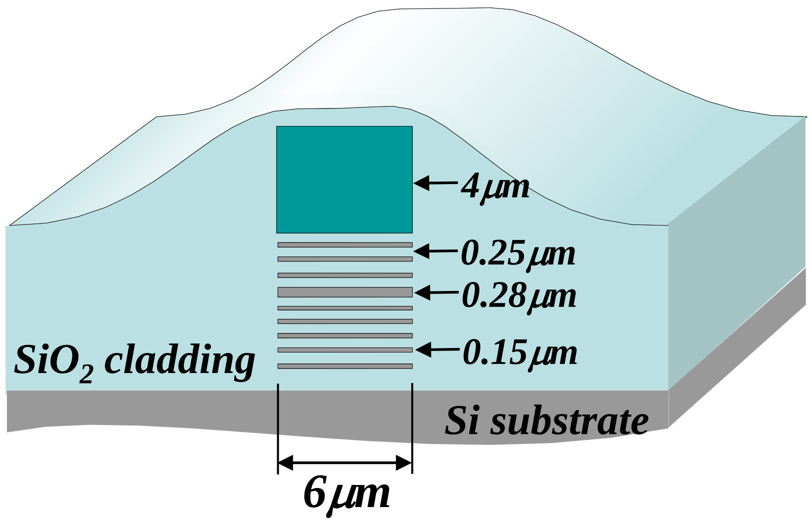


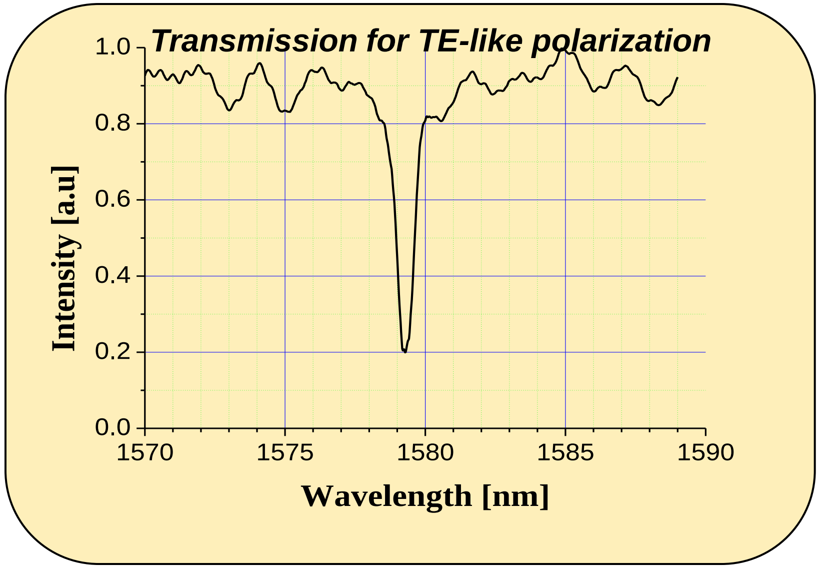


Strong dispersion → ***Wavelength selective devices***

Slow modes → ***Enhancement of nonlinear interactions***
Efficient compact amplifiers and lasers

Narrow-band directional-coupler filter: BRW waveguide + TIR waveguide



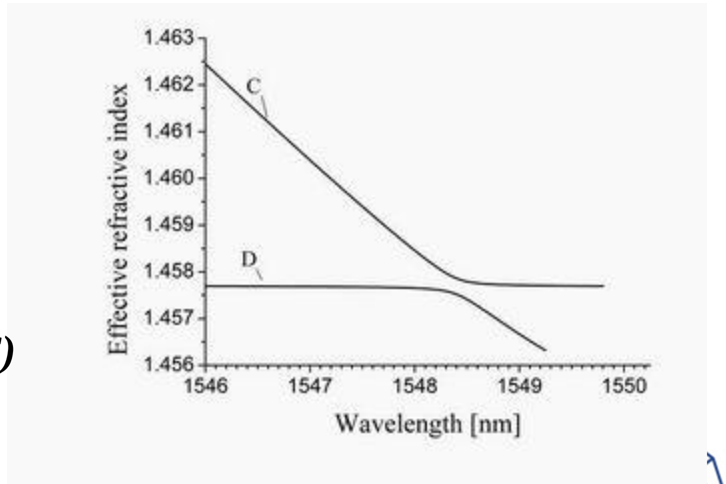


0.3 nm for 1.7 mm length demonstrated

KTH

OFC 2003

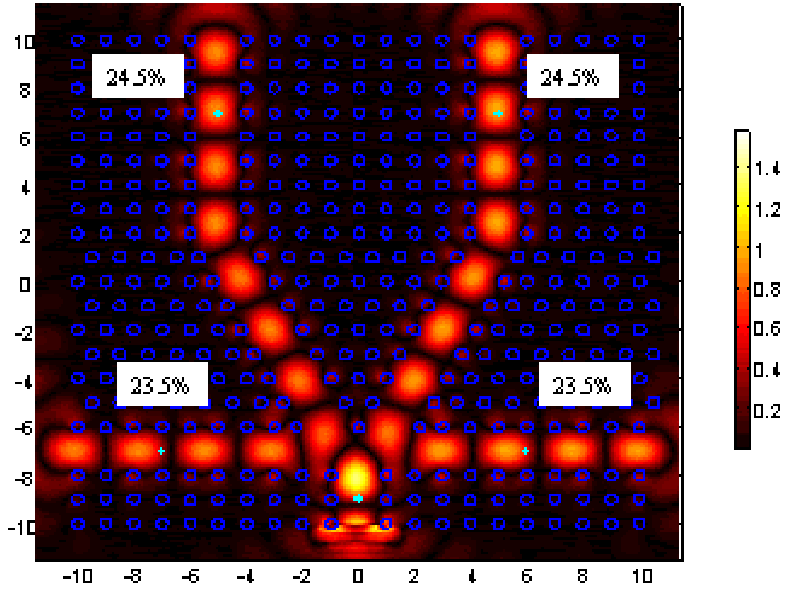
Optics Communications, vol 260/2 pp 514-521 (2006)



Defect structures in 2D photonic crystals

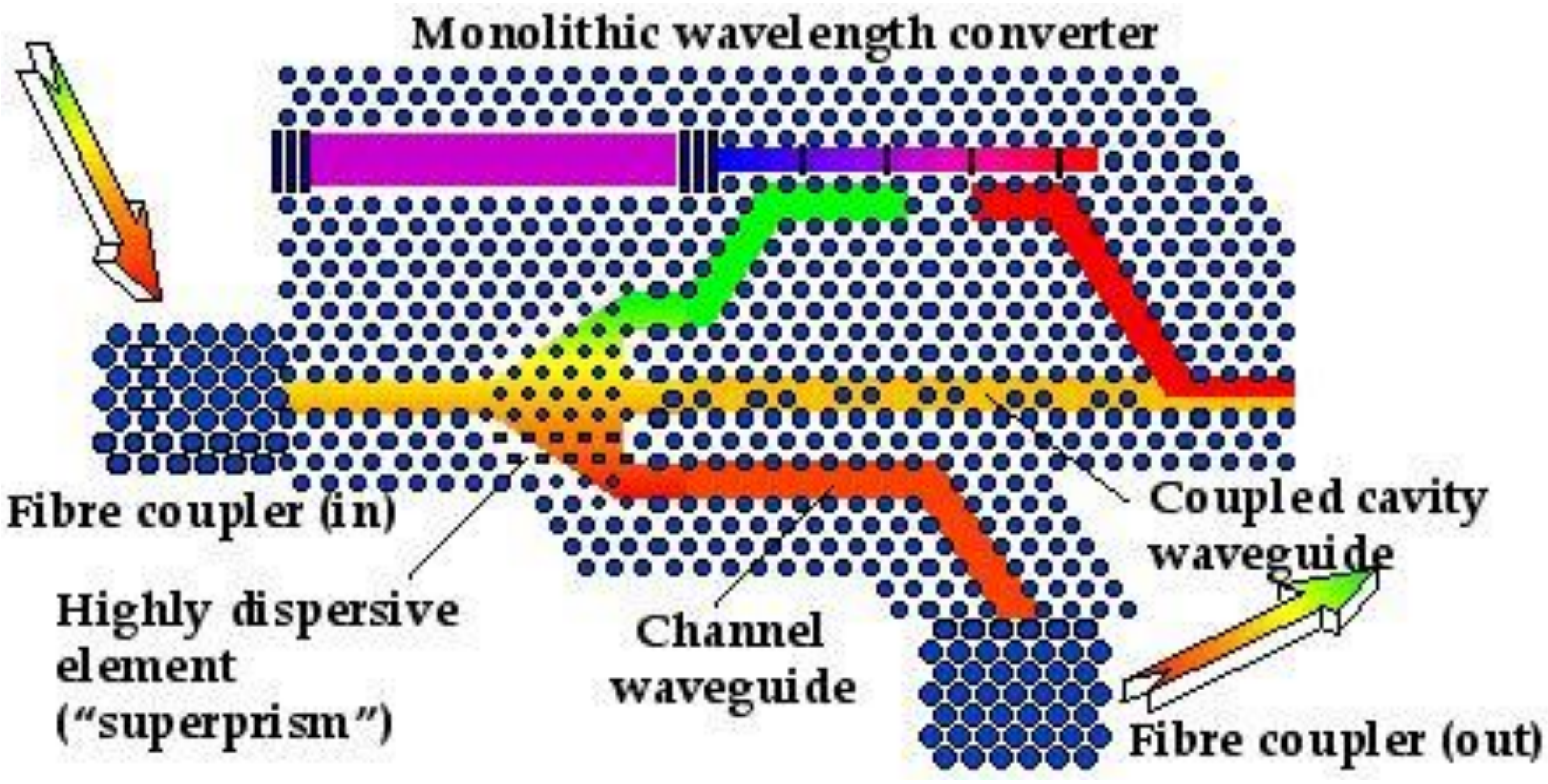
2D (and 3D) PCs open new routes for compact integrated optics

4 Way Hetro-Structure Beam Splitter



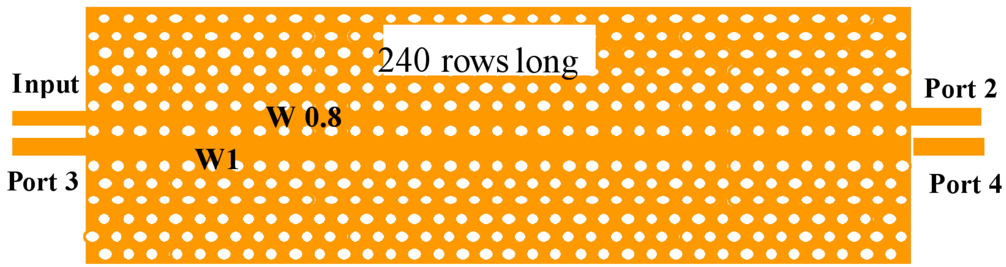
Parker, Delaware University

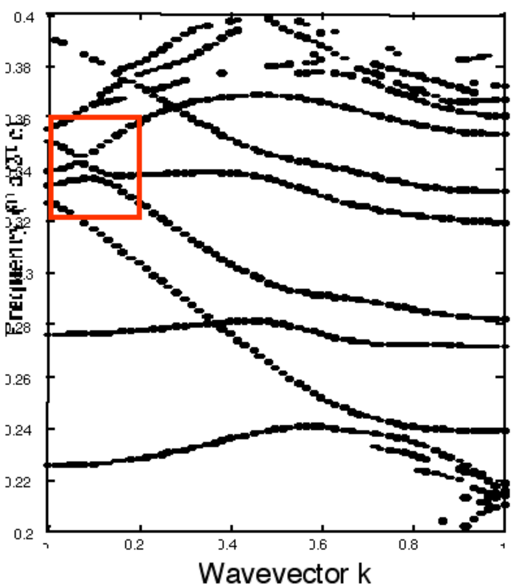
Photonic crystal integrated circuits?

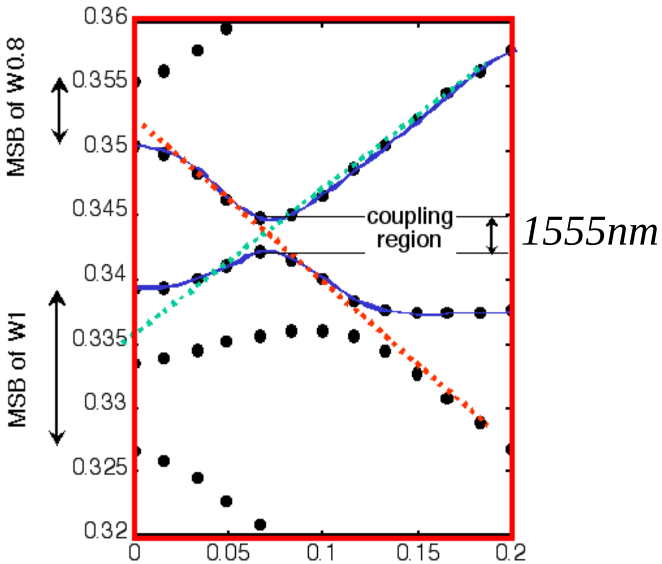


IST PICCO, Univ of St Andrews

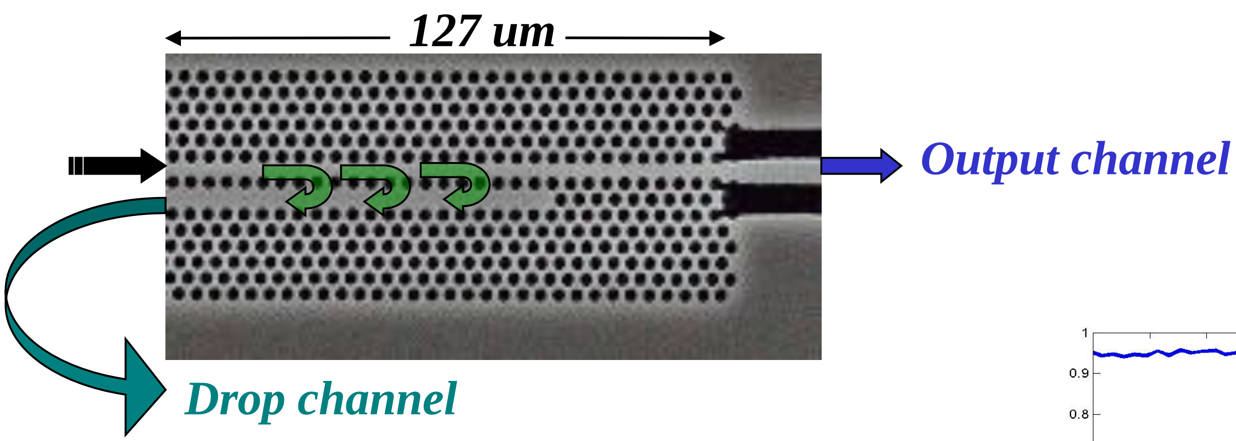
Contra-directional coupler drop-filter

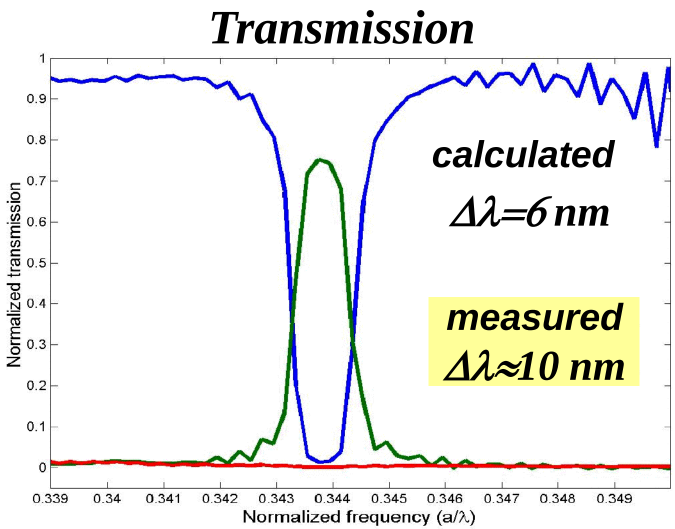






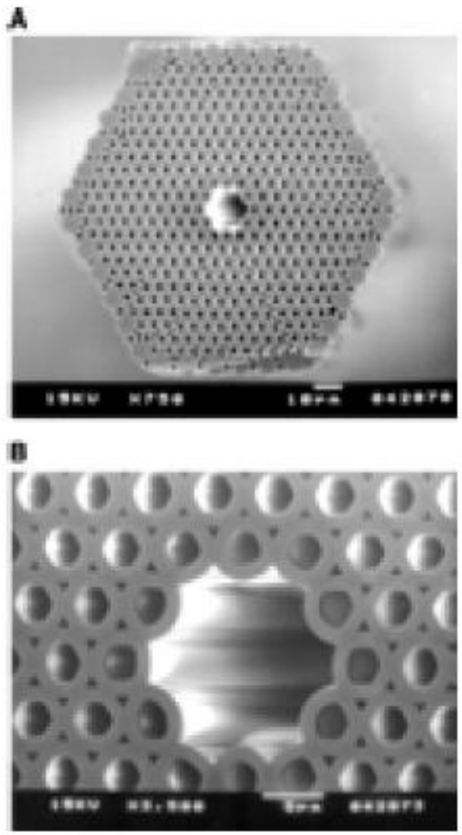
Demonstration of 2D PhC directional-coupler drop-filter

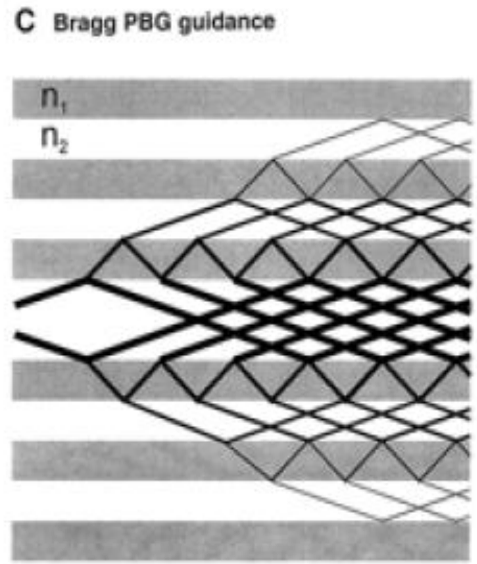




M Qiu, M Mulot, M Swillo, S Anand, B Jaskorzynska,
A Karlsson, M Kamp and A Forchel, Appl. Phys. Lett. 83, 5121 – 5123 (2003)

Photonic crystal fiber



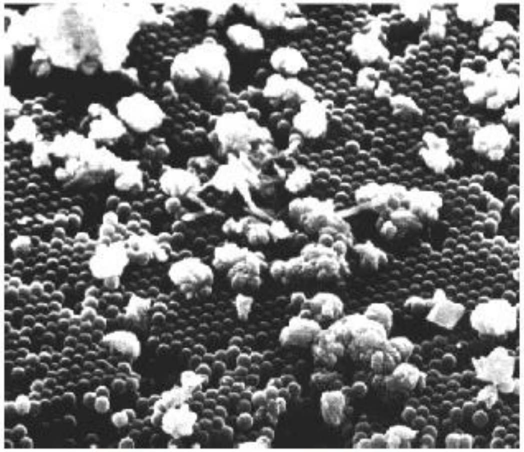


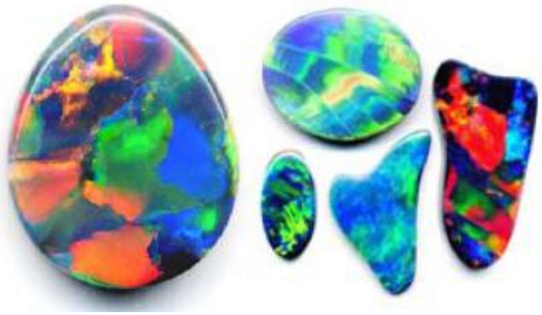
Out-of-plane propagation

Can be endlessly (at all wavelengths) single-mode regardless of the core size !

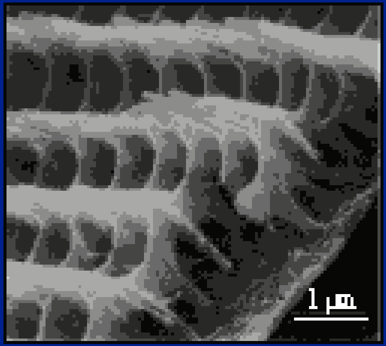
Photonic crystals in nature

Opal

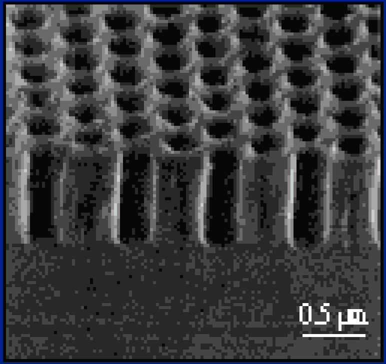




Adonis Blue Butterfly
Lysandra bellargus



SEM of wing scale
Zeuxidia amethystis



2D Photonic Crystal

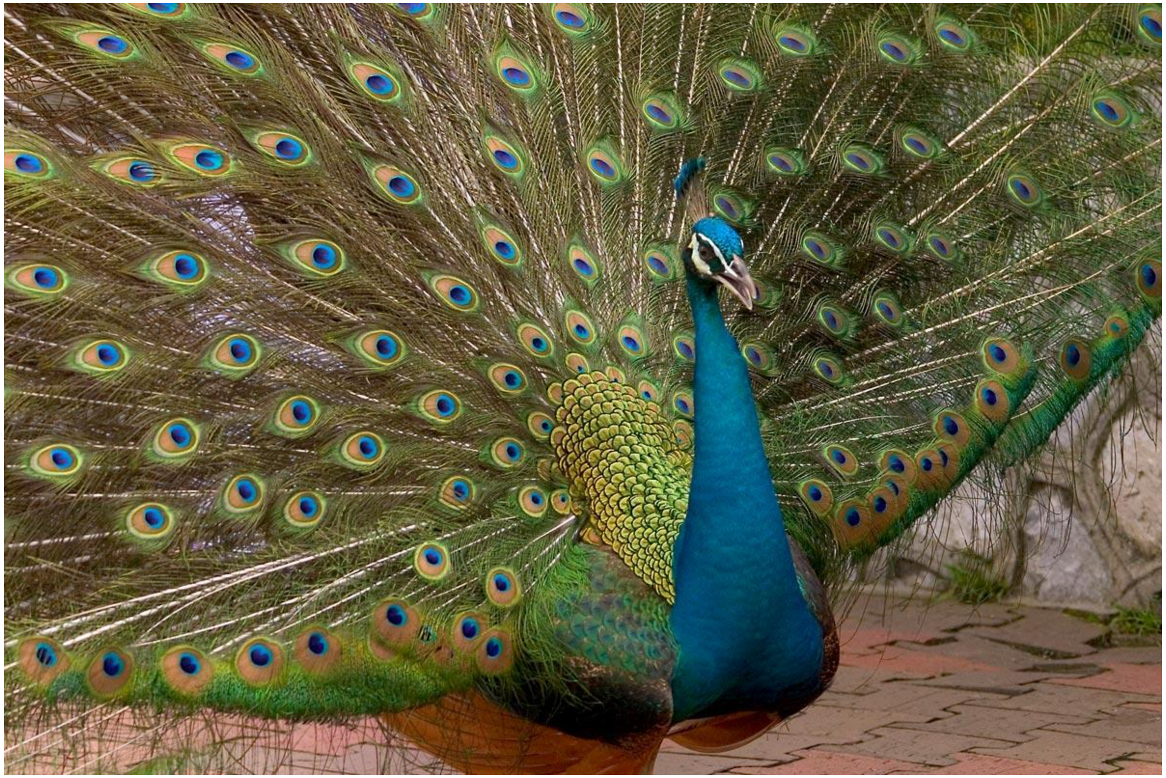
Photonic crystals in butterfly wings

The wings of the male Cyanophrys remus are bright metallic blue on one side, thought to attract mates, and a dull green on the other to act as camouflage.

The metallic blue colour is "produced" by scales that are photonic single crystals whereas the dull green is the result of a random arrangement of photonic crystals

http://technology.newscientist.com/article/dn10006

Peacocks wear photonic crystals



http://cr4.globalspec.com/thread/1248/Photonic-Crystals-in-Nature