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Characteristic periodicities of collective behavior at the foreign exchange market

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Abstract. As the result of empirical investigations into the foreign exchange market a group structure of characteristic periodic decisions of market participants is found. In order to explain this finding at the microscopic level the agent-based model of a financial market in which N market participants trade M financial commodities is considered. If different sources of periodic information exist then the relationship among these characteristic periodic behaviors may be associated with a special structure where market participants perceive such information in the foreign exchange market.

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1 Introduction

A generic feature of many complex systems is the presence of collective periodic oscillations of their interacting elements [1–12]. Such behavior is observed in various phenomena studied in both the natural and social sciences. Examples include not only well known collective excitations in crystals (phonons, magnons, charge-density waves, etc.) [13] or coupled laser systems [14,15] but also subsets of neurons in the striate and prestriate cortex [16–18], Belousov-Zhabotinskii chemical oscillators [19], flows of pedestrians and vehicles [20], social group decisions [9], various market cycles [21], or even log-periodic precursors of market crashes [22,23]. In order to explain these oscillations several kinds of synchronization mechanisms [1,2,24] and resonances have been introduced [25,26]. One of these is a stochastic resonance that occurs in nonlinear dynamical systems perturbed by noise and a weak periodic signal [26], where a signal amplification measure possesses a maximum by a nonzero value of noise strength.

It is important to stress that collective periodic motions may bring us information about the hidden structure of a corresponding system. For example, observing frequency and amplitudes of vibrations penetrating building constructions or machines, one can receive useful data about changes in their inner structure, while measurements of a spectrum of waves propagating in the ground allow us to model different Earth layers. Similarly, observing human brain waves makes it possible to infer perceptual cognitive states [27,28], wakefulness/sleep [29], and so on.

In the article we discuss a method to infer the structure of a human organization from participant behavior. Recently various human activities can be observed directly with high resolution, and collected data are stored by computers due to the development of Information and Communication Technology. Similarly, financial markets are computerized from communication to transaction and they can be observed at the microscopic level of human activities. Since the structure of human organizations is related to the pathways of communication, an unknown structure of financial markets can be realized through an interplay among the perceptions and actions of market participants.

Recently it has become possible to investigate the behavior of human activities at the microscopic level by using the high-frequency data of financial markets, known as *tick data* [30–32]. As a result it seems possible to establish a theoretical concept about the relationship among individuals based on empiricism, and it is expected to bring deep insights and understanding of human activities from a scientific point of view.

The foreign exchange market is the largest financial market worldwide. All market participants in the foreign exchange market communicate through electronic brokering systems and find other market participants who are able to exchange currencies with them. The electronic

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brokering system collects quotations from the market participants and broadcasts transaction information to the market participants who attend this system. The market participants perceive the information (market prices and news) through computer terminals connected to electronic communication devices, and determine their investment attitudes. Namely, there exists circular causality in the foreign exchange market.

One of the authors (JAH) introduced the concept of stochastic resonance for a human population dynamics using an Ising-like model of a financial market [33]. It was shown that if a financial market is driven by a sinusoidal signal then the signal-to-noise ratio of price changes depends on the noise strength. The other author (AHS) examined the quotation behavior of the foreign exchange market, defined as the number of quotations per unit of time. In previous works [34,35] it was reported that the periodic motions appear and disappear dependent on the observation periods, and that the characteristic frequencies, defined by the peak position of the power spectrum density of the tick frequency, depend on the currency pairs. This phenomenon is explainable if the periodic exogenous information which several market participants perceive exists.

In this article, the relationship between periodic motions of behavioral frequencies in the foreign exchange market are empirically observed and theoretically considered. A spectral analysis of the behavioral frequency for 15 currency pairs dealt with in the foreign exchange market is conducted through high-accuracy data of quotation. For all currency pairs characteristic periodic motions are confirmed, and several groups of currency pairs with the same periodicity are found. In order to understand the relationship among periodicities observed for behavioral frequency of currency pairs, an agent-based model is considered. Theoretical analysis of this agent-based model proposes the hypothesis that the relationship is related to the structure through which the market participants perceive information.

The remainder of this article is structured as follows. In Section 2 spectral analysis for 15 kinds of currency pairs is conducted. It is shown that there are several groups of currency pairs having the same periodic frequency. In Sections 3 and 4 an agent-based model is considered and theoretical analysis of the agent-based model is shown in order to explain the existence of several groups with the same periodicity. In Section 5 resonance phenomenon and the relationship among characteristic frequencies are discussed. Section 7 is devoted to concluding remarks.

2 Empirical analysis

15 kinds of currency pairs dealt with in the foreign exchange market contain 10 major currencies: EUR (Euro), NZD (New Zealand Dollar), CAD (Canadian Dollar), SEK (Swedish Krone), AUD (Australian Dollar), USD (United States Dollar), GBP (British Pound), NOK (Norwegian Krona), JPY (Japanese Yen), and CHF (Swiss Franc). The sampling periods of the data are from 1st September 2000 to 29th September 2000 for 15 kinds of currency pairs and from 1st January 1999 to 31st December 2004 for EUR/JPY. The data, which was provided by CQG Inc., contains quotations of market participants in the foreign exchange market [36]. One segment records bid or ask quotations and time stamps with 1 [min] resolution. Since the two-way quotation is adopted by market participants in the foreign exchange market, the number of ask quotations is nearly equal to that of the bid quotations. Therefore counting one side is sufficient. The tick frequency for the jth currency pair $A_j(k)$ is defined as the number of ask quotations per unit time,

$$A_j(k) = \frac{1}{\Delta t} C_j(k\Delta t, (k+1)\Delta t), \quad k = 1, 2, \ldots, \tag{1}$$

where $C_j(t_1, t_2)$ is the number of ask quotations of the jth currency pair between t_1 and t_2, and $\Delta t = 1$ [min] denotes sampling time.

By using periodgram estimator power spectrum for the jth currency pair $P_j(f_n)$ is computed with window length $N = 1440$ [min] (24 h) and averaged over 21 days. The Nyquist critical frequency $f_c = 1/(2\Delta t)$ is 0.5 [1/min] because $\Delta t = 1$ [min]. In order to eliminate contributions for low frequencies the spectra are multiplied by the low-cut filter function,

$$Q(f_n) = 2\left(1 - \cos\left(\frac{f_n}{f_c}\right)\right). \tag{2}$$

As shown in Figure 1 it is found that the tick frequencies for most currency pairs have several peaks at various characteristic frequencies, which are defined as the peak positions. This means that the behavior of the market participants in the foreign exchange market includes periodical components. The characteristic frequencies distribute into a range of a few minutes as shown in Figure 2 and Table 1.

It is found that there exist several groups of the currency pairs having the same characteristic frequency, as shown in Figure 2. The characteristic frequency of the major group is 0.4 [1/min]. This group contains the AUD/USD, NZD/USD, USD/CAD, USD/CHF, USD/JPY, USD/NOK, and USD/SEK. This group seems to be related to the USD. The second major groups are those having 0.429167 [1/min], and 0.5 [1/min]. These groups contain the EUR/CHF and EUR/GBP, and the EUR/NOK and GBP/CAD. The EUR/JPY, EUR/SEK, and EUR/USD each have a unique characteristic frequency, and in the GBP/AUD the characteristic periodicity is not detectable. These periodic motions of the tick frequency are only observed in the European time zone (9:00–17:00 (UTC)). The signal-to-noise ratio for a characteristic frequency $\tilde{f}$ is defined as

$$SNR = 10 \log P_j(\tilde{f})/N, \tag{3}$$

where N denotes the noise level, calculated from the power spectrum around $\tilde{f}$. Figure 3 shows the signal-to-noise ratio calculated from power spectra averaged over a month for USD/JPY as $\tilde{f} = 0.4$. The signal-to-noise ratio for USD/JPY depends on the period to average power spectra.

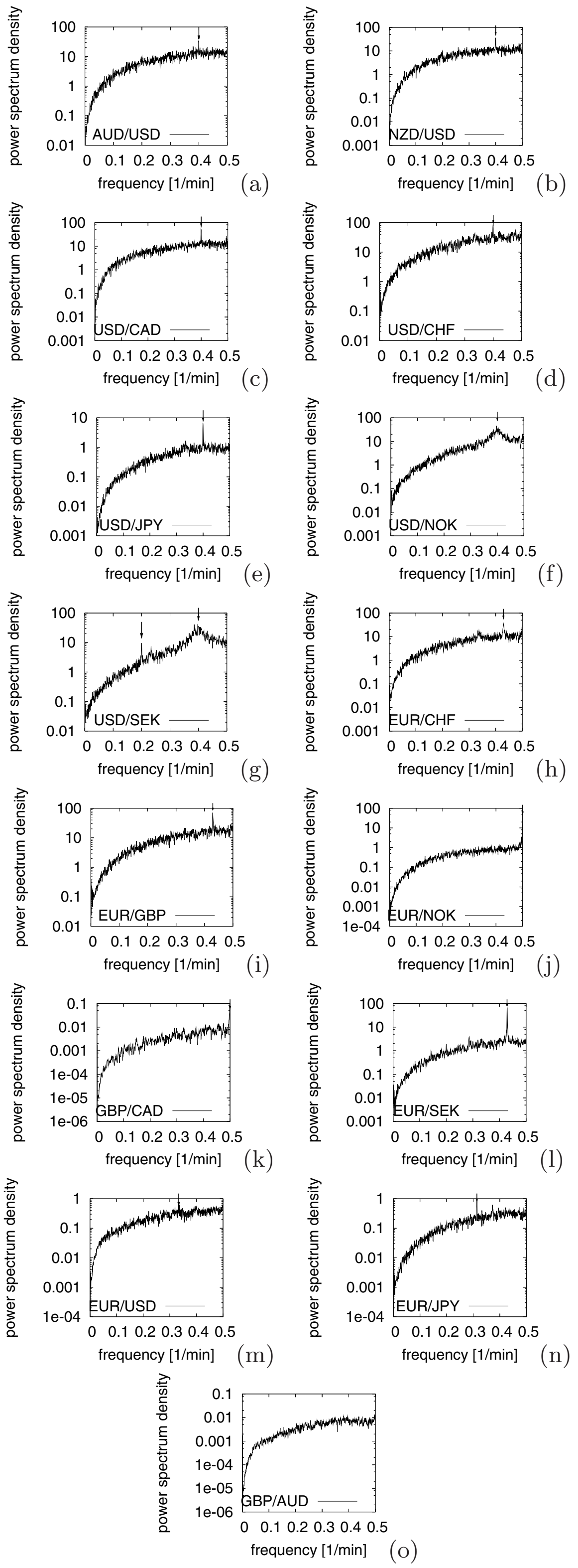

Fig. 1. Semi-log plots of the power spectrum density for the tick frequency during the period from 1st to 29th September 2000. (a) AUD/USD, (b) NZD/USD, (c) USD/CAD, (d) USD/CHF, (e) USD/JPY, (f) USD/NOK, (g) USD/SEK, (h) EUR/CHF, (i) EUR/GBP, (j) EUR/NOK, (k) GBP/CAD, (l) EUR/SEK, (m) EUR/USD, (n) EUR/JPY, (o) GBP/AUD.

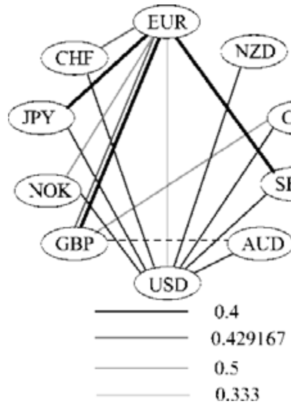

Fig. 2. Pictorial illustration of characteristic periodicity distribution on a currency network. Nodes represent currencies and links represent currency pairs. The same gray-scaled links exhibit pair with the same characteristic frequency. A bold/dashed links represent unique periodicity/non-detectable pair.

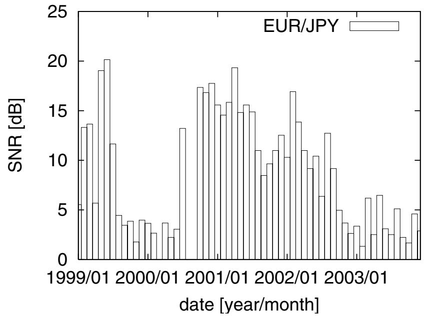

Fig. 3. The signal-to-noise ratio of tick frequencies calculated from power spectra averaged over a month for EUR/JPY during a period from January 1999 to December 2003.

Table 1. The characteristic frequencies of which periodic motions for behavioral frequencies are observed in the foreign exchange market.

currency pair	characteristic frequency [1/min]
AUD/USD	0.4
NZD/USD	0.4
USD/CAD	0.4
USD/CHF	0.4
USD/JPY	0.4
USD/NOK	0.4
USD/SEK	0.4, 0.2
EUR/CHF	0.429167
EUR/GBP	0.429167
EUR/NOK	0.5
GBP/CAD	0.5
EUR/SEK	0.428472
EUR/USD	0.333
EUR/JPY	0.313889
GBP/AU	ND

3 Model

In order to explain these periodic motions consider an agent-based model of a financial market where N market participants trade M currency pairs. Each agent perceives both periodic exogenous information and endogenous information and chooses his/her investment attitude from three kinds of actions (buying, selling, and waiting). The characteristics of their investment attitudes determine rate fluctuations and the tick frequency. This structure has some analogy to populations of nonlinear elements such as Josephson junction arrays and neuronal arrays [6,12,37].

Let $y_{ij}(t)$ denote an investment attitude of the ith market participant for the jth currency pair. The market participants are able to select their actions $y_{ij}(t)$ from three investment attitudes coded as 1 (buying), -1 (selling), and 0 (waiting).

For simplicity the ith market participant perceives scalar information $x_i(t)$. If $x_i(t) > (<) 0$ then he/she has a tendency to be a buyer (seller). This information builds a momentum when he/she decides his/her investment attitude. He/she makes a time-dependent interpretation based on the information. Since the possibility of interpretation is quite high, the interpretation is assumed to be described as $x_i(t) + \xi_i(t)$ by using a noise term. The interpretation drives the participant's feeling and his/her feeling about the feeling [38]. Since the feeling about the feeling sometimes becomes contrary to the feeling, it is expressed as $\Phi_{ij}(t) = a_{ij}(t)(x_i(t) + \xi_i(t))$ by using a multiplicative factor $a_{ij}(t)$. If $a_{ij}(t)$ is positive/negative then the feeling about the feeling supports/refutes the feeling.

The action is determined on the basis of the participant's feeling about the feeling. Since the decision and action have strong nonlinearity one may choose the Granovetter type of threshold dynamics [39].

$$y_{ij}(t) = \begin{cases} 1 & (\Phi_{ij}(t) \geq \theta^B_{ij}(t)) \\ 0 & (\theta^S_{ij}(t) < \Phi_{ij}(t) < \theta^B_{ij}(t)) \\ -1 & (\Phi_{ij}(t) \leq \theta^S_{ij}(t)), \end{cases} \tag{4}$$

where $\theta^B_{ij}(t)/\theta^S_{ij}(t)$ $(\theta^B_{ij}(t) > \theta^S_{ij}(t))$ denotes the threshold for the ith market participant to make a selling/buying decision for the jth currency pair.

The excess demand for the jth currency pair, $\sum_{i=1}^N y_{ij}(t)$, drives the market price $P_j(t)$ of the jth currency pair [40]: $R_j(t) = \log P_j(t + \Delta t) - \log P_j(t)$ and defines the log returns as the excess demand,

$$R_j(t) = \frac{\gamma}{N} \sum_{i=1}^{N} y_{ij}(t), \tag{5}$$

where γ is a positive constant to show the response of the return to the excess demand. Furthermore the tick frequency for the jth currency pair is defined as

$$A_j(t) = \frac{1}{\Delta t} \sum_{i=1}^{N} |y_{ij}(t)|. \tag{6}$$

Moreover it is assumed that the information which the ith market participant perceives at time t is described as the endogenous factor, moving average of log returns over $T_{ij}(t)$, plus the exogenous factor, $s_i(t)$:

$$x_i(t) = \sum_{k=1}^{M} c_{ik}(\theta^B_{ik}(t), \theta^S_{ik}(t)) \times \frac{1}{T_{ik}(t)} \sum_{\tau=1}^{T_{ik}(t)} R_k(t - \tau\Delta t) + s_i(t), \tag{7}$$

where $c_{ik}(\theta^B_{ik}(t), \theta^S_{ik}(t))$ represents the attention of the ith market participant for the jth currency pair. $T_{ij}(t)$ represents a time interval which the ith market participant uses in order to calculate the moving average for the jth currency pair at time t.

If $s_i(t)$ is positive/negative, then it builds the momentum when the ith market participant has a tendency to determine a buying/selling attitude. It seems reasonable to assume that $c_{ik}(x, y)$ has a tendency to be zero for $x \to \infty$ and $y \to -\infty$, $\lim_{x\to\infty, y\to-\infty} c_{ik}(x, y) \to 0$ since the market participants have a tendency to pay attention only to financial commodities which they frequently trade.

In this agent-based model each agent possesses $4M$ time-dependent behavioral parameters: $\theta^B_{ij}(t)$, $\theta^S_{ij}(t)$, $a_{ij}(t)$ and $T_{ij}(t)$. The whole model has $4MN$ behavioral parameters.

4 Theoretical analysis

For the sake of convenience assume that uncertainty of interpretation $\xi_i(t)$ is sampled from a zero-mean Gaussian distribution with a standard deviation σ. Moreover the correlation function is assumed to be

$$\langle \xi_i(t_1)\xi_j(t_2)\rangle = \sigma^2 \delta_{i,j}\delta_{t_1,t_2}, \tag{8}$$

where $\delta_{i,j} = 1(i = j)/ = 0(i \neq j)$. These assumptions mean that interactions among the interpretation of each individual are very weak in the financial market and that the interpretation of each individual randomly varies without correlation.

Then the probability for the ith agent to choose $y_{ij} = 1, 0, -1$ is given by

$$Q_{ij}(y_{ij}; t) = \begin{cases} \frac{1}{2}\mathrm{erfc}\Big(\frac{\theta^B_{ij}(t)/a_{ij}(t) - x_i(t)}{\sqrt{2}\sigma}\Big) & (y_{ij} = 1) \\ \frac{1}{2}\mathrm{erfc}\Big(\frac{x_i(t) - \theta^S_{ij}(t)/a_{ij}(t)}{\sqrt{2}\sigma}\Big) & (y_{ij} = -1) \\ 1 - Q_{ij}(1; t) - Q_{ij}(-1; t) & (y_{ij} = 0), \end{cases} \tag{9}$$

where $\mathrm{erfc}(x)$ is the complementary error function. From equation (9) one can describe

$$\langle y_{ij}(t)\rangle = Q_{ij}(1; t) - Q_{ij}(-1; t), \tag{10}$$

$$\langle |y_{ij}(t)|\rangle = Q_{ij}(1; t) + Q_{ij}(-1; t). \tag{11}$$

From equations (5), (6), (10), and (11) one obtains

$$\langle R_j(t)\rangle = \frac{\gamma}{N} \sum_{i=1}^{N} f\Big(x_i(t); \theta^B_{ij}(t)/a_{ij}(t), \theta^S_{ij}(t)/a_{ij}(t)\Big), \tag{12}$$

$$\langle A_j(t)\rangle = \frac{1}{\Delta t} \sum_{i=1}^{N} g\Big(x_i(t); \theta^B_{ij}(t)/a_{ij}(t), \theta^S_{ij}(t)/a_{ij}(t)\Big), \tag{13}$$

where

$$f(x;\alpha,\beta) = \frac{1}{2}\mathrm{erfc}\Big(\frac{\alpha - x}{\sqrt{2}\sigma}\Big) - \frac{1}{2}\mathrm{erfc}\Big(\frac{x-\beta}{\sqrt{2}\sigma}\Big), \quad (14)$$

$$g(x;\alpha,\beta) = \frac{1}{2}\mathrm{erfc}\Big(\frac{\alpha - x}{\sqrt{2}\sigma}\Big) + \frac{1}{2}\mathrm{erfc}\Big(\frac{x-\beta}{\sqrt{2}\sigma}\Big). \quad (15)$$

Introducing noises $\eta_j(t)$ and $\epsilon_j(t)$ we may approximate $R_j(t)$ and $A_j(t)$ as

$$R_j(t) = \langle R_j(t)\rangle + \eta_j(t), \quad (16)$$

$$A_j(t) = \langle A_j(t)\rangle + \epsilon_j(t). \quad (17)$$

According to the central limit theorem it is natural that $\eta_j(t)$ and $\epsilon_j(t)$ are assumed to be identically and independently distributed Gaussian noises. Therefore $R_j(t)$ and $A_j(t)$ are described as a nonlinear dynamical system with noise.

5 Discussion

If the exogenous information is assumed to be periodic

$$s_i(t) = q_l \sin(2\pi \Delta t f_l t), \quad (l = 1, 2, \ldots, K) \quad (18)$$

where q_l denotes an amplitude, f_l is a characteristic frequency of which the agents belonging to the lth group (W_l) perceive exogenous information then the characteristic periodic motion of $A_j(t)$ emerges. Suppose that the ith market participant belongs to the lth group ($i \in W_l$) and pays attention to the j_lth financial commodities ($j_l \in U_l$). Namely one can assume $\theta^B_{ij_l}(t) = \infty$ and $\theta^S_{ij_l}(t) = -\infty$ for $i \notin W_l$. Since one has $\lim_{\alpha\to\infty,\beta\to-\infty} f(0;\alpha,\beta) = 0$, $\lim_{\alpha\to\infty,\beta\to-\infty} g(0;\alpha,\beta) = 0$, and $\lim_{x\to\infty,y\to-\infty} c_{ik}(x,y) = 0$, using equations (7), (12), and (13) one can rewrite equations (16) and (17) as

$$\begin{aligned} R_{j_l}(t) = \frac{\gamma}{N}\sum_{i\in W_l}^{N} f\Big(&\sum_{k\in\{U_l\}} c_{ik}(\theta^B_{ik}(t), \theta^S_{ik}(t)) \\ &\times \frac{1}{T_{ik}(t)}\sum_{\tau=1}^{T_{ik}(t)} R_k(t-\tau\Delta t) + a_l \sin(2\pi\Delta t f_l t); \\ &\theta^B_{ij_l}(t)/a_{ij_l}(t), \theta^S_{ij_l}(t)/a_{ij_l}(t)\Big) + \eta_{j_l}(t), \end{aligned} \quad (19)$$

$$\begin{aligned} A_{j_l}(t) = \frac{1}{\Delta t}\sum_{i\in W_l}^{N} g\Big(&\sum_{k\in\{U_l\}} c_{ik}(\theta^B_{ik}(t), \theta^S_{ik}(t)) \\ &\times \frac{1}{T_{ik}(t)}\sum_{\tau=1}^{T_{ik}(t)} R_k(t-\tau\Delta t) + a_l \sin(2\pi\Delta t f_l t); \\ &\theta^B_{ij_l}(t)/a_{ij_l}(t), \theta^S_{ij_l}(t)/a_{ij_l}(t)\Big) + \epsilon_{j_l}(t). \end{aligned} \quad (20)$$

Therefore the j_lth financial commodities are contained in the lth group, and periodic motions with the same characteristic frequency appear for j_lth financial commodities.

The periodic exogenous information is thought of as periodic stimuli which market participants perceive through computer terminals as part of the electronic brokering system. Specifically, since these periodic motions are observed only for the European time zone, it is inferred that only European market participants perceive these stimuli.

From Appendix A it is clarified that the resonance depends on the behavioral parameters of the market participants. Namely the signal-to-noise ratio reflects the states of the behavioral parameters. If the behavioral parameters of the market participants vary in time, then the signal-to-noise ratio during observation periods depends on the observation period. This suggests the hypothesis that the signal-to-noise ratio is related to the states of market participants. In fact Figure 3 shows this dependency of the signal-to-noise ratio on the observation period.

Actually in psychophysiology it is known that in the method of constant stimuli the probability of decision-making is approximated as the logistic function if stimuli are given at random [41]. Since the facts that the same stimuli do not always lead to the same response and that the probability is well approximated as the error function which one of the candidates having the similar properties to the logistic function are consistent with equations (10) and (11), the assumption equation (8) seems to be adequate. Note that the probabilities of decision-making are dependent on the past interpretations or the past decisions and that $Q_{ij}(y;t)$ can be described as heterogeneous time-dependent functions in terms of $a_{ij}(t)$ and $x_i(t)$ if uncertainty of interpretation in equation (8) is not independent but correlated. If $\xi_i(t)$ is autocorrelated then both the noise strength and the autocorrelation times affect the signal-to-noise ratio as indicated in successive studies of stochastic resonance [42,43]. Furthermore if $\xi_i(t)$ is cross-correlated for each other then the effect of stochastic resonance weakens according to array stochastic resonance [44,45]. Detailed investigations on cases of autocorrelated and cross-correlated noises are reserved for future works.

6 Numerical simulation

In order to perform numerical simulation of the agent-based model, where 2 000 market participants trade two kinds of currency pairs we assume two groups (the number of the population is 1 000) of market participants who exchange a currency pair and perceive the periodic information. Each market participant belonging to the lth group perceives sinusoidal information with amplitude a_l and characteristic frequency f_l ($l = 1, 2$). In order to calculate this agent model we assumed that $c_{ik}(x,y) = m_{ik}/(x^2+y^2)$, where m_{ik} is a random matrix taking ± 1 with the same probability 1/2. Figure 4 shows the power spectra multiplied by the low-cut filter, equation(2). It is found that the periodic motion of the tick frequency appears. Namely the group of currency pairs with the same characteristic frequency may mean that the market participants who belong to the same group pay attention to and trade those currency pairs.

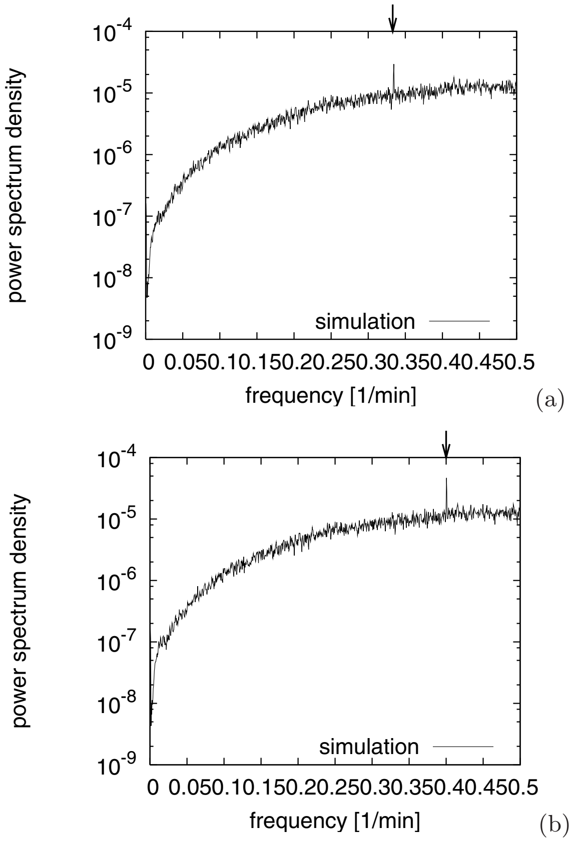

Fig. 4. Filtered power spectra obtained from numerical simulation at $\Delta t = 1$, $\theta_{ij}^{B}(t)$ which is initially sampled from a normal distribution with a range $[0.1, 0.2]$ for $1 < i \leq 1\,000$ and $j = 1$, and $1\,000 < i \leq 2\,000$ and $j = 2$, and fixed at 10 000 for otherwise, $\theta_{ij}^{S}(t)$ which is initially sampled from a normal distribution with a range $[-0.2, -0.1]$ for $1 < i \leq 1\,000$ and $j = 1$, and $1\,000 < i \leq 2\,000$ and $j = 2$, and fixed at $-10\,000$ for otherwise, $N = 2\,000$, $M = 2$, $\sigma = 0.23$, $q_1 = 0.1$, $q_2 = 0.1$, $f_1 = 0.333$ and $f_2 = 0.8$, and $T_{ij}(t) = 1$. Furthermore $a_{ij}(t)$ is assumed as $a_{ij}(t) = a + a'_{ij}(t)$, where $a = 0.1$ is a real number which represents a mean of $a_{ij}(t)$. $a'_{ij}(t)$ is assumed that they are sampled from an identical and independent zero-mean Gaussian distribution with standard deviation $\sigma_{a'} = 0.5$. (a) exhibits a power spectrum for the first group and (b) that for the second one.

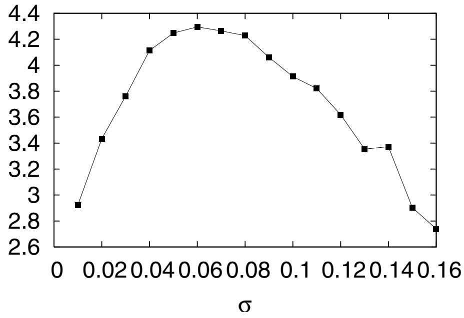

Fig. 5. A relation between uncertainty of interpretation σ and the signal-to-noise ratio of behavioral frequency of the first group f_1 obtained from numerical simulation at $\Delta t = 1$, $\theta_{ij}^{B}(t)$ which is initially sampled from a normal distribution with a range $[0.1, 0.2]$ for $i \leq 1\,000$ and $j = 1$, and $i > 1\,000$ and $j = 2$, and fixed at 10 000 for otherwise, $\theta_{ij}^{S}(t)$ which is initially sampled from a normal distribution with a range $[-0.2, -0.1]$ for $i \leq 1\,000$ and $j = 1$, and $i > 1\,000$ and $j = 2$, and fixed at $-10\,000$ for otherwise, $N = 2\,000$, $M = 2$, $a = 0.1$, $\sigma_{a'} = 0.5$, $q_1 = 0.1$, $q_2 = 0.1$, $f_1 = 0.333$ and $f_2 = 0.8$.

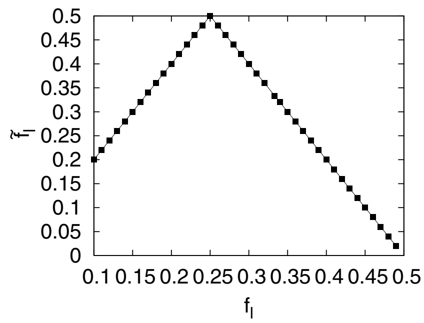

Fig. 6. A relation between the characteristic frequency of the first group $\tilde{f}_1$ and a frequency of exogenous periodic information f_1 obtained from numerical simulations at $\Delta t = 1$, $\theta_{ij}^{B}(t)$ which is initially sampled from a normal distribution with a range $[0.1, 0.2]$ for $i \leq 1\,000$ and $j = 1$, and $i > 1\,000$ and $j = 2$, and fixed at 10 000 for otherwise, $\theta_{ij}^{S}(t)$ which is initially sampled from a normal distribution with a range $[-0.2, -0.1]$ for $i \leq 1\,000$ and $j = 1$, and $i > 1\,000$ and $j = 2$, and fixed at $-10\,000$ for otherwise, $N = 2\,000$, $M = 2$, $a = 0.1$, $\sigma = 0.23$, $\sigma_{a'} = 0.5$, $q_1 = 0.1$, $q_2 = 0.1$ and $f_2 = 0.8$. The line represents a fitting curve: $\tilde{f}_l = (1 - |1 - 4f_l|)/2$.

As shown in Figure 5, the signal-to-noise ratio of behavioral frequencies defined as equation (3) is a function with an extreme value of σ. Namely stochastic resonance occurs in the agent-based model as predicted in the discussion section, Section 5. This result proposes the hypothesis that dependency of the signal-to-noise ratio on the observation period as shown in Figure 3 is related to resonance due to the variation of behavioral parameters of the market participants.

Peak positions of power spectra for tick frequencies $\tilde{f}$ depend on the frequency of exogenous periodic information f_l. As shown in Figure 6 it is confirmed that a characteristic frequency of lth group $\tilde{f}_l$ is described as

$$\tilde{f}_l = \frac{1}{2}\Big(1 - |1 - 4f_l|\Big). \tag{21}$$

This dependency comes from a second harmonic wave due to the nonlinearity of equations (19) and (20).

7 Conclusion

Empirical analysis of tick frequency data in the foreign exchange market showed that characteristic periodic motions are confirmed for all currency pairs and that there exist several groups with the same characteristic frequency. The signal-to-noise ratio of behavioral frequencies is dependent on observation periods.

Theoretical analysis of the agent-based model proposed the hypothesis that the relationship among characteristic periodicities is associated with the structure of which market participants pay attention to currency pairs. If different sources of periodic information exist and market participants pay attention to several currency pairs, then this finding is explainable. The dependency of the signal-to-noise ratio of the tick frequencies is derived from the variation of behavioral parameters of the market participants.

The applications of this finding are quantification of the states of market participants, and an influential structure through which the market participants perceive information. Quantifying the temporal structure of these groups is expected to lead to deep understanding of the foreign exchange market. It would be interesting to understand the meaning of the specific frequencies observed in our data for the foreign exchange markets. A broader data analysis and careful statistical tests are reserved for future work.

Analyzing the periodicities of collective motions is expected to provide insights into the coupling structure of individuals of many body systems in many fields of both the natural and social sciences.

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Appendix A: Linear response

Specifically, under the assumption that $x_i(t)$ is small and that $T_{ik}(t) = T$, the Taylor expansion of $\mathbf{R}(t) = [R_1(t) \dots R_M(t)]^t$ and $\mathbf{A}(t) = [A_1(t) \dots A_M(t)]^t$ and ignoring the higher order terms than the second order yields

$$\mathbf{R}(t) = \sum_{\tau=1}^{T} \mathbf{G}_\tau(t)\mathbf{R}(t-\tau\Delta t) + \mathbf{S}_R(t) + \eta'(t), \quad (22)$$

$$\mathbf{A}(t) = \sum_{\tau=1}^{T} \mathbf{D}_\tau(t)\mathbf{R}(t-\tau\Delta t) + \mathbf{S}_A(t) + \epsilon'(t), \quad (23)$$

where

$$(\mathbf{G}_\tau(t))_{jk} = \frac{\gamma}{NT}\sum_{i=1}^{N} f'\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big) \times c_{ik}\big(\theta_{ik}^B(t),\theta_{ik}^S(t)\big), \quad (24)$$

$$(\mathbf{D}_\tau(t))_{jk} = \frac{1}{T\Delta t}\sum_{i=1}^{N} g'\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big) \times c_{ik}\big(\theta_{ik}^B(t),\theta_{ik}^S(t)\big), \quad (25)$$

$$(\mathbf{S}_R(t))_j = \frac{\gamma}{NT}\sum_{i=1}^{N} f'\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big)s_i(t), \quad (26)$$

$$(\mathbf{S}_A(t))_j = \frac{1}{T\Delta t}\sum_{i=1}^{N} g'\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big)s_i(t), \quad (27)$$

$$(\eta'(t))_j = \frac{\gamma}{NT}\sum_{i=1}^{N} f\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big) + \eta_j(t),$$

$$(\epsilon'(t))_j = \frac{1}{T\Delta t}\sum_{i=1}^{N} g\big(0;\theta_{ij}^B(t)/a_{ij}(t),\theta_{ij}^S(t)/a_{ij}(t)\big) + \epsilon_j(t).$$

$f'(0;\alpha,\beta)$ and $g'(0;\alpha,\beta)$ are respectively calculated as

$$f'(0;\alpha,\beta) = 1/\sqrt{2\pi\sigma} \times\{\exp[-\alpha^2/2\sigma^2] + \exp[-\beta^2/2\sigma^2]\}, \quad (28)$$

$$g'(0;\alpha,\beta) = 1/\sqrt{2\pi\sigma} \times\{\exp[-\alpha^2/2\sigma^2] - \exp[-\beta^2/2\sigma^2]\}. \quad (29)$$

Furthermore the signal-to-noise ratio at f_l depends on behavioral parameters, $\theta_{ij}^B(t)/a_{ij}(t)$, $\theta_{ij}^S(t)/a_{ij}(t)$, and σ since coefficients for $s_i(t)$ of linear response, which are derived from equations (19) and (20) under $x_i(t) \ll 1$, depend on these behavioral parameters. This dependency of the signal-to-noise ratio on these behavioral parameters is estimated from the following relationship: $f'(0;\alpha,\beta)$ is a function which is a monotonically decreasing function for $|\alpha|$ and $|\beta|$ and has an extremal value at $\sigma = \sigma_1^*$, respectively. $g'(0;\alpha,\beta)$ is a function which is a monotonically decreasing function for $|\alpha|$ and a monotonically increasing function for $|\beta|$ and has an extremal value at $\sigma = \sigma_2^*$, respectively. Here σ_1^* and σ_2^* are solutions of $\frac{\partial}{\partial\sigma}f'(0;\alpha,\beta) = 0$ and $\frac{\partial}{\partial\sigma}g'(0;\alpha,\beta) = 0$, respectively. Specifically, if the signal-to-noise ratio depends on σ, then the resonance is classified as a stochastic resonance.

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