Quantum Electronics
Lecture 3

Light propagation in periodic media
Photonic crystals

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The book online:
http://ab-initio.mit.edu/book/

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Periodic media

Periodically repeated properties of the medium in analogy to a crystal lattice

\[ a_i \text{ – lattice constant} \]
Periodic media - scattering regimes

\[ a \gg \lambda \]

incoherent scattering

\[ a \ll \lambda \]

averaging

Constructive / destructive interference effects

\[ a \sim \lambda \]

coherent scattering

Bragg diffraction

Metamaterials
Bragg's law

**Bragg Condition:**

\[
\frac{\lambda}{n} = 2 a \quad \rightarrow \quad a = \frac{1}{2} \frac{\lambda}{n} \quad (x \text{m})
\]

Reflection outside

In phase

Exactly half a wavelength fits in one period of the lattice

Out of phase

Standing wave inside

**Formula for X-rays reflection from crystals (Bragg's law):**

Sir William Henry Bragg
and his son William Lawrence
Nobel Prize 1915

W.H. Bragg (1862-1942)

W.L. Bragg (1890-1971)
Bragg law for skew incidence

\[ 2k \sin \theta = m \frac{2\pi}{a} \]

Transverse resonance for projection of \( k \)-vector:

\[ 2a n \sin \theta = m \lambda \]

\[ a \sin \theta = m \frac{\lambda}{2n} \]
Periodic layered medium - Bragg reflector

Transmission spectra:
- solid line: index difference = 1.0; 5 layer pairs
- dashed line: index difference = 0.3; 14 layer pairs

Transmission forbidden around Bragg resonance

Photonic band gap!
**Band gap width in layered media**

**Band gap edge** - frequency at which the incoming and reflected waves in one of the layers are in phase

\[ \lambda_1 = 2 \frac{a}{2} n_1 = an_1 \]

\[ \lambda_2 = 2 \frac{a}{2} n_2 = an_2 \]

\[ \omega_1 - \omega_2 = \frac{2\pi c}{a} \left( \frac{1}{n_1} - \frac{1}{n_2} \right) \sim (n_2 - n_1) = \Delta n \]

\[ n_1 < n_2 \quad \implies \quad \lambda_1 < \lambda_2 \]
Dispersion in layered medium for $k_y=0$

Deviation from the straight-line dispersion curve of a homogeneous medium is to ensure Bragg reflection for $kz = \pm N(\pi/a)$ - the curve becomes horizontal.
Projected dispersion diagram – uniform dielectric

\[ |\vec{k}| \equiv k = \omega n / c \]

\[ k_t = \sqrt{\left(\frac{\omega n}{c}\right)^2 - \beta^2} \]

\[ \omega = \frac{c}{n} \sqrt{\beta^2 + |k_t|^2} \]

light cone: \( \omega = c\beta / n \)

light line: \( \beta = k \)

\( \beta > k \)

\( k_t \) IMAGINARY - no light propagation
Dispersion diagram – two uniform dielectrics

\[ k^i_t = \sqrt{\left( \frac{\omega n_i}{c} \right)^2 - \beta^2} \]

\[ \sin \theta_i = \frac{c \beta}{\omega n_i} \]

Light cone

Light propagates in dielectric 1,2

**light line 1:**\[ \omega = c \frac{\beta}{n_1} \]

Light propagates in dielectric 2 only

**light line 2:**\[ \omega = c \frac{\beta}{n_2} \]

No light propagation in dielectrics 1,2

TIR!!
Reflectance from dielectric interface

\[
\text{Reflectance} = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}
\]

As index contrast increase, reflectance increases.

\[
\theta_1 = \tan^{-1}\left(\frac{n_2}{n_1}\right)
\]

(Metallic mirror has low angular and polarization dependence, but very high loss for optical frequencies)
Dispersion diagram for 1D PhC (Bragg mirror)

\[ \beta = 0 \quad \text{-- propagation perpendicular to the layers} \]
Omnidirectional Reflection


In these $\omega$ ranges, there is no overlap between modes of air & crystal.

All incident light (any angle and polarization) is reflected from flat surface!!

needs sufficient index contrast & $n_{\text{high}} > n_{\text{low}} > 1$
Weak coupling – Coupled Mode Theory

**Co-directional**

\[
\frac{dA_m}{dz} = -i \kappa A_l e^{i\Delta \beta z}
\]

\[
\frac{dA_l}{dz} = -i \kappa^* A_m e^{-i\Delta \beta z}
\]

**Contra-directional**

\[
\frac{dA_m}{dz} = -i \kappa B_l e^{i\Delta \beta z}
\]

\[
\frac{dB_l}{dz} = i \kappa^* A_m e^{-i\Delta \beta z}
\]

**Coupling coefficient:**

\[
\kappa = \frac{\omega \epsilon_0}{4} \iint E^*_m \Delta n^2 E_l \, dx \, dy
\]

**Phase mismatch:**

\[
\Delta \beta = \beta_m - \beta_l
\]

\[
\Delta \beta = \beta_m + \beta_l
\]

*Two phase mismatched modes can be coupled when z-variation of $\Delta n^2$ compensates the mismatch.*
Grating assisted coupling

*Periodic index modulation for compensation of phase mismatch:* $K = \Delta \beta$

**Grating period:**
$$\Lambda = \frac{2\pi}{K} = \frac{2\pi}{\Delta \beta} = \frac{2\pi}{(2\pi / \lambda) \Delta n} = \frac{\lambda}{\Delta n}$$

- **Short period Bragg gratings (contra-directional coupling):**
  - Grating period shorter than $\lambda$:
  $$\Lambda = \frac{2\pi}{(\beta_2 + \beta_1)} = \frac{\lambda}{(n_2 + n_1)}$$

- **Long period Bragg gratings (co-directional coupling):**
  - Grating period longer than $\lambda$
  $$\Lambda = \frac{2\pi}{(\beta_2 - \beta_1)} = \frac{\lambda}{(n_2 - n_1)}$$
Fiber UV induced Bragg-Grating

Holographic UV Lithography with 244 nm UV light
Mode coupling in fiber grating

Core-core coupling

\[ \beta_{\text{core}} \]

\( \lambda_B = 2n_{\text{core}} \Lambda \)

Core-cladding coupling

\[ \beta_{\text{clad}} \]

\[ \beta_{\text{core}} \]

\[ \lambda_{\text{clad},i} = (n_{\text{core}} + n_{\text{clad},i}) \Lambda \]

Transmission (dB)

Wavelength (nm)

LP\(_{03} \)  LP\(_{01} \)
Bandgap in all 3 dimensions

Extension to 3D, full band gap 1987:
E. Yablonovitch, S. John

The first 3D photonic crystal made by E. Yablonovitch 1991 (microwaves)

“Woodpile” stack of Alumina rods

The first PhC for optical λs
2D Photonic Crystals

Simpler fabrication
Suitable for integrated optics
(slab waveguide confinement in vertical direction)
Theoretical description

Starting point: Maxwell Equations

\[ \nabla \cdot H = 0 \quad \nabla \cdot \varepsilon E = 0 \]

\[ \nabla \times H - \frac{\varepsilon}{c} \frac{\partial E}{\partial t} = 0 \quad \nabla \times E - \frac{1}{c} \frac{\partial H}{\partial t} = 0 \]

No sources

- Look for time-harmonic states: \( H(r,t) = H(r) e^{i\omega t} \), \( E(r,t) = E(r) e^{i\omega t} \)
- Eliminate the \( E \) fields \( \Rightarrow \) Hermitian eigenproblem:

\[ \nabla \times \left( \frac{1}{\varepsilon(r)} \nabla \times H(\vec{r}) \right) = \left( \frac{\omega}{c} \right)^2 H(\vec{r}) \]

- Real eigenvalues \( \omega \)
- Orthogonal eigenfunctions (modes) \( H \)

In general: The eigenproblem in infinite domain \( \Rightarrow \) Continuous \( \omega \) spectrum

But \( \varepsilon(r) \) is periodic! \( \Rightarrow \) Discrete set of \( \omega \)
Maxwell meets Bloch (& Floquet in 1D)

**Bloch theorem:**

\[ H_k(r) = e^{ikr} u_k(r) = e^{ikr} u_k(r+R) \]

Eigen-operator periodic \rightarrow Solutions: \( e^{ikr} x \) (periodic function)

plane wave spatially periodic amplitude ("envelope")

\( k \) is conserved, i.e. **no scattering** of Bloch waves on periodic index-modulation !!

Bloch waves are nonuniform plane waves with the envelope period = lattice period

F. Bloch 1905-1983
Bloch waves – ”interference optics”

Light is scattered from each of holes (rodes) in periodic media

But due to interference (Bragg diffraction) plane Bloch waves are formed
Huyghens interference

Constructive interference between two wavefronts in certain directions, + the destructive interference in other directions
Two ways of viewing Bloch waves

A Bloch wave is a plane wave with a modulation:

A

$\psi_{nk}(r) = e^{i\mathbf{k} \cdot r} u_{nk}(r)$

A Bloch wave consists of multiple wavevectors:

B

Plane Wave Expansion:

$\psi(r) = \sum_m c_m e^{i(k+m2\pi/a)r}$

Both pictures are correct and lead to the same results
Bloch eigenmodes

- **Propagating mode**: $\text{Im } k = 0$
- **Evanescent mode**: $\text{Im } k > 0$
- **Evanescent mode**: $\text{Im } k < 0$
Unusual dispersion relations in PhC

Equal frequency surface (EFS) plot of the band structure for frequency $w=0.56-0.635$

For frequency 0.60-0.64 EFSs are circles, but their radius shrinks for increasing $\omega$

$V_g = \nabla_k \omega$ inward $\rightarrow$ All-angle Negative refraction!

Close to $K$ propagation direction $V_g$
ultra-sensitive to incidence angle and wavelength
$\rightarrow$ Beam steering, Superprism effect

Dispersion in uniform medium and PhC

$$\bar{v}_g = \frac{\partial \omega}{\partial \vec{k}}$$

Isotropic non-dispersive medium:

$$k = n\omega/c$$

Photonic crystal for a given direction of $$\mathbf{k}$$.

Group velocity in photonic crystals can be inward directed due to negative slope of the dispersion!
“Negative Refraction” → Self Focusing

For some frequency regions, conventional refracted wave does not exist for any incidence angle – it falls into photonic band gap

Not true for weak index modulation

3D imaging

3D pictures?
Sub wavelength resolution imaging !!
(similar to that in Negative Index Materials – NIM)
Simulation of Self-focusing due to Negative Refraction

Without photonic crystal

With photonic crystal

PhC section  Focus points

Demonstration of Negative Refraction at optical wavelengths - KTH

Measured profile of the imaged output beam for different lens positions. Full-Width at Half-Maximum intensity shown with green points.

Wavelength = 1480 nm

Negative Refraction for polarization splitting - KTH

Operation range: 1530 – 1610 nm

TM polarization – positive refraction

TE polarization – negative refraction
Beam steering

Utilizing strong band-edge anisotropy in PhC

Equal-frequency surfaces in \((k_x,k_y)\) space:

Incident beams

Refracted beams

Air

Photonic crystal
Superprism effect

Conventional glass

3D photonic crystal

NEC Corp., Tohoku University, NTT

Quantum Electronics, Warsaw 2010
Superprism effect for de/multiplexing

\[ \lambda_1, \lambda_2, ..., \lambda_n \]


Potential applications:

De/Multiplexers for WDM optical communication systems

High-resolution spectral analyses, e.g. for biophotonics
Demonstration of PhC superprism demultiplexer

Defect structures in photonic crystals

Defects in PhC lattice trap light at frequencies within photonic band gap

*Line defects* → Waveguides, linear cavities
*Point defects* → Point cavities

Waveguide in 1D PhC

Cavity in 1D PhC

Bragg Reflection Waveguide

Distributed Bragg Reflectors

Confinement possible in a lower-index core – in contrast to index-guided waveguides (But in analogy to metal-mirror waveguides used for microwaves)

P. Yeh, A. Yariv, 1977
Defect states in 1D PhC - transmission spectrum

Transmission

http://ab-initio.mit.edu/book/
Unusual dispersion in PC waveguides

1D PC channel waveguide (KTH)

Strong dispersion $\rightarrow$ Wavelength selective devices

Slow modes $\rightarrow$ Enhancement of nonlinear interactions

Efficient compact amplifiers and lasers
Narrow-band directional-coupler filter: BRW waveguide + TIR waveguide

0.3 nm for 1.7 mm length demonstrated
KTH

OFC 2003
Defect structures in 2D photonic crystals

2D (and 3D) PCs open new routes for compact integrated optics

4 Way Hetro-Structure Beam Splitter

Parker, Delaware University
Photonic crystal integrated circuits?

IST PICCO, Univ of St Andrews
Contra-directional coupler drop-filter

[Diagram of a contra-directional coupler with labels and a 240 rows long section.]

[Graphs showing frequency versus wavevector and MSB of W0.8 and W1 with a 1555nm region highlighted.]
Demonstration of 2D PhC directional-coupler drop-filter

127 μm

Output channel

Drop channel

Transmission

calculated
$\Delta\lambda = 6$ nm

measured
$\Delta\lambda \approx 10$ nm

$M$ Qiu, $M$ Mulot, $M$ Swillo, $S$ Anand, $B$ Jaskorzynska,
Photonic crystal fiber

Can be endlessly (at all wavelengths) single-mode regardless of the core size!
Photonic crystals in nature

Opal

Adonis Blue Butterfly
*Lysandra bellargus*

SEM of wing scale
*Zeuxidia amethystis*

2D Photonic Crystal
Photonic crystals in butterfly wings

The wings of the male Cyanophrys remus are bright metallic blue on one side, thought to attract mates, and a dull green on the other to act as camouflage.

The metallic blue colour is “produced” by scales that are photonic single crystals whereas the dull green is the result of a random arrangement of photonic crystals.

http://technology.newscientist.com/article/dn10006
Peacocks wear photonic crystals