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Observations of deterministic chaos in financial time series by recurrence plots, can one control chaotic economy?

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Abstract. Several economical time series such as exchange rates US$/British Pound, USA Treasure Bonds rates and Warsaw Stock Index WIG have been investigated using the method of recurrence plots. The percentage of recurrence *REC* and the percentage of determinism *DET* have been calculated for the original and for shuffled data. We have found that in some cases the values of *REC* and *DET* parameters are about 20% lower for the surrogate data which indicates the presence of unstable periodical orbits in the considered data. A similar result has been obtained for the chaotic Lorenz model contaminated by noise. Our investigations suggest that real economical dynamics is a mixture of deterministic and stochastic chaos. We show how a simple chaotic economic model can be controlled by appropriate influence of time-delayed feedback.

PACS. 05.45.Tp Time series analysis – 05.45.Gg Control of chaos, applications of chaos

1 Introduction

Financial time series are usually considered from the point of view of stochastic processes [1], *i.e.* one assumes that the behaviour of financial markets is mostly driven by *unpredictable stochastic* variables. On the other hand it is well known that even simple nonlinear deterministic systems can exhibit the phenomenon of *deterministic chaos* [2–4] and it is not easy to distinguish between both types of dynamics especially when the observed deterministic system is high dimensional [5]. Here we use a recently developed method of nonlinear dynamics called recurrence plots (RPs) to investigate properties of several financial time series. In some cases we observe a large difference between values of characteristic parameters for RPs calculated from original and from surrogate data. These differences can follow from the presence of so called unstable periodic orbits that occur in chaotic deterministic systems. The presence of such orbits allows the chaotic system to change to a periodic one by use of appropriate control methods. We will illustrate this possibility using a chaotic microeconomical model controlled by time delayed feedback. In Section 2 we present the main principles of recurrence plots analysis, Section 3 is devoted to our investigation of financial time series and Section 4 includes an example of chaos control.

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2 Recurrence plots

The method of recurrence plots was used for the first time by Eckman, Kamphorst and Ruelle [6] to study recurrences and nonstationary behaviour occurring in dynamical systems. (For more recent review papers see [7,8].) The method allows the identification of system properties that cannot be observed using other linear and nonlinear approaches and is especially useful for analysis of nonstationary systems with high dimensional and/or noisy dynamics. Another advantage of RPs is the simplicity of the algorithm used during numerical calculations. RPs are constructed on the basis of mutual distances between points belonging to the same trajectory. According to the primary definition [6] the RP for a time series of N points $x(n)$ where n is time index, is a *matrix* $N \times N$ filled with white and dark points. A dark point, called a *recurrence point* is put at the position of coordinates (i, j) provided that the distance $\rho(i, j)$ between the system states at moments $n = j$ and $n = i$, is smaller than some value R. It follows that the resulting plot is dependent on the metric used and the assumed threshold value R. Usually one can observe only one component of an unknown multidimensional state and to reconstruct hidden variables one uses the Takens embedding method [2,5,9]. In such a case the system evolution is described by a d-dimensional vector $\mathbf{X}(n)$ where the k-component of $\mathbf{X}(n)$ is $x(n - (k - 1)\tau)$ while $k = 1, 2, 3, \ldots, d$ and τ is the delay time. The distance $\rho(i, j)$ is calculated using the Euclidean or the maximal matrix defined in the d-dimensional Cartesian space

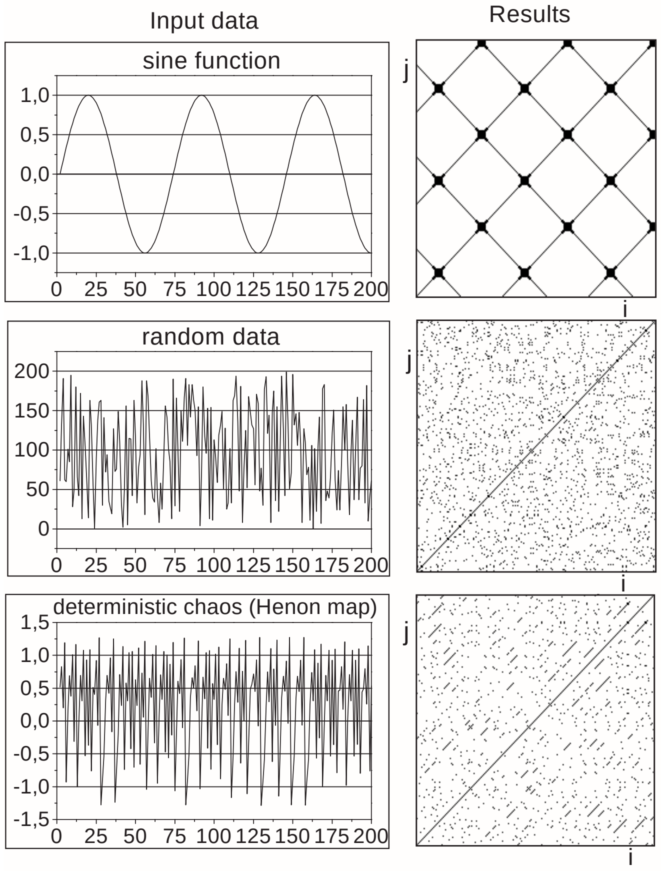

Fig. 1. Recurrence plots for sine function, for a data chain from random number generator and for the chaotic Henon map.

of vectors $\mathbf{X}(n)$. If the threshold value R is defined globally as a fixed number independent of the time index n, then the resulting plot is symmetric against the main diagonal $i = j$. This symmetry can be broken when $R = R(i)$ and the local value $R(i)$ depends on distances between the state i and a few nearest neighboring states j. In any case all points belonging to the main diagonal $i = j$ are recurrence points and the total number of recurrence points is an increasing function of the parameter R. The value of the globally defined R is sometimes assumed as a percentage of the standard deviation of the primary series $x(n)$ or a percentage of the maximal distances between all pairs of points $\mathbf{X}(i)$ and $\mathbf{X}(j)$.

Some typical examples of RPs are presented in Figure 1. One can see that in the RP corresponding to the deterministic chaotic system (Henon map [10]) there occur short lines that are parallel to the main diagonal. These are absent in the RP corresponding to the random data series. The presence of such short segments mirrors the deterministic character of the system and follow directly from the presence of so-called *unstable periodic orbits* (UPO-s) embedded in the chaotic attractor of a deterministic system. The value of the inverse of the maximal length $l_{\max}$ of these lines is related to the largest Lyapunov exponent of the chaotic system [8]. For periodic signals the largest Lyapunov exponent equals to zero and $l_{\max}$ equals to the length N of the observed signal.

Recurrence plots usually include many complex patterns and fine structure which is difficult to consider in detail. Thus in their quantitative analysis one applies several numbers. The most frequently used are: the recurrence percentage $REC = N_r/N_t$ where N_r is the number of all recurrence (dark) points, N_t is the number of all possible points (points belonging to the main diagonal $i = j$ are always excluded) and the percentage of determinism $DET = N_l/N_r$, where N_l is the number of recurrence points belonging to lines parallel to the main diagonal. One has to stress that values of REC and DET are strongly dependent on the parameters R, d, τ chosen for the construction of a RP. It follows that for the interpretation of results one needs to compare parameters REC and DET obtained from the original time series with corresponding results received from surrogate data [5].

3 RPs for financial data

We have used RPs to perform the analysis of the following financial data sets:

1. Exchange rates (US$/British Pound), monthly averages of daily figures, noon buying rates in New York City for cable transfers payable in foreign currencies, for the period January 1971–February 1999, 338 numbers.
2. A year constant maturity USA Treasury Bond rates, weekly data, for the period 05 January 1962–30 August 1996, 1809 numbers.
3. Standard & Poor's 500 Stock Index, close of month, for the period January 1945–January 1999, 649 numbers.
4. Warsaw Stock Index (WIG), daily data, for the period 02 February 1995–28 January 1999, 993 numbers.
5. *Okocim* shares (Polish beer company), daily data, for the period 02 February 1995–28 January 1999, 993 numbers.

To eliminate linear trends from the data sets we have used the simplest filter, *i.e.* an original time series $c(n)$ has been substituted by $x(n) = \log[c(n)/c(n-1)]$. A few resulting recurrence plots are shown in Figures 2, 3 and 4 where the plots on the left correspond to original time series, while those on the right correspond to surrogate data *surr1* and the bottom plots to surrogate data *surr2*. The lower part of all figures (below the main diagonal) correspond to the embedding dimension $d = 1$, upper parts to $d = 2$, time-delay equals $\tau = 1$, threshold parameter value equals to $R = 0.1\sigma$ where σ is a standard deviation calculated from the data chain $x(n)$ and the Euclidean matrix has been applied. We have used two types of surrogate data. To create *surr1* we have shuffled the data $x(n)$ randomly choosing a pair of points from the data chain and exchanging positions of such points. This procedure has been repeated N times where N is the number of all data points. The shuffling preserves the statistical distribution of the data but changes the correlation between points in the data chain. On the other hand data set *surr2* preserve

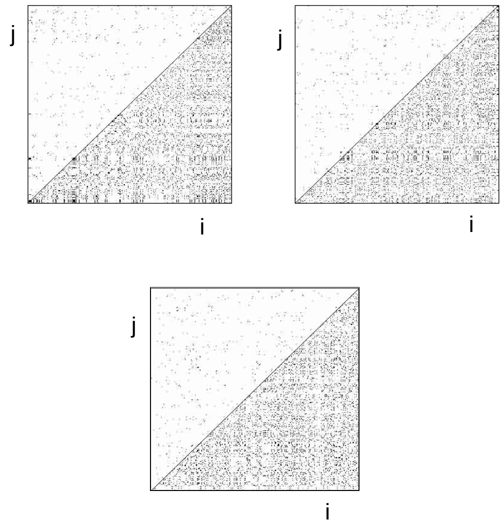

Fig. 2. Recurrence plots for US$/BP exchange rates.

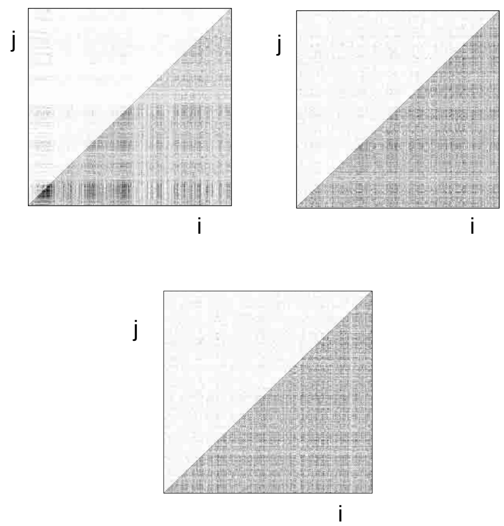

Fig. 3. Recurrence plots for USA Treasury Bond rates.

the correlations of the original data but has a different statistical distribution [5].

For the quantitative interpretation of these diagrams we have calculated the parameters REC and DET defined above. The values are presented in the Table 1 and Table 2 together with corresponding values of the surrogate data. All calculations presented in the tables have been performed using the embedding dimension $d = 2$ with the other values of parameters as given above.

Looking at Tables 1 and 2 one can see that the surrogate data possess lower values of the REC and DAT parameters as compared to the original data (with the exception of one result corresponding to *Okocim* shares). The typical percentage change of these parameters is between 10% and 20%. The last two rows of both tables describe REC and DAT values for a chain of $N = 900$ numbers corresponding to the chaotic Lorenz model [2] and to the Lorenz model contaminated by noise generated by a random numbers generator. The amplitude of the added noise

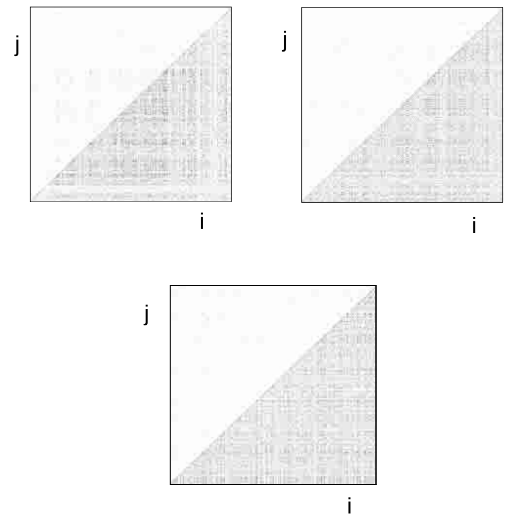

Fig. 4. Recurrence plots for Warsaw Stock Exchange index WIG.

Table 1. The percentage of recurrence (parameter REC) for various data series described in the text. TB stands for "USA Treasury Bond rates" and Lorenz+n stands for "Lorenz model contaminated by noise". The percentage changes of REC for surrogate data are written in brackets.

Data	REC				
	org.	$surr1$		$surr2$	
US$/BP	0.69	0.60	(−13%)	0.61	(−12%)
TB	0.78	0.69	(−12%)	0.70	(−10%)
S&P500	0.61	0.58	(−4.9%	0.55	(−9.8%)
WIG	0.82	0.71	(−13%)	0.70	(−15%)
Okocim	0.75	0.70	(−6.7%)	0.69	(−8.0%)
Lorenz	1.18	0.52	(−56%)		
Lorenz+n	0.58	0.48	(−17%)		

Table 2. The percentage of determinism (parameter DET) for various data series described in the text. TB stands for "USA Treasury Bond rates" and Lorenz+n stands for "Lorenz model contaminated by noise". The percentage changes of DET for surrogate data are written in brackets.

Data	DET				
	org.	$surr1$		$surr2$	
US$/BP	22.1	14.8	(−33%)	12.6	(−43%)
TB	19.3	15.7	(−19%)	15.8	(−18%)
S&P500	15.4	15.2	(−1.2%	13.1	(−15%)
WIG	20.0	15.8	(−21%)	17.2	(−14%)
Okocim	15.7	16.5	(5.2%)	13.9	(−11%)
Lorenz	67.4	13.0	(−81%)		
Lorenz+n	15.6	12.4	(−20%)		

was 50% of the amplitude of the original signal. One can see that the influence of shuffling on the purely deterministic Lorenz model is much larger than the corresponding influence on the analyzed economical data. For the former we have obtained a REC parameter that is lower by 56% and the DET parameter that is lower by 81% compared to the original data. On the other hand the corresponding changes observed for the noisy Lorenz model are milder (17% and 20% respectively) and are in the range observed for economic data. The results suggest that the analyzed economical data are mixtures of (unknown) deterministic and stochastic processes. As the largest impact of shuffling has been observed for exchange rates (US$/British Pound) these data can be considered as the most deterministic among the analyzed processes. It is interesting that according to our results a single share from the Polish stock market is much more noisy then the total Polish market index WIG. Such a behaviour can be expected because the total index is a weighted mean value of shares of individual firms and hence should be more robust against stochastic fluctuations.

4 Chaos control

If the economic evolution possesses at least in part a deterministic character then methods of chaos control [10,11] can be used to stabilized the market on the desired periodic orbit. Probably the most convenient approach for such a purpose would base on the Pyragas time-delayed feedback method [11]. A theoretical background of this method can be found in [12–14]. In the simplest case one needs to influence the system behaviour by the term

$$F_K = K(z_n - z_{n-m}) \quad (1)$$

where z_n is a system variable that can be observed (*e.g.* the exchange rate or stock market index), m is the period of the controlled orbit, and K is an appropriate control coefficient. Let us illustrate this approach using the microeconomic model of a chaotic market proposed by Behrens and Feichtinger [15]. The model describes the evolution of sales $x(n)$ and $y(n)$ of two firms X and Y competing on the same market of goods. Due to the active and asymmetric investment strategies of both firms sales evolve according to the equations

$$x_{n+1} = (1-\alpha)x_n + \frac{a}{1+\exp[-c(x_n - y_n)]} \quad (2)$$

$$y_{n+1} = (1-\beta)y_n + \frac{b}{1+\exp[-c(x_n - y_n)]} \cdot \quad (3)$$

The constants α and β (with $0 < \alpha, \beta < 1$) are the time rates at which the sales of both firms decay in the absence of investment while the second terms on the r.h.s of equations (2–3) describe the influence of investments at time n on the sales at time $(n+1)$, where positive constants a, b and c are investment parameters. The detailed analysis of control in this model by the Ott-Grebogi-Yorke method and by the Pyragas method can be found in references [16–18]. Here we present one example of the successful control of chaos in this model. Adding a control force $F_K = K_{yy}(y_n - y_{n-m})$ to the r.h.s of equation (3) can be interpreted as market control by the firm Y due to appropriate changes of its investment strategy [17]. Figure 5 shows the results of numerical simulations of the control of $m = 1$ and $m = 2$ periodic orbits. One can observe that changes of the control parameter K_{yy} induce substantial changes in the market evolution, *i.e.* the system switches between chaotic and period behaviour (chaos → period–one orbit → chaos → period–two orbit → chaos). We stress that the control is local in time, *i.e.* values of the control force F_K tend to zero after switching the system to the desired state. One can see that although the control of the market was introduced by the firm Y as the result the whole system has been stabilized and the sales x_n of the firm X have also become periodic. It is interesting that the sales of the firm X have decreased for the period-one orbit and increased for the period-two orbit similarly to the sales of firm Y. This effect is at the cost of all other firms acting on the market and influencing the dynamics of the firms X and Y due to nonzero values of decay parameters α, β in equations (2–3). The fact that the period-one orbit leads to a decrease of sales while period-two orbit leads to an increase of sales is a direct consequence of the location of these periodic orbits in the chaotic attractor [16–18].

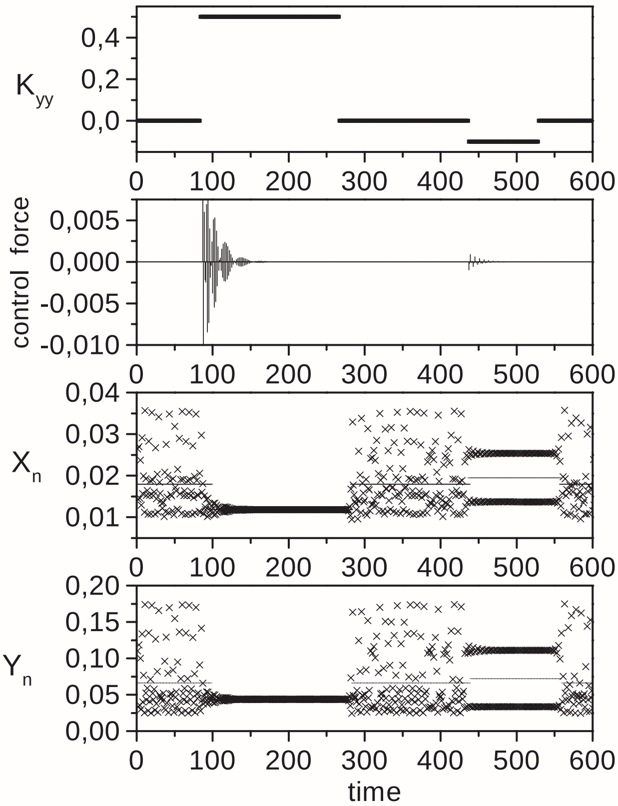

Fig. 5. Time dependence of the control amplitude K_{yy}, the control force F_K (Eq. (1)) and sales x_n, y_n. The results of the stabilization procedure are clearly seen. Thin lines depict mean values of the sales in various time intervals.

5 Conclusions

We have applied properties of recurrence plots to show that financial time series can be seen as a mixture of deterministic and stochastic behaviour because of the presence of unstable periodic orbits. The orbits can be used to control the chaos in economic dynamics by appropriate feedback methods.

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