

## OPINION EVOLUTION IN CLOSED COMMUNITY

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A simple Ising spin model which can describe a mechanism of making a decision in a closed community is proposed. It is shown via standard Monte Carlo simulations that very simple rules lead to rather complicated dynamics and to a power law in the decision time distribution. It is found that a closed community has to evolve either to a dictatorship or a stalemate state (inability to take any common decision). A common decision can be taken in a “democratic way” only by an open community.

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### 1. Introduction

The Ising spin system is undoubtedly one of the most frequently used models of statistical mechanics. Recently, this model has also become the most popular physics export article to “other branches of science” such as biology, economy or sociology.<sup>1–4</sup> There are two main reasons for that: first verbalized by Nobel prize winner Peter B. Medawar — “physics envy” (quoted e.g., by R. Dawkins in Ref. 5) — a syndrome appearing in some researchers who would like to have such beautiful and relatively simple models as physicists have (for example the Ising model). On the other hand, some physicists would like to be better understandable by non-physicists and create theories which could not be so univocally verified or falsified like in the classical areas of physics. It is rather obvious that Ising-type models cannot explain origins of very complicated phenomena observed in complex systems. However, it is believed that these kind of models could describe some universal behavior connected, for example, with selforganization of systems<sup>6,7</sup> or in other words, justify the existence of some power laws, which recently makes many scientists very happy.<sup>8</sup> Between many exotic applications of Ising spin models, these referring to

the problem of democratic choice of a one among two possibilities seem to be the most natural.<sup>2,4,9–11</sup>

## 2. Model

Let us consider a community which time and again should take a stand in some matter, for example vote on a president in a two-party system. If each member of the community can take only two attitudes ( $A$  or  $B$ ), then in several votes one expects some difference  $m$  of voters for  $A$  and against. We assume three limiting cases:

- (i) all members of the community vote for  $A$  (an “all  $A$ ” state),
- (ii) all members of the community vote for  $B$  (an “all  $B$ ” state),
- (iii) 50% vote for  $A$  and 50% vote for  $B$ ,

which should be the stable solutions of our model.

The aim of this paper is to analyze the time evolution of  $m$ . To model the above mentioned system, we consider an Ising spins chain ( $S_i; i = 1, 2, \dots, N$ ) with the following dynamic rules:

- if  $S_i S_{i+1} = 1$ , then  $S_{i-1}$  and  $S_{i+2}$  take the direction of the pair  $(i, i + 1)$ , (r1)
- if  $S_i S_{i+1} = -1$ , then  $S_{i-1}$  takes the direction of  $S_{i+1}$  and  $S_{i+2}$ , the direction of  $S_i$ . (r2)

These rules describe the influence of a given pair on the decision of its nearest neighbors. When members of a pair have the same opinion, then their nearest neighbors agree with them. On the contrary, when members of a pair have different opinions, then the nearest neighbor of each member disagrees with him (her). These dynamic rules lead to the three steady states above. However, the third steady state (50% for  $A$  and 50% for  $B$ ) is realized in a very special way. Every member of the community disagrees with his (her) nearest neighbor (it is easy to see that the Ising model with only next nearest neighbor interaction has such fixed points: ferro and antiferromagnetic state). This rule is in accordance with the well known sentence “united we stand divided we fall”. So, from now on, we will call our model — USDF.

## 3. Isolated System

To investigate our model, we perform a standard Monte Carlo simulation with random updating. We consider a chain of  $N$  Ising spins with free boundary conditions. In our simulations, we were taking usually  $N = 1000$ , but we have done simulations also for  $N = 10\,000$ . We start from a totally random initial state i.e., to each site of the chain, we assign an arrow with a randomly chosen direction: up or down (Ising spin). For this case, we obtain of course, as a final state, one of the three fixed points (1–3, i.e.,  $AAAA$ ,  $BBBB$ , and  $ABAB$ ) with probability 0.25, 0.25, and 0.5 respectively. The typical relaxation time for  $N = 1000$  is  $\sim 10^4$  Monte Carlo

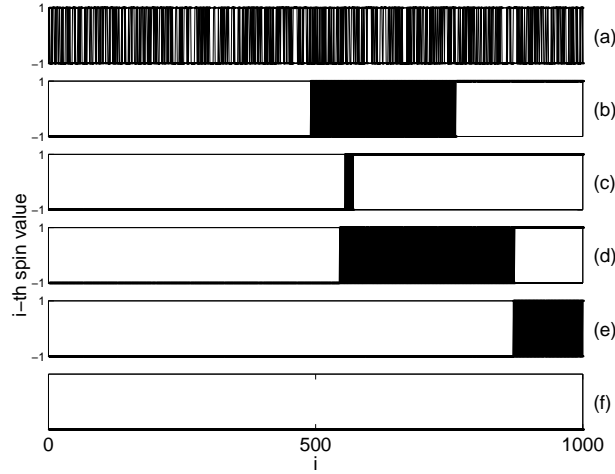


Fig. 1. Spatial distribution of spins for (a) the initial state, (b)–(e) intermediate states, (f) the final (steady) state; time interval between states is 10 000 MCS.

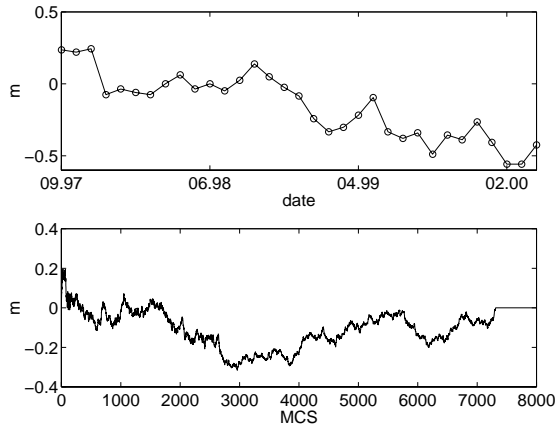


Fig. 2. Time evolution of decision  $m$  taken from empirical data<sup>12</sup> (upper) (question: “do you think future will be good?” asked to  $N = 1100$  adults) and simulation from a random initial state (lower) for  $N = 1000$ .

steps (MCS). The space distribution of spins from the initial to a steady state is shown in Fig. 1. For intermediate states, one can see the formation of clusters.

Let us define the decision as a magnetization, i.e.,

$$m = \frac{1}{N} \sum_{i=1}^N S_i. \tag{1}$$

In Fig. 2, we present typical time evolution of  $m$  and to compare certain empirical data on “social mood”.<sup>12</sup> Without any external stimulation, decision can change dramatically in a relatively short time. Such strongly nonmonotonic behavior of

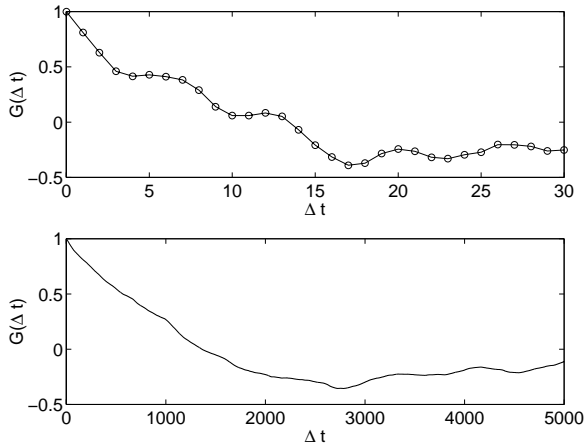


Fig. 3. Autocorrelation for empirical data (upper) and simulations (lower) evaluated from data shown on Fig. 2.

the change of  $m$  is typically observed in the USDF model when the system evolves towards the third steady state (total disagreement or in magnetic language, the antiferromagnetic state).

To measure the time correlation of  $m$ , we use classical autocorrelation function,

$$G(\Delta t) = \frac{\sum (m(t) - \langle m \rangle)(m(t + \Delta t) - \langle m \rangle)}{\sum (m(t) - \langle m \rangle)^2}. \tag{2}$$

Comparison of simulation results with empirical data is shown in Fig. 3.

It seems interesting to follow changes of one particular individual. The dynamic rules we have introduced lead to an amazing effect: if an individual changes his (her) opinion at time  $t$ , he (she) will probably change it also at time  $t + 1$ . Like in the Bak Sand-Pile model, one change can cause an avalanche.<sup>6,7</sup> On the other hand, an individual can stay for a long time without changing his (her) decision. Let us denote by  $\tau$  the time needed by an individual to change his (her) opinion. From Fig. 3, it can be seen that  $\tau$  is usually very short, but sometimes can be very long. The distribution of  $(\tau)(P(\tau))$  follows a power law with an exponent  $-3/2$  (see Fig. 4).

We have also analyzed the influence of initial conditions on the evolution of system. We have done it in two different ways: randomly and in clusters. In both cases, we start from an initial concentration  $c_B$  of opinion  $B$ . In the random setup,  $c_B * N$  individuals are randomly (uniformly) chosen out of all  $N$  individuals. In the cluster setup, simply the first  $c_B * N$  individuals are chosen.

It turns out that the distribution of decision time  $\tau$  still follows the power law with the same exponent as shown in Fig. 4. A nonmonotonic behavior of decision change is still typical and sometimes even much stronger. However, it is obvious that if initially there is more  $A$ 's than  $B$ 's, the final state should be more often “all  $A$ ”, then “all  $B$ ”. Dependence between initial concentration of  $B$  and the probability of steady state  $S$  ( $AAAA$ ,  $BBBB$ , or  $ABAB$ ) is shown in Fig. 5.

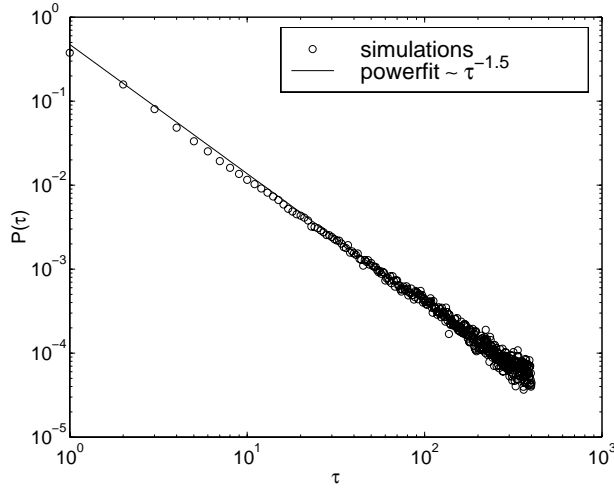


Fig. 4. Distribution of decision time  $\tau$  follows a power law.

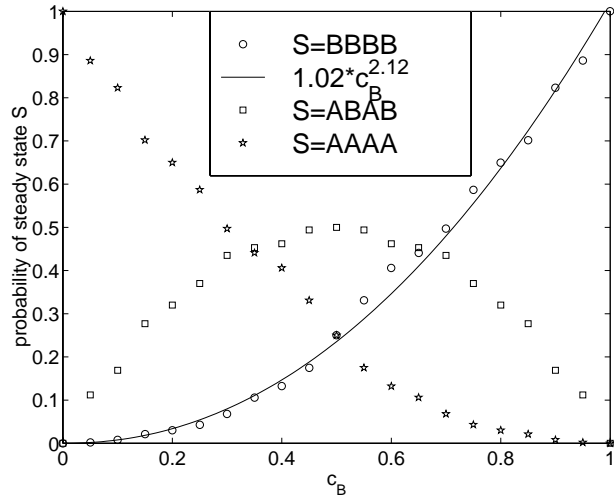


Fig. 5. Dependence between the initial concentration of  $B$  and the final state. Averaging was done over 1000 samples.

There is no significant difference between the “cluster” and “random case”. Although it should be noted that the “random case” is not well defined, because there is a number of random initial states. However, averaging over different initial random conditions gives a similar result to the “cluster case”.

Observe that  $c_B > 0.7$  is needed to obtain final state “all  $B$ ” with probability greater than 0.5 (see Fig. 5). Dependence between  $c_B$  and the probability of steady state “all  $B$ ” is well fitted by a power function with an exponent 2.12.

### 4. Information Noise

It is well known that the changes of opinion are determined by the *social impact*.<sup>13</sup> In the previous section, we have considered a community in which a change of an individuals opinion is caused only by a contact with its neighbors. It was the simplest *social impact* one can imagine. Now, we introduce to our model noise  $p$  (similar to the “social temperature”<sup>2</sup>), which is the probability that an individual, instead of following the dynamic rules, will make a random decision. We start from an “all  $A$ ” state to investigate if there is a  $p \in (0, 1)$  which does not throw the system out of this state. Time evolution of the decision from the “all  $A$ ” state is shown in Fig. 6.

It can be seen that for very small  $p \sim 10^{-6}$ , deviations from the steady state are slight and the system is almost totally ordered. If  $p$  increases, then the deviations are of course larger and the system goes to a completely disordered state. This suggests that there is some value of  $p = p^*$  below which the system is ordered. However, on the basis of our simulations, we can only determine that  $p^* \in [10^{-6}, 10^{-5}]$ . If we take totally random initial conditions and  $p < p^*$ , the system will reach, after some relaxation time, one of the three steady states.

In the previous section, we have shown that the distribution  $P(\tau)$  follows a power law with an exponent  $\sim -3/2$ . The same distribution for different values of  $p$  is shown in Fig. 7. In the limit  $p \rightarrow 0$ , distribution  $P(\tau)$  indeed follows a power law, whereas for  $p \rightarrow 1$ , the distribution is exponential. Between these two extreme

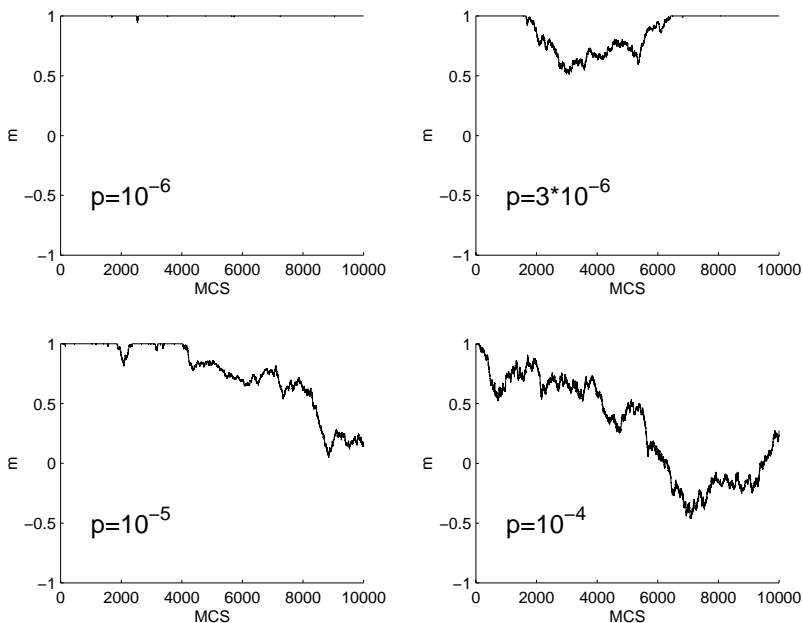


Fig. 6. Time evolution of decision  $m$  from an initial state “all  $A$ ” for different values of noise  $p$ .

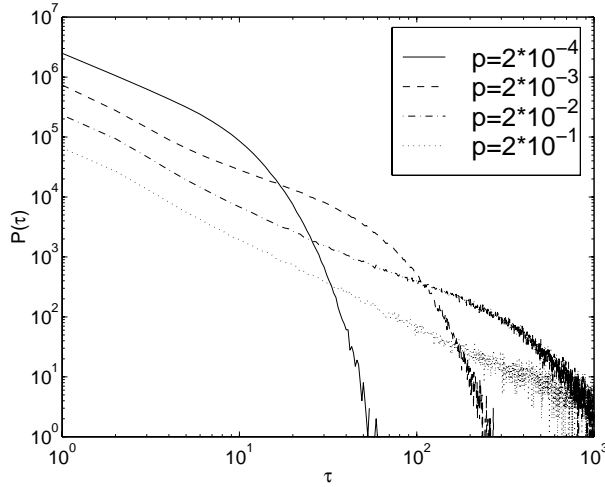


Fig. 7. Distribution of decision time  $P(\tau)$  for different values of noise  $p$ .

values of  $p$ , the distribution  $P(\tau)$  consists of two parts: exponential for large values of  $\tau$  and a power law with exponent  $\omega \sim -3/2$  for small values of  $\tau$ . Thus, we can write,

$$P(\tau) \sim \begin{cases} \tau^{-\omega} & \text{for } \tau < \tau^*, \quad \omega \sim \frac{3}{2}, \\ \exp(a\tau) & \text{for } \tau > \tau^*, \end{cases} \tag{3}$$

where  $\tau^*$  decreases with increasing  $p$ . For  $p = 0$ ,  $\tau^* = \infty$  and for  $p = 1$ ,  $\tau^* = 0$ .

### 5. Discussion

We have proposed a simple model called USDF, which can describe a “black and white” way of making a decision, where unanimity or disagreement of a given pair causes unanimity or disagreement to its nearest neighbors respectively. If such a mechanism of taking a decision by a community is correct, our model leads to the following conclusions:

- (i) In a closed (isolated) community, there are only two possibilities of a final state: dictatorship or stalemate. After a shorter or longer time, our model tends to one of the steady “ordered” states (1–3). It means either a total unanimity if the system goes to state 1(2) or inability to take any common decision if it goes to 3. However, small but finite information noise (open community) leads to disorder and the system does not go to any steady state. In this case, there is a possibility of taking a common decision in a democratic way.
- (ii) A change of opinion is followed by further changes. Periods of frequent changes of opinions are followed by periods of stagnancy.

- (iii) A relatively small group (a few percent of the whole population) by a favorable coincidence can bring to a stalemate. But in order to win, the group has to be quite large. For example, if the group wants to have a 50% chance of winning, it should consist of more than 70% of all individuals. It means that in order to change an existing law (for example that pornography is illegal), usually over 70% of the population have to vote for change. A similar effect was observed by Galam.<sup>10</sup>
- (iv) For finite information noise  $p$ , there is some characteristic time of a decision change  $\tau^*$  which depends on a value of  $p$ . For the decision time less than  $\tau^*$ , the distribution of decision time  $\tau$  follows a power-law.
- (v) The distribution of decision time  $\tau$  for  $p = 0$  and for  $p < 1$  and  $\tau < \tau^*$  follows the “universal” power-law with exponent  $\omega \sim 3/2$  independently of the initial conditions (totally random or clustered state).

The proposed very simple rules (r1, r2) leads to a rather complicated dynamics but one can doubt if these rules properly describe real mechanisms of taking a decision. There are of course other possibilities within the Ising spin model. For example, if the members of a given community are less prone to oppose nearest neighbors, then one should keep rule (r1) and skip rule (r2). This means that if  $S_i S_{i+1} = -1$ , then nothing is changed in the system. In this case, there are only two steady states “all  $A$ ” and “all  $B$ ”. Our simulations suggest that in such a closed community, there is a tendency to create two opposite clusters but the final state must be total unanimity (dictatorship). This result is rather obvious, it is easier to carry one’s opinion if the members of the community are peaceable or less active. For a while, this can be of profit to the community but finally it must lead to a dictatorship. It should be noted that also in this case, the distribution of decision time follows the same power-law like in USDF model with  $\omega \sim 3/2$ .

In another model which we call “if you do not know what to do, just do nothing”, an individual’s opinion depends on opinions of his (her) nearest neighbors. If the opinion of these neighbors is unanimity ( $S_{i-1} S_{i+1} = 1$ ), then the  $i$ th individual agrees with them, if not, the  $i$ th individual does not change his (her) opinion. In this case, the number of steady states is enormous. Namely, each state different from  $ABAB$  (antiferromagnetic) is a steady state. It means that if we start with a random distribution of opinions  $A$  and  $B$ , there will be a tendency to create small clusters.

It is rather obvious that in a real community, all mentioned and many others mechanisms can effect an opinion evolution.

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## References

1. D. Stauffer and T. J. P. Penna, *Physica A* **256**, 284 (1998).
2. F. Schweitzer and J. A. Holyst, *Eur. Phys. J. B* **15**, 723 (2000).
3. D. Derrida and P. G. Higgs, *J. Phys. A* **24**, L985 (1991).
4. R. Florian and S. Galam, *Eur. Phys. J. B* **16**, 189 (2000).
5. J. Brockman (ed.), *The Third Culture: Beyond the Scientific Revolution* (1995).
6. P. Bak, C. Tang, and K. Wiesenfeld, *Phys. Rev. Lett.* **59**, 381 (1987).
7. P. Bak and K. Sneppen, *Phys. Rev. Lett.* **71**, 4083 (1993).
8. D. Stauffer, “Exotic Statistical Physics”, eds. A. Pkalski and K. Sznajd-Weron, *Physica A* **285**, 121 (2000).
9. S. Solomon, G. Weisbuch, L. de Arcangelis, N. Jan, and D. Stauffer, *Physica A* **277**, 239 (2000).
10. S. Galam, *Physica A* **230**, 174 (1996); “Exotic Statistical Physics”, eds. A. Pkalski and K. Sznajd-Weron, *Physica A* **285**, 66 (2000).
11. J. A. Holyst, K. Kacperski, and F. Schweitzer, to be published in Vol. IX of Annual Reviews of Computational Physics, World Scientific, Singapore.
12. [www.cbos.com.pl](http://www.cbos.com.pl): Report of Public Opinion Research Center CBOS (104–115) “Social Mood”.
13. B. Latané, *Am. Psychologist* **36**, 343 (1981).