Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 317 (2003) 597–608
PHYSICA A
www.elsevier.com/locate/physa

Stochastic resonance as a model for financial market crashes and bubbles

A. Krawiecki[a,b], J.A. Hołyst[a,b,*]

[a] *Faculty of Physics, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland*
[b] *Institute for Economics and Traffic, Dresden University of Technology, D-01062 Dresden, Germany*

Received 8 July 2002

Abstract

A bistable model of a financial market is considered, aimed at modelling financial crashes and bubbles, based on the Ising model with thermal-bath dynamics and long-range interactions, subject to a weak external information-carrying signal and noise. In the ordered phase, opposite stable orientations of magnetization correspond to the growing and declining market before and after the crash or bubble, and jumps of magnetization direction correspond to crashes and bubbles. It is shown that the influence of an information-carrying signal, assumed to be too weak to induce magnetization jumps, can be enhanced by the external noise via the effect of stochastic resonance. It is argued that in real stock markets the arrival of a piece of information, considered a posteriori to be the cause for a crash or bubble, can be enhanced in a similar way, thus leading to price return whose value is unexpectedly large in comparison with relatively weak importance of this piece of information.

PACS: 89.90.+n; 05.40.−a; 05.45.−a

Keywords: Econophysics; Financial crashes; Stochastic resonance; Social impact models; Ising model

1. Introduction

In recent years there has been growing interest in the investigation of financial markets as complex systems in statistical physics [1–43]. These studies revealed that the empirical financial time series, e.g., for stock market price returns (forward in time

* Corresponding author. Faculty of Physics, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland. Tel.: +48-22-6607133; fax: +48-22-6282171.
E-mail address: jholyst@if.pw.edu.pl (J.A. Hołyst).

PII: S0378-4371(02)01375-4

changes of the logarithm of the stock price) can be perceived as complicated stochastic processes with strongly non-Gaussian characteristics [3–9]. It was also shown that the universal features of such time series, e.g., volatility clustering (large irregular bursts of price returns superimposed on otherwise noisy small-scale fluctuations, resulting in power-law tails in probability distributions of price returns and in the autocorrelation function of absolute values of price returns) can be modelled as collective phenomena in complex physical systems. The respective models can be based, e.g., on numerical simulations of behavior of many interacting agents whose reactions to price changes follow certain rational rules [10–14], on the theory of stochastic processes with multiplicative noise [15], percolation theory [16–18], Ising model [19–26], etc.

A significant part of research on financial time series is devoted to crashes, i.e., large negative price returns preceded by long-lasting phases of price growth. Similar, but less spectacular events of large positive price returns can be called financial bubbles. Crashes are atypical phenomena in the market, in the sense that they deviate from the statistics of price returns obtained at times when there were no spectacular price falls [27]. Empirical studies suggest that crashes are similar to phase transitions in physical systems, with the crash probability following a power scaling law in the vicinity of the critical time (which need not be the time at which the crash occurs). This point of view is also supported by observations of log-periodic in time oscillations of stock prices superimposed on the rising trend before the crash [28–39], although there were also doubts if such oscillations are common precursors of crashes [40,41]. The log-periodic oscillations are typical of phase transitions in systems with discrete-scale invariance [44]. As a result, several microscopic models of financial crashes based on the concepts of the theory of critical phenomena were proposed. For example, in the model of Ref. [42], the hierarchical structure of agents' actions leads directly to log-periodic oscillations. However, other models mostly neglect the problem of oscillations, while still considering the crash as a kind of a phase transition [16–18,23–26,43] in a system of many interacting agents. In particular, this can be a transition leading to ordering of the actions of all agents in the market, analogous to the paramagnetic–ferromagnetic transition in the Ising model [25,26], or the percolation transition [16–18]; since, if all agents place the same order (to sell stocks) at the same time, a sudden fall of the traded stock price will inevitably occur. A different point of view is adopted in the model of Kaizoji [23,24]. This is in fact the Ising model with non-zero temperature slightly below the critical temperature, with the agents treated as spins with two possible orientations imitating the decisions to sell (down) or buy (up) stocks, and with the magnetization (average orientation of agents) proportional to the stock price return. In this model, the two equivalent ordered (ferromagnetic) phases with opposite orientation of magnetization correspond to the growing and declining phases of the market before and after the crash, respectively, and the crash itself is seen as the jump of magnetization from the up-to-down orientation due to, e.g., thermal fluctuation. The jump in the opposite direction can be associated with a financial bubble.

A common feature of all crashes is that they are random events whose direct cause cannot usually be found, although a posteriori several reasons for the occurrence of a crash can be given [28]. In order to get some understanding of the origin of crashes, in this paper, we consider the effect of a weak external force acting on the agents in the above-mentioned microscopic spin model of the market of Refs. [23,24]. The force, modelling the a posteriori found cause for a crash (e.g., arrival of a piece of information accessible to all agents), is assumed to be too weak to cause the magnetization inversion (crash or bubble). However, we propose that its effect can be amplified by external noise and internal fluctuations so that the magnetization orientation can be highly correlated with the external force. The mechanism of amplification is the well-known phenomenon of stochastic resonance [45–47] (for review see Refs. [48,49]). Stochastic resonance occurs in certain, mainly non-linear systems driven by a combination of the information-carrying signal (usually assumed to be periodic) and noise. An important class of such systems are bistable systems, e.g., a particle in a bistable potential driven by additive signal and noise. The information-carrying signal is assumed too weak to induce particle jumps between the two potential wells; however, addition of noise enables jumps and the information-carrying signal modulates the probability of jump out of the two potential wells in an asymmetric way. It turns out that for an optimum non-zero noise intensity, the output signal from the system reflecting the location of the particle in the left or right well can be highly correlated with the external signal. In the case of a periodic signal, stochastic resonance is characterized, e.g., by a maximum of the output signal-to-noise ratio at the frequency of the external signal for the optimum noise intensity. In this paper we show that, analogously, the effect of weak external force can be amplified by external and internal noise in the bistable model for financial crashes and bubbles.

2. The model

In this paper we work with a microscopic model of a stock market [23,24] whose general features are common to many spin models aimed at modelling economic and social phenomena. We consider $i = 1, 2, \ldots, N$ agents (spins) with orientations $\sigma_i(t) = \pm 1$, corresponding to the decision to sell (-1) or to buy $(+1)$ a share of a traded stock at discrete time steps t. The orientation of the agent i at time $t + 1$ depends on the local field

$$I_i(t) = \frac{1}{z} \sum_{j=1}^{N} A_{ij} \sigma_j(t) + h_i(t) , \quad (1)$$

where z is the mean coordination number (the mean number of non-zero connections between agents), A_{ij} are interaction strengths per agent (possibly zero), and $h_i(t)$ are external fields reflecting the effect of environment (e.g., access to external information). The dynamics of the model consists in updating synchronously the orientations of all agents according to the probabilistic rule

$$\sigma_i(t+1) = \begin{cases} 1 & \text{with probability } p , \\ -1 & \text{with probability } 1 - p \end{cases} \quad (2)$$

describing uncertainty in decision-making of the agent i, where $p = 1/\{1 + \exp[-2I_i(t)]\}$. Thus, the model under study is a kind of Ising model with thermal bath dynamics (formally, with the "temperature" $k_B T = 1$); the application of such class of models in economics and sociology is theoretically and empirically supported by the social impact theory of opinion formation [50–56]. From the above-mentioned model, the dynamics of price $S(t)$ is obtained by introducing the average orientation of agents (magnetization) $x(t) = N^{-1}\sum_{i=1}^{N}\sigma_i(t)$ and using the equation $\mathrm{d}S/\mathrm{d}t \propto xS$, which follows from the fact that $x(t)$ is proportional to the difference between demand and supply. Thus, after discretizing time, the price returns $R(t) = \log S(t+1) - \log S(t)$ fulfill the proportionality relation $R(t) \propto x(t)$. In the mean-field approximation and in the thermodynamic limit, the dynamics of $x(t)$ can be obtained in a simple form, provided that $A_{ij} = A$ and $h_i(t) = h(t)$ for all i, j

$$x(t+1) = \tanh[Ax(t) + h(t)] \; . \tag{3}$$

The above equation remains true if the interaction strengths A_{ij} and external fields h_i are randomly and symmetrically distributed around their respective mean values, and if the number of agents N and the coordination number z are large enough; in most cases considered in this paper the above assumptions are fulfilled.

The first term on the right-hand side of Eq. (1) describes the influence of other agents on the decision of agent i at time $t+1$, and has a form of weighted average over the decisions at time t of all agents to whom the agent i is related. This term is responsible for the collective behavior of the agents: positive connections between agents $A_{ij} > 0$ mean that they will more probably follow the decisions of their partners, while negative connections $A_{ij} < 0$ prefer opposite decisions. Writing $A_{ij} = A + (A_{ij} - A)$, where A is the mean value of the interaction strengths A_{ij}, the interaction term contains a part $(NA/z)x(t)$ proportional to the price return $R(t)$. Thus the first term in Eq. (1) describes simultaneously the reaction of agents to the price changes, which emerges in this model as a result of averaging reactions to the past decisions of partner agents. It can be seen from Eq. (3) that for large N, z and $h(t) = 0$, if $0 < A < 1$ the state with $x = 0$ is a stable state of the system, while for $A > 1$ the system is bistable, and there are two symmetric stable states $x_+ > 0$, and $x_- < 0$ fulfilling the equation $x_\pm = \tanh(Ax_\pm)$. The former state corresponds to a disordered phase with no preferred orientation of agents (paramagnetic phase), while the two latter states correspond to equivalent ordered phases with opposite average orientations of agents (ferromagnetic phase). Since the states with $x = 0$, $x = x_+ > 0$, and $x = x_- < 0$ correspond to zero, positive and negative price returns, respectively, from the economical point of view, they can be interpreted as the market in equilibrium, the growing market, and the declining market. The crash in turn is the change of the average orientation of agents from positive to negative, while the bubble is the change of this orientation from negative to positive. In the mean-field limit of Eq. (3), for $A > 1$, the jumps of the average orientation of agents can occur under the influence of the external field $h(t)$ which points towards the preferred orientation of spins. In a system with large but finite N there are also thermal fluctuations of $x(t)$ which can cause thermally induced jumps of the average orientation of agents.

Concerning specific features of our model, we assume the interaction strengths in the form $A_{ij} = A(1 + a\zeta_{ij})$ with probability P, and $A_{ij} = 0$ with probability $1 - P$, where $A > 0$ is the mean strength of non-zero interactions, ζ_{ij} are random uncorrelated variables with uniform probability distribution in the interval $(-0.5, 0.5)$, and $a > 0$ is the relative range of distribution of interaction strengths between agents around their mean value. It follows that the mean coordination number $z = NP$. To model financial crashes and bubbles, $A > 1$ is assumed so that, at least in the mean-field approximation, the system is bistable, as desired. The case $P = 1$ corresponds to the fully connected model. No specific topology of connections is imposed since nowadays the interactions between agents are more and more determined by different kinds of long-range communications instead of spatial neighborhood, which makes the topological structure of the interaction network unimportant. Thus, provided that P is large enough, the conditions for the applicability of the mean-field approximation to our model are fulfilled. The external fields are assumed as

$$h_i(t) = (1 + b\chi_i)(s(t) + D\xi(t)) ,$$
$$s(t) = q \cos \omega_s t , \qquad (4)$$

where $s(t)$ is the information-carrying signal, $\xi(t)$ is Gaussian uncorrelated noise with variance one, χ_i are random uncorrelated variables with uniform distribution in the interval $(-0.5, 0.5)$, and b is the range of distribution of agents' reaction coefficients on external signals around their mean value one. Eq. (4) describes the "hidden" cause for crash, in the form of a weak external stimulus acting simultaneously on all agents, and the noisy influence of environment to each agent. At every time step t the direction of $h_i(t)$ indicates the preferred orientation of σ_i. The amplitude q of the signal $s(t)$ is assumed too small to enable, for given $A > 1$, the jumps of the average orientation of agents between the two equivalent states in the mean-field approximation. However, for moderate noise intensity D, the jumps are possible both in the microscopic model and in the mean-field dynamics of Eq. (3), and the average orientation of agents can be correlated with the signal $s(t)$. It should be emphasized that we assumed the external forcing in the form of a periodic signal only for convenience, as in most papers on stochastic resonance, which by no means suggests that there should be any hidden periodicity in the occurrence of financial crashes. In fact, it is well known that the effect of a weak aperiodic signal in a bistable potential can be enhanced by noise; this phenomenon is known as aperiodic stochastic resonance [57].

3. Methods of analysis

In the following, the response of the system of Section 2 to the information-carrying signal in Eq. (4) is investigated as a function of the external noise intensity D, for fixed $A > 1$, and for various densities P of connections between agents. The output signal $y(t)$ is defined as a sign of the average orientation of agents, $y(t) = \text{sign}(x(t))$. Such two-state approximation is typical in the investigation of stochastic resonance in bistable systems [46], and means that only the periodic component connected with the switching of the average orientation between equivalent states is taken into account, while the possibly periodic oscillations around the fixed points $x_\pm$ are neglected. Thus, only the occurrence of crashes (and bubbles) due to the arrival of some external information is analyzed, rather than small changes in the orientation of agents not related to large price returns. Qualitative theoretical analysis is performed in the mean-field approximation, Eq. (3), which after taking into account Eq. (4) yields $x(t+1) = \tanh[Ax(t) + q\cos\omega_s t + D\xi(t)]$. Numerical simulations are performed both in the mean-field approximation and with a full microscopic model with a large number of agents N. From the output signal $y(t)$ its power spectral density $S(\omega)$ is obtained; it consists of a broad noise background and peaks at the odd multiples of ω_s. As a measure of stochastic resonance the output signal-to-noise ratio (in dB) is used, defined as $\mathrm{SNR} = 10\log[S_P(\omega_s)/S_N(\omega_s)]$. Here, $S_P(\omega_s) = S(\omega_s) - S_N(\omega_s)$ is the height of the peak at $\omega = \omega_s$ and $S_N(\omega_s)$ is the noise background in the vicinity of ω_s. In our numerical simulations the signal-to-noise ratio is normalized to the frequency bandwidth $\Delta f = 2^{-12}$ Hz, i.e., the power spectral density is obtained by averaging results from many time series of $y(t)$, each containing 2^{12} points [46]. Stochastic resonance occurs if the curve SNR vs. D has a maximum for $D > 0$.

It is worth noting that stochastic resonance has been so far investigated in the Ising spin model in various dimensions [58–60], as well as in the Weidlich model of opinion formation [61] and in a model for financial markets [62], the latter being also certain extensions of the Ising model. In these papers, the response to weak periodic signals was investigated mostly as a function of strength of thermal fluctuations in the system (temperature), without external noise, and the analysis was performed using the linear response theory. As a result, for slowly varying signals the maximum signal-to-noise ratio was obtained at the critical temperature (where the system is at the border between the ordered and disordered phase), due to maximum susceptibility of the Ising model at the critical point. In this paper, the level of fluctuations is kept constant by assuming $A = \mathrm{const}$, and the noisy influence of the environment D is varied. Besides, assuming $A > 1$ ensures that the market shows a tendency towards spontaneous, collective ordering, so we usually work in a non-linear regime, far from the critical point. Thus, the mechanism of stochastic resonance here is typical of bistable systems with additive noise rather than that of the Ising model with thermal bath dynamics close to the critical point.

4. Theoretical analysis

In this section qualitative theoretical evaluation of the signal-to-noise ratio in our model is performed using the adiabatic theory of stochastic resonance in bistable systems subject to slowly varying periodic forces [46]. Only the mean-field approximation given by Eq. (3) for the dynamics of $x(t)$ is considered. The signal-to-noise ratio in a bistable system can be obtained if the dependence $r(D)$ of the escape rate from any of the two symmetric stable states to the opposite one on the noise intensity is known. Then it is assumed that the periodic signal modulates asymmetrically the escape rates from the two states, thus making them time dependent; so that during half of the period, the escape from, e.g., the left state $r_+(D,t)$ is higher than that from the right one, $r_-(D,t)$, and during the next half the asymmetry is inverse. For small periodic signal amplitudes the rates can be expanded in the Taylor series to the first order

$$r_\pm(D,t) = \tfrac{1}{2}r_0(D) \pm \tfrac{1}{2}r_1(D)q\cos\omega_s t\ , \qquad (5)$$

where $r_0(D)$ is the escape rate (equal for the two states) in the absence of the periodic forcing for a given noise intensity. In the first approximation, the signal-to-noise ratio is [46]

$$\mathrm{SNR} = \frac{\pi r_1^2 q^2}{4 r_0 \Delta f} , \tag{6}$$

where the dependence of the signal-to-noise ratio on the bandwidth Δf has been included. In bistable potentials with additive noise the rate $r(D)$ is usually given by the Kramers formula. However, in the case of Eq. (3) the noise is non-additive, thus making the evaluation of the escape rates difficult [46]. Hence, only qualitative estimations of the signal-to-noise ratio can be easily given.

For $h(t) = 0$, the system (3) stays close to $x = x_\pm$, and addition of small noise and periodic modulation causes only that $x(t)$, apart from performing large jumps, fluctuates mostly in the neighborhood of these fixed points. Hence, in order to obtain the estimator for the signal-to-noise ratio it can be assumed that for $h(t) \neq 0$, the jumps of the mean orientation of agents occur if and only if the zero of the function $f(x) = \tanh[Ax(t) + q\cos\omega_s t + D\xi(t)]$ is located right to x_+ (crash) or left to x_- (bubble). This underestimates the true probability of jump by neglecting the possibility of a jump under the influence of a sequence of several pulses of noise. The condition for the jump of $x(t)$ from the positive to negative orientation becomes then $\xi < \xi_+ = (-Ax_+ - q\cos\omega_s t)/D$, and the escape rate $r_+ = \Pr(\xi < \xi_+)$; similarly the jump of $x(t)$ from the negative to positive orientation occurs when $\xi > \xi_- = (-Ax_- - q\cos\omega_s t)/D$, and the escape rate $r_- = \Pr(\xi > \xi_-)$. It can be seen that both escape rates are modulated by the periodic signal in an asymmetric way. Thus the coefficients in Eqs. (5) and (6) can be evaluated as

$$r_0 = 1 - \mathrm{erf}\left(\frac{Ax_+}{\sqrt{2}D}\right), \quad r_1 = \frac{2}{\sqrt{2\pi}D}\exp\left(-\frac{A^2 x_+^2}{2D^2}\right) . \tag{7}$$

Another estimator of the signal-to-noise ratio can be obtained by assuming that the jump of $x(t)$ out of a given orientation occurs always if $h(t)$ is such that the respective fixed point x_+ or x_- disappears. This overestimates the true probability of jump since, e.g., in the next step the fixed point can reappear due to a smaller value of the external field, and the jump does not take place. The conditions for the disappearance of x_+ and x_- are, respectively, $\xi < (-h_0 - q\cos\omega_s t)/D$ and $\xi > (h_0 - q\cos\omega_s t)/D$, where

$$h_0 = -\mathrm{artanh}\sqrt{1 - A^{-1}} + A\sqrt{1 - A^{-1}} . \tag{8}$$

Similarly, as in the previous case, we arrive at

$$r_0 = 1 - \mathrm{erf}\left(\frac{h_0}{\sqrt{2}D}\right), \quad r_1 = \frac{2}{\sqrt{2\pi}D}\exp\left(-\frac{h_0^2}{2D^2}\right) . \tag{9}$$

It should be noted that in the mean-field approximation the condition for the periodic signal to be unable to induce switching of the mean orientation of agents without external noise is $q < h_0$, since then any of the two fixed points $x_\pm$ never disappears. Under this condition, and provided that $A > 1$, inserting both estimations, Eqs. (7) and (9) into Eq. (6) yields curves SNR vs. D with a maximum at $D > 0$ (Fig. 1(a)), thus suggesting possibility to observe stochastic resonance also in the multi-agent simulations.

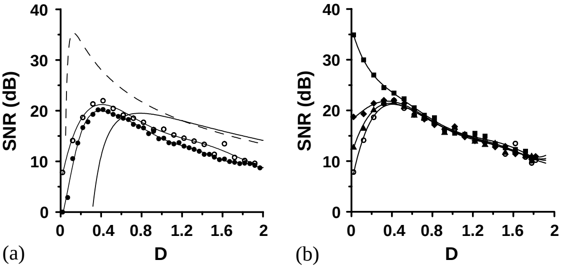

Fig. 1. (a) Signal-to-noise ratio SNR vs. the noise intensity D for a model with $A = 1.5$, $a = 2.0$, $b = 2.0$, $q = 0.2$: solid line—theoretical estimation using Eqs. (6) and (7), dashed line—theoretical estimation using Eqs. (6) and (9), dots—numerical simulation in the mean-field approximation, circles—numerical simulation of a system of $N = 250$ agents with $P = 1$; in the case of numerical simulations the solid lines are guides to the eyes. (b) As in (a), but only results of numerical simulations for a system of $N = 250$ agents with $P = 0.05$ (squares), $P = 0.25$ (diamonds), $P = 0.50$ (triangles), $P = 1$ (circles); the solid lines are guides to the eyes.

5. Numerical results

The signal-to-noise ratio obtained from numerical simulations of the multi-agent model with $N = 250$ agents and the probability $P = 1$, as well as from simulations in the mean-field approximation, are shown in Fig. 1(a) and compared with theoretical estimations of Section 4. In all cases stochastic resonance is observed, and the estimators of the signal-to-noise ratio yield theoretical results comparable with the numerical ones. The results from the multi-agent simulation fit well those from the mean-field simulation, despite a relatively small number of agents N and large values of a, b (note, however, that for the parameters in Fig. 1 all interaction strengths A_{ij} and agent's reactions to external signals are positive). For this simulation, a relatively large value of $A = 1.5$ was assumed (the system was deeply in the bistable regime) in comparison with the value $A = 1.05$ proposed by Kaizoji for modelling the Japanese stock market crash in 1997 [23]. This is because for A very close to unity and finite N frequent jumps of the mean orientation of agents resulting from internal system fluctuations occur, making comparison with the mean-field limit possible only for very large N and for prohibitively long simulation times.

In Fig. 1(b), the effect of dilution of connections between agents is shown. It can be seen that decreasing P leads to some increase of the maximum of the signal-to-noise ratio. However, for small P stochastic resonance disappears and the curve SNR vs. D is monotonously decreasing. This is because for decreasing density of connections between agents and for zero external field there is order–disorder transition at $P_c \approx 0.01$. Thus for small P the system is either in, or close to, the paramagnetic (disordered) state, so the agents can be easily oriented by a weak periodic signal without the help of external noise. Besides, it should be noted that decreasing P is not equivalent to increasing temperature in the Ising spin models with stochastic resonance [58–62] since it does not affect the external fields $h_i(t)$. Thus the maximum value of the signal-to-noise

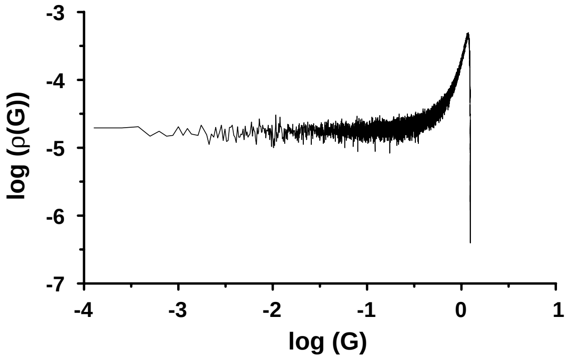

Fig. 2. Cumulative distribution of normalized price returns $G=(x-\langle x\rangle)/\sqrt{\langle x^2\rangle-\langle x\rangle^2}$, where the angular brackets denote the time average, obtained from the numerical simulation of the mean-field model with parameters given in Fig. 1 and $D=0.4$ (close to the maximum of the signal-to-noise ratio); the distribution was evaluated from $x>0$ only, due to symmetry of Eq. (3) there is $\langle x\rangle=0$.

ratio at $D=0$ turns out to be almost constant for a range of small P instead of showing maximum as in Refs. [58–62] close to P_c as a function of P.

It should be emphasized that permanent switching between symmetric states (growing and declining market), with or without a periodic component, is not a realistic model for a true market. This can be seen from the cumulative probability distribution $\rho(G)$ of normalized price returns G obtained from the mean-field simulation of our model (Fig. 2). For real markets this distribution has a pronounced maximum at $x=0$ and tails decaying according to a power law [5], which reflects the fact that the market remains mostly in equilibrium where the returns are small, while large returns are seldom. In Fig. 2, the distribution is, in contrast, bimodal (only positive part is shown), which is obvious from the assumed bistability of our model, and which also indicates a high probability of crash or bubble (large negative or positive return). We stress once more that we chose the bistable model, and the information-carrying signal in the form of a periodic function, only in order to obtain good statistics for the signal-to-noise ratio, which is a measure of enhancement of the external signal by noise in most papers on stochastic resonance. In fact, it seems that from time to time the actions of the agents in the market become highly synchronized; this situation can be modelled by, e.g., increasing the parameter A with time as long as an ordered phase occurs, corresponding to a growing or declining market. A similar mechanism for market ordering was proposed, e.g., in Ref. [25], where, however, the occurrence of ordered phase was identified with an immediate crash; since then all agents place simultaneously the same orders. According to our interpretation, the market can remain in the ordered phase for some time. Then the arrival of, maybe, an unimportant piece of information causes a sudden inverse of the average orientation of agents (crash or bubble) due to the enhancement of the effect of information-carrying signal by noise. This piece of information can be a posteriori recognized as, e.g., a cause for crash; after the crash the market can continue declining, but after some time it usually returns to equilibrium via, e.g., decreasing the mean interaction strength A with time. A symmetric situation can take place before and after a bubble. However, modelling the buildup of ordered phase before the crash or bubble is beyond the scope of the present model.

6. Summary and conclusions

We considered a microscopic model of a stock market, in which the agents were treated as spins in the Ising model with thermal-bath dynamics and long-range interactions, subject to the external information-carrying signal and noise. In order to model financial crashes and bubbles, strong enough average interaction strength between agents was assumed, leading to bistability of the average orientation of agents with two equivalent orientations corresponding to the growing and declining market. The crashes and bubbles were modelled as jumps of the mean orientation of agents between the two stable orientations, under the influence of the information-carrying signal and noise. It was shown that even if the information-carrying signal is not strong enough to induce crashes, its effect can be enhanced by optimum noise via stochastic resonance. Stochastic resonance was observed both in multi-agent and mean-field simulations, and qualitatively explained theoretically using the adiabatic theory valid in bistable systems with external noise.

The main outcome of this investigation is that the apparently weak stimuli from outside can have potentially strong effect on financial markets by creating large price returns if they are enhanced by noise. Such stimuli can be a posteriori identified with, e.g., the causes for crashes, although at their arrival nobody usually expects that crash occur. For this purpose the market has to be far from equilibrium, with most agents sharing the same orientation just before the crash or bubble. The possible role of stochastic resonance in the occurrence of stock market crashes and bubbles does not exclude, of course, that the crash or bubble can appear either under the influence of a very strong signal, able to invert the average orientation of agents without the help of noise, or due to some internal or external fluctuation, when even a posteriori the possible cause for crash or bubble is difficult to find.

Acknowledgements

The authors would like to thank the Quandt Foundation of the ALTANA AG for financial support of the project “Nonlinear Dynamics in Models of Complex Systems”. The authors would like to thank Prof. Dr. D. Helbing for his hospitality during their stay in Dresden, and P. Łagodziński for helpful discussions.

References

[1] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, Cambridge, UK, 1999.

[2] J.-P. Bouchaud, M. Potters, Theory of Financial Risk, Cambridge University Press, Cambridge, UK, 1999.

[3] R.N. Mantegna, H.E. Stanley, Nature 376 (1995) 46;
R.N. Mantegna, H.E. Stanley, Nature 383 (1996) 587.
[4] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner, Y. Dodge, Nature 381 (1996) 767.
[5] P. Gopikrishnan, V. Plerou, L.A. Nunes Amaral, M. Meyer, H.E. Stanley, Phys. Rev. E 60 (1999) 5305.
[6] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A. Nunes Amaral, H.E. Stanley, Phys. Rev. Lett. 83 (1999) 1471.
[7] F. Lillo, R.N. Mantegna, Phys. Rev. E 62 (2000) 6126.
[8] M. Ausloos, Physica A 285 (2000) 48.
[9] J.A. Hołyst, M. Żebrowska, K. Urbanowicz, Eur. Phys. J. B 20 (2001) 531.
[10] M. Levy, H. Levy, S. Solomon, Econ. Lett. 45 (1994) 103;
M. Levy, H. Levy, S. Solomon, J. Phys. I France 5 (1995) 1087.
[11] P. Bak, M. Paczuski, M. Shubik, Physica A 246 (1997) 430.
[12] M. Youssefmir, B. Huberman, J. Econ. Behav. Organ. 32 (1997) 101.
[13] T. Lux, M. Marchesi, Nature 297 (1999) 498.
[14] M. Raberto, S. Cincotti, S.M. Focardi, M. Marchesi, Physica A 299 (2001) 319.
[15] A. Sato, H. Takayasu, Physica A 250 (1998) 231.
[16] R. Cont, J.-P. Bouchaud, Macroecon. Dyn. 4 (2000) 170.
[17] V.M. Eguíluz, M.G. Zimmermann, Phys. Rev. Lett. 85 (2000) 5659.
[18] R. D'Hulst, G.J. Rodgers, Eur. Phys. J. B 20 (2001) 619.
[19] J. Cremer, Physica A 246 (1997) 377.
[20] G. Iori, Int. J. Mod. Phys. C 10 (1999) 1149.
[21] D. Chowdhury, D. Stauffer, Eur. Phys. J. B 8 (1999) 477.
[22] S. Bornholdt, Int. J. Mod. Phys. C 12 (2001) 667.
[23] T. Kaizoji, Physica A 287 (2000) 493.
[24] T. Kaizoji, Physica A 299 (2001) 279.
[25] B.M. Roehner, D. Sornette, Eur. Phys. J. B 16 (2000) 729.
[26] D. Sornette, Physica A 284 (2000) 355.
[27] A. Johansen, D. Sornette, Eur. Phys. J. B 1 (1998) 141.
[28] D. Sornette, A. Johansen, J.-P. Bouchaud, J. Phys. I France 6 (1996) 167.
[29] J.A. Feigenbaum, P.G.O. Freund, Int. J. Mod. Phys. B 10 (1996) 3737.
[30] D. Sornette, A. Johansen, Physica A 245 (1997) 411.
[31] N. Vandewalle, Ph. Boveroux, A. Minguet, M. Ausloos, Physica A 255 (1998) 201.
[32] N. Vandewalle, M. Ausloos, Ph. Boveroux, A. Minguet, Eur. Phys. J. B 4 (1998) 139.
[33] A. Johansen, D. Sornette, O. Ledoit, J. Risk 1 (1999) 5.
[34] A. Johansen, D. Sornette, Eur. Phys. J. B 9 (1999) 167.
[35] A. Johansen, D. Sornette, Int. J. Mod. Phys. C 10 (1999) 563.
[36] N. Vandewalle, M. Ausloos, Ph. Boveroux, A. Minguet, Eur. Phys. J. B 9 (1999) 355.
[37] S. Drożdż, F. Ruf, J. Speth, M. Wójcik, Eur. Phys. J. B 10 (1999) 589.
[38] A. Johansen, O. Ledoit, D. Sornette, Int. J. Theor. Appl. Finance 3 (2000) 2.
[39] A. Johansen, D. Sornette, Eur. Phys. J. B 17 (2000) 319.
[40] L. Laloux, M. Potters, R. Cont, J.-P. Aguilar, J.-P. Bouchaud, Europhys. Lett. 45 (1999) 1.
[41] K. Ilinski, Int. J. Mod. Phys. C 10 (1999) 741.
[42] D. Sornette, A. Johansen, Physica A 261 (1998) 581.
[43] J.-P. Bouchaud, R. Cont, Eur. Phys. J. B 6 (1998) 543.
[44] D. Sornette, Phys. Rep. 297 (1998) 239.
[45] R. Benzi, A. Sutera, A. Vulpiani, J. Phys. A 14 (1981) L453.
[46] B. McNamara, K. Wiesenfeld, Phys. Rev. A 39 (1989) 4854.
[47] P. Jung, P. Hänggi, Phys. Rev. A 44 (1991) 8032.
[48] L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. 70 (1998) 223.
[49] V.S. Anishchenko, A.B. Neiman, F. Moss, L. Schimansky-Geier, Phys.—Usp. 42 (1999) 7 [Usp. Fiz. Nauk 169 (1999) 7].
[50] B. Latané, Am. Psychol. 36 (1981) 343.
[51] W. Weidlich, Phys. Rep. 204 (1991) 1.

[52] M. Lewenstein, A. Nowak, B. Latané, Phys. Rev. A 45 (1992) 763.
[53] D. Helbing, Quantitative Sociodynamics, Kluwer Academic, Dordrecht, 1995.
[54] K. Kacperski, J.A. Hołyst, J. Stat. Phys. 84 (1996) 169.
[55] J.A. Hołyst, K. Kacperski, F. Schweitzer, Physica A 285 (2000) 199.
[56] K. Kacperski, J.A. Hołyst, Physica A 287 (2000) 631.
[57] J.J. Collins, C.C. Chow, A.C. Capela, T.T. Imhoff, Phys. Rev. E 54 (1996) 5575.
[58] J. Javier Brey, A. Prados, Phys. Lett. A 216 (1996) 241.
[59] U. Siewert, L. Schimansky-Geier, Phys. Rev. E 58 (1998) 2843.
[60] K. Leung, Z. Néda, Phys. Rev. E 59 (1999) 2730.
[61] P. Babinec, Phys. Lett. A 225 (1997) 179.
[62] P. Babinec, Chaos Solitons & Fractals 13 (2002) 1767.