PHYSA 6127
pp: 1-7 (col.fig.: nil)
PROD. TYPE: COM
ED: Rohini
PAGN: Murthy.N – SCAN: Shobha

ELSEVIER

Physica A 000 (2001) 000–000

PHYSICA A

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A simple model of bank bankruptcies

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Abstract

Interbank deposits (loans and credits) are quite common in banking system all over the world. Such interbank co-operation is profitable for banks but it can also lead to collective financial failures. In this paper, we introduce a new model of directed percolation as a simple representation for contagion process and mass bankruptcies in banking systems. Directed connections that are randomly distributed between junctions of bank lattice simulate flows of money in our model. Critical values of a mean density of interbank connections as well as static and dynamic scaling laws for the statistics of avalanche bankruptcies are found. Results of computer simulations for the universal profile of bankruptcies spreading are in a qualitative agreement with the third wave of bank suspensions during The Great Depression in USA. © 2001 Published by Elsevier Science B.V.

Keywords: Random directed percolation; Interbank deposits; Mass bankruptcies

1. Introduction

Making a short review of latest publications on the percolation phenomenon one could come to the conclusion that the percolation theory [1,2] is a universal paradigm for physics, sociology and economy. In fact, percolating systems composed of large number of interacting units can be simply adopted for simulations of complex behaviours and environments. A number of such adoptions have been done so far including microscopic simulations of the stock market [3–7], social percolation models [8,9] and marketing percolation describing diffusion of innovations [10]. Here we propose a simple model based on the intuitive similarity between percolation and banking networks.

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PII: S0378-4371(01)00296-5

At present, economists agree that the robustness of a country is financial system is related to the strength of a domestic economy. Bank bankruptcies usually follow dramatic changes in the banking capital, assets as well as liabilities and can be socially costly. In general, two factors may cause a bank failure: bad credits and rapid withdrawing of deposits. Economic researches confirm that solvent and insolvent banks alike can experience withdrawals for reasons unrelated to the bank failure risk in circumstances of a banking panic [11]. The same investigations emphasize the importance of withdrawal rates. In fact, sudden withdrawals can have dramatic effects on the bank stability and may force a bank to bankruptcy in a short time if it does not receive assistance from other banks. On the other hand, a bankruptcy of a single bank can start an avalanche of other bank failures due to the *domino effect*.

2. The model

In our model, banks are represented by vertices in a lattice, which for simplicity has a square or cubic symmetry. Directed connections that are randomly distributed between banks simulate flows of money. Banking capital consists of assets and liabilities as in reality. Arrows entering into vertices represent liabilities (deposits of other banks). Branches with opposite direction reflect assets (investments and given credits). It follows that an average number of arrows entering into a vertex is equal to an average number of exiting arrows. We assume that even one withdrawal or bad credit can force the bank to bankruptcy and one failing bank can cause bankruptcies of other banks. Only *interbank* credit connections are considered, i.e., bank deposits and investments are neglected and no insurance system is assumed in our model.

Dynamical rules governing time evolution of the model are as follows. Initially, each bank is solvent. The first bankrupt is selected at random and we do not specify the reason for this bankruptcy that can be a bad credit or sudden deposit withdrawal. During the next time step, neighbouring banks lose their solvency if they gave a loan to the bankrupt. This process is repeated until no bank survives that gave a bad interbank credit. Above mentioned rules become comprehensible after tracing Fig. 1. The figure presents a system with $N = 25$ banks. All possible flows of money (connections between vertices) are realized in this pattern. Let us choose the seventh vertex as the

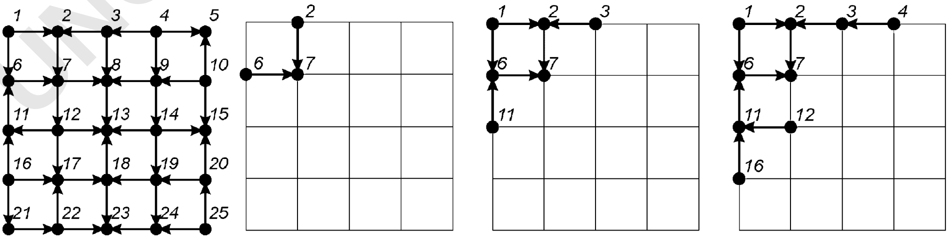

Fig. 1. Bankruptcy spreading in banking network based on square lattice.

first bankrupt. According to the rules assumed earlier the collapse of this bank forces suspension of two other banks with numbers $\{2, 6\}$. During the next step three other banks are swept $\{1, 3, 11\}$. At the end, the avalanche originating from the bank with the number 7 includes nine banks $\{1, 2, 3, 4, 6, 7, 11, 12, 16\}$.

Despite the seeming similarity of our model to the well known *directed percolation* [1] it is based on a new approach to this phenomenon. In the traditional directed percolation directions in space are not equal, i.e., the system is anisotropic and one direction, which is called the growth direction, is special. In our model, all directions are equal. There is also another feature distinguishing the presented model from the standard percolation. In both cases of the traditional site and bond percolation, each occupied site/bond belongs to *only one* cluster. In our model, this condition is not valid and the same bank can be included to *various* avalanches depending on the first bankrupt.

3. Computer simulations

We investigated statistics of bankrupt avalanches in systems characterized by different mean concentrations p of existing interbank deposits. It follows that the system parameter is the same as in the percolation theory. In analogy with the traditional percolation one can expect a critical value p_c when an avalanche composed of bankrupts can spread *all over* the banking network. This phenomenon is related to the percolation phase transition. We performed numerical calculations in order to estimate p_c and based on the finite size scaling law $p(L) - p_c \sim L^{-1/\nu}$ (where L is the linear size of the system) we found that critical values p_c in our model are approximately two times larger than in the usual bond percolation, i.e., $p_c^{2\mathrm{D}} \approx 1.00 \pm 0.01$ and $p_c^{3\mathrm{D}} \approx 0.51 \pm 0.02$ for the square and the cubic lattice, respectively.

We observed that the distributions of avalanche lengths have the same properties as statistics of cluster numbers in the usual percolation system. At the percolation threshold, the probability that a random bank causes l-avalanche (failures of l other banks) fulfills the power law $P_l(p_c) \sim l^{1-\tau}$, where τ is the Fisher exponent. In both two- and three-dimensional systems numerically calculated Fisher exponents are consistent with their equivalents taken from the literature (Fig. 2). For p near p_c and for $l \to \infty$ we found a good agreement with the scaling law describing avalanche distribution $P_l(p) = l^{1-\tau} f[(p - p_c)l^{\sigma}]$, where f is a universal scaling function. Fig. 3 illustrates this law for a square lattice when the scaling exponent $\sigma_{2\mathrm{D}} = \frac{36}{91}$ has been used.

Dynamical properties of our model are described by the number of banks $n(l,t)$ swept during the bankruptcy avalanche up to the moment t, where l is the total avalanche length ($\lim_{t\to\infty} n(l,t) = l$). A typical plot of $n(l,t)$ (Fig. 4) has two regions separated by a *crossover time* t_x [12]. Initially, when $t \ll t_x$ the number of bankruptcies increases as $n(l,t) \sim t^{\beta} l^{\gamma}$. In the dynamic scaling theory of surface growth, the analogous exponent β is called the *growth exponent*. Fortunately, for bank shareholders,

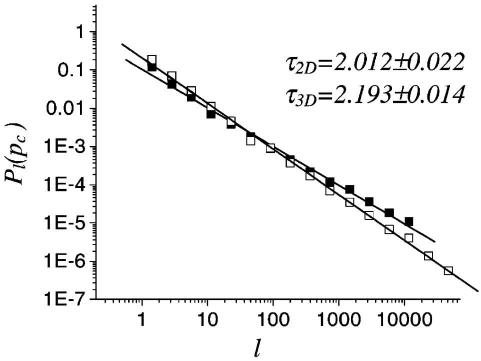

Fig. 2. Avalanche length distribution at p_c in square lattice (solid squares) and cubic lattice (open squares).

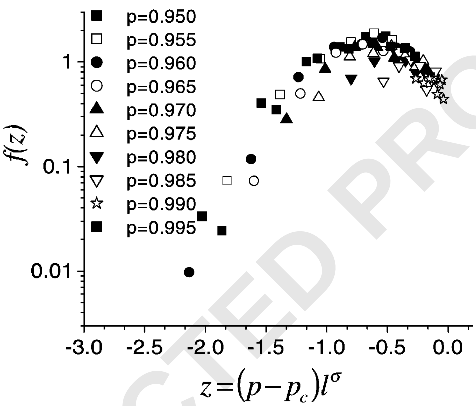

Fig. 3. Scaling behaviour for the renormalized avalanche statistics described by $f(z) = P_l(p)/P_l(p_c)$.

the power law increase is followed by the saturation regime for $t \gg t_x$. The saturation time t_x depends on the avalanche length as $t_x \sim l^z$. By analogy to the standard terminology [12], we call z as the *dynamic exponent*. We found that the avalanche growth in our model fulfills the Family–Vicsek scaling relation $n(l,t) \sim lg(t/l^z)$, where g is a universal scaling function (Fig. 4). The scaling exponents β, γ, z are connected by the equation $\gamma + z\beta = 1$. According to our numerical studies for the square lattice the exponents account to $\beta = 1.60 \pm 0.01$, $\gamma = 0.08 \pm 0.02$, $z = 0.56 \pm 0.01$ and do not depend on the system parameter p.

Fig. 5 shows time distributions of bankruptcies belonging to avalanches presented in Fig. 4, i.e., the curves in Fig. 5 are the first derivatives of those in Fig. 4. Observing the speed of avalanche spreading we find a clear maximum which corresponds to the highest probability of bankruptcy. Figs. 4 and 5 clearly show that there is a unique mechanism governing avalanche growth in our model. The mechanism is independent of the system parameter as well as the avalanche length. Our preliminary

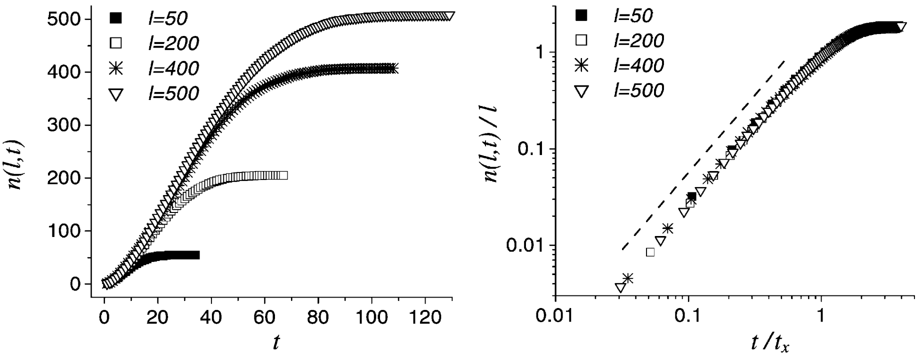

Fig. 4. Time evolution of avalanche growth in the square lattice with $L = 512$ for different avalanche lengths (l). The number of banks that became bankrupt until the time t is presented at the vertical axis. Both right and left plots present the same data. Data on the right plot correspond to the data from the left plot rescaled according to the Family–Vicsek scaling relation.

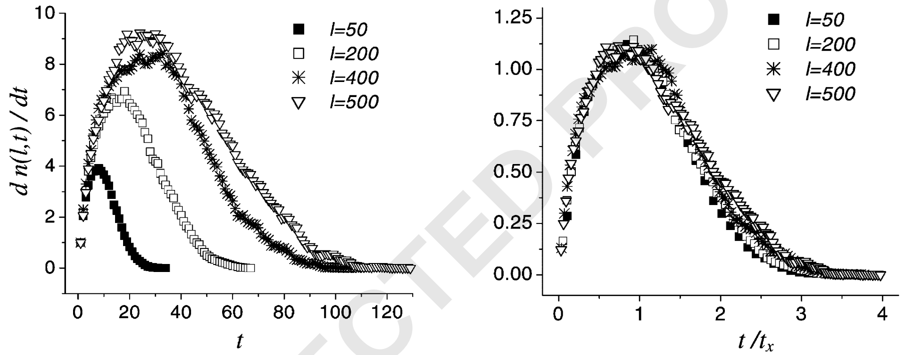

Fig. 5. Speed of avalanche spreading for the same data as in Fig. 4. The right plot presents rescaled data.

studies on cubic lattices prove that the same mechanism governs the avalanche growth in three-dimensional systems.

According to our knowledge, this work is the first one connecting problems of bank failures with the statistical physics. Although exact data concerning spatial and time evolution of mass bankruptcies are hard to receive, it is known that such bankruptcies were quite frequent in the 19th and 20th centuries [13]. The banking crisis that accompanied The Great Depression was probably the most dramatic. Economists distinguish three waves of bank failures during this period and the third wave (starting in May 1932) can be seen as qualitatively consistent with our directed percolation model. In fact, during the period May–September 1932 distributions of total bank suspensions in Illinois, the Chicago Federal Reserve District and the USA have shapes (Fig. 6) similar to the time profile observed in our model (Fig. 5). Contrary to the situation during the earlier massive bank collapses in USA there was no significant interventions

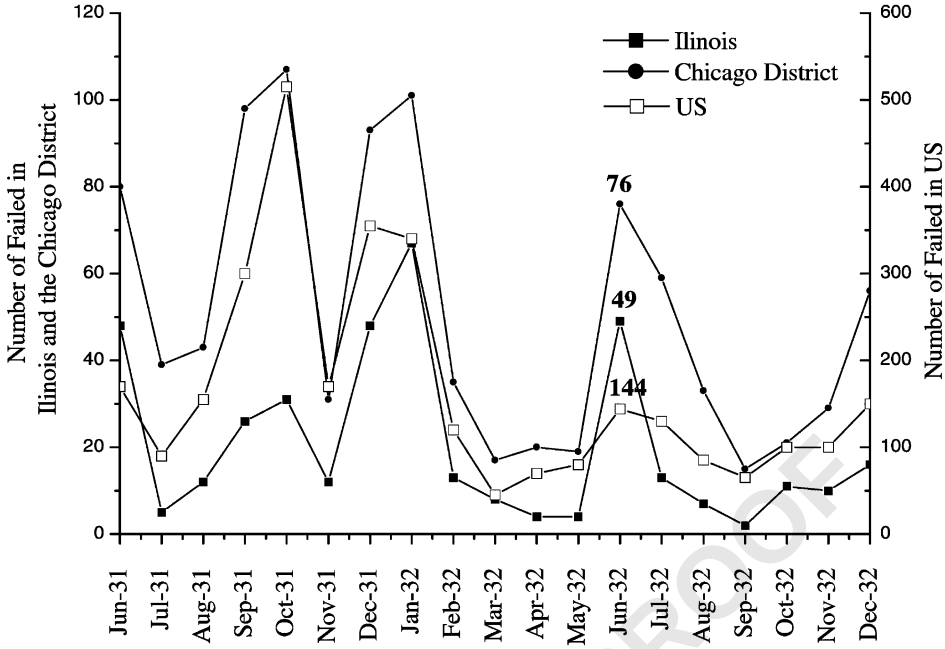

Fig. 6. Total bank suspensions in Illinois, the Chicago Federal Reserve District, and the US, monthly, June 1931–December 1932. After Ref. [5], courtesy of Ch.W. Calomiris and J.R. Mason.

from government institutions in order to stop the contagion of banking system in this time [11].

At present, government institutions guard security of banking system, therefore, the black scenario known from The Great Depression seems incredible but it can repeat. It is necessary to emphasize that the proposed model would be more realistic if it were widened to the whole financial system composed not only of banks but also other financial institutions like trust or pension funds, insurance companies and firms. Although each institution enumerated above possesses a different capital structure, but all of them suffer from risks related to bad investments/credits and are connected to one another.

4. Conclusions

The model presented here has been thought to reflect the cooperative behaviour of banking systems. We have shown that avalanches of bankruptcies can be related to clusters in the random directed percolation problem. It follows that a large number of interbank credits can lead to the percolation phase transition when bankruptcies can spread all over the banking network. Static and dynamic properties of this model are in a good agreement with the percolation theory. As observed in the numerical simulations, the shape of avalanche spreading is in a qualitative agreement with data from The Great Depression.

Acknowledgements

One of us (JAH) is thankful to Prof. Dirk Helbing for his hospitality during the stay in Dresden. The work has been in part supported by the ALTANA AG due to the Herbert Quandt-Programme.

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