Very peculiar properties of kinks in a driven damped anisotropic spin chain

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The motion of kink solitons in a classical Heisenberg chain with a composite anisotropy (an easy-magnetization-plane anisotropy with an additional easy-magnetization axis along that plane) is studied. Both the Gilbert damping and a spatially uniform external field applied along the easy axis are taken into account. For applied fields lower than a certain critical value the velocity of the kink is constant and its magnitude results from a balance between the external field and the damping. For fields larger than critical the motion of the kink is a nontrivial superposition of the forward translational and of the oscillatory motions. The special case of large anisotropy is analyzed in detail. The effect of damping and of the external field on the motion of the kink is discussed.

I. INTRODUCTION

Recently, one-dimensional nonlinear systems with conservative forces as well as dissipative effects have been of great interest. The coexistence of nonlinearity and dissipation may cause some very interesting phenomena, e.g., the appearance of chaos in completely deterministic systems,[1] or the occurrence of inertial modes.[2] On the other hand, it is a well-known fact that in one-dimensional nonlinear systems without damping, large-amplitude excitations in the form of pulses, kinks, or other solitary waves can propagate with constant velocities and constant shapes. For systems which are completely integrable, these excitations are called solitons.[3] It seems interesting to investigate the motion of such solitary waves in one-dimensional nonlinear systems with damping and external forces included.

In this paper we investigate the motion of kinks in a classical Heisenberg chain with a special double anisotropy consisting of an easy-magnetization-plane anisotropy and a uniaxial anisotropy with the easy-magnetization axis within that plane,[4-6] as well as with both the Gilbert damping and an external field along the easy axis taken into account. This problem is formally identical to the very-well-known problem of domain-wall dynamics in bulk ferromagnetic materials[7-9] (also see Refs. 10 and 11). The solutions proposed for domain-wall dynamics may be easily applied to the problem of the driven and damped motion of solitons. In such a way some very peculiar behavior of solitons may be obtained.

II. EQUATIONS OF MOTION

We consider a spin chain with the Hamiltonian

$$\mathscr{H}=\sum_i[-\widetilde{J}\mathbf{S}_i\cdot\mathbf{S}_{i+1}+\widetilde{A}(S_i^x)^2-\widetilde{C}(S_i^z)^2-\widetilde{B}\gamma S_i^z]\ , \tag{1}$$

where the spins $\mathbf{S}_i$ are treated classically, $|\mathbf{S}_i|=S$, $\gamma=-g|\mu_B|\hbar^{-1}$ with g the Landé factor, μ_B the Bohr magneton, and $\hbar$ the reduced Planck constant. The constants $\widetilde{J}$, $\widetilde{A}$, and $\widetilde{C}$ are assumed positive. The first term in (1) is the nearest-neighbor exchange interaction, the next two represent the easy-plane and the easy-axis anisotropy, respectively, while the last term gives the energy of the spin in the external field B. The spin coordinates are written in the form

$$\mathbf{S}_i=S(\sin\theta_i\cos\varphi_i,\ \sin\theta_i\sin\varphi_i,\ \cos\theta_i)\ .$$

The continuum version of the Hamiltonian (1) is then obtained if θ_i and φ_i are changed to $\theta(x,t)$ and $\varphi(x,t)$:

$$\mathscr{H}=\int\frac{dx}{a}\left[\frac{\widetilde{J}a^2}{2}\left[\frac{\partial\mathbf{S}}{\partial x}\right]^2+\widetilde{A}[S^x(x,t)]^2-\widetilde{C}[S^z(x,t)]^2-\widetilde{B}\gamma S^z(x,t)\right]\equiv\int\frac{dx}{a}\widetilde{h}[\mathbf{S}(x,t)]\ , \tag{2}$$

where a is the lattice constant. The classical equation of motion for the spin vector $\mathbf{S}$,

$$\dot{\mathbf{S}}=-\mathbf{S}\times\frac{\delta\widetilde{h}}{\delta\mathbf{S}}-\frac{\alpha\mathbf{S}\times\dot{\mathbf{S}}}{S} \tag{3}$$

(where α is the Gilbert damping parameter, $0<\alpha\ll 1$), written with the use of the angle variables $\theta(x,t)$ and $\varphi(x,t)$, has the form

$$\dot{\theta}\sin\theta=\frac{\delta\widetilde{h}}{\delta\varphi}S^{-1}+\alpha\dot{\varphi}\sin^2\theta\ , \tag{4}$$

$$-\dot{\varphi}\sin\theta=\frac{\delta\widetilde{h}}{\delta\theta}S^{-1}+\alpha\dot{\theta}\ . \tag{5}$$

Using the Hamiltonian density $\widetilde{h}[\mathbf{S}(x,t)]$ expressed in terms of the angles θ and φ,

$$\widetilde{h}[\mathbf{S}(x,t)]=\frac{\widetilde{J}a^2S^2}{2}(\theta_x^2+\varphi_x^2\sin^2\theta)+\widetilde{A}S^2\sin^2\theta\cos^2\varphi-\widetilde{C}S^2\cos^2\theta-\gamma\widetilde{B}S\cos\theta\ , \tag{6}$$

one finds

$$\dot\theta\sin\theta-\alpha\dot\varphi\sin^2\theta=-J\frac{\partial}{\partial x}(\sin^2\theta\,\varphi_x)-2A\sin^2\theta\sin\varphi\cos\varphi\ , \quad (7)$$

$$-\dot\varphi\sin\theta-\alpha\dot\theta=-J\theta_{xx}+J\sin\theta\cos\theta(\varphi_x)^2+2A\sin\theta\cos\theta\cos^2\varphi+2C\sin\theta\cos\theta+B\sin\theta\ , \quad (8)$$

where $J=\tilde{J}Sa^2$, $A=\tilde{A}S$, $C=\tilde{C}S$, $B=\tilde{B}\gamma$, the subscript x signifies a spatial derivative along the chain, a dot over a symbol denotes the derivative with respect to time.

III. SOLITARY WAVES: π KINKS

For $B=0$ and $\alpha=0$ the system (7) and (8) is completely integrable[6] and one-soliton solutions were found in the form of π kinks:[4–6]

$$\cos\theta=\pm\tanh\left[\frac{\mu}{d}(x-vt)\right], \quad (9)$$

$$\mu=\left[1+\frac{A}{C}\cos^2\varphi_0\right]^{1/2},\quad v=\frac{A}{\mu}d\sin(2\varphi_0)\ , \quad (10)$$

with $\varphi=\varphi_0=\text{const}$ and $d=(J/2C)^{1/2}$.

If the damping parameter α is different from zero with B still zero, one may expect that, after adiabatically switching on the dissipative effects, the solutions (9) and (10) should evolve to the solutions corresponding to the static π kinks with $\varphi_0=(2n+1)(\pi/2)$ (such kinks possess the minimum energy). If, however, the external field B is also not zero, it may be conjectured that a stationary solution may be found only if there is compensation of the dissipative effects by the driving force connected with the external field, exactly as in the case of Walker's solution for domain-wall dynamics in bulk ferromagnetic materials[7] (also see Refs. 8—10). To show that this is really the case, we seek the solution of Eqs. (7) and (8) in the form of the solitary waves (9) and (10), i.e., we now assume that $\varphi=\varphi_0=\text{const}$ and that $\theta(x,t)=\theta(x-vt)$. Then Eqs. (7) and (8) lead to the following:

$$v\frac{d\theta}{du}=2A\sin\theta\sin\varphi_0\cos\varphi_0\ , \quad (11)$$

$$-J\theta_{xx}+2(A\cos^2\varphi_0+C)\sin\theta\cos\theta+B\sin\theta-\alpha v\frac{\partial\theta}{\partial x}=0\ , \quad (12)$$

where $u=x-vt$. Assuming $v\neq0$, Eq. (11) may be integrated directly, giving as a result π kinks with the same characteristics as (9):

$$\cos\theta(x,t)=-\tanh\left[\frac{A}{v}(x-vt)\sin(2\varphi_0)\right]. \quad (13)$$

Together with (13), Eq. (12) leads to the following:

$$B-\alpha A\sin(2\varphi_0)=\cos\theta\left[\frac{JA^2}{v^2}\sin^2(2\varphi_0)-2(A\cos^2\varphi_0+C)\right]. \quad (14)$$

The left-hand side of (14) is a constant and—for $v=\text{const}$—the right-hand side is just another constant multiplied by $\cos\theta$. Hence Eq. (14) has a nontrivial solution (i.e., one for which $\cos\theta$ is not a constant everywhere) only if both constants in this equation are equal to zero. One then obtains

$$B=\alpha A\sin(2\varphi_0)\ , \quad (15)$$

$$v^2=\frac{JA^2}{2}\frac{\sin^2(2\varphi_0)}{A\cos^2\varphi_0+C}\ . \quad (16)$$

Formula (16) means that the relation (10) also holds when both the damping and the drive are turned on. From (15) it is seen that the stationary motion solution (13) may be obtained for the π kink only if $|B|\leq B_c=\alpha A$. The critical field B_c is then the limiting field above which no solutions in the form of solitary waves propagating with a constant velocity exist.

For $|B|<B_c$ there are two solutions with $\varphi_0=\text{const}$, namely $\varphi_0^{(1)}$ and $\varphi_0^{(2)}=\pi/2-\varphi_0^{(1)}$, both given by

$$\cos^2(\varphi_0)=\frac{1}{2}\left\{1\mp\left[1-\left[\frac{B}{\alpha A}\right]^2\right]^{1/2}\right\}. \quad (17)$$

The lower sign in (17) corresponds to an unstable solution for π-kink motion, i.e., such that any small perturbation will cause the value of the angle φ_0 [given initially by (17) with the minus sign] to change (see Sec. VI of this paper for details). Equations (15) and (16) together yield the dependence of the velocity v on the external field B. One obtains, for the stable solutions,

$$v=\pm B\left[\frac{J}{\alpha^2\{A[1-(1-B^2/\alpha^2A^2)^{1/2}]+2C\}}\right]^{1/2}. \quad (18)$$

For $|B|\ll\alpha A$, or for $C\gg A$, the π-kink velocity v is thus roughly proportional to B, so that in such a case the motion of the kink is similar to the Newtonian motion of a forced damped classical particle. Substituting (15) and (18) into (13), one obtains

$$\cos\theta=-\tanh\left[\pm\frac{x-vt}{\delta}\right], \quad (19)$$

where

$$\delta=\left[\frac{J}{A[1-(1-B^2/\alpha^2A^2)^{1/2}]+2C}\right]^{1/2} \quad (20)$$

denotes the width parameter of the π kink propagating with the velocity (18). It is interesting that the sign in (19) is a consequence of the sign in (18), i.e., for a given sign of the field B lower than the critical field B_c, the kink moves in one direction (favoring one region of uniform magnetization lying along the easy axis), while an antikink moves in the opposite direction. Note also that for $B=0$ and $\alpha\neq0$, the only solitary-wave solution is for $v=0$ and

$\varphi_0=(2n+1)(\pi/2)$, n an integer.

For the critical field $B=B_c'=\alpha A$, the corresponding velocity is

$$|v_c|=[JA^2/(A+2C)]^{1/2} .$$

Note that v_c does not depend on the parameter α. Also, v_c is not the maximum velocity of π kinks, which is equal to

$$|v_{\max}|=[(1+A/C)^{1/2}-1](2CJ)^{1/2} ,$$

and except for the case of $A=0$, when $|v_c|=|v_{\max}|=0$, one always has $|v_c|<|v_{\max}|$.

Independent of the above solutions, representing π kinks propagating, in general, with constant velocities, there is also a static solution representing a 2π kink at rest. To obtain this solution it suffices to set $v=0$ in Eq. (12). As a result, the static version of the double-sine-Gordon equation is found, which for $B\neq 0$ has a solution in the form of two coupled π kinks[12] (for $B\to 0$ the distance between these tends to infinity). Equation (11) shows that in this case $\varphi_0=(2n+1)(\pi/2)$, but the solutions with $\varphi_0=2n\pi$ correspond to unstable configurations.[4] Below we discuss only the dynamic solutions in the form of π kinks.

IV. TRANSIENT MOTION OF π KINKS

Now it would be interesting to know how π kinks behave in the driven and damped case when the external field is suddenly switched on. To answer this question we first assume that $B(t)=B\,\epsilon(t)$, where $\epsilon(t)$ is the Heaviside step function equal to 1 for $t>0$ and 0 otherwise. In this case we look for a solution in the form

$$\cos\theta(x,t)=-\tanh[\pm s(t)],\quad \varphi=\varphi(t) , \tag{21a}$$

$$s(t)=\frac{x-q(t)}{\delta(t)} , \tag{21b}$$

i.e., in the form (19), but with both the position q and the width-parameter δ functions of the time. Because our equations of motion are formally identical to those used by Schryer and Walker for the dynamics of a domain wall, one may apply the results obtained numerically by them[8] directly to our problem. Thus, if $|B|\leq B_c$, then the velocity of π kinks increases monotonically from zero for the static solution to a value given by (18) corresponding to the stationary-motion solution at $t\to\infty$. Simultaneously, both the wall width parameter δ as well as the angle φ change from the values they have in the static solution to those corresponding to a given magnitude of the field B during stationary motion.

V. OSCILLATORY MOTION OF π KINKS

The nature of the solutions which one obtains for $B(t)=B\,\epsilon(t)$ with $|B|>B_c$ is certainly very peculiar. No motion with a constant angle φ is then possible. The numerical analysis of Schryer and Walker[8] mapped to our situation shows that, in this case, the motion of π kinks consists of a running motion with an oscillatory motion superimposed. Namely, after the external field is turned on at $t=0$, the velocity of the kink—assumed zero initially—quickly increases to a maximum value. Simultaneously, the width-parameter value decreases and the azimuthal angle φ changes monotonically to the value $\pi/4+n\pi$, where n is an integer. In the next phase of the motion, the kink velocity slowly begins to decrease, which is accompanied by a further decrease of the width parameter and a further monotonical change of the azimuthal angle φ. Next, the velocity decreases, rapidly reaching zero, or even negative values down to a certain minimum value. During the whole of this last phase of the motion, the kink moves in the direction *opposite to that* in the earlier phases. After that, the kink velocity begins to increase again, which is accompanied by an increase of the width parameter and quick changes of the angle φ. At a certain moment in time, say $t=T$, the instantaneous velocity of the kink attains zero, the width parameter has the value corresponding to the static kink and the value of the azimuthal angle φ differs from the initial by π, exactly. At that moment the kink looks the same as the beginning of the motion except for two features. First, if the kink had at $t=0$ a right (left) handedness, at $t=T$ it now has a left (right) handedness. Second, the kink has moved forward under the influence of the external field from its starting point to a new position. Between $t=T$ and $t=2T$ the kink again moves forward under the influence of the field—in much the same way as between $t=0$ and $t=T$—however, its left (right) handedness changes to the right (left) one. The motion of the kink then has, to some extent, an oscillatory character; however, a net forward motion results. The physical reason for such a character of the motion lies in the interplay between the external field, the anisotropy, and the damping. The situation here is exactly the same as in the case of domain-wall dynamics in bulk ferromagnetic materials. It is interesting that the very peculiar behavior of domain walls in bulk materials was confirmed experimentally.[13]

VI. APPROXIMATE ANALYTICAL SOLUTIONS OF TRANSIENT AND OSCILLATORY MOTION OF π KINKS FOR A SPECIAL CASE

As an illustration of both Secs. IV and V, we consider a special case which is easy to treat using an approximate analytical method. We assume that, during nonstationary motion of a kink, the angle φ is constant across the kink ($\varphi_x=0$), similar to the case of solitons without damping and no external field,[4–6], or for the undercritical stationary solutions for the driven damped kink (or a domain wall in bulk ferromagnetic materials). We now look for a solution in the form (21) where, for simplicity, we choose a definite sign—the positive one. Substituting this solution into Eqs. (7) and (8) gives two new equations,

$$\sin^2\theta\dot{s}(t)-\alpha\dot{\varphi}\sin^2\theta=-A\sin^2\theta\sin(2\varphi) , \tag{22a}$$

$$-\dot{\varphi}\sin\theta-\alpha\dot{s}(t)\sin\theta=-\frac{J\sin(2\theta)}{2[\delta(t)]^2}+B\sin\theta+(A\cos^2\varphi+C)\sin(2\theta) , \tag{22b}$$

with

$$\dot{s}=\frac{-\delta(t)\dot{q}(t)+\dot{\delta}(t)q(t)}{[\delta(t)]^2} . \tag{23}$$

Setting the coefficient of $\sin^2\theta$ in Eq. (22a) and the coefficients of both $\sin\theta$ and $\sin(2\theta)$ in Eq. (22b) simultaneously equal to zero, we obtain

$$-\dot{s}+\alpha\dot{\varphi}=A\sin(2\varphi) , \tag{24}$$

$$\dot{\varphi}+\alpha\dot{s}=-B , \tag{25}$$

$$\delta(t)=\left[\frac{J}{2(A\cos^2\varphi+C)}\right]^{1/2} . \tag{26}$$

Equation (26) shows that for nonstationary motion the dependence of the width parameter δ on the angle φ is the same as for the soliton solution described by Eqs. (9) and (10). Taking into account (23), we see that both Eqs. (24) and (25) are not consistent because φ does not depend on the variable x but $\dot{s}$ does. However, these equations may be made consistent if one neglects, in (23), the term proportional to $\dot{\delta}(t)$, i.e., if one treats the kink as a rigid object. This is acceptable if $C/A \gg 1$ because, due to the relation (26), $\delta(\varphi(t))$ in this case is practically a constant, i.e., $\delta(\varphi(t))\approx d=(J/2C)^{1/2}$ (a more rigorous condition $2C/\alpha A \gg 1$ was considered in Ref. 8, but it is consistent with ours as we have assumed $\alpha \ll 1$). With such an approximation it is easy to transform the pair of equations (24) and (25) to the following:

$$\dot{\varphi}=\frac{-B+\alpha A\sin(2\varphi)}{1+\alpha^2} , \tag{27}$$

$$q=q_0+\frac{d}{\alpha}[Bt+\varphi(t)] , \tag{28}$$

where we set $\dot{s}\approx-\dot{q}/\delta\approx-\dot{q}/d$. Integrating (27), we obtain, for $|B|<B_c=\alpha A$,

$$\varphi=\tan^{-1}\{b^{-1}+(b^{-2}-1)^{1/2}\times\text{cotanh}[(t-t_0)/\tau_1]\}+n\pi \tag{29a}$$

or

$$\varphi=\tan^{-1}\{b^{-1}+(b^{-2}-1)^{1/2}\times\tanh[(t-t_0)/\tau_1]\}+n\pi , \tag{29b}$$

where

$$\tau_1=\frac{1+\alpha^2}{B_c(1-b^2)^{1/2}} , \tag{29c}$$

and $b=B/B_c$, while n is an integer [depending on the initial conditions, we obtain (29a) or (29b)]; for $|B|=B_c=\alpha A$,

$$\varphi=\tan^{-1}\left[1+\frac{1+\alpha^2}{(t-t_0)B_c}\right]+n\pi , \tag{30}$$

and for $|B|>B_c=\alpha A$,

$$\varphi=\tan^{-1}\left[b^{-1}+(1-b^{-2})^{1/2}\text{cotan}\left[\frac{t-t_0}{\tau_2}\right]\right] , \tag{31}$$

where

$$\tau_2=(1+\alpha^2)/[B_c(b^2-1)^{1/2}] .$$

In all the above formulas t_0 is an arbitrary constant. Equations (29)–(31) combined with Eq. (28) describe the motion of the kink completely. From (29a) or (29b) in the limit of $t\rightarrow\infty$ we obtain for the angle φ a value

$$\varphi_s=\tan^{-1}\{[1+(1-b^{-2})^{1/2}]/b\}+n\pi ,$$

exactly the same as Eq. (17) taken with the negative sign—which corresponds to the stable stationary motion. Assuming formally that $t\rightarrow-\infty$ in the formulas (29a) and (29b), we obtain a different limiting value for the angle φ:

$$\varphi_{ns}=\tan^{-1}\{[1-(1-b^{-2})^{1/2}]/b\}+n\pi ,$$

which is equal to the solution (17) taken with the positive sign and corresponds to the unstable stationary motion. Relation (29) is depicted in Fig. 1 for two values of the reduced external field: $b=0.5$ (bold curve in Fig. 1) and $b=0.95$ (the thin line). It can be seen that for both values of the field, the curves join at the common value of $\varphi=\varphi_{ns}$ for $t-t_0\rightarrow-\infty$ and then separate into two branches which, for $t-t_0\rightarrow\infty$, achieve either the value φ_s (lower branch) or the value $\varphi_s+\pi$ (upper branch).[14] For a given value of the reduced applied field b, the lower branch of the curve in Fig. 1 describes the evolution of the angle φ of a kink for which, at the moment when the field b was applied (i.e., at a certain moment $t-t_0$), the angle φ was at an initial value ϕ_0 between φ_s and φ_{ns} [Eq. (29a)]. On the other hand, the upper branch of each curve in Fig. 1 describes the evolution of the angle φ for a kink for which the initial value ϕ_0 was between φ_{ns} and $\varphi_s+\pi$ [Eq. (29b)]. If, before the given value of the external field b was applied to the kink, no field had been applied to it, then ϕ_0 is at the stable value of $-\pi/2$. After the field is switched on, the angle φ evolves along the part of the

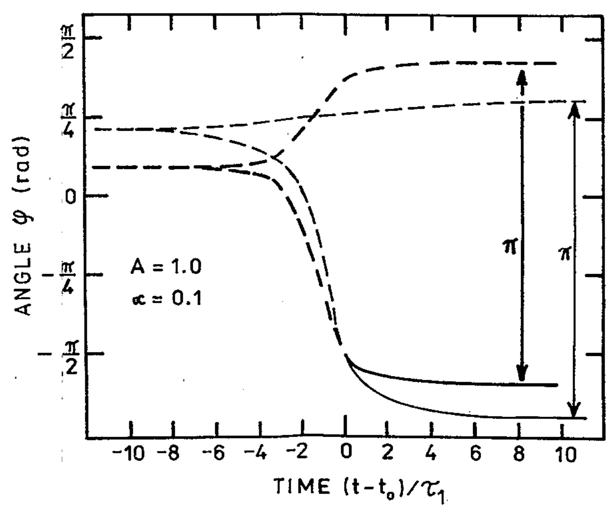

FIG. 1. Undercritical motion: azimuthal angle φ as a function of the time. Bold curve is for $b=0.5$ and thin line is for $b=0.95$. τ_1 taken with $b=0.5$. For the explanation of the solid and dashed lines, see text.

curve appropriate for the given value of b—this is marked in Fig. 1 by the solid line. On the other hand, if the previous history of the kink was such that, at the moment the external field was applied, the value of $\phi_0 \neq -\pi/2$, then the ensuing evolution of the angle φ also follows the curve proper for the given value of b, but the branch of that curve, along which the evolution will take place, depends on the range into which ϕ_0 falls. In any case the end result of this evolution is that $\varphi=\varphi_s$ modπ.

φ_s is then, for a given subcritical value of b, the angle for stable stationary motion, while φ_{ns} corresponds to an unstable stationary motion. To see the latter it suffices to assume that at the moment in time at which the external field was applied, the initial value $\phi_0=\varphi_{ns}$. From the shape of the two branches of each curve in Fig. 1, it is seen that any small perturbation of the angle φ will cause an evolution away from φ_{ns} along one of the two branches. Which branch the kink will follow depends on the sign of the perturbation.

Equation (27) may be treated as a one-dimensional equation of motion of a dynamical system.[15] The solutions φ_s and φ_{ns} are then the fixed points of the system. Their stability is given by the sign of the derivative with respect to φ of the right-hand side of (27) taken at these points. It is easy to show that at $\varphi=\varphi_{ns}$ the sign is positive, while at $\varphi=\varphi_s$ it is negative. Thus φ_{ns} is an unstable fixed point and φ_s is a stable fixed point, i.e., an attractor of the system.

The time dependence of the kink position $q(t)$ given by Eq. (28) is depicted in Fig. 2 for the same two values of the reduced field b as were used for Fig. 1. In both parts of Fig. 2, curve a describes the motion of the kink due to the changes of the angle φ which occur according to Eq. (29a) and curve b—according to Eq. (29b). The value of q_0 in Fig. 2 is zero, so that at $t-t_0=0$ the position of the wall is defined by $\varphi(t-t_0=0)$. It can be seen that there is a difference in behavior between a kink following Eq. (29a) and one which obeys Eq. (29b). In the case of the latter, the motion is monotonic, while otherwise a single oscillation occurs before that kink also attains saturation velocity. From (28) it follows that this velocity is equal to dB/α, in agreement with Eq. (18) for $C \gg A$. The difference in the value of the saturation velocity for the two values of the drive used is evident [Figs. 2(a) and 2(b)]. Note that, when $|b| \to 1-$, the relaxation time τ_1 tends to infinity and the angle φ given by Eq. (29b) becomes a constant.

For $|B|=B_c$ one has $\varphi_s=\varphi_{ns}=\pi/4+n\pi$ and stationary motion is unstable. To show this, assume that, at the moment the external field $|B|=B_c$ was applied to the kink, the angle φ was $\phi_0=\pi/4+n\pi+\epsilon$, where $|\epsilon|<\pi$. Then, for $\epsilon>0$, the angle φ will evolve towards φ_s as the time derivative of φ is negative [cf. Eq. (30)]. On the other hand, for a negative ϵ the angle φ will tend to $\pi/4+(n-1)\pi$ and the system is unstable with respect to a small perturbation of this sign. The same result may be obtained by checking the sign of the derivative of the right-hand side of (27) with respect to φ for $\phi_0\pi/4+n\pi\pm|\epsilon|$ and $]B|=B_c$.

For overcritical motion of the kink ($|b|>1$) there is no limiting value for the angle φ. This may be seen in

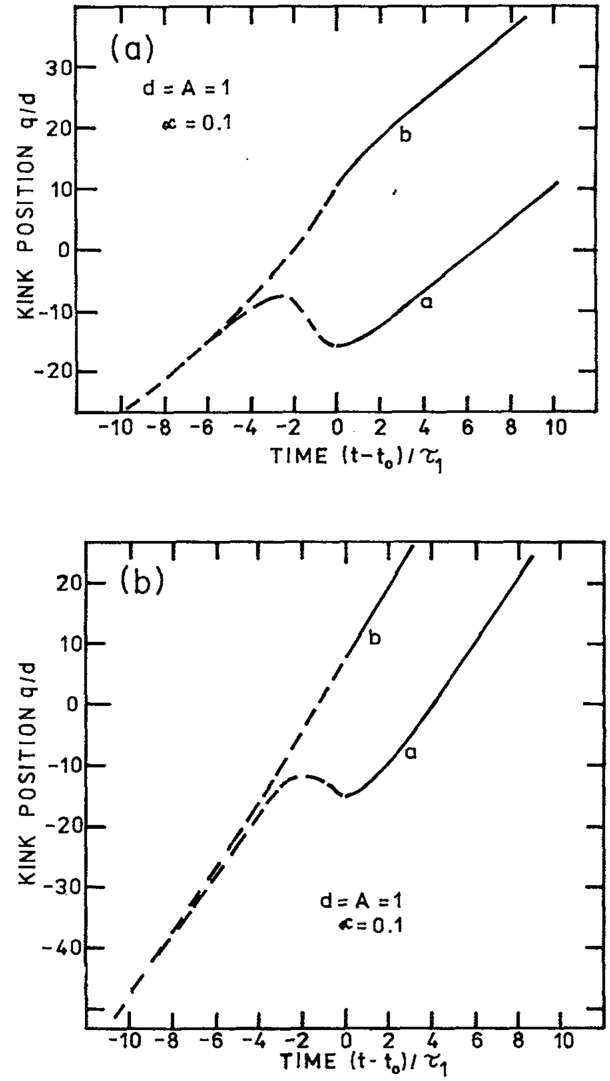

FIG. 2. Undercritical motion: reduced position of the kink as a function of the time. $b=0.5$ in (a) and $b=0.95$ in (b). In both figures, curve a denotes kink position due to $\varphi(t)$ given by Eq. (29a) and curve b denotes that due to $\varphi(t)$ given by Eq. (29b). τ_1 taken with $b=0.5$.

Fig. 3, where the dependence of the angle φ on the time given by Eq. (31) is depicted for several values of the reduced field b. It can be seen that, for $|b|=1$, the angle φ attains the value $-\pi/4$ asymptotically. For all $|b|>1$ the changes of the angle φ with time are running periodic. Large oscillations are visible in Fig. 3 for $|b|=1.1$, much smaller for $|b|=3$ and smaller than the thickness of the line used to draw the curve for $|b|=10$.

The behavior of φ for overcritical values of the external field are reflected in the dependence of the position of the kink q on the time (Fig. 4). Except for the critical motion $|b|=1$, for which stationary motion with constant velocity occurs, for all other values of b in Fig. 4 a running oscillatory motion may be seen. The period of the oscillations together with their amplitude is seen to diminish with increasing $|b|$, so that for $b=10$ only a straight line is shown in Fig. 4. The mean velocity of the overcritical motion is given by

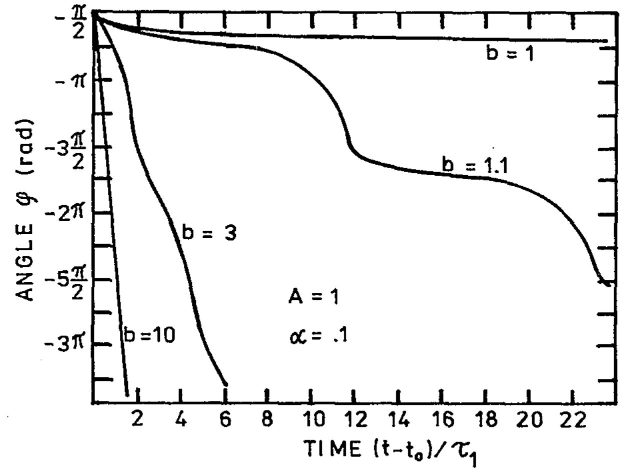

FIG. 3. Nonstationary motion $b>1$ and critical motion $b=1$: azimuthal angle φ as a function of the time. τ_1 taken with $b=0.5$.

$$\dot{q}=\frac{dB}{\alpha}-\frac{dA}{1+\alpha^2}\left[\left[\frac{B}{\alpha A}\right]^2-1\right]^{1/2} . \quad (32)$$

For all $|b|>1$, $\varphi(t+\pi\tau_2)=\varphi(t)-\pi$. When $|b|\to 1_+$, τ_2 becomes infinite, while in the limit of large $|b|$, τ_2 vanishes. The lack of stationary motion for all $|b|>1$ corresponds to the disappearance of the fixed points of the dynamical system described by Eq. (27). From the behavior of the fixed points of this system for different values of the parameters of the motion, it is concluded that a tangential bifurcation[15] occurs at $|b|=1$ (Fig. 5—compare also Ref. 16 for a similar result).

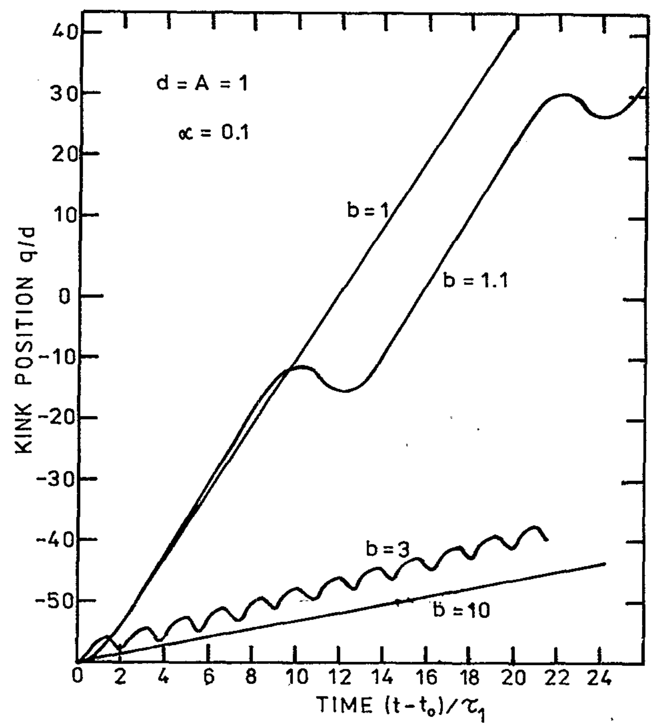

FIG. 4. Reduced kink position during nonstationary and critical motion as a function of the time. τ_1 taken with $b=0.5$.

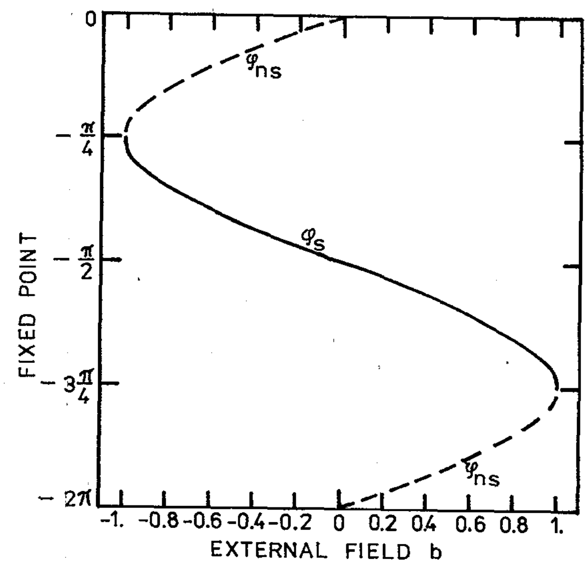

FIG. 5. Tangential bifurcation at the critical value of the external field $|b|=1$.

The dependence of the velocity of the kink on the field is shown in Fig. 6. In this figure, for $|b|<1$ the stable stationary-motion velocity is depicted, while for $|b|>1$ the mean velocity (32) was used. In the low-field range a linear mobility is manifest. For external fields larger than critical, the velocity of the kink first decreases sharply and then slowly increases with the value of the field. The minimum of the average velocity of the kink occurs at the "overcritical" field

$$B_m=\frac{\alpha A(1+\alpha^2)}{[(1+\alpha^2)^2-1]^{1/2}} . \quad (33)$$

An interesting limiting case occurs for zero damping. Equations (24) and (25) are then reduced to $\dot{q}=dA\sin 2\varphi$ and $\dot{\varphi}=-B$, respectively. Hence, $\varphi(t)=\phi_0-Bt$ and

$$q(t)=q_0+dA\cos[2(\phi_0-Bt)]/2B ,$$

and only pure oscillations of the kink are obtained with no net motion.

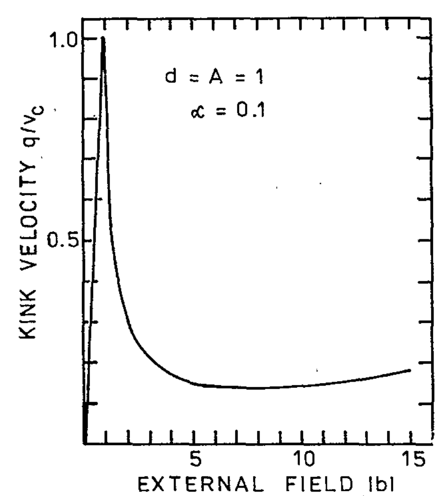

FIG. 6. Velocity of the kink as a function of the external field b.

VII. CONCLUSIONS

We have analyzed the equations of motion of π-kink solitons in a classical Heisenberg chain with a double anisotropy, in the presence of a constant and uniform external magnetic field and in the presence of damping.

Two different regions of kink motion exist. For external fields below a certain critical value, there is a stationary motion of the kink with a constant velocity resulting from a balance between the effect of the external field and that of the damping. This is in contrast to the behavior of kinks with no external field and no damping, where, depending on the initial conditions, the kink may move with any velocity from a certain range $(-v_{\max}, v_{\max})$. For the analysis of transient and nonstationary motion of kinks, we used an approximate method in which the kink is treated as a rigid object. We found that, after an external field is applied smaller than the critical, the kink accelerates or decelerates (depending on the initial condition), attaining a new equilibrium velocity of stationary motion after a certain time. This relaxation time depends on the difference between the value of the applied field and that of the critical field. The relaxation time tends to infinity when the external field tends to the critical value.

When the external field is equal to the critical value, stationary motion is unstable.

For external fields larger than the critical, the motion of the kink is a nontrivial superposition of the forward translational and of the oscillatory motions. The period of oscillations tends to infinity when the external field is diminished to the critical value and this period tends to zero when the field tends to infinity. The mean velocity of the kink has a minimum for a certain value of the field in the overcritical range. In the case of zero damping, for nonzero external fields only pure oscillations of the kink are obtained.

Apart from solutions in the form of moving π kinks, static 2π kinks, which may be treated as a pair of π kinks coupled by the external field, were also found to be solutions of the problem. When the external field tends to zero, the 2π kink dissociates into a pair of two uncoupled π kinks.

One of the most interesting results of this paper is that some features of the motion of π kinks in the system described here may be expressed in terms of the theory of nonlinear dynamical systems. In particular, stable stationary motion of the kinks is shown here to correspond to the existence of a fixed-point attractor, and the phenomenon of the critical drive is shown to correspond to a tangential bifurcation.

The problem of the motion of the π kinks described here is also shown to be equivalent to the well-known problem of the motion of a one-dimensional domain wall in ferromagnetic bulk materials. As a result the kinks are shown to have certain interesting features known from theoretical and experimental analysis of domain-wall dynamics. Although the initial idea for this work was to transplant the solutions for domain-wall motion into the language of solitons in one-dimensional magnetic systems, we stress that the results obtained through this procedure vastly exceed those found in the analysis of domain walls.

Note added in proof. The main points of this paper have been presented at the ICM-85 in San Francisco and a digest of that presentation will appear in J. Magn. Magn. Mater. **54-57** (1986).

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