

# Modeling defaults of companies in multi-stage supply chain networks

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## Abstract

The interest in supply chain networks and their analysis as complex systems is rapidly growing. The physical approach to the topic of production networks relies on the concept of heterogeneous interacting agents. The interaction among agents is considered as a repeated process of orders and production. The dynamics of production in the supply chain network which we observe is nonlinear due to the random failures in processes of orders and production. We introduced a modified model of a supply chain network which represents in more detail the real economic environment in which firms are operating. We focused on the influence of local processes on the global economic behavior of the system and studied how the proposed modifications change the general properties of the model. To simulate the economic environment of companies realistically, we introduced the following features to the model: evolution of a supply chain network with the reconfiguration of links, price dispersion and the dynamics of prices and costs of production.

*Keywords:* Multi-agent systems, Supply chain networks, Company defaults.

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## 1 Introduction

The current crisis of the world's financial markets has clearly shown that the structure of connections among interacting market participants plays a crucial role in the valuation of default risk. Uncertainty in the economic environment in which companies are operating has a significant impact on the

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financial condition- even of highly profitable and well-known companies. Analysts now claim that there is more than 95% probability that one of the three biggest automakers in the US (GM, Ford and Chrysler) will go bankrupt in the next 5 years. A default of one of the Big Three would trigger extreme losses in the \$1.2 trillion market for collateralized debt obligations (CDOs), since 80% of them include credit-swaps on GM or Ford (according to Standard and Poor's). The impact of default of one of these car manufacturers on their supply chain could be even more devastating. The implications of the insolvency can propagate through different interaction channels, as each firm is a part of a complex supply chain network, composed of parts suppliers (like Visteon, Lear, TRW), car manufacturers and car dealers (like Autonation, United Technologies, Lithia) (Schellhorn and Cossin 2004). In order to understand the means by which defaults propagate in the complex structures of modern supply chain networks many techniques have been used, including linear and nonlinear programming, stochastic processes modeling, and traditional discrete-event programming. In this paper we will consider the supply chain network as a system of agents (raw materials providers, suppliers, manufacturers, retailers) connected through production relationship in form of orders and production flows. We will use agent-based modeling (ABM) in order to analyze how local defaults of supply chain participants propagate through the dynamic supply chain network and form avalanches of bankruptcy.

This paper is organized as follows. In Section 2 we give an overview of the literature. Section 3 describes the features of the model. In section 4 we present the results. In section 5 we discuss our findings and the managerial implications. We draw the conclusions and recommend further research ideas in section 6.

## 2 Literature overview

### 2.1 Supply chain networks

As a result of the fast developing technology and growing complexity of today's business, logistical systems are becoming densely interconnected. The complicated structure of the supply chains forms a network consisting of multiple parts. The Council of Supply Chain Management Professionals (CSCMP) defines a supply chain network as a network of interconnected elements of manufacturing plants, distribution centers, points-of-sale, as well as raw materials, relationships among product families, and other factors. Following Melo et al. (2009) supply chain network design and optimization problems can be divided into four groups, based on the following features:

- (1) number of stages (single, multiple)
- (2) number of commodities (single, multiple)
- (3) number of periods (single, multiple)
- (4) economic environment (deterministic, stochastic)

All of these factors influence the complexity of problems studied in Supply Chain Management (SCM). For instance, the complexity of production dynamics in a supply chain network increases rapidly with the number of agents, stages in the network, connections between suppliers and producers, and the number of uncertain parameters, such as customers' demand or quality and frequency of deliveries. In this paper we will focus on the multi-stage supply chain network with single commodity and multi-period observations. Examples of the similar problem's setup can be found in Ambrosino and Scutell (2005) or Troncoso and Garrido (2005), where the authors study intra-layer flows in the deterministic environment. The proposed distribution network design includes facility location, warehousing, transportation and inventory decisions. The major weakness of this line of research is that many crucial parameters of these models are set to be deterministic. In the real world however, managers need to make decisions, taking into account many sources of operational, market, business or political risk (for the overview see Bouchaud and Potters 2000). These risks are associated with the stochasticity in customer demand, prices, exchange rates, frequency and quality of deliveries and others. Uncertainty in network dynamics plays a critical role in the design and operation of supply chain networks.

There are relatively few papers dealing on the uncertainties in distribution problems (Cheung and Powell 1996) or demand planning (Van Landeghem and Vanmaele 2002; Yu and Li 2000). Santoso et al. (2005) propose a stochastic programming methodology for the large-scale supply chain network design problems, including uncertainty. They use the sample average approximation (SAA) scheme, with an accelerated Benders decomposition algorithm for efficient solving of a potentially infinite number of scenarios. The configuration of the network is one of the crucial factors, which enables the efficient operations of the supply chain network. Goetschalckx et al. (2002) present an overview of algorithms for the design of global logistics systems and propose an efficient decomposition algorithm for multiperiod production distribution networks. Almeder et al. (2009) observe that combining complex simulation models and abstract optimization models allows the modeling and solving of more realistic problems, which include dynamics and uncertainty. Chopra et al. (2007) show that bundling two sources of risk (i.e. disruption and delays of supply) can lead to overutilization of the cheaper and less reliable supplier and underutilization of the most reliable supplier. They propose decoupling the two sources of risk. The mitigation strategy to hedge against the increased risk from disruption is to use the reliable supplier. The mitigation strategy in case of increased recurrent risk is to choose the less reliable, but cheaper supplier.

In order to investigate the supply chain network in an uncertain environ-

ment, we will apply the agent-based modeling technique described in the next subsection.

## *2.2 Agent-based modeling and bankruptcies*

Agent-based modeling and simulation is a powerful computational technique, which is very useful in gaining insights about decision making in large interconnected systems. ABM has attracted a great deal of attention in recent years and has already found many applications in supporting business decisions, for developing optimized systems for organizations, improving manufacturing and operations and managing supply chain networks (North and Macal 2007). The idea originally comes from complex adaptive systems. The main question behind the application of this technique (Surana et al. 2005) is: how does the collective behavior of a system arise from interactions among autonomous agents? Supply chain networks belong to a large class of complex economic systems, in which agent-based modeling has proven capable of delivering new knowledge, developing new business policies and improving processes of decision-making (Macal 2003; Macal et al. 2003). Swaminathan et al. (1998) in cooperation with IBM developed a multi-agent supply chain simulation and reengineering tool for developing customized applications. Chatfield et al. 2007 combined the agent simulation with a process-oriented approach for supply chain simulation modeling and presented an open information standard to assist supply chain modeling, analysis, and decision support (Chatfield et al. 2009). Another framework was proposed by Julka et al. (2002b,a); they analyze business policies in terms of different situations arising in the supply chain, using the example of a refinery supply chain. Akanle and Zhang (2008) propose a methodology for manufacturing organizations to optimize supply-chain configurations to cope with customer demand over a future period. Cruz et al. (2006) developed a supernetwork model (in both static and dynamic forms) that integrated global supply chain networks with social networks using methods from financial engineering. Qi and Shen (2007) introduced an integrated supply chain design model that considers unreliable supply in a three-tiered supply chain network with one supplier, one or more facilities and retailers.

Many researchers from the natural sciences are interested in applying ABM to supply chain networks, mainly by focusing on the dynamics of these complex systems. Weisbuch and Battiston (2007) proposed the simple agent-based model of supply chain network, which Battiston et al. (2007) subsequently examined in more detail. The former paper analyzed the geographical aspects of a supply chain network whereas the latter provided an analysis of the known qualitative facts of the firms' demography, like size distribution and aggregate output time series as well as the emergence of spatio-temporal patterns for output, growth and failures. The dynamics of the supply chain network resembles the reaction-diffusion systems studied in chemical physics. The reac-

tion part of the process is the growth of the total capital coupled with failures in deliveries and bankruptcies, whereas the diffusion part of the process is the propagation of orders and production across the network. The result of this multiplicative stochastic process is the occurrence of the metastable regions of higher and lower production activity.

The multi-layer supply chain network is a dynamic system in which local interactions between suppliers and producers unfold with stochastic shortages. These local instabilities, if accumulated, can amplify and give rise to non-linear processes leading to avalanches of bankruptcy. One of the first papers dealing with this approach to modeling supply chains is by Bak et al. (1993), who study the connections among firms related by production activity. The triggering events in their model are stochastic changes of demand at the customer stage of a supply chain network. The occurrence of avalanches on a supply chain network represented by the lattice structure was also studied by Delli Gatti et al. (2005). Aleksiejuk and Holyst (2001) proposed an agent-based model of collective bank bankruptcies, where the properties of the simulated avalanches are in a good agreement with the percolation theory. In the related paper Aleksiejuk et al. (2002) observe the scaling behavior of collective bankruptcies, which are common for the self-organizing complex systems. The most common strategy of risk diversification is the expansion of the portfolio of suppliers, known as the multi-source strategy (Kleindorfer and Saad 2005). Battiston et al. (2007) have more recently examined the influence of the network density on the amplification of systemic risk in trade-credit networks. They have shown that a trade-off emerges between the local risk diversification and the global systemic risk. Wagner et al. (2009) studied the positive supplier default dependencies using the copula approach. They indicate that default dependencies are the important factor in risk mitigation strategies and should be considered in constructing supplier portfolio. Another phenomenon which might lead to shortages in deliveries and production and in extreme cases to bankruptcies is the bullwhip effect (Lee et al. 2004). This effect was reproduced using the ABM approach, for example by Ahn and Lee (2004), who focused on the efficiency of supply chain networks and the minimization of bullwhip effects via information sharing between agents. Another promising way of dealing with information distortion and amplification of fluctuations in complex supply chain networks are concepts from statistical physics and nonlinear dynamics of complex systems (Peters et al. 2007). Nagatani and Helbing (2004) showed that forecasting of future stock levels can stabilize the linear supply chain and hedge against the bullwhip effect. In another model originally developed for transporting and traffic dynamics Helbing (2003) proposes a framework in which instabilities of supply chains may lead to business cycles. Helbing et al. (2004a,b) show that although nodes of the supply chain network behave in an overdamped way, the system itself displays damped oscillations. They use equations of material flows to model supply chain networks. The bullwhip effect in their model can be explained as a convective instability phenomenon based on a resonance effect. The authors point out,

that the network theory makes a useful contribution to planning and to the design of stable, robust and adaptive supply chain networks.

Our objective is to find out, how sources of uncertainty observed in economy affect the dynamics of the supply chain network. We focus on the emergence of global patterns of the network dynamics, resulting from local dependencies. To achieve this objective we will build an agent-based modeling framework of a supply chain network along with the visualization tool, which will allow us to monitor online the dynamics of the system. This will enable us to understand the dependencies between suppliers and producers both on local and on global scales. The results have implications for improved risk hedging and optimized decision making for managers of the large-scale supply chain networks.

### 3 Model

The model is based on the framework proposed by Weisbuch and Battiston (2007) that was mentioned in the previous section. In our model we make the framework more realistic by adding features, which convey the complex character of a supply chain network in the real economic environment. We will discuss the following enhancements of this model:

- (1) Price dispersion
- (2) Evolution of supply chain topology
- (3) Network reconfiguration
- (4) Production dynamics
- (5) Dynamics of costs of production

Following the framework introduced by Weisbuch and Battiston (2007) our network consists of  $N$  nodes organized in  $S$  stages (fig.1).

We assume the network has  $S = 5$  stages and  $N = 150$  nodes. Nodes represent production facilities which are connected by undirected links. Links represent information flow in one direction (from consumers to suppliers) and the material flow in the opposite direction. No reverse activities and closed-loops are allowed. The stages of the network are numbered in the direction of the information flow, i.e. from the output stage  $s = 0$  (retail sector) to the input stage  $s = 4$  (suppliers providing raw materials). We assume that final goods are fully absorbed by the consumers and that primary producers have unlimited access to raw materials.

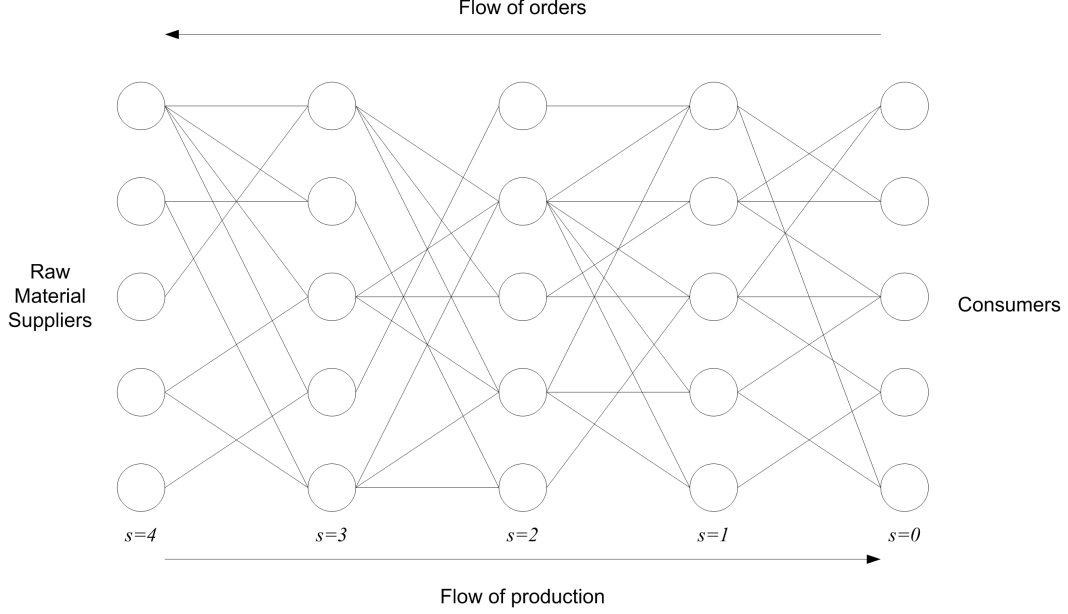


Fig. 1. The structure of the supply chain network.

### 3.1 Price dispersion

Price dispersion (or the violation of the law of one price) is the phenomenon often observed in the markets. It captures the fact that prices of homogenous products may vary from one seller to another – a common fact, used by price comparison services in the Internet. A study of price dispersion in automobile industry can be found in Delgado and Waterson (2003). Examples for many other industry sectors and the theoretical analysis can be found in Baye et al. (2006). To include this phenomenon in our model, at the beginning of the simulation we assign sales prices to all nodes from the log-normal distribution (1).

$$f(x, \mu, \sigma) = \frac{1}{x\sqrt{2\pi}\sigma} \exp\left(\frac{-\ln^2(x/\mu)}{2\sigma^2}\right) \quad (1)$$

for  $x > 0$ , where  $\mu$  is the expected value, and  $\sigma$  is the standard deviation of the logarithm of  $x$ . The log-normal distribution is widespread in financial mathematics, for instance for simulations of stock prices. It is convenient to model price dispersion using log-normal distribution, since values generated from this distribution have no negative values. Price dispersion will allow us to introduce a mechanism of the dynamic growth of the supply chain network (linking algorithm).

### 3.2 Evolution of supply chain topology

The first introduced element of stochasticity is the variable number of incoming/outgoing connections, reflecting the continuous search for suppliers which offer more favourable prices (Wagner and Friedl 2007). It has been accomplished by introducing the linking algorithm. The algorithm assumes that the main objectives of firms are profit maximization, cost reduction and hedging against risk. If firm  $i$  has  $k_i$  suppliers, then if one of them cannot deliver the ordered goods, then the risk of taking a profit loss would be proportional to  $1/k_i$ . The linking algorithm is constructed as follows. In each time step:

- Starting from the output stage  $s = 0$ 
  - (1) choose randomly node  $i$  from the stage  $s$
  - (2) choose randomly three nodes  $j_k$ ,  $k \in (1, 2, 3)$  from stage  $s + 1$
  - (3) check, which node  $j_k$  has the lowest sales price ( $j_k^{min}$ )
  - (4) connect node  $i$  with node  $j_k^{min}$ , if the sales price of the node  $j_k^{min}$  is lower than the sales price of the node  $i$
- go to the stage  $s + 1$  and repeat steps (1) to (4)
- the linking process finishes when the connections between the last two stages are established.

### 3.3 Network reconfiguration

To limit the density of the network and keep the number of connections at the predefined, realistic level we used a method of network reconfiguration. Production facilities are allowed to change the supplier if a new supplier with the lower sales price was found. Additionally, if the firm has more suppliers (higher in-degree  $k_{in}$ ) the probability of switching to the new supplier increases. Firms with few suppliers face a higher risk of supply and production shortage in the periods following a switch to the new supplier. To factor in this assumption in our model we have chosen a *tgh* function as probability function of reconfiguration depicted in figure 2.

$$P_{reconf} = tgh(k_{in}/a), \quad (2)$$

where  $a > 0$  is the parameter,  $k_{in} \geq 0$ .

In simulations we used function (2) with parameter  $a = 5$ , which gives the probability value  $P_{reconf} \approx 0.95$  when  $k_{in} = 12$ .



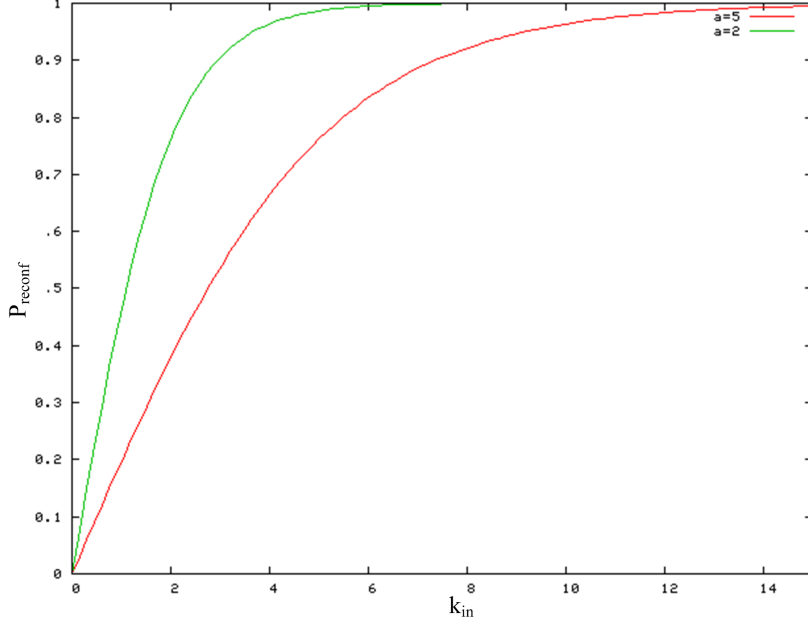


Fig. 2. Probability function of link reconfiguration for two values of parameter  $a$ .

### 3.4 Production dynamics

The production dynamics is an enhancement of the framework proposed by Weisbuch and Battiston (2007) with adjustments required by the variable input and output connections and the stochasticity of prices. At the beginning of each time step we start the linking algorithm. Next, starting from the output stage, orders to the next stages are placed. When the order flow reaches the input stage, production starts. We assume the linear technology of production of homogenous goods, which means that the production function is linear to the firm's working capital  $A$ . The orders from the output stage ( $s = 0$ ) are only limited by production capacity. Under these assumptions retailers place their orders  $Y$  according to Equation:

$$Y_{0,i}(t) = qA_{0,i}(t), \quad (3)$$

where  $q = 1$ , is the technological proportionality coefficient, which relates the working capital  $A_{0,i}(t)$  of firm  $i$  to the quantity of products  $Y_{0,i}(t)$  it is able to produce. The orders are then distributed proportionally to all suppliers actually connected to node  $(0, i)$ , i.e. in the amount of  $\frac{1}{k_{in}^i(t)}Y_{0,i}(t)$ . In all other stages  $s$  firms place their orders to stage  $s + 1$  based on the orders coming from stage  $s - 1$ , following Equation:

$$Y_{s,i}(t) = \min \left( qA_{s,i}(t), \sum_{j \in \nu} \frac{Y_{s-1,j}(t)}{k_{in}^j(t)} \right), \quad (4)$$

where  $\nu$  is the actual set of customers of the firm  $(s, i)$  from stage  $s - 1$ . Equation 4 reflects the assumption that if the incoming orders exceed the production capacity, the total production of the firm is limited by its working capital (multiplied by  $q$ ). This procedure ends up at the input stage. The providers of raw materials now start the production flow. The delivered production is given by

$$Y_{s,i}^d(t) = \left( \sum_{i' \in \nu'} Y_{s+1,i'}^d(t) \frac{Y_{s,i}(t)}{\sum_{j \in \nu} Y_{s-1,j}} \right) \epsilon(t), \quad (5)$$

where  $\epsilon(t)$  is the stochastic coefficient, which with probability  $P$  leads to the shortage in the firm's production. Formally,  $\epsilon(t) = 0$  with probability  $P$  and  $\epsilon(t) = 1$  with probability  $1 - P$ . Thus, the delivered production of firm  $(s, i)$  depends on the production received upstream from suppliers in neighborhood  $\nu'$  from stage  $s + 1$  and is distributed proportionally downstream, according to the orders placed by the customers in the neighborhood  $\nu$  (Figure 3).

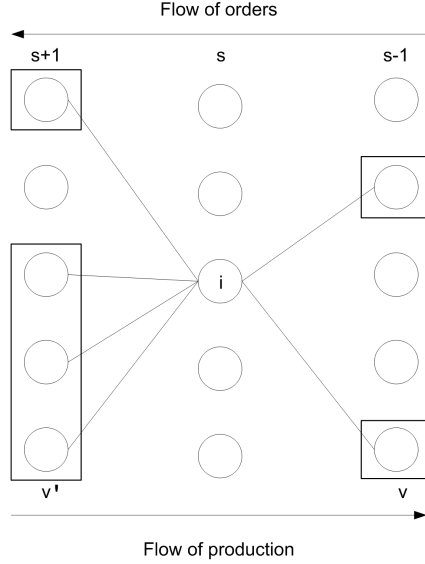


Fig. 3. An example of the operating environment of the firm  $(s, i)$ . The actual suppliers and customers belong to the corresponding neighborhood  $\nu'$  and  $\nu$  (highlighted with squares).

The next step is the update of profits and losses. All profits are calculated using Equation

$$\Pi_{s,i} = p_{s,i}(t)Y_{s,i}^d - c_{s,i}(t)Y_{s,i}^d - \lambda A_{s,i}, \quad (6)$$

where  $p_{s,i}(t)$  is the sales price at time  $t$ ,  $\lambda$  is the exogenously given interest rate constant, which expresses the cost of capital and  $c_{s,i}(t)$  is the cost of

production of a unit of production at time  $t$

$$c_{s,i}(t) = \left( \frac{\sum_{i' \in \nu'} p_{s+1,i'}(t) Y_{s+1,i'}^d(t)}{\sum_{i' \in \nu'} Y_{s+1,i'}^d} \right), \quad (7)$$

where  $\nu'$  is the neighbourhood of node  $i$  in stage  $s + 1$ . Margin of firm  $(s, i)$  is variable and depends on the sales prices of all suppliers which are actually connected

$$m_{s,i}(t) = p_{s,i}(t) - c_{s,i}(t). \quad (8)$$

Margins of firms in the input stage are given as a constant parameter  $m_{inp}$ .

### 3.5 Dynamics of costs of production

We also introduced the dynamics of costs of production. We used the assumption that after each 5 time steps firms are changing their costs of production (as a result of changes in the technology of production or changes in the labor market). The costs of production consist now of two parts. The first term  $c_{s,i}(t)$  includes the deterministic costs consequent to the sales prices offered by the suppliers. The second part is a stochastically changing term  $q(t)$ , which will be updated after the predefined number of time steps. The total cost of production  $\bar{c}_{s,i}(t)$  can be written as follows

$$\bar{c}_{s,i}(t) = c_{s,i}(t) + q(t), \quad (9)$$

where  $c_{s,i}(t)$  is given by Equation (7). We assume that the change of the logarithm of  $q$  is a random variable with the normal distribution.

$$q(t+1) = q(t) \exp(\eta), \quad (10)$$

where  $\eta$  is a random variable, with mean value  $\mu_2 = -0.1$  and standard deviation  $\sigma_2 = 0.1$ . This set of parameters causes that the negative values are more probable than the positive, that is, firms are more frequently lowering than raising their costs of production (for instance, thanks to investments in technology of production). Now we need to rewrite Equation (6), in order to include the dynamically changing costs of production

$$\Pi_{s,i} = p_{s,i}(t) Y_{s,i}^d - \bar{c}_{s,i}(t) Y_{s,i}^d - \lambda A_{s,i}. \quad (11)$$

### 3.6 Bankruptcies and recovery

In this paper we will consider the firm as bankrupt when it cannot meet its short-term commitments (i.e. it has insufficient cash flows to continue its operations). Since we do not include inventories and credit in our model, the financial default occurs when the working capital  $A$  of the company hits the threshold level. This level is assumed to be a fraction of the average working capital of all firms in the supply chain network  $A_{default} = \frac{1}{100} < A >$ . It follows the assumption that bankruptcy processes evolve with the state of economy. It implies that there are no fixed bankruptcy thresholds. After a number of time steps  $\tau$  the defaulted firms are replaced with new ones with the working capital equal to a fraction of the firms' average working capital  $A_{recovery} = \frac{1}{10} < A >$ .

## 4 Results

### 4.1 Influence of delivery failures on the growth of the total working capital of the supply chain

We used C++ programming language to build an agent-based modeling environment for simulations. The first characteristic of the system, which we focused on was the influence of the probability of delivery failures on the total working capital of the supply chain. The growth of the working capital aggregated over all production facilities in a supply chain network  $A_{total}$  is exponential with the value of the growth constant strongly dependent on the probability of delivery failure. For the sake of simplicity we analyzed this dependency by simulating the production on the supply chain. Let's consider the chain of production facilities with one facility in each of the  $S$  stages. The average growth rate of the working capital of firm  $k$  at time  $t$  is given by Equation:

$$< 1 + m > = \left( \frac{A_k(t)}{A_k(0)} \right)^{\frac{1}{t}}, \quad (12)$$

where  $A_k(0)$  is the working capital of the firm  $k$  at time 0. The average growth rate can be calculated from the geometric average

$$< 1 + m > = \sqrt[t]{(1 + m)^{n_1} \cdot (1)^{n_2}}, \quad (13)$$

where  $n_1$  is the number of periods with full profits (margin  $m > 0$ ) and  $n_2$  is the number of periods without profits (margin  $m = 0$  due to delivery failures)

after  $t$  iterations ( $t = n_1 + n_2$ ). Assuming that the delivery failure occurs with probability  $P$ , we can rewrite  $n_1$  and  $n_2$  as:

$$n_1 = (1 - P)t, \quad (14)$$

$$n_2 = Pt. \quad (15)$$

Because failures are independent and can occur in every stage,  $n_1$  and  $n_2$  take the following form

$$n_1 = (1 - P)^S t, \quad (16)$$

$$n_2 = (1 - (1 - P)^S)t, \quad (17)$$

where  $S$  is the number of stages in the supply chain. After substituting  $n_1$  and  $n_2$  into (13) we obtain

$$\langle 1 + m \rangle = \sqrt[t]{(1 + m)^{(1-P)^S t} \cdot (1)^{(1-(1-P)^S)t}} = (1 + m)^{(1-P)^S}. \quad (18)$$

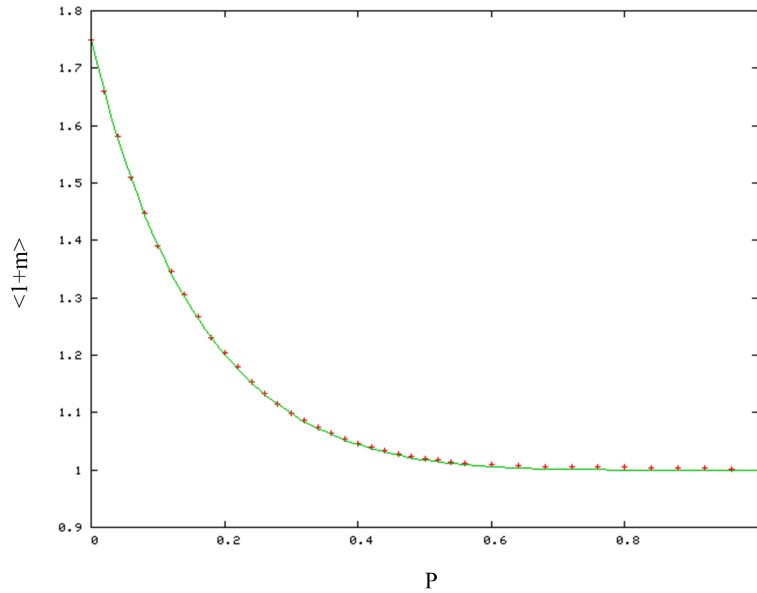


Fig. 4. Growth constant as a function of  $P$  for the production chain ( $S = 5$ ) (crosses). Solid line represents the fit given by function (18).

This expression is true for the average growth rate of the working capital of all firms in the chain, since all nodes are independently exposed to delivery failure with probability  $P$  and if one facility fails to deliver, the margin  $m$  is equal to zero for the whole production chain. The analytic solution for the

supply chain network with the dynamical reconfiguration of links is a subject of the future research. The results of simulations and the theoretical function's fit are shown in figure 4.

#### 4.2 Performance of the network

A supply chain network with the dynamically changing links between nodes is a nonstationary system. The performance of the system is characterized by the turbulences during the first period of the network's growth. This process is illustrated in figure 5, which shows the performance as a function of time. The performance of the system is given by:

$$M(t) = \frac{A_{total}(t)}{Y_{total}^d(t)} * 100\%, \quad (19)$$

where  $A_{total}(t)$  is the aggregated production capacity of the whole system,  $Y_{total}^d(t)$  is the total delivered production at time  $t$ .  $M(t)$  is the measure of the percentage of the working capital utilized in production (for the static network of production  $M(t) = const = 100\%$ ). The mean value of system's

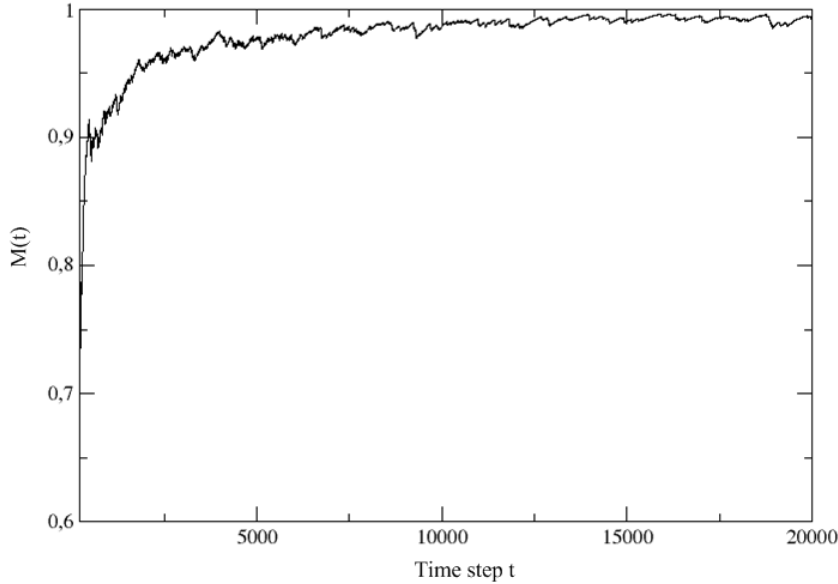


Fig. 5. Performance of the network.

performance  $\langle M \rangle = 97.88\%$  corresponds to the value of average growth rate of working capital  $\langle 1 + m \rangle = 1.009788$ . The observed fluctuations are

caused by the stochastic process of the network reconfiguration. Our result indicates that the reconfiguration process leads to the periods of higher and lower production performance, which have a negative effect on the average growth of the total working capital of the supply chain network.

#### 4.3 Evolution of the network topology and aggregated working capital

For the simulations we used the following set of values:  
(for parameters characterizing price dispersion)

- (1)  $\mu = 1.0$  - mean value
- (2)  $\sigma = 0.01$  - standard deviation of  $\ln x$

(for global parameters)

- (1)  $m_{inp} = 0.01$  - margin level of the input stage
- (2)  $\lambda = 0.002$  - interest rate constant
- (3)  $P = 0.05$  - delivery failure probability

These values of parameters ensure an exponentially growing working capital of the supply chain network. We have chosen a relatively low value of  $\lambda$  in order to track the evolution of the system for the longer periods of time.

The online monitoring of the network dynamics was accomplished by writing an application for visualization using GLUT - The OpenGL Utility Toolkit libraries.

At the beginning, we iterate 200 times to establish connections between the nodes. The state of the network after the test period is shown in figure 6.

Afterwards we switch on the production. The firms with the best profit/cost ratio are growing and adding new suppliers, whereas the working capital of the firms whose sales price is higher than the mean price of the given stage is slowly decaying and results in the defaults of firms. After sufficient time a large portion of the firms goes out of the market and the meta-stable network configuration is formed. The state of the supply chain network in this phase is shown in figure 7. The bigger the node, the more working capital a firm has.

The time series of the total capital and total delivered production are depicted in figure 8. The system is characterized by significant turbulences at the beginning of the simulation. By applying Equation (20)

$$t = \frac{\log(A_{k,i}(t)) - \log(A_{k,i}(0))}{\log(1 + m)} = \frac{\log \frac{A_{k,i}(t)}{A_{k,i}(0)}}{\log(1 + m)}, \quad (20)$$

we calculate that after approximately 2350 time steps most of the firms with the unprofitable set of suppliers or customers have defaulted and at the same

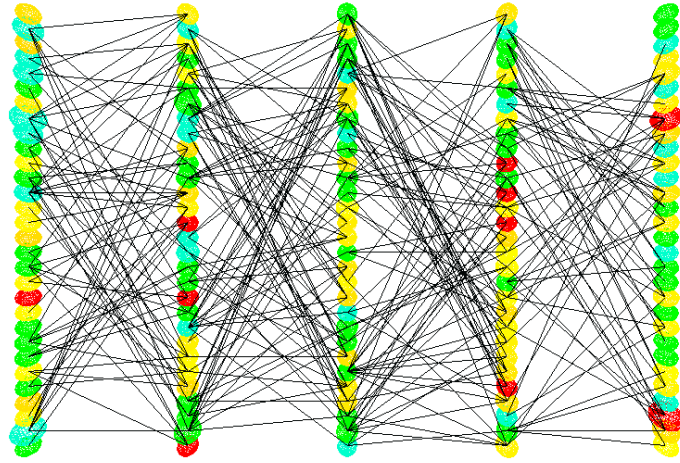


Fig. 6. State of the network after the test period.

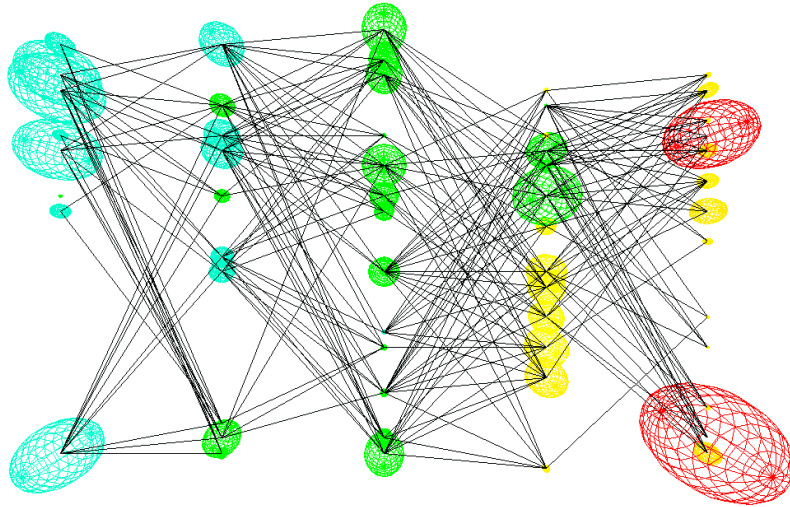


Fig. 7. State of the network after reaching the (meta)stable configuration.

time the total production of the system starts to grow exponentially with the growth constant proportional to the mean margin of the firms which survived in the network. This process is depicted in figure 9.



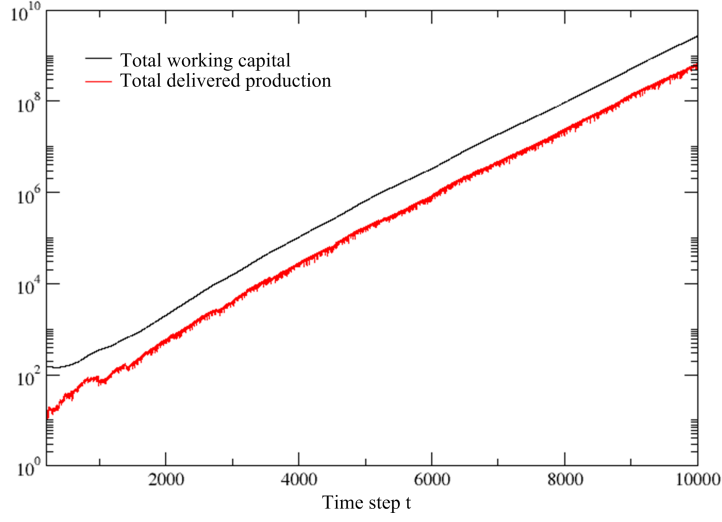


Fig. 8. Total working capital and total delivered production.

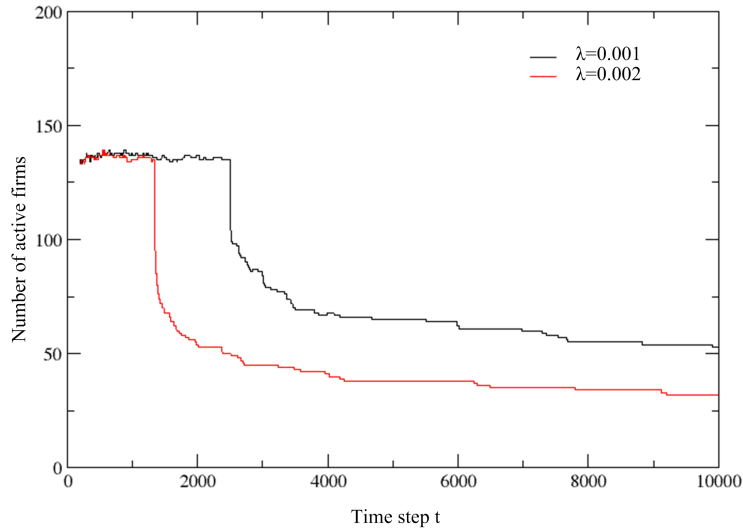


Fig. 9. Number of active firms for two different values of  $\lambda$ .

#### 4.4 Evolution of the aggregated working capital in presence of stochastic changes in costs of production

In order to study the behavior of the system in presence of stochastic changes in costs of production, we considered two adaptive strategies of the firms. After each 5 time steps the stochastic term  $q(t)$  of Equation (9) is

updated and one of the following actions can be undertaken:

- (1) firms are lowering their sales price by the factor equal to the change of the factor  $q$ ,
- (2) firms are lowering their sales price by the factor equal to the half of the change of the factor  $q$ .

In figure 10 we see the influence of each of the strategies on the evolution of the total working capital. Both time series are characterized by the periodical

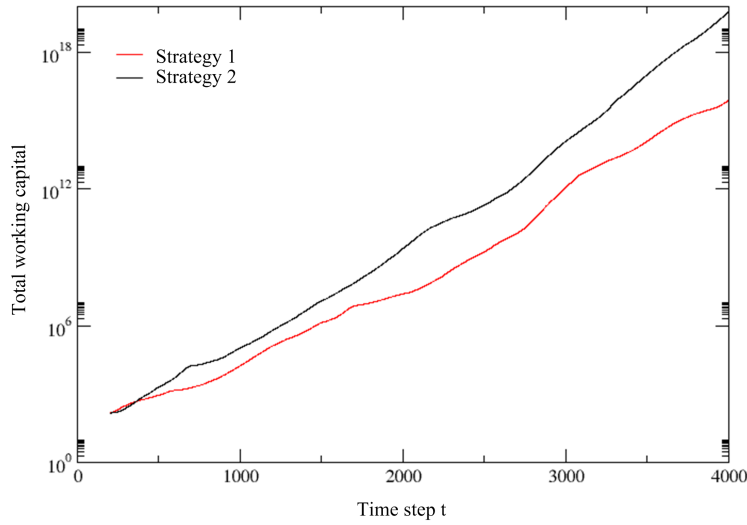


Fig. 10. Evolution of the total working capital of the supply chain network after choice of one of two adaptive strategies.

changes of growth. It is the result of the stochastic changes of production costs and sales prices, which are not correlated in time and space and they are independent of the size of the firms. We observe that strategy 1 is less effective than strategy 2.

## 5 Discussion and implications

In our study we extended the agent-based model of the supply chain network proposed by Weisbuch and Battiston (2007) by adding features which reflect some of the uncertainties that are present in the real market environment. Thanks to the graphic engine we gained more insight into mechanisms which can lead to avalanches of bankruptcies. The contribution of this paper is to establish an agent-based framework, which reproduces the sources of

stochasticity in the operations of supply chain networks.

In contrast to the static topology studied by Bak et al. (1993); Weisbuch and Battiston (2007); Battiston et al. (2007); Delli Gatti et al. (2005) our approach includes the dynamic reconfiguration of the network based on the simple algorithm of agent behavior. We note that at a certain point of the system's evolution, the meta-stable structures of the network emerge. As a result of the dynamics of prices and costs of production, we observed both the emergence of highly profitable supply chains with the high market share and the avalanches of bankruptcies. This process displays the self-organization effects of the supply chain network and contributes to the understanding of the complex network design problems. ABM is a promising alternative to the commonly used mathematical programming optimizations techniques (Ambrosino and Scutell 2005) thanks to its ability to implement sophisticated behavioral rules of agents on the local level and relate them to the global outcomes.

Our observations and quantitative results lead to the following conclusions which are important for managerial practice. The first implication is that during the process of assessment of the company's risk exposure, managers should keep their focus on the global structure of the supply chain network instead of being restricted to the own portfolio of suppliers and customers. Based on the analytical solution derived in subsection 4.1 we conclude, that the average growth of the working capital of a supply chain depends on the number of stages and decays quickly as the probability of shortage in production grows. Since this probability does not depend on time and location in the supply chain, managers should be aware of the risk coming from the whole structure of the supply chain network.

Secondly, as a result of the dynamics of the topology of the supply chain network, strong competition in prices and fast changing technology, even the most reliable firms should be monitored and constantly re-evaluated in terms of their production capacity and risks associated with their structure of connections. In simulations we observed, that if a company is not able to quickly adapt to the changing environment, it might be exposed to the risk of collective defaults of suppliers, which can give rise to disruptions and delays in production. After a period of time, even the biggest and most reliable suppliers may default as they cannot meet the demand they face from the consumer's stage. In other words, managers should be aware of the inertia risk arising from the connections among suppliers in the next tiers of a supply chain network. This observation adds to the understanding of the risk of collective bankruptcies of suppliers, as highlighted by Wagner et al. (2009).

Third and most important, managers should find ways to cut costs and reinvest the free cash flows in new technology of production, which will allow further cost reductions and the development of new innovative and cheaper products. Especially in times of recession, investments made in R&D will prepare companies better for the economic rebound. At the same time, we found that if decision makers focus on bringing down the sales price in order to gain competitive advantage, it turns out to be a strategy which leads to a lower

performance of the global supply chain network.

## 6 Conclusions

In this paper we studied the collective dynamics of a supply chain network using the agent-based modeling approach. We introduced stochastic elements to our model and observed how local events affect the global performance of the system. We observed that although network reconfiguration leads to the turbulent performance of the system, after a period of time, meta stable structures are formed, resulting from the high profitability of selected portfolios of suppliers and consumers. Thanks to the graphic engine, we could monitor the behavior of the supply chain network online. We think, that visualization is a powerful tool for analysis of complex supply chain networks as it gives insight into underlying network dynamics.

The limitation of our model is the simplified agent's behaviour. Future research should model the choice of optimal suppliers and formation of most robust link topology more realistically (Wagner and Friedl 2007). As computational power rises, it is possible to implement complex behavioral rules, based on the risk aversion of agents and the commitment to establish long-term relationships based on the quality of deliveries. This could be modeled, for instance, by introducing correlations between the production reliability and the cost of production. The micro-correlations in production failures among large set of suppliers may lead to the significant supply chain disruptions occurring with the low probability. Quantifying this process is an open field for investigations. Another research path is to study the emerging meta-stable network structures which we observed in simulations. It is interesting to see which configurations of the network prove the most robust when production processes are severely disrupted.

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