

PUBLIC TRANSPORT SYSTEMS IN POLAND:  
FROM BIAŁYSTOK TO ZIELONA GÓRA BY BUS  
AND TRAM USING UNIVERSAL STATISTICS OF  
COMPLEX NETWORKS\*

JULIAN SIENKIEWICZ AND JANUSZ A. HOŁYST

Faculty of Physics and Center of Excellence for Complex Systems Research  
Warsaw University of Technology  
Koszykowa 75, 00-662 Warsaw, Poland

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*Dedicated to Professor Andrzej Fuliński on the occasion of his 70th birthday*

We have examined a topology of 21 public transport networks in Poland. Our data exhibit several universal features in considered systems when they are analyzed from the point of view of evolving networks. Depending on the assumed definition of a network topology the degree distribution can follow a power law  $p(k) \sim k^{-\gamma}$  or can be described by an exponential function  $p(k) \sim \exp(-\alpha k)$ . In the first case one observes that mean distances between two nodes are a linear function of logarithms of their degrees product.

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## 1. Introduction

Have you ever been confused and tired using city public transport? Have you studied city maps during your holiday's excursions looking for the best connection from a railway station to your hotel? Do you think there is any regularity in complex objects called *public transport systems*? During the last few years several transport networks have already been investigated using various concepts of statistical physics of complex networks. A study of Boston underground transportation system (MBTA) [1, 2] has taken into account physical distances and has been focused on problems of links efficiency and costs. In [3] a part of the Indian Railway Network (IRN) has been considered and a new topology describing the system as a set of train

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lines, not stops has been introduced. In [4] data from MBTA and U-Bahn network of Vienna have been compared to predictions of random bipartite graphs. Another class of transport systems form airport and airlines networks: world-wide [5, 6], Chinese [7] or Indian [8] airline networks — here such properties as links weights, correlations among different airports and their centrality have been investigated.

In the present paper we have studied a part of data for public transport networks in 21 Polish cities and we find that among apparent differences there are also universal features of these systems. As far as we know, our results are the first comparative survey of several individual transport systems in the same country.

## 2. Network topology: space L and space P

It is clear that distances for city travelers are not the same as physical distances if he/she needs to come from a city point A to a city point B using existing public transport media. Sometimes it occurs that a physical distance between points A and B is not very large but the travel between these points in the city is unfortunately time consuming since either a direct bus makes a lot of loops on its way or we need to change buses or tramways several times. It follows that one can introduce at least two different representations of city transport networks where a network is a set of nodes (vertices) and links (edges) between them. The first representation is the space L which consists of nodes being bus or tramway stops while a link between two nodes exists provided they are consecutive stops on a bus or a tramway line. The distance in such a space is measured by the total number of stops passed on the shortest path between two nodes. However, the distance measured

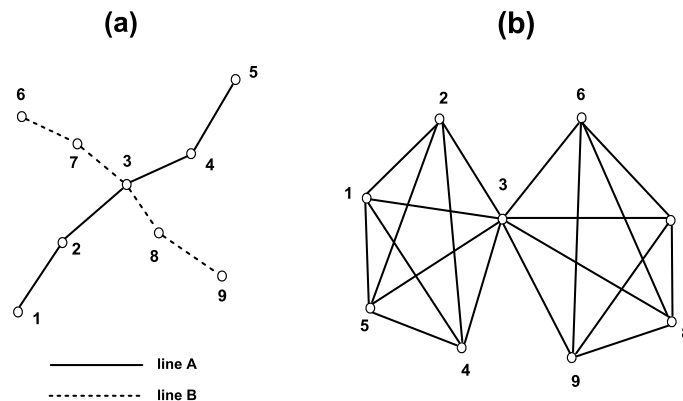


Fig. 1. Transformation from space L (a) to space P (b) using an example of two public transport lines A and B.

in such a way does not reflect the necessity of transfer during the trip. This factor is taken into account in the second space  $P$  [3]. Nodes in such a space are the same as in the previous one but now an edge connecting two nodes means that there is a link by a *single* bus or tramway between them. It follows that in the space  $P$  the distances are numbers of *transfers* (plus one) needed during the trip. It is obvious that distances defined in the space  $P$  are much shorter than in the space  $L$  and there is no universal relation between them. Both spaces are presented in Fig. 1.

### 3. Explored systems

We have studied data collected from 21 public transport networks in Polish cities that are depicted in Fig. 2 and listed in Table I. The first analyzed features are degree distributions for networks represented in both spaces. A degree of node  $i$  is the number  $k_i$  of its nearest neighbors. In regular networks (*e.g.* crystals) all nodes (*e.g.* atoms) can have the same degree. In complex networks [9] there is a wide spectrum of degrees and a large interest on such systems started from a discovery of scale-free distributions in several real networks [10–13].



Fig. 2. Map of twenty one examined cities in Poland.

Fig. 3 shows typical plots for degree distribution in the space  $L$ . In all studied plots we neglected the point  $k = 1$  that corresponds to line ends. Remaining parts of degree distributions can be approximately described by

TABLE I

Number of nodes  $N$ , coefficients  $\gamma$  and  $\alpha$  with their standard errors  $\Delta\gamma$ ,  $\Delta\alpha$  and Pearson's coefficients  $R_\gamma^2$ ,  $R_\alpha^2$ , for considered cities. The last column  $R_l^2$  represents Pearson's coefficient for the scaling (4). Fitting to the scaling relations (1) and (4) has been performed at whole ranges of degrees  $k$ . Fitting to (2) has been performed at approximately half of available ranges to exclude large fluctuations occurring for higher degrees (see Fig. 4).

City	$N$	$\gamma$	$\Delta\gamma$	$R_\gamma^2$	$\alpha$	$\Delta\alpha$	$R_\alpha^2$	$R_l^2$
Białystok	559	3.0	0.4	0.945	0.0211	0.0002	0.997	0.873
Bydgoszcz	276	2.8	0.3	0.961	0.0384	0.0004	0.996	0.965
Częstochowa	419	4.1	0.4	0.974	0.0264	0.0004	0.992	0.976
Gdańsk	493	3.0	0.3	0.952	0.0304	0.0006	0.981	0.980
Gdynia	406	3.04	0.2	0.983	0.0207	0.0003	0.990	0.967
GOP	2811	3.46	0.15	0.987	0.0177	0.0002	0.988	0.885
Gorzów Wlkp.	269	3.6	0.3	0.983	0.0499	0.0009	0.994	0.984
Jelenia Góra	194	3.0	0.3	0.979	0.038	0.001	0.984	0.994
Kielce	414	3.00	0.15	0.992	0.0263	0.0004	0.991	0.963
Kraków	940	3.77	0.18	0.992	0.0202	0.0002	0.996	0.977
Łódź	1023	3.9	0.3	0.968	0.0251	0.0001	0.998	0.983
Olsztyn	268	2.95	0.21	0.980	0.0226	0.0004	0.986	0.985
Opole	205	2.29	0.23	0.978	0.0244	0.0004	0.989	0.992
Piła	152	2.86	0.17	0.990	0.0310	0.0006	0.989	0.989
Poznań	532	3.6	0.3	0.978	0.0276	0.0003	0.994	0.976
Radom	282	3.1	0.3	0.960	0.0219	0.0004	0.989	0.991
Szczecin	467	2.7	0.3	0.963	0.0459	0.0006	0.995	0.979
Toruń	243	3.1	0.4	0.964	0.0331	0.0006	0.990	0.979
Warszawa	1530	3.44	0.22	0.980	0.0127	0.0001	0.998	0.985
Wrocław	526	3.1	0.4	0.964	0.0225	0.0002	0.993	0.983
Zielona Góra	312	2.68	0.20	0.979	0.0286	0.0003	0.995	0.996

a power law

$$p(k) \sim k^{-\gamma} \quad (1)$$

with a characteristic exponent  $\gamma$  between 2.4 and 4.1. Values of exponents  $\gamma$  are different from the value  $\gamma = 3$  which is characteristic for Barabási–Albert model of evolving networks with preferential attachment and one can suppose that a corresponding model for transport network evolution should include several other effects. One can see also that larger exponents  $\gamma$  correspond usually to larger numbers  $N$  of nodes in the network (Table I).

Quite different situation is in the space P. Corresponding cumulative degree distributions for selected cities  $P(k) = \int_k^{k_{\max}} p(k') dk'$  are presented in Fig. 4. The distributions  $P(k)$  and  $p(k)$  are well fitted by exponential

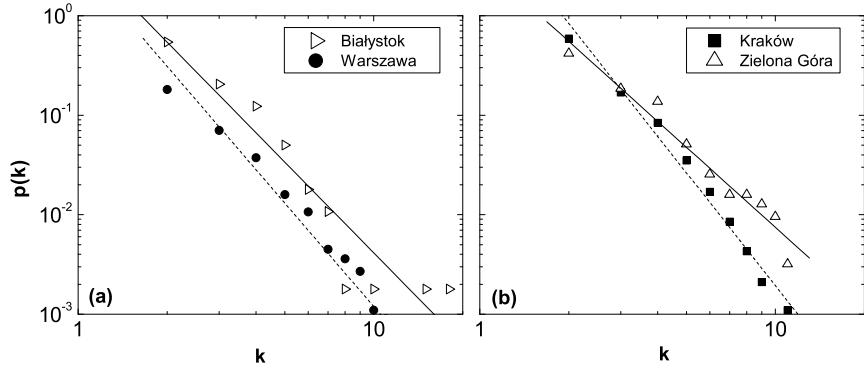


Fig. 3. Degree distribution in space L with power law  $k^{-\gamma}$  fit. (a) Białystok  $\gamma = 3.0 \pm 0.4$  (solid line) and Warszawa  $\gamma = 3.44 \pm 0.22$  (dotted line). (b) Kraków  $\gamma = 3.77 \pm 0.18$  (solid line) and Zielona Góra  $\gamma = 2.68 \pm 0.20$  (dotted line).

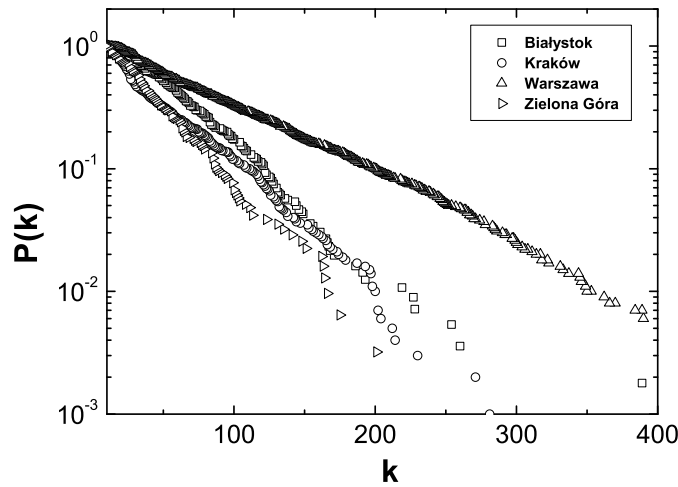


Fig. 4. Cumulative distribution  $P(k)$  in space P for Białystok, Kraków, Warszawa and Zielona Góra.

representation

$$p(k) \sim \exp(-\alpha k). \tag{2}$$

As it is well known [14] the exponential distribution (2) can occur when a network evolves but nodes are attached completely *randomly*. We are surprised that such a random evolution could well correspond to the topology of urban transport networks.

Table I presents exponents  $\gamma$  and  $\alpha$  for investigated cities. The values have been received from the standard linear regression method.

### 3.1. Path length as function of product $k_i k_j$

In [15] an analytical model of average path length was considered and it was shown that the shortest path length between nodes  $i$  and  $j$  possessing degrees  $k_i$  and  $k_j$  in a random graph characterized by its degree distribution  $p(k)$  can be described as:

$$l_{ij}(k_i, k_j) = \frac{-\ln k_i k_j + \ln(\langle k^2 \rangle - \langle k \rangle) + \ln N - \gamma}{\ln(\langle k^2 \rangle / \langle k \rangle - 1)} + \frac{1}{2}, \quad (3)$$

where  $\gamma = 0.5772$  is Euler constant while  $\langle k \rangle$  and  $\langle k^2 \rangle$  are corresponding first and second moments of  $p(k)$  distributions. In [16–18] a random tree (a random graph with no loops) was studied and it was shown that

$$l_{ij}(k_i, k_j) = A - B \log k_i k_j, \quad (4)$$

where coefficients  $A$  and  $B$  depend on an average branching factor  $\kappa$  of the considered tree and on a total number of its edges  $E = N\langle k \rangle/2$

$$A = 1 + \frac{\log(2E)}{\log \kappa}, \quad (5)$$

$$B = \frac{1}{\log \kappa}. \quad (6)$$

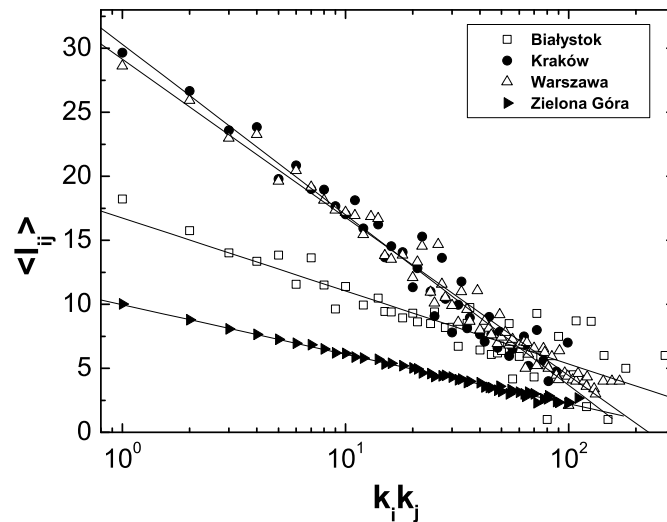


Fig. 5. Dependence of  $l_{ij}$  on  $k_i k_j$  in space L for Białystok, Kraków, Warszawa and Zielona Góra.

We have found that if distances are measured in the space L then the scaling (4) is well fulfilled for considered public transport networks (Fig. 5). Table I presents corresponding Pearson's coefficients. One can see that except the cases of Białystok and GOP all other  $R_t^2$  coefficients are above 0.96 and the best fit to the scaling relation (4) has been found for Zielona Góra, where  $R^2 = 0.996$ . Observed values of  $A$  and  $B$  coefficients differ as much as 20 percent (in average) from theoretical values received for random graphs where contribution from clustering and node degree correlations are taken into account (see [16, 17, 19]).

It is useless to examine the relation (4) in the space P because of the structure of this space. In fact the set  $l_{ij}$  contains usually only 3 points what means that one needs just two changes of a bus or a tram to come from one city point to another [19].

#### 4. Conclusions

In conclusion we have observed that public transport networks in many Polish cities follow universal scalings. The degree distribution  $p(k)$  fits to a power law in the space L where a distance is measured in numbers of bus or tram stops. A corresponding distribution in the space P where a distance is measured in a number of transfers between different vehicles follows an exponential distribution. Distances in the space L are a linear function of logarithms of corresponding nodes degrees.

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