Eur. Phys. J. Special Topics **222**, 1335–1345 (2013)

DOI: 10.1140/epjst/e2013-01929-5

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Regular Article

Stochastic resonance for information flows on hierarchical networks

Agnieszka Czaplicka[1,a], Janusz A. Hołyst[1,b], and Peter M.A. Sloot[2,3,4,c]

[1] Faculty of Physics, Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland
[2] Computational Science, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands
[3] National Research University of Information Technologies, Mechanics and Optics (ITMO), Kronverkskiy 49, 197101 Saint Petersburg, Russia
[4] Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore

Received 3 June 2013 / Received in final form 10 July 2013
Published online 13 September 2013

Abstract. A simple model of information flows represented by package delivery on networks with hierarchical structures is considered. The packages should be transferred from one network node to another and the delivery process is influenced by two types of noise. The first type of noise is related to a partially false knowledge of network topology (topological noise), i.e. membership of nodes in communities in a shipping algorithm include a number of errors corresponding to a random rewiring of a fraction of network links. The second type of noise (dynamical noise) is related to a diffusive part in packet dynamics, i.e. package paths do not follow from completely deterministic rules. In the case of a pure topological noise and in the case of combination of both types of noises, we observe a resonance-like phenomenon for communication efficiency. The system performance measured as a fraction of packages that are delivered in a certain time period or as an inverse of time of a package delivery is maximal for intermediate levels of noise. This effect resembles the phenomenon of stochastic resonance that exists in many complex systems where a noise can enhance the information transfer.

1 Introduction

The issue of an optimal distribution of information flows transferred between individuals or communities is one of the most challenging problems related to communication efficiency [1–4]. Here we shall consider the quality of information transfer in a simple package delivery model on networks with hierarchical structures. This kind of topology corresponds to connections observed in many real systems, including metabolic

[a] e-mail: andagaczapl@if.pw.edu.pl
[b] e-mail: jholyst@if.pw.edu.pl
[c] e-mail: p.m.a.sloot@uva.nl

networks [5,6], protein interaction networks [7–9], the World Wide Web [10], and some social networks [11–13].

The meaning of information and information spreading varies considerably in such fields as mathematics [14], physics [15], biology [16,17], social science, or communication and computer sciences [18]. When the information is transmitted by a complex network, the process can be represented as an evolution of a distributed information field that is related to states of network nodes (and/or links) or as paths of localized packages traveling through the network. In the first case, nodes and links form systems of coupled threshold devices [16,18–22] or oscillators [23] that make a signal propagation from a sender to a receiver possible. In the second case, the nodes possess some binary attributes that can be transmitted from one node to another (see next paragraphs).

A challenging issue is understanding the role of network topology in the information transfer. Different attributes of topology dependence have been considered. Networks can consist of heterogeneous (routers and peripheral nodes) [24–31] or homogeneous [16,18–20,22,31–39], nodes. One can also focus on the heterogeneity of edges representing weighted networks [32,39,40]. Several parameters have been introduced to quantify and to compare the efficiency of information transfer in various networks [41]. Influence of network parameters on such a performance (e.g. the question if the information flow on scale-free network is more efficient than on the regular lattice) has been studied by several authors [27,31,35,39,42–52].

An important attribute of the information flow models is the dynamics of the information. One can consider dynamics of binary packets in the specific way the packets are sent to the next nodes. The rule can be as simple as a random walk [31,34,36,38,49,53] or more elaborate, like a biased random walk with some local and/or global navigation rules [25–28,30,32,35,39,45,47,50–52,54–56]. It is also possible to implement epidemic models like SIR [17,27,33,48,57–59]. Moreover, we can consider either single particles [26,27,30,35,36,38,47,49] or interacting particles systems [21,24,28,31,32,34,51,53,56]. The decision of which types of dynamics we should choose strongly depends on the interpretation of the modeled information.

The other way one can classify models of information flows is to take into account the aim of the model, i.e. what kind of information transfer property is modeled. One can measure entropy of the network [45,52,55,60], mutual information [19,20,61], transmitted information [16,18,21,43], first passage time [53], or congestion in a network [25,30–32,35,36,39,44,46,47,51,56]. There are also models tailored to reconstruct collected data from real systems [17,29,44,62–64].

In this paper, we consider a simple model of information flows represented by packages traveling on networks with hierarchical structures. We take into account two types of noise in the system. The first type corresponds to misinformation about a community structure and the second type describes a non-deterministic part of *system dynamics*. Numerical simulations show a resonance-like behavior of information transfer efficiency in the presence of both types of noises in investigated networks.

The paper is organized as follows. In Sect. 2, we introduce the topology of considered hierarchical networks. In Sect. 3, we describe assumed rules of packet dynamics. Numerical results are reported in Sect. 4 and conclusions are presented in Sect. 5.

2 Networks topology and topological noise

In this section, we focus on the information flow on networks with hierarchical structures. What is known from previous studies is that this kind of topology is very common in real-world systems such as social and biological networks ([5–13]). One can also find several examples of hierarchical network models in other literature

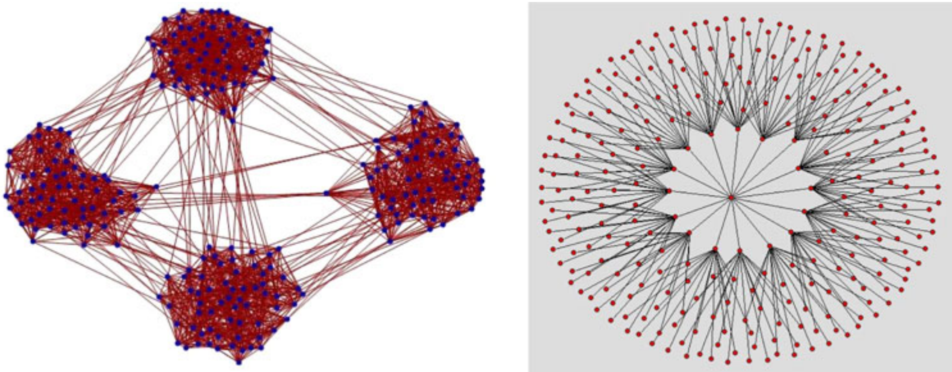

Fig. 1. Topology of hierarchical networks. The left picture represents a network with Poisson-like degree distribution and the right picture represents a tree structure.

(e.g. [35,65]). A common feature of hierarchical networks is a high value of clustering coefficient and its dependency on node degree. Based on this attribute, we can extract information about the hierarchical community structure.

We consider hierarchical networks with an additional feature that we call *topological noise*. This noise corresponds to the false information that we have in the network topology and can result, for example, from the process of links rewiring if the data on this process are not stored in an appropriate vector of membership of nodes in communities. In such a case, the package that is sent from node i to node j can fail to be delivered in a time comparable to a network size since its path is traced using partially error data.

2.1 Network with Poisson degree distribution

We take into account networks with hierarchical structures proposed in [66] (see Fig. 1) and examined in [34] with respect to efficiency of information transport as a function of community structure in such a network topology. We divide $\mathrm{N} = 256$ nodes into 16 communities (16 nodes each). A community of 16 nodes forms the first level of the hierarchical structure. Next, we merge every 4 communities in one group and define the group as the second level of hierarchy structure. All individual nodes form the third hierarchical level. In the first hierarchical level, any node possesses z_1 connections to neighbors in its community. Similarly, any node possesses z_2 and z_3 links in the second and third hierarchical level, respectively. We keep $z_1+z_2+z_3 = 18$, therefore $\langle k \rangle = 18$. We call type of networks *Poisson-like networks* since their degree distributions resemble the Poisson distribution.

We study how the topological noise influences network's delivery properties. In our model, we assume that, with probability p a single edge can be randomly rewired, i.e. we change the link between nodes i and j to a link between i and k. Adjacency matrix is updated for every p-level, but vector of membership of nodes in communities is defined for network without noise ($p = 0$) and we keep it constant during topology changes. For that reason when we increase p it is more difficult to find receiver of a packet when we based on its false community belonging. Although the total number of edges is constant for any level of p, we observed some changes in a node degree distribution $P(k)$. The most important difference occurs in the first hierarchical level. As one can expect, at some high level of noise, we loose the hierarchical structure, and reach random networks with a mean degree of node $\langle k \rangle \approx 18$.

2.2 Network with tree structure

A network with a tree structure can be considered as the simplest hierarchical model (see Fig. 1). We build our tree in the following way: at the first hierarchical level, we have a single node. The second level consists of n offspring nodes that branch to the third level and third level consists of $n \cdot n$ nodes, etc. It is easy to identify communities that correspond to groups of nodes having the same 'mother-node' (nodes which are connected to common node in a higher hierarchy level). In our simulations we study a tree with $n = 16$ and $h = 3$ levels of hierarchy, so we have a network with $N = 273$ nodes. Similarly as in the case of the network 2.1 we perform a random rewiring with a probability p of all existing links.

2.3 Connected networks

Let us consider two identical networks A and B with N nodes in each of them and let us consider the influence of topological noise (defined in Sect. 2) on them. When the noise level is zero, i.e. $p = 0$, there are no connections between these networks. When the noise level increases, links between nodes in A and B appear. For $p \approx 1$, we have a network AB with $2N$ nodes and a topology that is similar to an Erdos-Renyi random network topology [67].

3 Definition of model dynamics

We investigate the dynamics of packets displacements on networks. The process represents an information transfer between network nodes. We consider a journey of a single package from its sender to its receiver in a finite time, T, with a noise level q. This means that, with a probability q, the dynamics of the packets is similar to a random walk and with the probability $1 - q$, the packet is traveling according to a deterministic algorithm described below. In every time step, the packet can change its position no more than once. We consider the packets separately, i.e. we do not take into account any interactions between packets on a network. The algorithm of packet dynamics is as follows:

- Firstly, we randomly choose sender and receiver nodes for the packet (they must be different nodes).
- For consecutive time steps $t = 0, ...T$, we observe how a packet travels from a node to a node and we guide it towards its destination node. Let us consider a packet a in the node i in the time step t.
  - (i) We check if $t < T$. If the condition is fulfilled, we continue dynamics, i.e. we go the the step (ii). Otherwise, the loop is completed and the packet is considered as undelivered.
  - (ii) We check if, in the nearest neighborhood of node i, there is a receiver of packet a. If so, then with the probability $1 - q$, the packet a is delivered to its receiver. If not, we go to the step (iii).
  - (iii) We check if, in the nearest neighborhood of the node i, there is a node that belongs to the same community as the receiver of the packet a. If so, then with the probability $1 - q$, the packet a is moved to this node. Otherwise, we go to the step (iv).
  - (iv) We randomly choose one of the nearest neighbours of the node i and send the packet a there.
  - (v) If the packet a finds its destination, the loop is completed. If not, we continue iterations starting from the step (i).

Important is that we take into account the membership of nodes in communities defined from the original network structure, i.e. for the topological noise level $p = 0$ (when links were NOT rewired). This leads to misinformation (false information) about *a current* community structure for $p > 0$ in packets' dynamics. Particularly for the case of p close to 1, the packets can never reach their destination because of incorrect navigation rules. An interesting phenomenon is the influence of both types of noise on each other. We discuss these effects in detail in the next Section.

4 Results

We examine the system efficiency for different levels of noise contaminating the network topology and the packets' dynamics. We assume that the maximal acceptable time T of packet delivery equals to a network size $T = N$ for a single network and $T = 2N$ for the case of two connected networks described in Sect. 2.3. For each pair of parameters p and q, we consider $L = 1000$ packets and we average our results over $Q = 1000$ realizations, i.e. during the journey of L packets we keep our topology constant and we take into consideration Q network realizations for every p level. As measures of system efficiency, we take into consideration the following quantities:

- the ratio of delivered packets $l_p = \frac{L_p}{L}$, where L_p describes the number of delivered packets.
- the inverse of packet delivery time averaged over all packets $\frac{1}{\tau} = \frac{1}{L}\sum_a \frac{1}{\tau_a}$, where τ_a is the number of time steps after the packet a reaches its destination node. If, after T time steps, the packet a is still undelivered, we take $\frac{1}{\tau_a} = 0$.

One can see representative results of our simulations in Figs. 2–5.

Let us start with a case when there is no false information in the network, i.e. $p = 0$ and the packet receives correct information about a network community structure described in Sect. 2 but the packet travel is randomly perturbed by the dynamical noise described by the parameter q. When we increase the intensity of the dynamical noise, one can observe a similar decline of information transfer efficiency for all studied networks, see Fig. 2. As one can expect when the randomness in the system dynamics is higher, the packets need more time to reach their well defined destination nodes. As a consequence, we also observe a decrease of ratio of delivered packets l_p as a function of the parameter q. However, it is worth noting that the network topology with a Poisson-like degree distribution is more efficient for information transfer than a tree-like structure in this case (compare values at plots presented in upper and lower rows at Fig. 2).

Now let us consider an opposite case when the dynamical noise is absent ($q = 0$) and a packet follows a deterministic dynamics but its destination address can include incorrect information about a community structure since network links are randomly rewired due to the presence of topological noise ($p > 0$). Figure 3 shows that for a majority of studied systems the fraction of delivered packets $l_p(p)$ decreases monotonically with the noise intensity p. A remarkable exception is the behavior of two coupled Poissonian networks (Fig. 3(B)) where a maximum of $l_p(p)$ is for $0 < p_c \ll 1$. The emergence of this maximum can be understood as follows: for the case of two separate and identical networks, A and B, the maximal value of the total number of delivered packets can be only $L/2$ since a receiver and a sender are randomly selected with equal probability from the set of $2N$ nodes. It gives the value $l_p(p = 0) = 0.5$. When we start the links rewiring, cross-links can emerge between A and B networks and as a result some packets can be delivered *between* both networks. This effect increases the ratio l_p that reaches its maximum $l_p(p) \approx 1$ for a small value of noise p_c. An increasing noise level causes however obstacles for a packet delivery *inside* every network thus

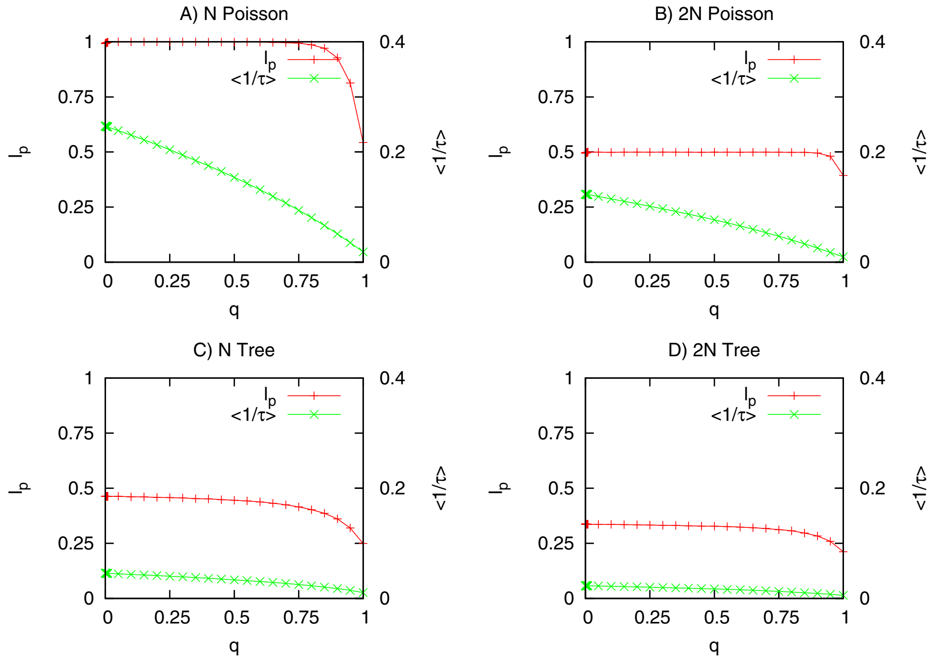

Fig. 2. Ratio $l_p(q)$ of delivered packets and averaged inverse time of packet delivery $\frac{1}{\tau}(q)$ as functions of dynamical noise q in the absence of topological noise ($p = 0$).

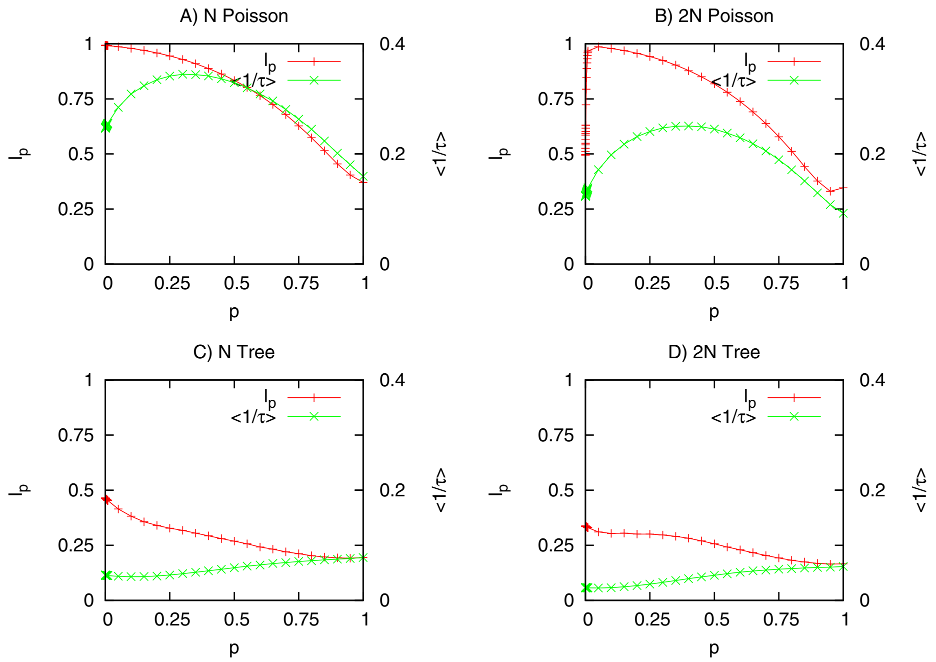

Fig. 3. Ratio $l_p(p)$ of delivered packets and averaged inverse time of packet delivery $\frac{1}{\tau}(p)$ as functions of topological noise p in the absence of dynamical noise ($q = 0$).

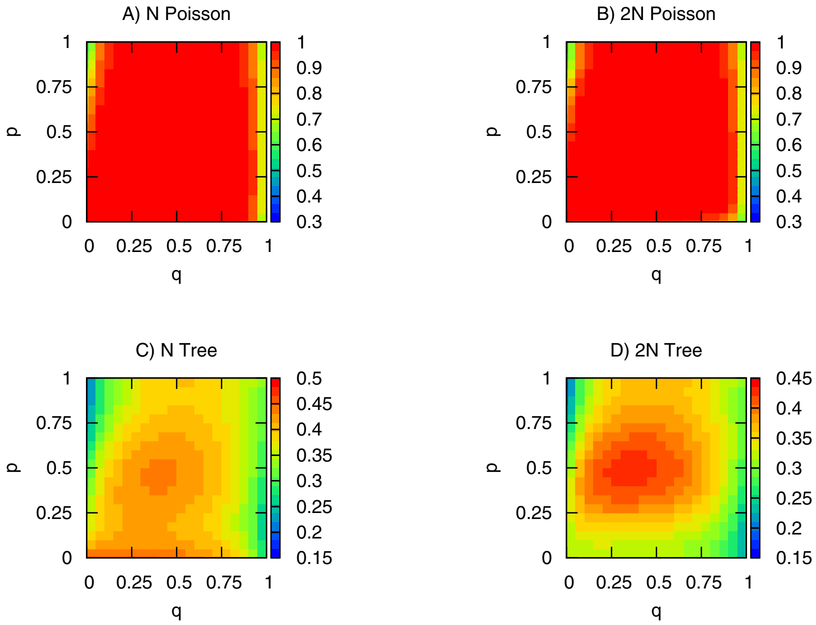

Fig. 4. Ratio $l_p(q, p)$ of delivered packets as a function both types of noises.

the fraction l_p diminishes for $p > p_c$ similarly to the case of a single network (see Fig. 3(A)). It is interesting that the effect of increasing of l_p due to cross-links is not observed in the case of coupled trees (see Fig. 3(D)).

Figure 3 shows also that for some intermediate levels of noise p networks with a Poisson-like degree distribution are most efficient in packets delivery when transfer times τ are considered, i.e. $\frac{1}{\tau}$ has a maximum for $0 < p < 1$ which means that the average time of packets delivery is minimal. The effect is observed for a single network (Fig. 3(A)) as well as for a pair of networks (Fig. 3(B)) and is probably a result of an increasing clustering coefficient due to random rewiring.

When we take into account networks with tree-like structures Fig. 3(C), (D) the system performance is lower in comparison to Poisson-like networks. Similar to previous cases, we also observed a negative influence of noise on the ratio of delivered packets l_p. What is interesting in Fig. 3(C), (D) one can see a monotonically increasing average inverse of packet delivery time. This means that, even for completely wrong information about communities of the neighbors, packet can find its destination. Similarly to the Poisson-like networks it can be caused by an increase of clustering coefficient as a function of rewiring parameter p.

One can ask how the system transmission performance changes when we take into consideration two noises at one time? The results are presented at Figs. 4–5. The first observation is that the combination of very high values of parameters p and q has a negative influence on the system efficiency since the point $(p = 1, q = 1)$ is never a maximum of functions $l_p(p, q)$ or $1/\tau(p, q)$. The next observation is a clear domination of red color areas at Figs. 4(A), (B). The areas correspond to these combinations of noise parameters p, q when the value of the ratio $l_p(p, q) \cong 1$, i.e. when all or nearly all of the sent packets reach their destination thus the system transmission performance is very high. As can be expected, for this reason, in Fig. 5(A), (B) the maximal value of average inverse of delivery time is high $((1/\tau(p, q))_{max} \approx 0.35)$, thus, the average time of packet travel was around 3 time steps.

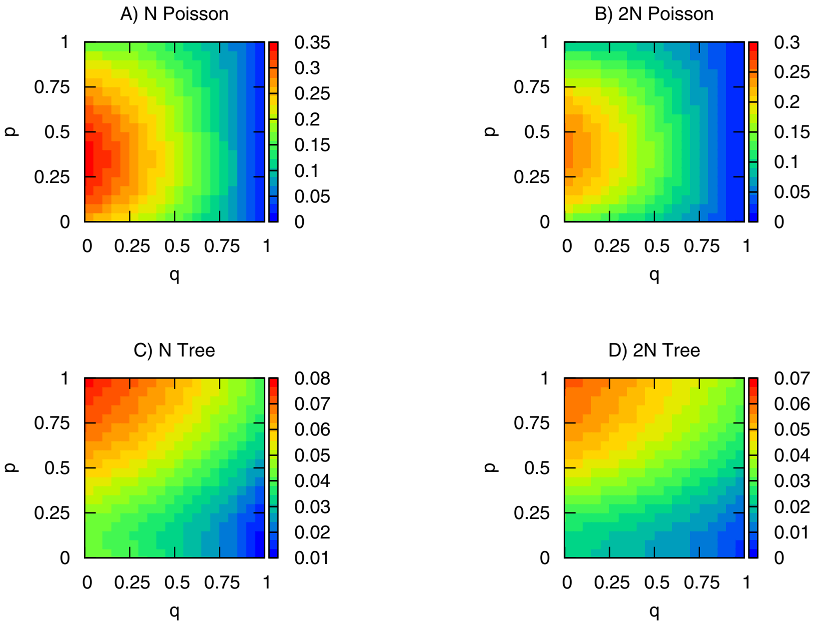

Fig. 5. Averaged inverse time $\frac{1}{\tau}(q, p)$ of packet delivery as a function of both types of noises.

On the other hand at the Fig. 4(C), (D) representing the delivery ratio $l_p(p, q)$ for a pair of Poisson-like networks the red color corresponding to $l_p(p, q)_{max} \approx 0.45$ is less common. When we look at Fig. 5(C), (D), we notice that, in this case, the maximal average time of packet delivery is more than 4 times larger than in the cases of networks presented at 5A, B.

A very striking result is shown in Fig. 4(C), (D), that for a tree and a pair of coupled tree networks the most efficient packet delivery measured by the ratio $l_p(p, q)$ is for some intermediate levels of two noises. This phenomenon could be never deducted from strictly monotonic functions $l_p(q)$, $l_p(p)$ presented at Figs. 2(C), (D), 3(C), (D) and is a very interesting manifestation of a specific synergy effect between two kinds of stochastic influences. The effect corresponds to the well known phenomenon of *stochastic resonance* (SR), see e.g. [68–70,72–79]. SR is a peculiar effect in which the transmission of a signal by certain nonlinear systems can be improved by the addition of noisy component. In systems with SR a selected measure of information transmission performance shows maximum (or maxima) as a function of noise intensity. SR can take place under various forms, according to the types considered for the noise, for the information-carrying signal, for the nonlinear system realizing the transmission or detection, and for the quantitative measure of performance receiving improvement from the noise [70]. This paradoxical effect was reported in bistable and monostable systems [79], including sensory neurons [71,72], optical systems [73], signal transmission in ion channels [74], electronic circuits [75] or models of financial crises [76]. SR was also observed in noise-free systems [77,78] when the role of stochastic component is played by deterministic chaos. For reviews about SR see e.g. [79].

The last observation from our numerical simulations follows from Fig. 5. For all studied networks when we increase the dynamical noise level q the traveling packets need more time to find their destination thus the optimal point for the system performance measured by the maximum of the function $1/\tau(p, q)$ is always at the boundary $q = 0$.

5 Conclusions

We studied the efficiency of packet delivery in several models of complex networks. The system communication efficiencies were quantified by the fraction of delivered packages l_p and the inverse of the delivery time $1/\tau$. If the systems are free from a topological noise, corresponding to a random rewiring of some network links, then both communication efficiency parameters are maximal with the absence of the dynamical noise (Fig. 2). On the other hand, if the systems are free from the dynamical noise, the role of topological noise is very ambiguous (Fig. 3): for a hierarchical Poisson-like network and coupled Poisson-like networks, the mean inverse of delivery time is maximal for intermediate values of the topological noise. In the case of coupled Poisson-like networks, this type of noise enhances also the ratio of delivered packages. The topological noise decreases the delivery time of packages traveling in tree-networks and for coupled tree-networks since its presence increases the delivery time and decreases the fraction of delivered packages in such systems. When both types of noise can mutually interfere, the most striking effect is for a system of trees (Fig. 4(C), (D)). In such a case, the maximal fraction of delivered packages occurs in the middle of the noise plane, i.e. for non-zero values of topological and dynamical noise. In other words, a randomized packet delivery helps to avoid paths received from the false topological information. In fact a tourist given a wrong hotel address can faster reach the destination if he tries a random walk strategy instead of permanent recurrences to a wrong site. This constructive effect of the noise interference can be identified as the phenomenon of Stochastic Resonance, which is common in several physical, biological and technical systems where the noise enhances the information transfer efficiency.

The work was supported by FP7 FET Open project *Dynamically Changing Complex Networks – DynaNets* EU Grant Agreement Number 233847, European COST Action MP0801 *Physics of Competition and Conflicts*, and Polish Ministry of Science Grant 1029/7.PR UE/2009/7 and Grant 578/N-COST/2009/0. PMAS also acknowledges the support from a grant from the "Leading Scientist Program" of the Government of the Russian Federation, under contract 11.G34.31.0019 and the support from the FET-Proactive grant TOPDRIM, number FP7-ICT-318121 as well as the Complexity program of the NTU in Singapore.

References

1. E. Agiliari, R. Burioni, D. Cassi, F.M. Neri. Phys. Rev. E **73**, 046138 (2006)
2. M. Rosvall, A. Transina, P. Minnhagen, K. Sneppen, Phys. Rev. Lett. **94**, 028701 (2005)
3. M. Boguna, D. Krioukov, Phys. Rev. Lett. **102**, 058701 (2009)
4. J. Vragovic, E. Louis, A. Diaz-Gouliera, Phys. Rev. E **71**, 036122 (2005)
5. H. Jeong, B. Tombor, R. Albert, Z. Oltvai, A.L. Barabasi, Nature (London) **407**, 651 (2000)
6. A. Wagner, D.A. Fell, Proc. R. Soc. London Ser. B **268**, 1803 (2001)
7. H. Jeong, S. Mason, A.L. Barabasi, Z.N. Oltvai, Nature (London) **411**, 41 (2001)
8. A. Wagner Mol. Biol. Evol. **18**, 1283 (2001)
9. D. van Dijk, G. Ertaylan, C.A.B. Boucher, P.M.A. Sloot, BMC Systems Biology **4**, 1 (2010)
10. R. Albert, H. Jeong, A.L. Barabasi, Nature (London) **401**, 130 (1999)
11. M.E.J. Newman, Proc. Natl. Acad. Sci. U.S.A. **98**, 404 (2001)
12. M.E.J. Newman, Phys. Rev. E **64**, 016131 (2001)
13. A.L. Barabasi, H. Jeong, Z. Neda, E. Ravasz, A. Schubert, T. Vicsek, Physica A **311**, 590 (2002)

14. T.M. Cover, J.A. Thomas, *Elements of Information Theory*, 2nd edition (Willey, 2006)
15. M. Karnani, K. Paakkonen, A. Annila, Proc. R. Soc. A **465**, 2155 (2009)
16. H. Mino, D.M. Durand, Biol. Cybern. **103**, 227 (2010)
17. B. Voelk, R. Noe, Behav. Ecol. Sociobiol. **64**, 1449 (2010)
18. M.D. McDonnell, D. Abbott, E.M. Pearce Ch, Microelectronics J. **33**, 1079 (2002)
19. A. Nikitin, N.G. Stocks, R.P. Morse, Phys. Rev. E **75**, 021121 (2007)
20. L. Kostal, P. Lansky, Phys. Rev. E **81**, 050901(R) (2010)
21. N.G. Stocks, Phys. Rev. E **63**, 041114 (2001)
22. D. Rousseau, F. Duan, F. Chapeau-Blondeau, Phys. Rev. E **68**, 031107 (2003)
23. S. Barbay, G. Giacomelli, F. Marin, Phys. Rev. E **63**, 051110 (2001)
24. E. Agliari, R. Burioni, D. Cassi, F.M. Neri, Phys. Rev. E **73**, 046138 (2006)
25. S. Meloni, J. Gómez-Gardenes, Phys. Rev. E **82**, 056105 (2010)
26. M. Boguna, D. Krioukov, Phys. Rev. Lett. **102**, 058701 (2009)
27. M. Nekovee, Y. Moreno, G. Bianconi, M. Marsili, Physica A **374**, 457 (2007)
28. X. Ling, M.B. Hu, R. Jiang, R. Wang, X.B. Cao, Q.S. Wu, Phys. Rev. E **80**, 066110 (2009)
29. M. Barthelemy, B. Gondran, E. Guichard, Physica A **319**, 633 (2003)
30. Z. Zhou, Z.G. Huang, L. Yang, D.S. Xue, Y.H. Wang, J. Stat. P08001 (2010)
31. R. Germano, A.P.S. de Moura, Phys. Rev. E **74**, 036117 (2006)
32. A. Arenas, A. Díaz-Guilera, R. Guimera, Phys. Rev. Lett. **86**, 3196 (2001)
33. Y. Moreno, M. Nekovee, A.F. Pacheco, Phys. Rev. E **69**, 066130 (2004)
34. A.P.S. de Moura, Phys. Rev. E **71**, 066114 (2005)
35. L. Danon, A. Arenas, A. Díaz-Guilera, Phys. Rev. E **77**, 036103 (2008)
36. J. Duch, A. Arenas, Phys. Rev. Lett. **96**, 218702 (2006)
37. P. Cappellaro, C. Ramanathan, D.G. Cory, Phys. Rev. Lett. **99**, 250506 (2007)
38. E. Agliari, R. Burioni, D. Cassi, F.M. Neri, Phys. Rev. E **75**, 021119 (2007)
39. A. Fronczak, P. Fronczak, Phys. Rev. E **80**, 016107 (2009)
40. A. Baronchelli, R. Pastor-Satorras, Phys. Rev. E **82**, 011111 (2010)
41. M. Lungarella, A. Pitti, Y. Kuniyoshi, Phys. Rev. E **76**, 056117 (2007)
42. A. Trusina, M. Rosvall, K. Sneppen, Phys. Rev. Lett. **94**, 238701 (2005)
43. M. Nykter, N.D. Price, A. Larjo, T. Aho, S.A. Kauffman, O. Yli-Harja, I. Shmulevich, Phys. Rev. Lett. **100**, 058702 (2008)
44. I. Vragovic, E. Louis, A. Díaz-Guilera Phys. Rev. E **71**, 036122 (2005)
45. J. Gómez-Gardenes, V. Latora, Phys. Rev. E **78**, 065102(R) (2008)
46. D. Helbing, H. Ammosera, Ch. Kuhnert, Physica A **363**, 141 (2006)
47. E. Estrada, Phys. Rev. E **80**, 026104 (2009)
48. L. Huang, K. Park, Y.Ch. Lai, Phys. Rev. E **73**, 035103(R) (2006)
49. B. Tadic, S. Thurner, Physica A **332**, 566 (2004)
50. M. Rosvall, K. Sneppen, Europhys. Lett. **74**, 1109 (2006)
51. B. Tadic, Adv. Complex Syst. **5**, 445 (2002)
52. R. Sinatra, J. Gomez-Gardenes, R. Lambiotte, V. Nicosia, V. Latora, Phys. Rev. E **83**, 030103(R) (2011)
53. L. Lizana, M. Rosvall, K. Sneppen, Phys. Rev. Lett. **104**, 040603 (2010)
54. M. Rosvall, P. Minnhagen, K. Sneppen, Phys. Rev. E **71**, 066111 (2005)
55. Y. Yokoya, Phys. Rev. E **69**, 016121 (2004)
56. W.X. Wang, B.H. Wang, Ch.Y. Yin, Y.B. Xie, T. Zhou, Phys. Rev. E **73**, 026111 (2006)
57. B. Karrer, M.E.J. Newman, Phys. Rev. E **82**, 016101 (2010)
58. J. Gu, W. Li, X. Cai, Eur. Phys. J. B **62**, 247 (2008)
59. P.S. Dodds, D.J. Watts, Phys. Rev. Lett. **92**, 218701 (2004)
60. K. Anand, G. Bianconi, Phys. Rev. E **80**, 045102(R) (2009)
61. A.S. Ribeiro, S.A. Kauffman, J. Lloyd-Price, B. Samuelsson, J.E.S. Socolar, Phys. Rev. E **77**, 011901 (2008)
62. M. Rosvall, A. Trusina, P. Minnhagen, K. Sneppen, Phys. Rev. Lett. **94**, 028701 (2005)
63. D. Liben-Nowell, J. Kleinberg PNAS **105**, 4633 (2008)
64. J. Zhao, J. Wu, K. Xu, Phys. Rev. E **82**, 016105 (2010)

65. E. Ravasz, A.L. Barabasi, Phys. Rev. E **67**, 026112 (2003)
66. M.E.J. Newman, E. Girvan, Phys. Rev. E **69**, 026113 (2004)
67. P. Erdos, A. Renyi, Publ. Math. Debrecen **6**, 290 (1959)
68. Gammaitoni, P. Hänggi, P. Jung, P. Marchesoni, Rev. Mod. Phys. **70**, 223 (1998)
69. R. Benzi, A. Sutera, A. Vulpiani, J. Phys. A **14**, L453 (1981)
70. F. Chapeau-Blondeau, *in Noise, Oscillators and Algebraic Randomness. In: From Noise in Communication Systems to Number Theory*, edited by M. Planat, Lect. Notes Phys., 550 (Springer, Berlin, 2000), p. 137
71. K. Wiesenfeld, D. Pierson, E. Pantazelou, Ch. Dames, F. Moss, Phys. Rev. Lett. **72**, 2125 (1994)
72. B. NcNamara, K. Wiesenfeld, R. Roy, Phys. Rev. Lett. **60**, 2626 (1988)
73. G. Vemuri, R. Roy, Phys. Rev. A **39**, 4668 (1989)
74. A. Fuliński, Phys. Rev. Lett. **79**, 4926 (1997)
75. V.S. Anishchenko, M.A. Safonova, L.O. Chua, Int. J. Bifurcation Chaos Appl. Sci. Eng. **4**, 441 (1994)
76. A. Krawiecki, J.A. Hołyst, Physica A **317**, 597 (2003)
77. S. Matyjaskiewicz, A. Krawiecki, J.A. Hołyst, L. Schimansky-Geier Phys. Rev. E **68**, 016216 (2003)
78. A. Krawiecki, S. Matyjaśkiewicz, K. Kacperski, J.A. Hołyst. Phys. Rev. E **64**, 041104 (2001)
79. L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Stochastic resonance. Rev. Mod. Phys. **70**, 223 (1998); Reports on Progress in Physics Vol. 67 Number 1, Thomas Wellens et al., Rep. Prog. Phys. 67, 45, (2004), doi:10.1088/0034-4885/67/1/R02