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Modeling defaults of companies in multi-stage supply chain networks

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ABSTRACT

The interest in supply chain networks and their analysis as complex systems is rapidly growing. The physical approach to the topic draws on the concept of heterogenous interacting agents. The interaction among agents is considered as a repeated process of orders and production. The dynamics of production in the supply chain network which we observe is nonlinear due to the random failures in processes of orders and production. We introduce an agent-based model of a supply chain network which represents in more detail the real economic environment in which firms operate. We focus on the influence of local processes on the global economic behavior of the system and study how the proposed modifications change the general properties of the model. We observe collective bankruptcies of firms, which lead to self-emerging network structures. Our results give insight into the dynamics of default processes in supply chain networks, which have important implications both for risk managers and policy makers. Based on the simulations we show that agent-based modeling is a powerful tool for optimization of supply chain networks.

1. Introduction

The recent crisis of the world's financial markets has clearly shown that the structure of connections among interacting market participants plays a crucial role in the valuation of default risk. Uncertainty in the economic environment in which companies are operating has a significant impact on the financial condition—even of highly profitable and well-known companies. Before the bankruptcy filing of Chrysler in April 2009 and General Motors in June 2009, analysts claimed that there was more than 95% probability that one of the three biggest automakers in the US (GM, Chrysler, and Ford) would go bankrupt in the next 5 years. They estimated that a default of one of the Big Three could trigger extreme losses in the $1.2 trillion market for collateralized debt obligations (CDOs), since 80% of them included credit-swaps on GM or Ford (projected by Standard & Poor's and UniCredit in 2008). The impact of default of one of these car manufacturers on their supply chain could be even more devastating. The implications of the insolvency can propagate through different interaction channels, as each firm is part of a complex supply chain network, composed of parts suppliers (like Visteon, Lear, TRW), car manufacturers and car dealers (like Autonation, United Technologies, Lithia) (Schellhorn and Cossin, 2004). In order to understand the means by which defaults propagate in the complex structures of modern supply chain networks many techniques have been used, including linear and nonlinear programming, stochastic processes modeling, and traditional discrete-event programming. The major shortcomings of these methods are that they either look at the supply chain network from the static perspective or simulate the dynamics as a random process. In reality however each of the companies has its own decision making units and uses them to gain competitive advantage. In this article we will consider the supply chain network as a system of agents (suppliers, manufacturers, retailers) connected through production relationships in the form of orders and production flows. We will use agent-based modeling (ABM) in order to analyze how local defaults of supply chain participants propagate through the dynamic supply chain network and form avalanches of bankruptcy.

This article is organized as follows. In Section 2 we give an overview of the literature. Section 3 describes the features of the model. In Section 4 we present the results. In Section 5 we discuss our findings and the managerial implications. We draw the conclusions and recommend further research ideas in Section 6.

2. Literature overview

2.1. Supply chain networks

As a result of the fast developing technology and growing complexity of today's business, logistical systems are becoming densely interconnected. The complicated structure of supply chains forms a network consisting of multiple parts. The Council of Supply Chain Management Professionals (CSCMP) defines a supply chain network as a network of interconnected elements of

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manufacturing plants, distribution centers, points-of-sale, as well as raw materials, relationships among product families, and other factors. Following Melo et al. (2009) supply chain network design and optimization problems can be divided into four groups, based on the following features:

(1) number of stages (single, multiple);
(2) number of commodities (single, multiple);
(3) number of periods (single, multiple);
(4) economic environment (deterministic, stochastic).

All of these factors influence the complexity of problems studied in supply chain management (SCM). For instance, the complexity of production dynamics in a supply chain network increases rapidly with the number of agents, stages in the network, connections between suppliers and producers, and the number of uncertain parameters, such as customers' demand or quality and frequency of deliveries. In this article we will focus on the multi-stage supply chain network with single commodity and multi-period observations. Examples of similar problem setups can be found in Ambrosino and Scutellà (2005) or Troncoso and Garrido (2005), where the authors study intra-layer flows in the deterministic environment. The proposed distribution network design includes facility location, warehousing, transportation, and inventory decisions. The major weakness of this line of research is that many crucial parameters of these models are set to be deterministic. In the real world however, managers need to make decisions, taking into account many sources of operational, market, business or political risk (for an overview see Bouchaud and Potters, 2000). These risks are associated with the stochasticity in customer demand, prices, exchange rates, frequency and quality of deliveries and others. Uncertainty in network dynamics plays a critical role in the design and operation of supply chain networks.

There are relatively few papers dealing with the uncertainties in distribution problems (Cheung and Powell, 1996) or demand planning (Van Landeghem and Vanmaele, 2002; Yu and Li, 2000). Santoso et al. (2005) propose a stochastic programming methodology for large-scale supply chain network design problems, including uncertainty. They use the sample average approximation (SAA) scheme, with an accelerated Benders decomposition algorithm for efficient solving of a potentially infinite number of scenarios. The configuration of the network is one of the crucial factors, which enables the efficient operation of the supply chain network. Goetschalckx et al. (2002) present an overview of algorithms for the design of global logistics systems and propose an efficient decomposition algorithm for multi-period production/distribution networks. Almeder et al. (2009) observe that combining complex simulation models and abstract optimization models allows the modeling and solving of more realistic problems, which include dynamics and uncertainty. Chopra et al. (2007) show that bundling two sources of risk (i.e. disruption and delays of supply) can lead to overutilization of the cheaper and less reliable supplier and underutilization of the most reliable supplier. They propose decoupling the two sources of risk. The mitigation strategy to hedge against the increased risk from disruption is to use the reliable supplier. The mitigation strategy in case of increased recurrent risk is to choose the less reliable, but cheaper supplier.

In order to investigate the dynamics of the supply chain network in an uncertain environment, we will apply the ABM technique described in the next subsection.

2.2. Agent-based modeling and bankruptcies

ABM is a powerful computational technique, which is very useful in gaining insights about decision making in large interconnected systems. ABM has attracted a great deal of attention in recent years and has already found many applications in supporting business decisions, for developing optimized systems for organizations, improving manufacturing and operations and managing supply chain networks (North and Macal, 2007). The idea originally comes from complex adaptive systems. The main question behind the application of this technique (Surana et al., 2005) is: How does the collective behavior of a system arise from interactions among autonomous agents? Supply chain networks belong to a large class of complex economic systems, in which ABM has proven capable of delivering new knowledge, developing new business policies, and improving processes of decision-making (Macal, 2003; Macal et al., 2003). The complex systems approach, although well established in many science disciplines, such as physics, engineering, computer and social science has not been given much attention in operations and supply chain management (Pathak et al., 2007a; Amaral and Uzzi, 2007). Below we present some publications devoted to this field of research and its applications to supply chain problems. Swaminathan et al. (1998) in cooperation with IBM developed a multi-agent supply chain simulation and reengineering tool for developing customized applications. Chatfield et al. (2007) combined the agent simulation with a process-oriented approach for supply chain simulation modeling and presented an open information standard to assist supply chain modeling, analysis, and decision support (Chatfield et al., 2009). Another framework was proposed by Julka et al. (2002a,b) who analyzed business policies in terms of different situations arising in the supply chain, using the example of a refinery supply chain. Akanle and Zhang (2008) proposed a methodology for manufacturing organizations to optimize supply chain configurations to cope with customer demand over a future period. Anosike and Zhang (2009) presented an agent-based approach for optimized utilization of resources under demand uncertainty, whereas Fox et al. (2000) provided a prototype of an agent-oriented software architecture for supply chain management. Their framework is an optimization software package capable of solving constraint-based problems and supports decisions under stochastic inventory perturbations. Sawaya (2006) presented a complex adaptive systems approach to analyze the benefits of sharing point-of-sale data in supply chain networks. He also addressed the issue of validation and verification methods for agent-based models with empirical data.

The multi-layer supply chain network is a dynamic system in which local interactions between suppliers and producers unfold with stochastic shortages. These local instabilities, if accumulated, can amplify and give rise to nonlinear processes leading to avalanches of bankruptcy. One of the first papers dealing with this approach to modeling supply chains is by Bak et al. (1993), who study the connections among firms related by production activity. The triggering events in their model are stochastic changes of demand at the customer stage of a supply chain network. The occurrence of avalanches on a supply chain network represented by the lattice structure was also studied by Gatti et al. (2005). Aleksiejuk and Holyst (2001) proposed an agent-based model of collective bank bankruptcies, where the properties of the simulated avalanches are in a good agreement with the percolation theory. In the related paper Aleksiejuk et al. (2002) observe the scaling behavior of collective bankruptcies, which are common for self-organizing complex systems. The most common strategy of risk diversification is the expansion of the portfolio of suppliers, known as the multi-source strategy (Kleindorfer and Saad, 2005). Battiston et al. (2007) have examined the influence of the network density on the amplification of systemic risk in trade-credit networks. They have shown that a trade-off emerges between the local risk

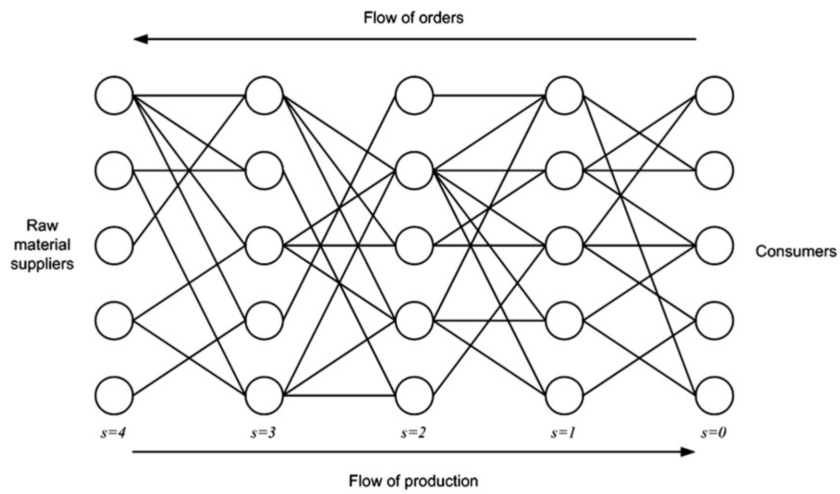

Fig. 1. The structure of the supply chain network.

diversification and the global systemic risk. Wagner et al. (2009) studied the positive supplier default dependencies using the copula approach. They indicate that default dependencies are the important factor in risk mitigation strategies and should be considered in constructing supplier portfolios. Another phenomenon which might lead to shortages in deliveries and production and in extreme cases to bankruptcies is the bullwhip effect (Lee et al., 2004). This effect was reproduced using the ABM approach, for example by Ahn and Lee (2004), who focused on the efficiency of supply chain networks and the minimization of bullwhip effects via information sharing between agents. Nagatani and Helbing (2004) showed that forecasting of future inventory levels can stabilize the linear supply chain and hedge against the bullwhip effect. In another model originally developed for transportation and traffic dynamics Helbing (2003) proposes a framework in which instabilities of supply chains may lead to business cycles. Helbing et al. (2004a,b) show that although nodes of the supply chain network behave in an overdamped way, the system itself displays damped oscillations. They use equations of material flows to model supply chain networks. The bullwhip effect in their model can be explained as a convective instability phenomenon based on a resonance effect. The authors point out that the network theory makes a useful contribution to planning and to the design of stable, robust and adaptive supply chain networks.

Our objective is to find out, how sources of uncertainty observed in practice affect the performance of the supply chain network. We focus on the emergence of global patterns of the network dynamics, resulting from local dependencies. To achieve this objective we build an ABM framework of a supply chain network along with the visualization tool, which allows us to monitor online the dynamics of the system. This enables us to understand the dependencies between suppliers and producers both on local and on global scales. We show that accumulation of the uncorrelated production disruptions leads to cascades of bankruptcies. They are the result of the dynamically changing buyer–supplier relationships, which form a complex supply chain network. By introducing the dynamically changing interconnectedness of the network we show that even the companies with the highest working capital are exposed to the risk of bankruptcy, which is much higher than in the static version of the model. The results have implications for improved risk hedging and optimized decision making for managers of large-scale supply chain networks.

3. Model

The model is based on the framework proposed by Weisbuch and Battiston (2007) which is a simple agent-based supply chain network model to analyze the geographical aspects of a supply chain network. One of the basic assumptions and the limitation of their model is the static structure of the supply chain network. Every producer has three suppliers and three customers. The production dynamics resembles the reaction–diffusion systems studied in chemical physics. The ‘reaction’ part of the process is the growth of the total capital coupled with failures in deliveries and bankruptcies, whereas the ‘diffusion’ part of the process is the propagation of orders and production across the network. The result of this multiplicative stochastic process is the occurrence of meta-stable regions of higher and lower production activity. Additionally their model reproduces some common stylistic facts about production economics, such as:

- scale-free distributions of wealth and production;
- regional organization of wealth and production;
- avalanches of shortage and resulting bankruptcies.

These results confirm that the framework is robust and open for further investigations. We propose a more general framework and extend the original approach in several dimensions. All basic assumptions remain the same as in the original model. In our model we make the framework more realistic by adding features, which convey the complex character of a supply chain network in the real economic environment, but we try to keep it as simple as possible.[1] We will discuss the following enhancements of this model:

(1) Price dispersion.
(2) Evolution of supply chain topology.
(3) Network reconfiguration.
(4) Production dynamics.
(5) Costs of production.
(6) Bankruptcies and recovery.

Following the framework introduced by Weisbuch and Battiston (2007) our network consists of N nodes organized in S stages (Fig. 1).

[1] According to Einstein’s famous quote “everything should be made as simple as possible, but not simpler.”

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We assume the network has $S=5$ stages and $N=150$ nodes. Nodes represent production facilities which are connected by undirected links. Links represent information flow in one direction (from consumers to suppliers) and the material flow in the opposite direction. No reverse activities and closed-loops are allowed. The stages of the network are numbered in the direction of the information flow, i.e. from the output stage $s=0$ (retail sector) to the input stage $s=4$ (suppliers providing raw materials). We assume that final goods are fully absorbed by the consumers and that primary producers have unlimited access to raw materials.

3.1. Price dispersion

Price dispersion (or the violation of the law of one price) is the phenomenon often observed in the markets. It captures the fact that prices of homogenous products may vary from one seller to another—a common fact, used by price comparison services in the Internet. A study of price dispersion in the automobile industry can be found in Delgado and Waterson (2003). Examples for many other industry sectors and the theoretical analysis can be found in Baye et al. (2006). To include this phenomenon in our model, at the beginning of the simulation we assign sales price to all nodes from the log-normal distribution

$$f(x,\mu,\sigma)=\frac{1}{x\sqrt{2\pi}}\sigma\exp\left(\frac{-\ln^2(x/\mu)}{2\sigma^2}\right) \tag{1}$$

for $x>0$, where μ is the expected value, and σ is the standard deviation of the logarithm of x. The log-normal distribution is widespread in financial mathematics, for instance for simulations of stock prices. It is convenient to model price dispersion using log-normal distribution, since values generated from this distribution are always positive. Price dispersion will allow us to introduce a mechanism of the dynamic growth of the supply chain network (linking algorithm).

3.2. Evolution of supply chain topology

The first element of stochasticity is the variable number of incoming/outgoing connections, reflecting the continuous search for suppliers which offer more favorable prices (Wagner and Friedl, 2007). It has been accomplished by introducing the linking algorithm. The algorithm assumes that the main objectives of firms are profit maximization, cost reduction, and hedging against risk. Since we are in a dynamic regime, it does not automatically imply that the system is going to end up in the optimal state. The three objectives mentioned above refer only to the way how we construct the linking algorithm. These objectives are not used to formulate the multi-criteria optimization problem, which is frequently done in the equilibrium analysis. The choice of the supplier must include the sales price-variable that differentiates suppliers in the market and contributes to the overall profit of the company. The linking algorithm is constructed as follows. In each time step:

- starting from the output stage $s=0$:
  (1) choose randomly node i from stage s;
  (2) choose randomly three nodes j_k, $k\in(1,2,3)$ from stage $s+1$;
  (3) check, which node j_k has the lowest sales price (j_k^{min});
  (4) connect node i with node j_k^{min}, if the sales price of node j_k^{min} is lower than the sales price of node i;
- go to stage $s+1$ and repeat steps (1)–(4);
- the linking process finishes when the connections between the last two stages are established.

3.3. Network reconfiguration

The network reconfiguration process imitates the agent's multi-sourcing strategy to hedge against risk on one side, but on the other side it also reflects the constant search for suppliers, who offer the best prices in the market. If firm i has k_i suppliers, then if one of them cannot deliver the ordered goods, the risk of taking a profit loss would be proportional to $1/k_i$. To limit the density of the network and keep the number of connections at the predefined, realistic level we used a method of network reconfiguration. Production facilities are allowed to change the supplier if a new supplier with a lower sales price was found. Additionally, if the firm has more suppliers (higher in-degree k_{in}) the probability of switching to the new supplier increases. Firms with few suppliers face a higher risk of supply and production shortage in the periods following a switch to the new supplier. To factor in this assumption in our model we have chosen a *tgh* function as probability function of reconfiguration depicted in Fig. 2.

$$P_{reconf}=tgh(k_{in}/a), \tag{2}$$

where $a>0$ is a parameter, and $k_{in}\geq 0$.

In simulations we used function (2) with parameter $a=5$, which gives the probability value $P_{reconf}\approx 0.95$ when $k_{in}=12$.

3.4. Production dynamics

The production dynamics is an enhancement of the framework proposed by Weisbuch and Battiston (2007) with adjustments required by the variable input and output connections and the stochasticity of prices. Working capital is in our model the variable which describes the state of each node in the network. We have N nodes in the network, which implies that if we were able to solve the N differential equation, we could describe the state of the whole system at every point in time. The situation is complicated by the fact that the companies are in a dynamic process of interaction. The topology of the network is evolving and some random failures of production take place. This is why we choose the numerical approach to see how these nonlinear phenomena influence the performance of the supply chain network. At the beginning of each time step we run the linking algorithm. Next, starting from the output stage, orders to the next stages are placed. When the order flow reaches the input stage, production starts. We assume linear production of homogenous goods, which means that the production function is linear to the firm's working capital A. The orders from the output stage ($s=0$) are only limited by production capacity. All basic assumptions are

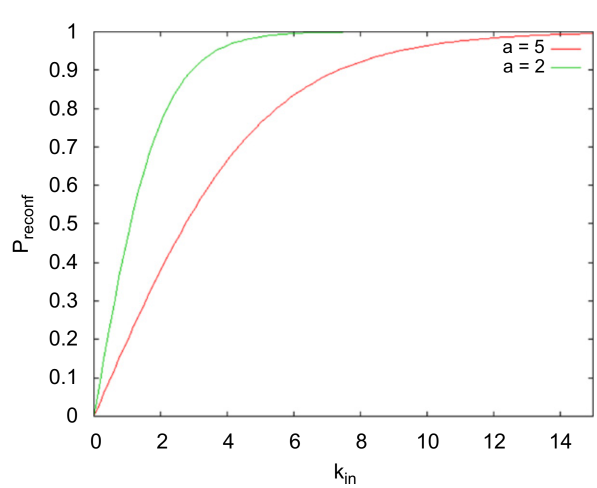

Fig. 2. Probability function of link reconfiguration for two values of parameter a.

the same as in Weisbuch and Battiston's (2007) model, which we first tested to obtain the same results and later extended to include our enhancements. Under these assumptions retailers place their orders Y according to equation:

$$Y_{0,i}(t) = qA_{0,i}(t), \tag{3}$$

where q is the technological proportionality coefficient, which relates the working capital $A_{0,i}(t)$ of firm i to the quantity of products $Y_{0,i}(t)$ it is able to produce. The orders are then distributed proportionally to all suppliers actually connected to node (0, i), i.e. in the amount of $(1/k^i_{in}(t))Y_{0,i}(t)$. In all other stages s firms place their orders to stage $s+1$ based on the orders coming from stage $s-1$, following equation:

$$Y_{s,i}(t) = \min\left(qA_{s,i}(t), \sum_{j \in v} \frac{Y_{s-1,j}(t)}{k^j_{in}(t)}\right), \tag{4}$$

where v is the actual set of customers of the firm (s,i) from stage $s-1$. Eq. (4) reflects the assumption that if the incoming orders exceed the production capacity, the total production of the firm is limited by its working capital (multiplied by q). This procedure ends up at the input stage. The providers of raw materials now start the production flow. The delivered production is given by

$$Y^d_{s,i}(t) = \left(\sum_{i' \in v'} Y^d_{s+1,i'}(t) \frac{Y_{s,i}(t)}{\sum_{j \in v} Y_{s-1,j}}\right)\varepsilon(t), \tag{5}$$

where $\varepsilon(t)$ is the stochastic coefficient, which with probability P leads to the shortage in the firm's production. Formally, $\varepsilon(t)=0$ with probability P and $\varepsilon(t)=1$ with probability $1-P$. Thus, the delivered production of firm (s,i) depends on the production received upstream from suppliers in neighborhood v' from stage $s+1$ and is distributed proportionally downstream, according to the orders placed by the customers in the neighborhood v (Fig. 3).

The next step is the update of profits and losses. The main variable, which describes the state of each node in the network is its working capital $A_{s,i}$. The evolution of this variable is described by equation

$$A_{s,i}(t+1) = A_{s,i}(t) + \Pi_{s,i}, \tag{6}$$

where $\Pi_{s,i}$ is the profit in time t. All profits are calculated using equation

$$\Pi_{s,i} = p_{s,i}(t)Y^d_{s,i} - c_{s,i}(t)Y^d_{s,i} - \lambda A_{s,i}, \tag{7}$$

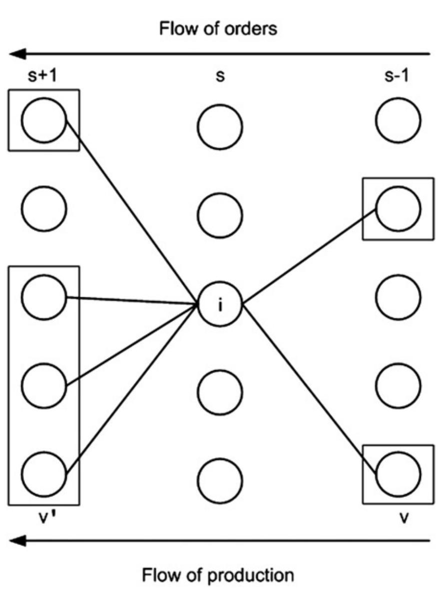

Fig. 3. Example of the operating environment of the firm (s,i). The actual suppliers and customers belong to the corresponding neighborhood v' and v (highlighted with squares).

where $p_{s,i}(t)$ is the sales price at time t, λ is the exogenously given interest rate constant, which expresses the cost of capital, and $c_{s,i}(t)$ is the cost of production of a unit of production at time t

$$c_{s,i}(t) = \left(\frac{\sum_{i' \in v'} p_{s+1,i'}(t) Y^d_{s+1,i'}(t)}{\sum_{i' \in v'} Y^d_{s+1,i'}}\right), \tag{8}$$

where v' is the neighborhood of node i in stage $s+1$. Margin of firm (s,i) is variable and depends on the sales price of all suppliers which are actually connected

$$m_{s,i}(t) = p_{s,i}(t) - c_{s,i}(t). \tag{9}$$

Margins of firms in the input stage are given as a constant parameter m_{inp}.

3.5. Costs of production

We also introduced the costs of production which are changing over time as a result of advances in the technology of production or changes in the labor market. We used the assumption that after every five time steps firms are changing their costs of production. The costs of production consist now of two parts. The first term $c_{s,i}(t)$ includes the deterministic costs consequent to the sales price offered by the suppliers. The second part is a stochastically changing term $q(t)$, which will be updated after the predefined number of time steps. The total cost of production $\bar{c}_{s,i}(t)$ can be written as follows:

$$\bar{c}_{s,i}(t) = c_{s,i}(t) + q(t), \tag{10}$$

where $c_{s,i}(t)$ is given by Eq. (8). We assume that the change of the logarithm of q is a random variable with a normal distribution:

$$q(t+1) = q(t)\exp(\eta), \tag{11}$$

where η is a random variable, with mean value $\mu_2 = -0.1$ and standard deviation $\sigma_2 = 0.1$. This set of parameters causes that negative values are more probable than positive values, that is, firms are more frequently lowering than raising their costs of production (for instance, thanks to investments in technology of production). Now we need to rewrite Eq. (7), in order to include the dynamically changing costs of production

$$\Pi_{s,i} = p_{s,i}(t)Y^d_{s,i} - \bar{c}_{s,i}(t)Y^d_{s,i} - \lambda A_{s,i}. \tag{12}$$

3.6. Bankruptcies and recovery

In this article we will consider the firm as bankrupt when it cannot meet its short-term commitments (i.e. it has insufficient cash flows to continue its operations). Since we do not include inventories and credit in our model, the financial default occurs when the working capital A of the company hits the threshold level. This level is assumed to be a fraction of the average working capital of all firms in the supply chain network $A_{default} = \frac{1}{100}\langle A \rangle$. It follows the assumption that bankruptcy processes evolve with the state of economy. It implies that there are no fixed bankruptcy thresholds. After a number of time steps τ the defaulted firms are replaced with new ones with the working capital equal to a fraction of the firms' average working capital $A_{recovery} = \frac{1}{10}\langle A \rangle$.

4. Results

4.1. Influence of delivery failures on the growth of the total working capital of the supply chain

We used C++programming language to build an ABM environment for simulations. The first characteristic of the system, which we focused on was the influence of the probability of delivery failures on the total working capital of the supply chain. The growth of the working capital aggregated over all production facilities in a supply chain network A_{total} is exponential with the value of the growth constant strongly dependent on the probability of delivery failure. For the sake of simplicity we analyzed this dependency by simulating production in the supply chain. We consider the chain of production facilities with one facility in each of the S stages. The average growth rate of the working capital of firm k at time t is given by equation

$$\langle m \rangle = \left(\frac{A_k(t)}{A_k(0)}\right)^{1/t}, \quad (13)$$

where $A_k(0)$ is the working capital of the firm k at time 0. The average growth rate can be calculated from the geometric average

$$\langle m \rangle = \sqrt[t]{(m)^{n_1} \cdot (1)^{n_2}}, \quad (14)$$

where n_1 is the number of periods with full profits (margin $m>0$) and n_2 is the number of periods without profits (margin $m=0$ due to delivery failures) after t iterations ($t=n_1+n_2$). Assuming that the delivery failure occurs with probability P, we can rewrite n_1 and n_2 as the expected value of respective occurrences in time t:

$$n_1 = (1-P)t, \quad (15)$$

$$n_2 = Pt. \quad (16)$$

Because failures are independent and can occur in every stage, n_1 and n_2 take the following form:

$$n_1 = (1-P)^S t, \quad (17)$$

$$n_2 = (1-(1-P)^S)t, \quad (18)$$

where S is the number of stages in the supply chain. After substituting n_1 and n_2 into (14) we obtain

$$\langle m \rangle = \sqrt[t]{(m)^{(1-P)^S t} \cdot (1)^{(1-(1-P)^S)t}} = (m)^{(1-P)^S}. \quad (19)$$

This expression is true for the average growth rate of the working capital of all firms in the chain, since all nodes are independently exposed to delivery failure with probability P and if one facility fails to deliver, the margin m is equal to zero for the whole supply chain. The analytic solution for the supply chain network with the dynamical reconfiguration of links is subject to future research. The results of simulations and the theoretical function's fit are shown in Fig. 4.

4.2. Performance of the network

A supply chain network with dynamically changing links between nodes is a nonstationary system. The performance of the system is characterized by the turbulences during the first period of the network's growth. This process is illustrated in Fig. 5, which shows the performance as a function of time. The performance of the system is given by

$$M(t) = \frac{A_{total}(t)}{Y^d_{total}(t)} \times 100\%, \quad (20)$$

where $A_{total}(t)$ is the aggregated production capacity of the whole system and $Y^d_{total}(t)$ is the total delivered production at time t. $M(t)$ is the measure of the percentage of the working capital utilized in production (for the static network of production $M(t)=const=100\%$). The mean value of the system's performance $\langle M \rangle = 97.88\%$ corresponds to the value of the average growth rate of working capital $\langle m \rangle = 1.009788$. The observed fluctuations are caused by the stochastic process of the network reconfiguration. Our result indicates that the reconfiguration process leads to periods of higher and lower production performance, which have a negative effect on the average growth of the total working capital of the supply chain network.

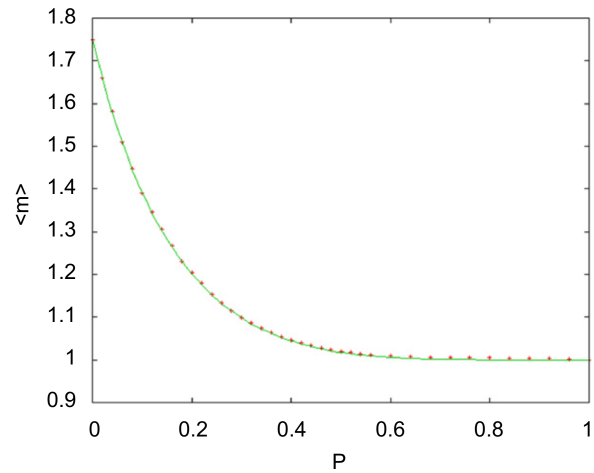

Fig. 4. Growth constant as a function of P for the supply chain ($S=5$) (crosses). Solid line represents the fit given by function (19).

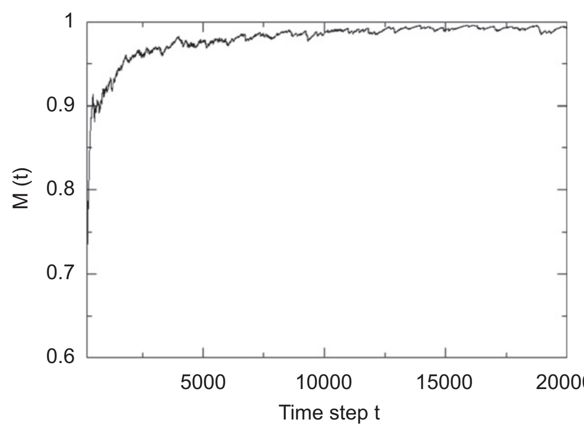

Fig. 5. Performance of the network.

4.3. Evolution of the network topology and aggregated working capital

The model was first calibrated with the values of parameters which reproduce the stylistic facts described in Weisbuch and Battiston (2007). In case of no delivery failures we ensured an exponentially growing working capital of the supply chain network (growth regime). The values of coefficients describing the moments of price distributions were taken from empirical research, or followed by logical reasoning (e.g. price dispersion coefficients). We have chosen a relatively low value of λ in order to track the evolution of the system for longer periods of time. Assigning $q=1$ implies constant returns to scale in terms of production technology. The parameter values are summarized in Table 1.

After having confirmed the results with the original model we started to add our extensions stepwise and we observed how the behavior of the system changed. Online monitoring of the network dynamics was accomplished by writing an application for visualization using GLUT—The OpenGL Utility Toolkit libraries.

Table 1
Parameter values.

Parameter	Value
Margin level of the raw materials suppliers	$m_{inp}=0.01$
Technological proportionality constant	$q=1.0$
Interest rate constant	$\lambda=0.002$
Mean value of price dispersion	$\mu=1.0$
Standard deviation of price dispersion	$\sigma=0.01$
Probability of delivery failure	$P=0.05$

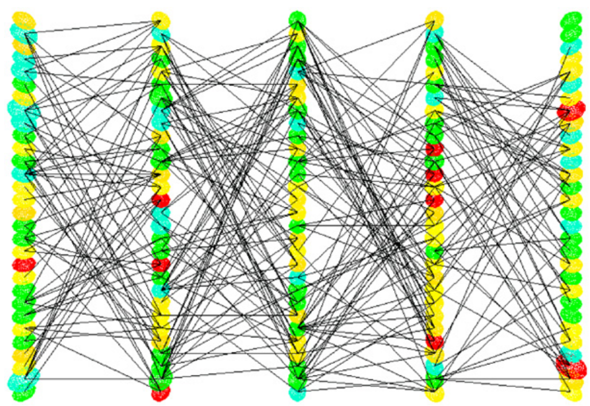

Fig. 6. State of the network after the test period.

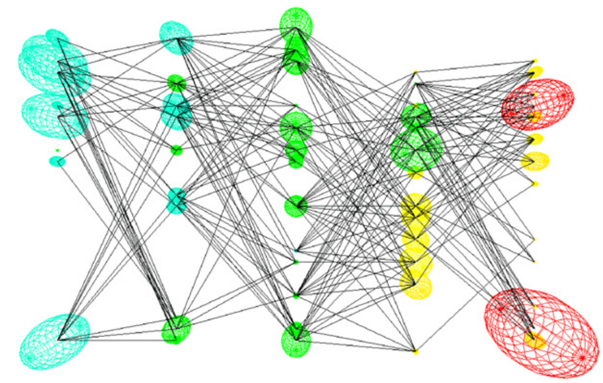

Fig. 7. State of the network after reaching the (meta)stable configuration.

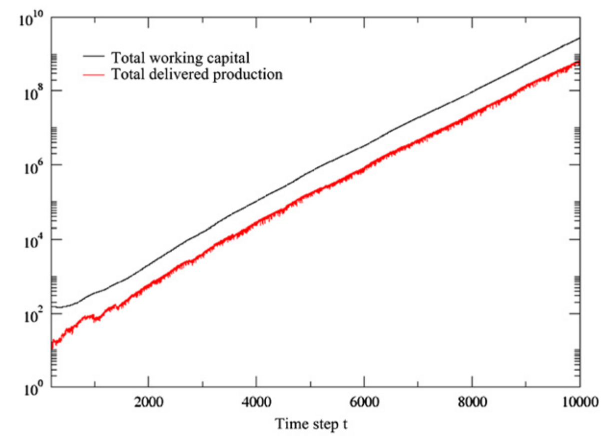

Fig. 8. Total working capital and total delivered production.

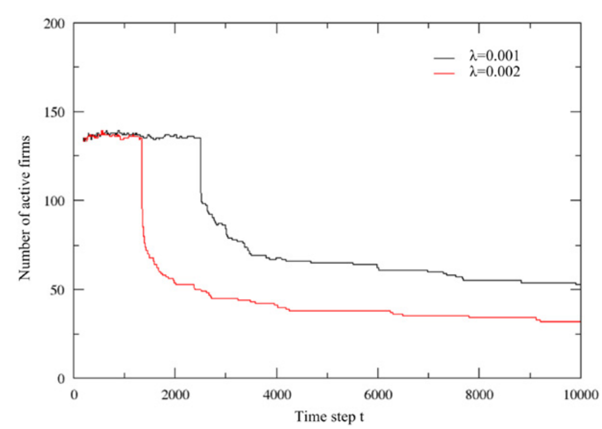

Fig. 9. Number of active firms for two different values of λ.

The number of time steps can be days or weeks, or any other scale which is representative for one cycle of order and production. At the beginning, we iterate 200 times to establish connections between the nodes. The state of the network after the test period is shown in Fig. 6. Afterwards we switch on the production. The firms with the best profit/cost ratio are growing and adding new suppliers, whereas the working capital of the firms whose sales price is higher than the mean price of the given stage is slowly decaying and results in defaults of firms. After sufficient time a large portion of firms goes out of the market and the meta-stable network configuration is formed. The state of the supply chain network in this phase is shown in Fig. 7. Colors represent different sales prices of end products in every stage (blue–green–red = low–medium–high). The bigger the node, the more working capital a firm has. One of the interesting observations is the self-emergence of the price levels between subsequent stages, which corresponds to what we see in reality. Most of the companies with the lowest sales price in a given stage survive. Nevertheless, due to the limited search area (the linking algorithm searches three randomly selected companies in the nearest neighborhood) it may happen that a company with a medium or even high sales price remains, and quickly generates profits, attributed to the higher than average margins.

The time series of the total working capital and total delivered production are depicted in Fig. 8. The system is characterized by significant turbulences at the beginning of the simulation. By applying equation

$$t=\frac{\log(A_{k,i}(t))-\log(A_{k,i}(0))}{\log(1+m)}=\frac{\log\frac{A_{k,i}(t)}{A_{k,i}(0)}}{\log(1+m)}, \tag{21}$$

we calculate that after approximately 2350 time steps most of the firms with an unprofitable set of suppliers or customers have defaulted and at the same time the total production of the system starts to grow exponentially with the growth constant proportional to the mean margin of the firms which survived in the network. This process is depicted in Fig. 9.

A next logical step was the introduction of inventories, since inventories are generally considered a means of reducing uncertainty (Chopra and Meindl, 2010). As expected, our analysis shows that fluctuations in the volume of products delivered are reduced, which eliminates the effect of random failures to a certain degree. Since buffering of inventories is well documented (Chikán, 2009) and these new insights are marginal, the level of inventories and fluctuations is out of the scope of this work. Therefore we do not report it in this article.

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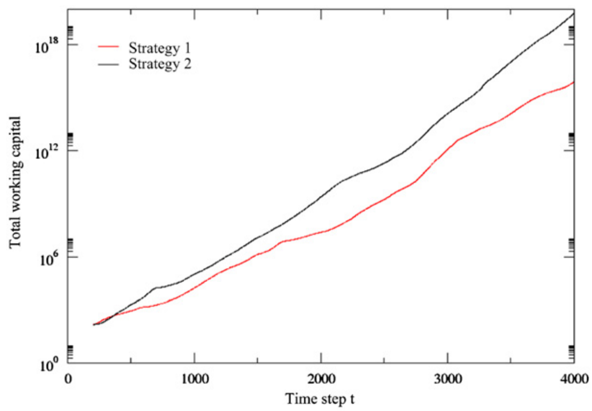

Fig. 10. Evolution of the total working capital of the supply chain network after choice of one of the two adaptive strategies.

4.4. *Evolution of the aggregated working capital in the presence of stochastic changes in costs of production*

In order to study the behavior of the system in the presence of stochastic changes in costs of production, we considered two adaptive strategies of the firms. After each 5 time steps the stochastic term $q(t)$ of Eq. (10) is updated and one of the following actions can be undertaken:

(1) Strategy 1: firms are lowering their sales price by the factor equal to the change of the factor q.
(2) Strategy 2: firms are lowering their sales price by the factor equal to half of the change of the factor q.

In Fig. 10 we see the influence of each of the strategies on the evolution of the total working capital. Both time series are characterized by the periodical changes of growth. It is the result of the stochastic changes of production costs and sales price, which are not correlated in time and space and are independent of the size of the firms. We observe that strategy 1 is less effective than strategy 2.

5. Contributions and implications

5.1. *Contributions to the literature*

The main contribution of this research is to include two very important factors, which constitute the complexity of modern supply chain networks into one model: the dynamics of the complex network topology and the stochasticity of the economic variables. In our study we extended the agent-based model of the supply chain network proposed by Weisbuch and Battiston (2007) by adding features which reflect some of the uncertainties that are present in the real market environment. We proposed an agent-based framework, which is capable of simulating the production dynamics and leads to the self-emergence of supply chain network structures. Thanks to the graphic engine we observed the evolution of the network and we gained more insight into mechanisms which can lead to avalanches of bankruptcies. As outlined by Choi et al. (2001), modeling of global supply chain networks needs a sophisticated approach, which more accurately reflects its true underlying complexity and dynamism. We added some assumptions which bring the model closer to reality, but we still keep the number of parameters as low as possible. The advantage of our approach is that it is based on the model of dynamics developed to study certain phenomena in physics. In other words, there exist ready-to-use analytical solutions for the set of governing equations, which can be solved in order to describe the behavior of the system at any given point in time. Our model is aligned with the research of Pathak et al. (2007a), who highlighted the importance of the integration of complexity theory into the existing supply chain management research and presented their research in more detail in Pathak et al. (2007b). Based on the OR and the complex systems literature, we proposed a framework, which can be used to simulate large-scale supply chain networks and can be easily adapted to any particular industry.

In contrast to the static topology studied by Bak et al. (1993), Weisbuch and Battiston (2007), Battiston et al. (2007), and Gatti et al. (2005) our approach includes the dynamic reconfiguration of the network based on the simple algorithm of agent behavior. We note that at a certain point of the system's evolution, the meta-stable structures of the network emerge. As a result of the dynamics of prices and costs of production, we observed both the emergence of highly profitable supply chains with high market shares and avalanches of bankruptcies. This process displays the self-organization effects of the supply chain network and contributes to the understanding of complex network design problems. ABM is a promising alternative to the commonly used mathematical programming optimization techniques (Ambrosino and Scutellà, 2005) due to its ability to implement sophisticated behavioral rules of agents on the local level and its ability to relate them to the global outcomes.

5.2. *Managerial implications*

Our observations and quantitative results lead to the following conclusions which are important for managerial practice. The first implication is that during the process of assessment of the company's risk exposure, managers should keep their focus on the global structure of the supply chain network instead of being restricted to the own portfolio of suppliers and customers. Based on the analytical solution derived in Section 4.1 we conclude that the average growth of the working capital of a supply chain depends on the number of stages and decays quickly as the probability of shortage in production grows. Since this probability does not depend on time and location in the supply chain, managers should be aware of the risk coming from the whole structure of the supply chain network.

Second, as a result of the dynamics of the topology of the supply chain network, strong competition in prices and fast changing technology, even the most reliable firms should be monitored and constantly re-evaluated in terms of their production capacity and risks associated with their structure of connections. In simulations we observed that if a company is not able to quickly adapt to the changing environment, it might be exposed to the risk of collective defaults of suppliers, which can give rise to disruptions and delays in production. After a period of time, even the biggest and most reliable suppliers may default as they cannot meet the demand they face from the consumer's stage. In other words, managers should be aware of the inertia risk arising from the connections among suppliers in the next tiers of a supply chain network. This observation adds to the understanding of the risk of collective bankruptcies of suppliers, as highlighted by Wagner et al. (2009).

Third and most important, managers should find ways to cut costs and reinvest the free cash flows in new technology of production, which will allow further cost reductions and the development of new innovative and cheaper products. Especially in times of recession, investments made in R&D will prepare companies better for the economic rebound. At the same time, we

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found that if decision makers focus on bringing down the sales price in order to gain competitive advantage, it turns out to be a strategy which leads to a lower performance of the global supply chain network.

6. Conclusions

In this article we studied the collective dynamics of a supply chain network using the ABM approach. We introduced stochastic elements to our model and observed how local events affect the global performance of the system. We observed that although network reconfiguration leads to turbulent performance of the system, after a period of time, meta-stable structures are formed, resulting from the high profitability of selected portfolios of suppliers and consumers. The development of the graphic engine allowed us to monitor the behavior of the supply chain network online. We think that visualization is a powerful tool for analysis of complex supply chain networks as it gives insight into underlying network dynamics.

The limitation of our model is the simplified agent's behavior. Future research should model the choice of the most optimal supplier portfolios more realistically (Wagner and Friedl, 2007), whereas the collection of real-world data would allow us to empirically test our model. As computational power rises, it is possible to implement complex behavioral rules, based on the risk aversion of agents and the commitment to establish long-term relationships based on the quality of deliveries. This could be modeled, for instance, by introducing correlations between the production reliability and the cost of production. Another shortcoming of our model is the lack of competition among companies, in terms of decision making, based on the anticipation or reaction to the decisions of other companies. Behavioral game theory applied to the complex networks is a promising approach. It is out of the scope of this article, but would be the next step in adding more realism to the model. The micro-correlations in production failures among large sets of suppliers may lead to significant supply chain disruptions occurring with low probability. Quantifying this process is also an open field for investigations. Another research path is to study the emerging meta-stable network structures which we observed in simulations. It is interesting to see which configurations of the network prove the most robust when production processes are severely disrupted.

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