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Networks of companies and branches in Poland

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Abstract

In the present study we consider relations between companies in Poland taking into account common branches they belong to. It is clear that companies belonging to the same branch compete for similar customers, so the market induces correlations between them. On the other hand two branches can be related by companies acting in both of them. To remove weak, accidental links we shall use a concept of threshold filtering for weighted networks where a link weight corresponds to a number of existing connections (common companies or branches) between a pair of nodes.

Keywords: Complex networks; Firms statistics; Weighted networks; Bipartite graphs

1. Introduction

Modelling behavior of economical agents is a challenging issue that can be studied from a network point of view. The examples of such studies are models of financial networks [1], supply chains [2,3], collective bank bankruptcies [4,5] and interbank money market [6,7]. Relations between different companies have been already analyzed using several methods: as networks of shareholders [8], networks of correlations between stock prices [9] or networks of board directors [10]. In several cases scaling laws for network characteristics have been observed.

2. Bipartite graph and companies and branches networks

We have used the commercial database "Baza Kompass Polskie Firmy B2B" [11] from September 2005. It contains information over 50 000 large and medium size Polish companies belonging to one or more of 2150 different branches. We have constructed a bipartite graph (Fig.1) with two kinds of objects: branches $A = 1, 2 \ldots N_b$ and companies $i = 1, 2 \ldots N_f$, where $N_b = 2150$—total number of branches and $N_f = 48\,158$—total number of companies.

Let us define a *branch capacity* $|Z(A)|$ as the cardinality of companies set belonging to the branch A. The largest capacity of a branch in our database was 2486 (construction executives), the second largest was 2334 (building materials). Let $B(i)$ be a set of branches a given company i belongs to. We define a *company diversity*

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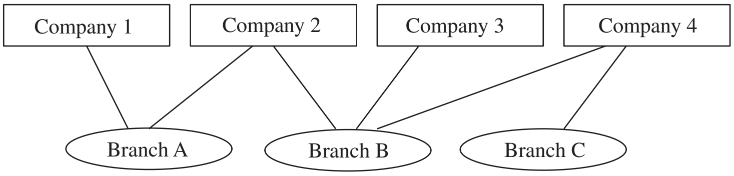

Fig. 1. Bipartite graph of companies and trades.

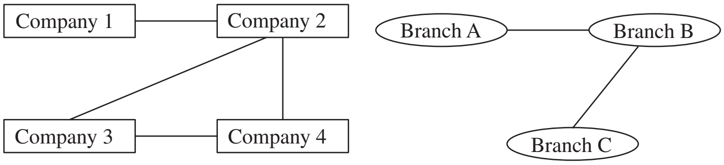

Fig. 2. Companies network on the left, branches network on the right.

as $|B(i)|$. An average company diversity μ is given as $\mu = (1/N_f)\sum_{i=1}^{i=N_f}|B(i)| = 5.99$. Similarly an average branch capacity v is given as $v = (1/N_b)\sum_{A=1}^{A=N_b}|Z(A)| = 134$. It is obvious that the following relation is fulfilled for our bipartite graph: $v/N_f = \mu/N_b$. The bipartite graph from Fig. 1 has been transformed to create a *companies network*, where nodes are companies and a link means that two connected companies belong to at least one common branch. If we use the example from Fig. 1 we obtain a companies network presented in Fig. 2 (left). Similarly a *branches network* has been constructed where nodes are branches and an edge represents a connection if at least one company belongs to both branches.

3. Weight, weight distribution and networks with cutoffs

We have considered link-weighted networks. In the branches network the link weight means the number of companies that are active in the same pair of branches and it is formally a cardinality of a common part of sets $Z(A)$ and $Z(B)$, $w_{AB} = |Z(A) \cap Z(B)|$. The weight distribution $p(w)$, meaning the probability p to find a link with a given weight w, is presented in Fig. 3. The distribution is well approximated by a power function $p(w) \sim w^{-\gamma}$ where the exponent $\gamma = 2.46 \pm 0.07$. One can notice the existence of edges with large weights. The maximum weight value is $w_{max} = 764$, and the average weight equals $\langle w \rangle = 4.67$.

A weight in the companies network is defined in a similar way as in the branches networks, i.e., it is the number of common branches for two companies—formally it is equal to the cardinality of a common part of sets $B(i)$ and $B(j)$, $w_{ij} = |B(i) \cap B(j)|$. The maximum value of observed weights $w_{max} = 207$ is smaller in this networks than in the branches network while the average value equals $\langle w \rangle = 1.48$. The weight distribution is not a power law in this case and it shows an exponential behavior in a certain range. Using cutoffs for link weights we have constructed networks with different levels of filtering. In such networks nodes are connected only when their edge weight is no less than an assumed cutoff parameter w_0. In Fig. 4 we present average degrees of nodes and maximum degrees as functions of the cutoff parameter w_0. We have observed a power law scaling $\langle k \rangle \sim w_0^{-\beta}$, $k_{max} \sim w_0^{-\alpha}$ where for branches networks $\alpha_b = 1.069 \pm 0.008$ and $\beta_b = 0.792 \pm 0.005$ while for companies networks $\alpha_f = 2.13 \pm 0.07$ and $\beta_f = 1.55 \pm 0.04$.

4. Degree distribution

We have analyzed the degree distribution for networks with different cutoff parameters. In Fig. 5 we present the degree distributions for companies networks for different values of w_0. The distributions change qualitatively with increasing w_0 from a non-monotonic function with an exponential tail (for $w_0 = 1$) to a power law (for $w_0 > 6$).

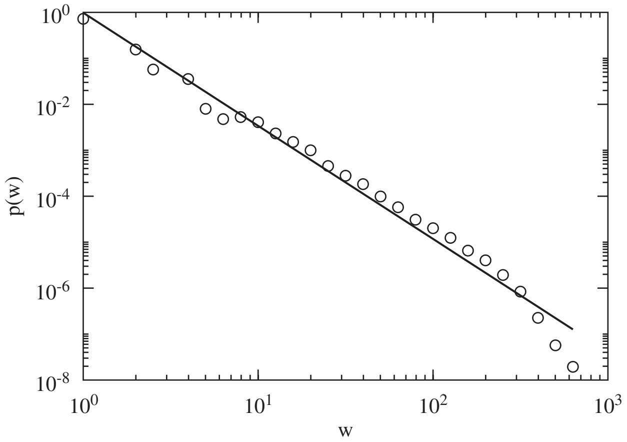

Fig. 3. Weight distribution in branches network.

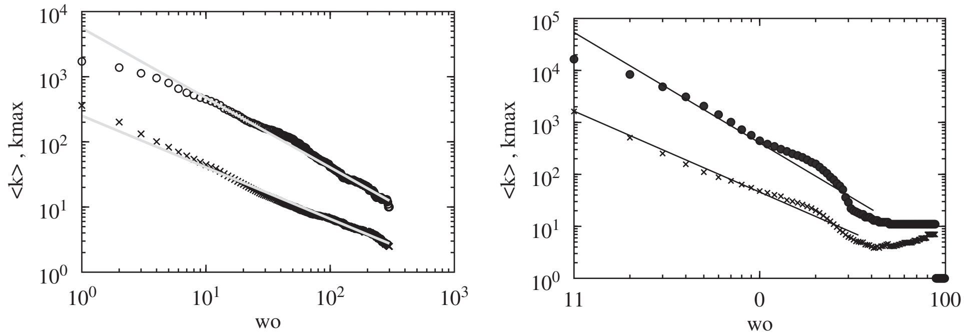

Fig. 4. Dependence of $\langle k \rangle$ (circles) and k_{max} (crosses) on cutoff parameter w_0 for branches networks (left) and companies networks (right).

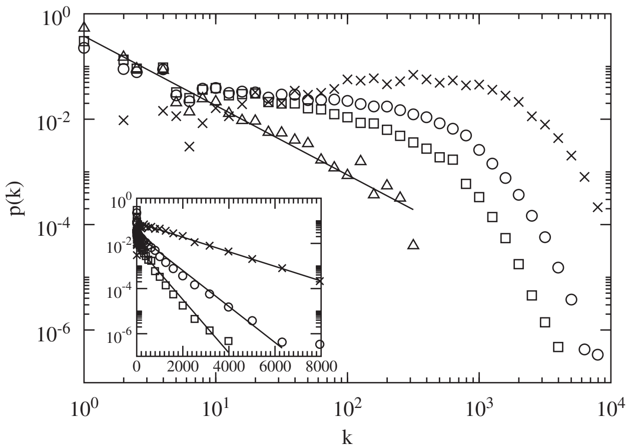

Fig. 5. Degree distributions for companies networks for different values of w_0. Crosses are for $w_0 = 1$, circles are for $w_0 = 2$, squares are for $w_0 = 3$ and triangles are for $w_0 = 12$.

In Fig. 6 we present a degree distribution for $w_0 = 1$ for branches network. We observe a high diversity of node degrees—vertices with large values of k occur almost as frequent as vertices with a small k. For a properly chosen cutoff values the degree distributions are described by power laws. For $w_0 = 4$ we see two

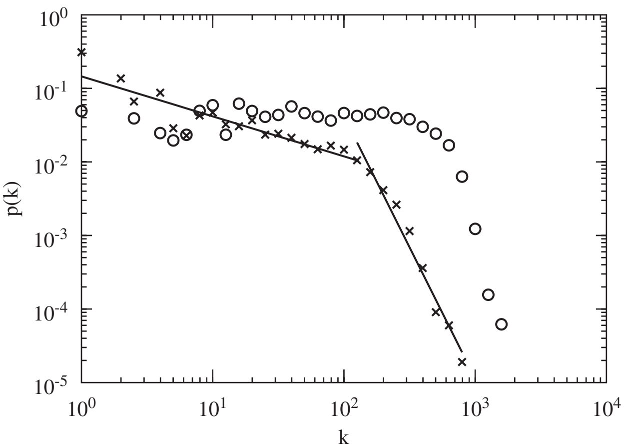

Fig. 6. Degree distribution in branches network for different values of w_0. Circles are for $w_0 = 1$, crosses are for $w_0 = 4$.

regions of scaling with different exponents γ_1 and γ_2 while a transition point between both scaling regimes appears at $k \approx 100$. Branches belonging to the first regime of scaling are more specific, for example production, and branches on the right are more general like ‘‘import an export general’’, ‘‘network of supermarket’’.

It is important to stress that in both networks (companies and branches) the scaling behavior for degree distribution occurs only if we use cutoffs for links weights, compare Figs. 5 and 6. It follows that such cutoffs act as filters for the noise present in the complex network topology.

5. Conclusions

In this study, we have collected and analyzed data on companies in Poland. 48 158 medium/large firms and 2150 branches form a bipartite graph that allows to construct weighted networks of companies and branches. Link weights in both networks are very heterogenous and a corresponding link weight distribution in the branches network follows a power law. Removing links with weights smaller than a cutoff (threshold) w_0 acts as a kind of filtering for network topology. The method results in recovery of hidden scaling relations present in the network. The degree distribution for companies networks changes with increasing w_0 from a non-monotonic function with an exponential tail (for $w_0 = 1$) to a power law (for $w_0 > 6$). For a filtered ($w_0 > 4$) branches network we see two regions of scaling with different exponents and a transition point between both regimes.

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