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Fluctuation scaling of quotation activities in the foreign exchange market

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We study the scaling behavior of quotation activities for various currency pairs in the foreign exchange market. The components' centrality is estimated from multiple time series and visualized as a currency pair network. The power-law relationship between a mean of quotation activity and its standard deviation for each currency pair is found. The scaling exponent α and the ratio between common and specific fluctuations η increase with the length of the observation time window Δt. The result means that although for $\Delta t = 1$ (min), the market dynamics are governed by specific processes, and at a longer time scale $\Delta t > 100$ (min) the common information flow becomes more important. We point out that quotation activities are not independently Poissonian for $\Delta t = 1$ (min), and temporally or mutually correlated activities of quotations can happen even at this time scale. A stochastic model for the foreign exchange market based on a bipartite graph representation is proposed.

1. Introduction

The complexity of economic and social systems has attracted a lot of attention from physicists recently [1–8]. Collective behavior among interacting agents shows different properties from particles governed by Newtonian laws. However, intriguing universal properties could be found and mathematical models should be considered. This movement, called socio/econo-physics, is expected to bridge a gap between physics and our societies [9].

Financial markets are complex systems which consist of many interacting agents. The progress of understanding information flows among agents sheds light on the states of financial markets, i.e. the states of market participants. The recent accumulation of a massive amount of data about financial markets due to both the development and spread of information and communication technology allows us to quantify the states of financial markets in detail [10,11]. In fact, the correlation structure of high-frequency financial time series is exhaustively and quantitatively investigated [12,13]; however, the further the dimension of multiple time series increases, the more difficult it becomes to compute cross-correlations and to recognize them.

On the other hand, several studies in both socio/econo-physics and engineering were focused on the structure of corresponding complex networks, their internal dynamics and the flows of the constituents on them [14–17]. Menezes and Barabási studied the scaling behavior of constituents' flows on several constructions such as river networks (water flows), transportation systems (car flows), and computer networks (information flows) [18,19]. As a result, scaling properties are found for flow fluctuations in such systems. This relationship is known as a fluctuation scaling or Taylor's power law [20,21]. Taylor's power law is known as the scaling relationship between the mean of populations and their standard deviation in ecological systems. The ubiquity of Taylor's power-law slopes between 1/2 and 1 suggests that there exists an underlying fundamental mechanism affecting the transportation of constituents.

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Eisler and Kertész found that there is such a power-law relationship between a mean of traded volumes of stocks and their standard deviation on the New York Stock Exchange, and that the power-law exponent takes nontrivial values between 1/2 and 1 [22]. Jian et al. investigated trade volumes of stocks in the Chinese stock market [23]. They also found a non-universal scaling exponent of different fluctuations from 1/2 and 1. We think that their results provide a method to quantify the states of agents with multiple time series in the financial markets. This method is also useful to quantify the states of market participants from the viewpoint of information flows in financial markets.

The aim of this paper is to investigate information flows in the foreign exchange market (FX market) by means of quotation activities, measured as arrival rates of quotations on brokerage systems. We investigate the statistical properties of quotation activities in the FX market and quantify the total states of the market participants through fluctuation scaling.

The organization of this paper is as follows. Section 2 is a short overview of high-frequency financial data taken for our studies from the FX market. Section 3 is a brief summary of the power-law relationship (Taylor scaling) between a mean of constituents' flow on a graph and their standard deviations. In Section 4 an empirical analysis of quotation activities is performed. In Section 5 the dependence of scaling exponents on a time window length is examined, and the relationship between the states of market participants in the FX market and the scaling exponents is discussed. Section 6 is devoted to concluding remarks, and addresses possible future studies.

2. Foreign exchange market

The foreign exchange market is the largest financial market in the world. It is a network consisting of brokers, bank traders, and investors. Recent developments in Information and Communication Technology have led to the spread of electronic trading systems all over the world. As a result, many market participants can directly access the FX market by using computer terminals. Moreover, trading activities are recorded in the computer servers, which perform matching operations among quotations from the market participants, and one can analyze a large amount of data about market activities with high resolution.

In the analysis we use Time & Sales (T&S) data provided by CQG Inc. [24]. The data contain time stamps, rates, and indicators to show ask or bid quotations with a 1 min resolution. The database includes 45 currency pairs[1] consisting of 24 currencies.

Quotation activities for each currency pair are extracted from the database. Since the two-way quotation is adopted in the FX market, it is enough to count the number of the bid or ask quotations.

Let $X_{i,\Delta t}(t)$ $(i = 1, \ldots, N; t = 0, \Delta t, 2\Delta t, \ldots, (Q-1)\Delta t)$ denote the number of all incoming ask quotations for a period between t and $t+\Delta t$ for the currency pair i. $\Delta t(>0)$ denotes the time window to compute the number of quotations. Examples of quotation activities for EUR/JPY, USD/JPY and EUR/USD are shown in Fig. 1. We shall treat the FX market as a complex system consisting of 45 sites i with an unknown number of internal variables, where every site i is a corresponding currency pair from our database and its internal activity is given by the number of quotations $X_{i,\Delta t}(t)$. It is confirmed that the quotation activities exhibit an intraday pattern related to the rotation of the earth, so that they have a strong regional dependence due to entering and leaving of market participants. We focus on the short-term behavior of quotation activities under the assumption of local stationarity. In the context of financial markets, the number of quotations or transactions is known to represent a proxy variable of the latent number of information arrivals in the system [25], where the term "information" is defined by Bateson as "a difference which makes a difference" [26].

3. Theory

3.1. Fluctuation scaling

It is known [18–20,22,21,27] that for complex systems consisting of many sites i, the mean values of their internal activities $X_{i,\Delta t}(t)$

$$\langle X_{i,\Delta t}\rangle = \frac{1}{Q}\sum_{j=0}^{Q-1} X_{i,\Delta t}(j\Delta t), \tag{1}$$

and their standard deviations

$$\sigma_{i,\Delta t} = \sqrt{\frac{1}{Q}\sum_{j=0}^{Q-1}\Bigl(X_{i,\Delta t}(j\Delta t) - \langle X_{i,\Delta t}\rangle\Bigr)^2}, \tag{2}$$

[1] The database contains AUD/CHF, AUD/JPY, AUD/NZD, AUD/SGD, AUD/USD, CAD/CHF, CAD/JPY, CHF/JPY, EUR/AUD, EUR/CHF, EUR/CZK, EUR/GBP, EUR/HUF, EUR/JPY, EUR/NOK, EUR/PLN, EUR/SEK, EUR/TRL, EUR/USD, GBP/AUD, GBP/CAD, GBP/CHF, GBP/INR, GBP/JPY, GBP/NZD, GBP/USD, NZD/USD, USD/BRL, USD/CAD, USD/CHF, USD/DKK, USD/HKD, USD/HUF, USD/INR, USD/JPY, USD/KRW, USD/MXN, USD/NOK, USD/PKR, USD/PLN, USD/SEK, USD/SGD, USD/THB, USD/TRL, and USD/ZAR. These currency names are described based on ISO 4217.

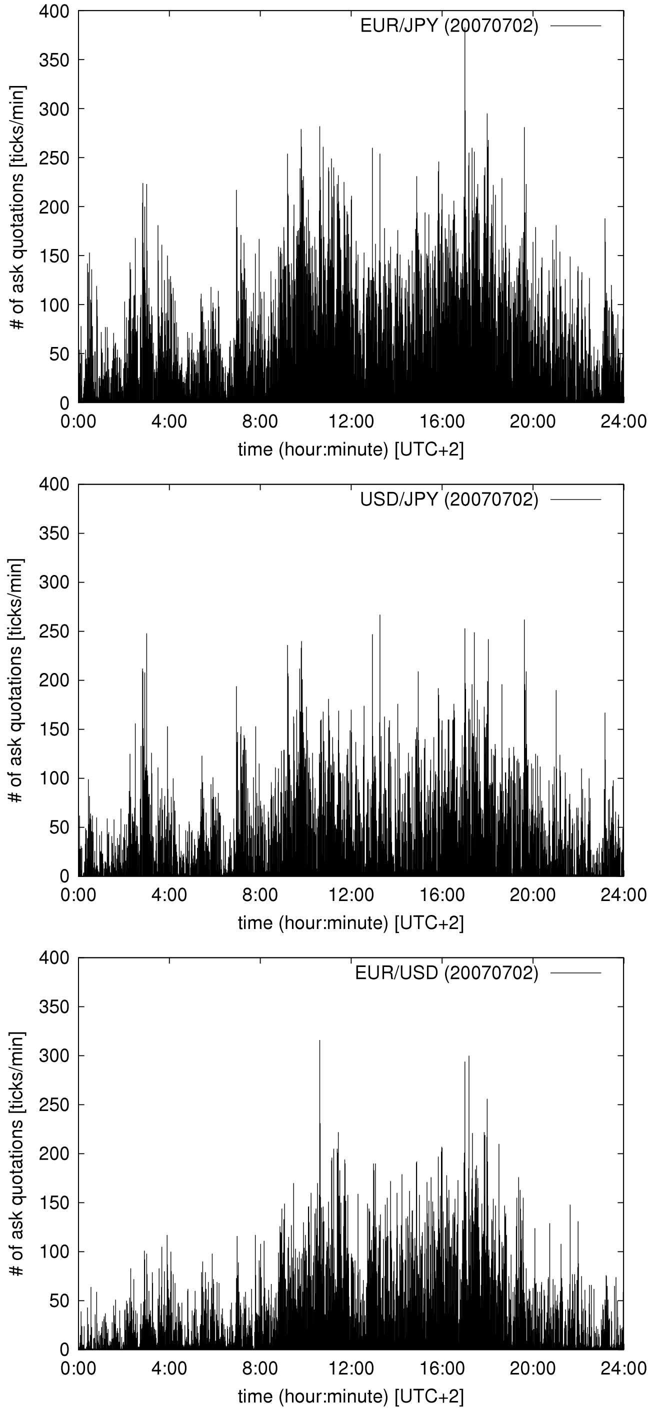

Fig. 1. Time series of quotation activities at $\Delta t = 1$ (min) for EUR/JPY, USD/JPY, and EUR/USD in 2nd July 2007.

can follow a power-law relationship

$$\sigma_{i,\Delta t} = C\langle X_{i,\Delta t}\rangle^{\alpha}, \tag{3}$$

where C is a positive constant, and $\alpha \quad (1/2 \le \alpha \le 1)$ denotes a scaling exponent. This power-law relationship is referred to as fluctuation scaling, or Taylors' law [20,21].

The limiting values $\alpha = 1/2$ and $\alpha = 1$ can result from several scenarios (see [21] for a detailed discussion). For $\alpha = 1/2$ the first possibility is thermal fluctuations observed in systems described by equilibrium statistical physics, e.g. the dispersion volume of the total magnetization scaling against the system size as the power $1/2$. The second possibility can be a Poissonian-like process with very low activity where one can neglect the square of the mean of activity compared to the mean activity itself. The third case is when the activity for every site i is a sum of the activities of identical and independent constituents. The case $\alpha = 1$ can result from the situation when every site i is driven by a universal (common) random variable and the dynamics of different sites varies only due to time-independent multiplicative constants f_i. The second possible scenario for $\alpha = 1$ is when the activity of every site i is a sum of the activities of identical and completely synchronized constituents [28,29]. If $1/2 < \alpha < 1$ one can have a mixture of different above scenarios, a nonhomogenous impact for various nodes, partial correlations between conveyances of constituents in the system, as well as the situation when α depends on the time window Δt because of Hurst exponents' heterogeneity [21]. We will show that the last case seems to be present in the observed FX data.

3.2. Specific fluctuations and common fluctuations

According to Ref. [19] we can separate specific and common fluctuations of complex systems as follows. This can be based on the assumption that a common fluctuation may originate from exogenous factors of elements and that specific ones come from endogenous factors. Of course the common fluctuations are not always originated from a common exogenous factor, e.g. they can occur due to synchronization of interactions among elements [28,29]. We assume that we can separate the two contributions by writing

$$X_{i,\Delta t}(t) = X^{\rm spe}_{i,\Delta t}(t) + X^{\rm com}_{i,\Delta t}(t). \tag{4}$$

It is proposed that a time-independent fraction A_i of the total quotation is determined by the component's centrality [30]. We shall write the currency pair centrality A_i as the ratio of the total quotations that have arrived on matching engines for the currency pair i in the time interval $t \in [0, Q-1]$, and the total quotations over all observed components during the same time interval

$$A_i = \frac{\sum_{j=0}^{Q-1} X_{i,\Delta t}(j\Delta t)}{\sum_{j=0}^{Q-1}\sum_{i=1}^{N} X_{i,\Delta t}(j\Delta t)}. \tag{5}$$

Therefore, the common fluctuations that contribute to the activity of the currency pair i are expected to be

$$X^{\rm com}_{i,\Delta t}(t) = A_i \sum_{i=1}^{N} X_{i,\Delta t}(t). \tag{6}$$

From Eqs. (4) and (6), the specific fluctuations are given by

$$X^{\rm spe}_{i,\Delta t}(t) = X_{i,\Delta t}(t) - \left(\frac{\sum_{j=0}^{Q-1} X_{i,\Delta t}(j\Delta t)}{\sum_{j=0}^{Q-1}\sum_{i=1}^{N} X_{i,\Delta t}(j\Delta t)} \right) \sum_{i=1}^{N} X_{i,\Delta t}(t). \tag{7}$$

Furthermore, by means of Eqs. (6) and (7), we can determine whether the fluctuations observed in a system are synchronous or asynchronous, so that they may be affected by endogenous or exogenous contributions. From each currency pair, we calculate the standard deviations of both specific and common fluctuations,

$$\sigma^{\rm spe}_{i,\Delta t} = \sqrt{\langle X^{\rm spe\,2}_{i,\Delta t}\rangle - \langle X^{\rm spe}_{i,\Delta t}\rangle^2}, \tag{8}$$

$$\sigma^{\rm com}_{i,\Delta t} = \sqrt{\langle X^{\rm com 2}_{i,\Delta t}\rangle - \langle X^{\rm com}_{i,\Delta t}\rangle^2}, \tag{9}$$

and their ratio

$$\eta_{i,\Delta t} = \frac{\sigma^{\rm com}_{i,\Delta t}}{\sigma^{\rm spe}_{i,\Delta t}}. \tag{10}$$

From definition $\langle X^{\rm spe}_{i,\Delta t}\rangle = 0$. If $\eta_{i,\Delta t} \gg 1$, the common fluctuations are dominant in the dynamics of the currency pair i. Otherwise, the specific fluctuations are dominant. Moreover, due to definition (6) the common fluctuations scale as a power law with the exponent $\alpha = 1$, when we plot the standard deviation $\sigma^{\rm com}_{i,\Delta t}$ as the function of the mean $\langle X_{i,\Delta t}\rangle$. The scaling of the specific fluctuation provides us with a more detailed understanding of the system's dynamics.

4. Data analysis

For each currency pair, quotation activities are extracted from a T&S database. We calculate a mean of the quotation activities and their standard deviation for each currency pair following Eqs. (1) and (2). As shown in Fig. 2, a power-law

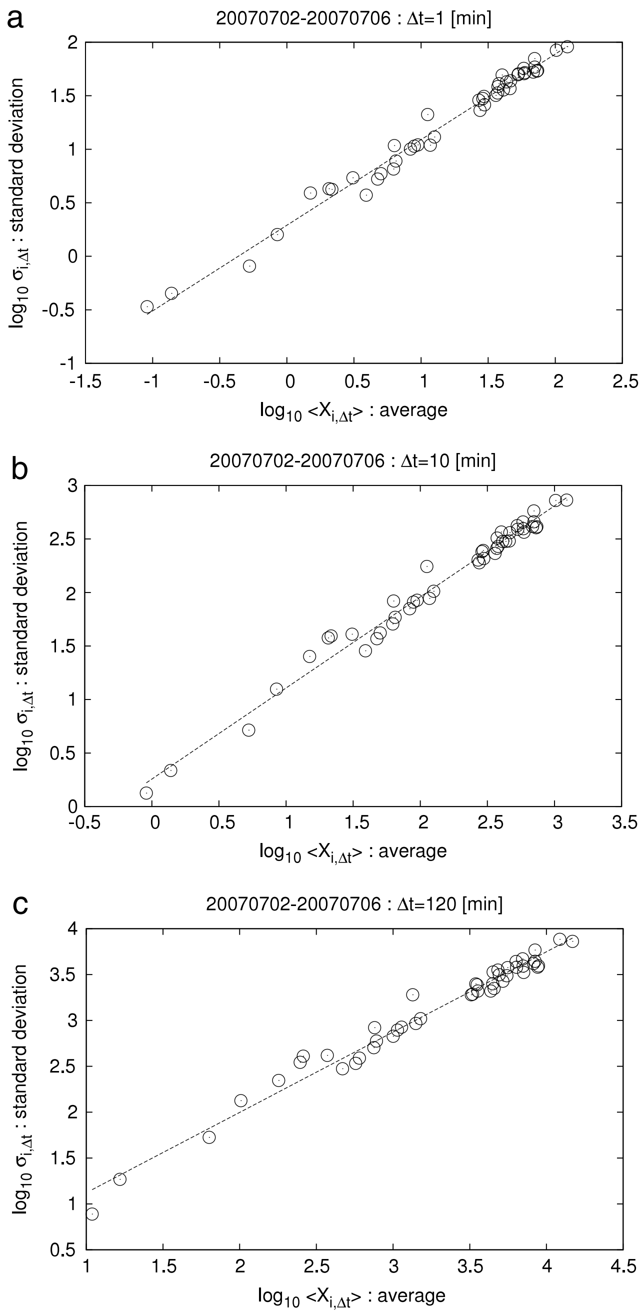

Fig. 2. Log–log plots of a mean of quotation activities and their standard deviation for each currency pair at $\Delta t = 1$ (min) (a), $\Delta t = 10$ (min) (b), and $\Delta t = 120$ (min) (c) for a period from 2nd to 6th July 2007. The x-axis represents a mean value, and the y-axis a standard deviation. The straight line is estimated by using the ordinary least squares. $\log_{10} C = 0.29 \pm 0.02$ and $\alpha = 0.80 \pm 0.02$ at $\Delta t = 1$ (min), $\log_{10} C = 0.26 \pm 0.04$ and $\alpha = 0.85 \pm 0.02$ at $\Delta t = 10$ [min], and $\log_{10} C = 0.25 \pm 0.07$ and $\alpha = 0.88 \pm 0.02$ at $\Delta t = 120$ (min).

relationship between a mean and standard deviation in log–log plots exists. In order to estimate C and α, we used the least-squares method for $\log_{10}\langle X_{i,\Delta t}\rangle$ and $\log_{10}\sigma_{i,\Delta t}$. From a logarithmic form of Eq. (3), the fitting function is set as

$$\log_{10}\sigma_{i,\Delta t} = \alpha \log_{10}\langle X_{i,\Delta t}\rangle + \log_{10} C. \tag{11}$$

Following this procedure, the power-law exponent is estimated as $\alpha = 0.80 \pm 0.02$ at $\Delta t = 1$ (min), $\alpha = 0.85 \pm 0.02$ at $\Delta t = 10$ (min), and $\alpha = 0.88 \pm 0.02$ at $\Delta t = 120$ (min) for a period from the 2nd to 6th July, 2007.

Furthermore, the dependences of the α on Δt are computed for four periods in July 2007 (see Fig. 3). One can see that the exponent α increases as the function of the time window Δt. The mean slope of this increase

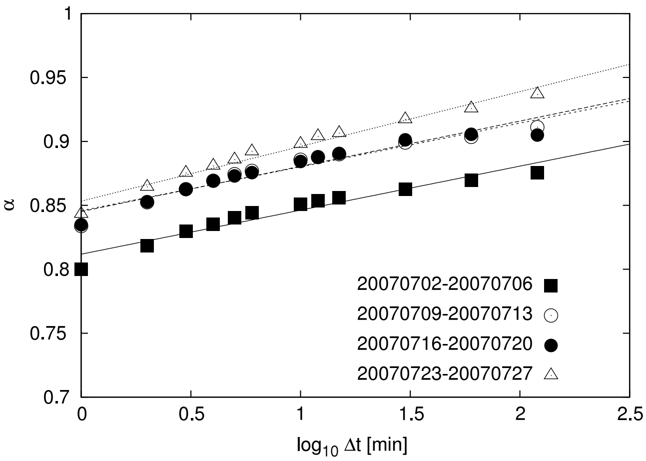

Fig. 3. A semi-log plot of a relation between α and Δt for a period from 2nd to 6th (filled square), 9th to 13th (unfilled circle), 16th to 20th (filled circle), and 23rd to and 27th (unfilled triangle) July 2007. By means of the least-squares method slopes are calculated as $\gamma = 0.034 \pm 0.003$ from 2nd to 6th July 2007, $\gamma = 0.035 \pm 0.002$ from 9th to 13th July 2007, $\gamma = 0.034 \pm 0.003$ from 16th to 20th July 2007, and $\gamma = 0.043 \pm 0.002$ from 23rd to 27th July 2007, respectively.

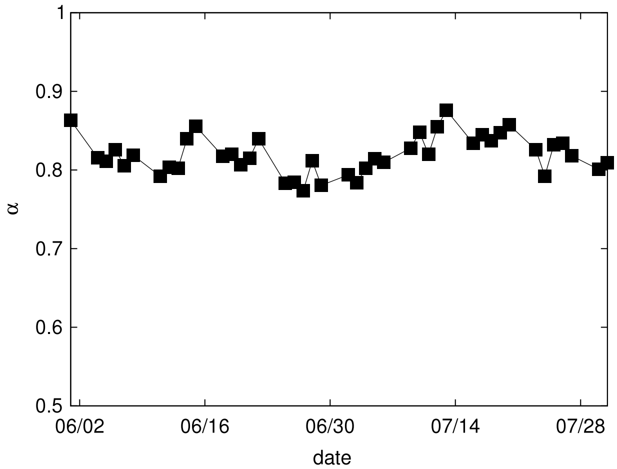

Fig. 4. Temporal variation of the scaling exponents α for every trading day ($Q\Delta t = 1440$ (min)) at $\Delta t = 1$ (min) for periods from 1st June to 31st July 2007.

$$\gamma = \frac{\partial \alpha}{\partial(\log_{10} \Delta t)} \tag{12}$$

is about $\gamma = 0.04$ and there is a saturation of this dependence for long time windows $\Delta t > 100$ (min). Since the larger values of α correspond to more synchronous behavior, we can conclude that such synchronization effects are visible for longer time windows Δt. As shown in Fig. 4, the α value estimated for each trading day ($Q\Delta t = 1440$ (min)) varies temporally in a range from 0.8 to 0.9 at $\Delta t = 1$ (min). This temporal variation can be related to the temporal dependence of the synchronous and desynchronous activities of the quotation for each currency pair arriving on brokerage systems on trading days.

Following Eq. (5), we calculated the centrality A_i of each currency pair for $Q\Delta t = 1440$ (min) and drew the currency pair network with links corresponding to its centrality as shown in Fig. 5. The pairs USD/CHF, AUD/JPY and CHF/JPY possess the largest centrality values.

As shown in Fig. 6, the distribution of the ratio $\eta_{i,\Delta t}$ between common and specific fluctuations of the currency pair i is dependent on the time window Δt. As Δt increases, the distribution moves to larger values. This observation means that for longer time windows Δt the fluctuations are more and more driven by common impacts, which is in agreement with the data presented in Fig. 3 as well in the Figs. 7 and 10. Table 1 shows the values of $\eta_{i,\Delta t}$ for each Δt. We see that for the following currency pairs with small centrality A_i, the specific fluctuations are dominant: EUR/CZK, GBP/INR, USD/BRL, USD/HKD, USD/INR, USD/KRW, USD/MXN, USD/PKR, USD/SGD, USD/THB, USD/TRL, and USD/ZAR. The currency pairs in which common fluctuations are dominant (large centrality nodes) are USD/CHF, AUD/JPY, CAD/CHF, CHF/JPY, EUR/AUD, EUR/GBP, EUR/JPY, EUR/USD, GBP/AUD, GBP/CAD, GBP/CHF, GBP/JPY, GBP/NZD, GBP/USD, USD/CHF, USD/DKK, USD/HUF, USD/JPY, USD/PLN, and USD/SEK. Consequently, rare currency pairs tend to be subject to specific fluctuations, while hard

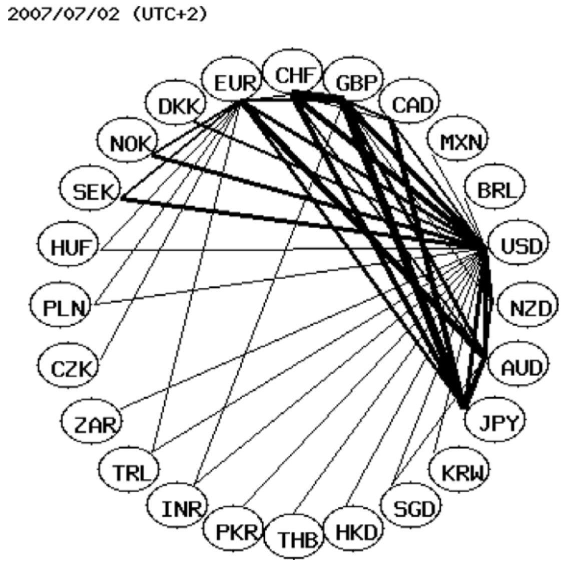

Fig. 5. Graphical illustration of 45 currency pairs in the foreign exchange market. Each node represents a currency, and a link between two nodes represents currency pairs. The link width corresponds to its centrality at 2nd July 2007.

Table 1
The descending order of ratios of specfic fluctuation to common fluctuation $\eta_{i,\Delta t}$ for each currency pair at $\Delta t = 1$ (min), 10 (min), and 120 (min), respectively. For $\eta_{i,\Delta t} \gg 1$ the common fluctuation is dominant, and for $\eta_{i,\Delta t} \ll 1$ the specific fluctuation is dominant.

Currency pair	$\eta_{i,\Delta t} = \sigma^{\text{com}}_{i,\Delta t}/\sigma^{\text{spe}}_{i,\Delta t}$			Centrality	Currency pair	$\eta_{i,\Delta t} = \sigma^{\text{com}}_{i,\Delta t}/\sigma^{\text{spe}}_{i,\Delta t}$			Centrality
	$\Delta t =$ 1 (min)	$\Delta t =$ 10 (min)	$\Delta t = 120$ [min]			$\Delta t =$ 1 (min)	$\Delta t =$ 10 (min)	$\Delta t =$ 120 (min)	
GBP/CHF	1.968670	2.932370	4.097840	0.073483	USD/HUF	1.069590	1.723470	2.504850	0.003848
GBP/JPY	1.921390	2.880750	4.616420	0.083473	EUR/SEK	1.056040	1.342030	1.516210	0.018920
CHF/JPY	1.788030	2.927300	4.288900	0.050316	AUD/NZD	1.027500	1.575420	2.618750	0.027027
EUR/AUD	1.687810	2.742090	4.224280	0.045738	EUR/PLN	0.964241	1.490540	1.978740	0.007835
GBP/AUD	1.665740	2.690220	4.100750	0.025256	AUD/USD	0.914931	1.440190	2.241430	0.030047
EUR/JPY	1.567680	2.139110	2.471270	0.058396	EUR/CHF	0.879273	1.522200	2.918620	0.021515
AUD/CHF	1.501580	2.585130	4.080710	0.018153	EUR/HUF	0.854864	1.333680	1.947080	0.005011
GBP/NZD	1.488660	2.432650	4.288810	0.019091	AUD/SGD	0.821126	1.384470	2.263380	0.003037
USD/NOK	1.480580	2.212470	2.749590	0.027128	NZD/USD	0.724408	1.023540	1.238160	0.024964
AUD/JPY	1.449240	1.991590	2.285760	0.048812	USD/CAD	0.690279	1.001620	1.429220	0.027763
EUR/USD	1.431830	2.580360	5.546850	0.037189	USD/HKD	0.618552	0.859312	1.176710	0.003615
USD/SEK	1.391260	1.802250	2.041910	0.028913	USD/SGD	0.600880	0.812786	0.966003	0.005496
GBP/CAD	1.380080	2.044050	2.765770	0.023929	GBP/INR	0.479731	0.699536	0.716067	0.000071
EUR/GBP	1.321160	2.059820	2.965900	0.029086	USD/ZAR	0.479609	0.661205	0.839839	0.005222
CAD/JPY	1.273120	1.956630	2.877350	0.038632	USD/TRL	0.460390	0.642635	0.813392	0.002803
USD/CHF	1.260280	2.016300	2.824350	0.046142	USD/MXN	0.400749	0.456217	0.456219	0.010174
GBP/USD	1.256070	1.873100	2.830940	0.048574	USD/INR	0.370464	0.428502	0.437336	0.000626
USD/DKK	1.218130	1.857580	2.583080	0.016821	USD/BRL	0.341083	0.327208	0.335608	0.002162
CAD/CHF	1.212150	1.947910	2.599770	0.009310	USD/KRW	0.305093	0.302344	0.329875	0.000662
USD/PLN	1.156600	1.843390	2.563440	0.009219	EUR/CZK	0.285527	0.436140	0.632806	0.001291
USD/JPY	1.148990	1.757520	2.369350	0.039418	USD/THB	0.210386	0.361743	0.425551	0.000073
EUR/TRL	1.088990	1.936500	2.926600	0.002623	USD/PKR	0.187039	0.402792	0.713309	0.000050
EUR/NOK	1.071960	1.584290	1.978870	0.018088					

currency pairs with the large values of the centrality parameter A_i tend to be affected by common fluctuations. Fig. 7 shows that indeed the $\eta_{i,\Delta t}$ parameter increases with increasing centrality A_i.

5. Discussion

The nontrivial values of α presented in Fig. 3 may imply that the market participants are affected by information both from different origins relating to microscopic dynamics and from common sources relating to macroscopic dynamics, or may imply that market participants have strong interactions. In other words the arrival rates of quotations on the brokerage systems are not independently Poissonian.

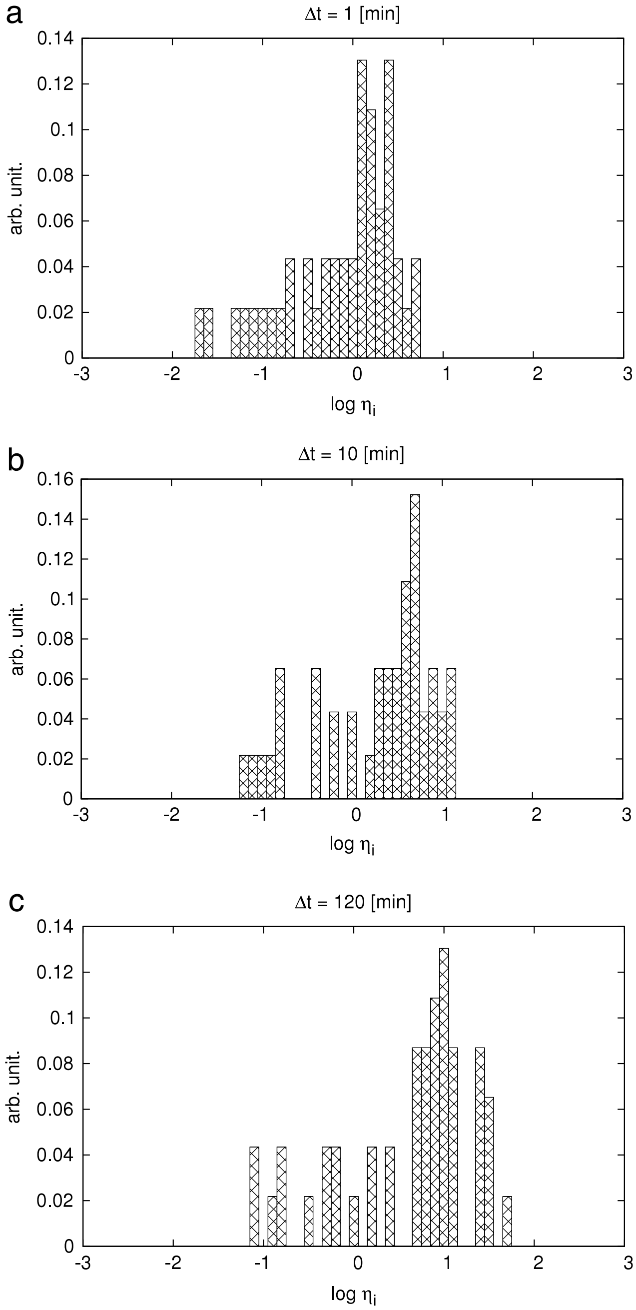

Fig. 6. Histograms of the contribution of specific and common fluctuations ($\log \eta_{i,\Delta t}$) at $\Delta t = 1$ (min) (a mean $= -0.113$, a standard deviation $= 0.627$) (a), $\Delta t = 10$ (min) (a mean $= 0.358$, a standard deviation $= 0.634$)(b), and $\Delta t = 120$ (min) (a mean $= 0.807$, a standard deviation $= 0.763$) (c) on 2nd July 2007.

The market participants communicate via electronic brokerage systems and perceive both endogenous and exogenous information through electronic communication (e-mails, telephones, video-chat systems, and so on). Therefore, it is natural that they are affected by both specific and common fluctuations and/or have strong interactions with one another. When α descends toward 1/2, the quotations arriving on the brokerage system tend to be random. On the other hand, when α ascends toward 1, the arrival tends to be synchronous. Therefore, the values of α are useful to evaluate the states of information flows in the whole FX market.

According to Ref. [22], exponents α calculated for systems with strong temporal correlations can become time-scale dependent. Specifically if Hurst exponents are dependent on the activity $\langle X_{i,\Delta t} \rangle$ for each currency pair i, then the α values depend on the time window Δt. This behavior has been observed in our case and a quantitative agreement between both

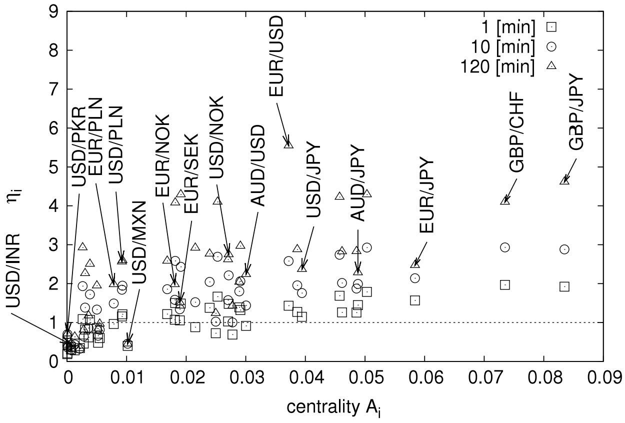

Fig. 7. Relationship between centrality of the ith currency pair A_i and its ratio of specific fluctuation to common one for each time window Δt for four periods in July 2007. The x-axis represents the centrality and the y-axis the ratio of end-exo. Unfilled squares represent the values of 46 kinds of currency pairs for $\Delta t = 1$ (min), unfilled circles those for $\Delta t = 10$ (min), and unfilled triangles those for $\Delta t = 120$ (min).

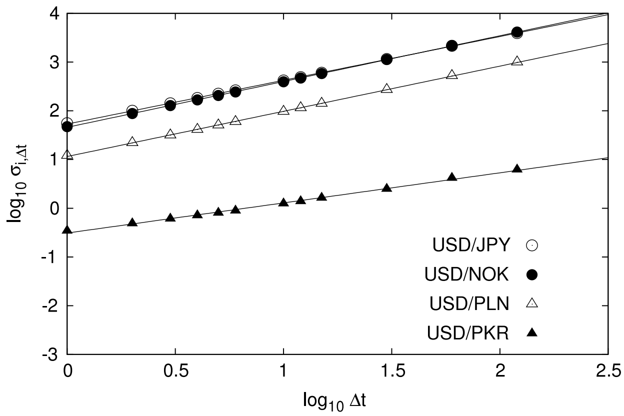

Fig. 8. A semi-log plot of a relation between standard deviation $\sigma_{i,\Delta t}$ and Δt of USD/JPY (unfilled circle), USD/NOK (filled circle), USD/PLN (unfilled triangle), and USD/PKR (filled triangle) for a period from 2nd to 8th July 2007. The Hurst exponents are calculated as $H_i = 0.898 \pm 0.004$ (USD/JPY), $H_i = 0.940 \pm 0.002$ (USD/NOK), $H_i = 0.928 \pm 0.004$ (USD/PLN), and $H_i = 0.618 \pm 0.011$ (USD/PKR), respectively.

dependences has been found. In fact the following relation should be valid [21]:

$$\frac{\partial^2 (\log \sigma_{i,\Delta t})}{\partial (\log_{10} \Delta t) \partial (\log_{10} \langle X_{i,\Delta t} \rangle)} = \gamma \approx \gamma' = \frac{\partial H_i}{\partial (\log_{10} \langle X_{i,\Delta t} \rangle)} = \frac{\partial^2 (\log_{10} \sigma_{i,\Delta t})}{\partial (\log_{10} \langle X_{i,\Delta t} \rangle) \partial (\log_{10} \Delta t)}, \tag{13}$$

where γ is defined in Eq. (12) and we used the standard definition of the Hurst exponent:

$$H_i = \frac{\partial (\log_{10} \sigma_{i,\Delta t})}{\partial (\log_{10} \Delta t)}. \tag{14}$$

The Hurst exponents were calculated from plots similar to those presented at Fig. 8. Then from Fig. 9 we got that $\gamma' = 0.034$, which is in good agreement with $\gamma = 0.04$ received from Fig. 3.

By plotting, for each currency pair, the standard deviations $\sigma^{com}_{i,\Delta t}$ and $\sigma^{spe}_{i,\Delta t}$ as a function of the average $\langle X_{i,\Delta t} \rangle$, we can get more insight into the origin of these fluctuations. As shown in Fig. 10, the common fluctuations scale with $\alpha_{com} = 1$ (as is expected from the definition Eq. (6)). By using Eq. (11), it is found that the specific fluctuation scales with $\alpha_{spe} = 0.69 \pm 0.03$ at $\Delta t = 1$ (min), $\alpha_{spe} = 0.68 \pm 0.03$ at $\Delta t = 10$ (min), and $\alpha_{spe} = 0.66 \pm 0.04$ at $\Delta t = 120$ (min).

The exponent α_{spe} of specific fluctuations is thus much smaller than the exponent α of total fluctuations and it is nearly independent from the length of the time window Δt. It follows that as observed in Fig. 3, the increase of the exponent α with Δt can result from the growing part of common fluctuations that possess the scaling exponent $\alpha_{com} = 1$ and give a larger contribution to the total fluctuations for longer time windows Δt (see Fig. 6 and Table 1).

It is known that the asymptotic probability of finding a walker at the site i in unweighted uncorrelated networks is related to the node degree k_i if the walker moves randomly with the same probability on a network [30]. For weighted networks, this

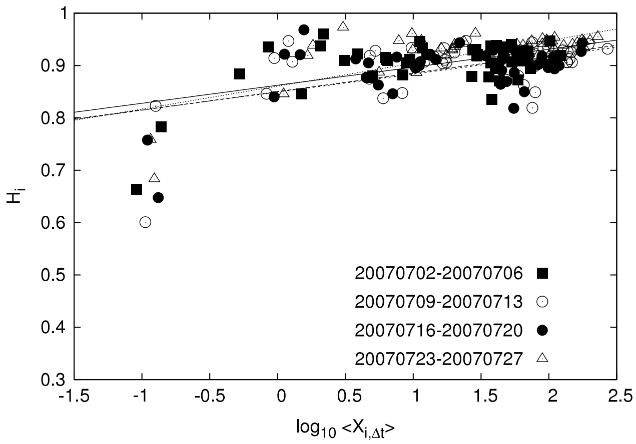

Fig. 9. Scatter diagram between Hurst exponents of each currency pair and its mean value calculated from activities during four periods in July 2007. The *x*-axis represents a mean value and the *y*-axis a Hurst exponent. By means of the least-squares method slopes are calculated as $\gamma' = 0.034 \pm 0.008$ from 2nd to 6th July 2007, $\gamma' = 0.035 \pm 0.009$ from 9th to 13th July 2007, $\gamma' = 0.034 \pm 0.008$, and $\gamma' = 0.043 \pm 0.007$ from 23rd to 27th July 2007, respectively.

probability is proportional to the node strength s_i [31,32]. In the FX market, the relationship among market participants and currency pairs based on conveyance of quotations can be described as a bipartite network. If we can accept the assumption that quotations are randomly created by market participants, then the component's centrality estimated by Eq. (5) is related to the number of market participants connected to it.

This intuitive understanding of the FX market can be mathematically displayed as follows. Let N and M denote the number of currency pairs and the number of market participants, respectively. C_{jk} is a weighted adjacency matrix of a bipartite graph assumed to be taken as non-negative integer values, *i.e.* $C_{jk} = 0, 1, 2, \ldots,$ and represents a link from the jth market participant to the kth currency pair. Then the total strength of outgoing links of the jth market participant is given by $s_j = \sum_{k=1}^{N} C_{jk}$.

Furthermore, we assume that the jth market participant generates a quotation to the kth currency pair with the same probability proportional to the ratio C_{jk}/s_j. Then the kth currency pair's centrality may be approximated as

$$A_k = \sum_{j=1}^{M} \frac{C_{jk}}{s_j} q_j \Big/ \left(\sum_{j=1}^{M} q_j\right), \tag{15}$$

where q_j is a probability for the jth market participants to enter a quotation during Δt. Practically, A_k can be estimated by using Eq. (5) with multiple time series in the observed time period $[0, (Q-1)\Delta t]$.

6. Conclusions

We found the power-law relationship between means of quotation activities for currency pairs at the FX and their standard deviation. The scaling exponents α take values in a range from 0.8 to 0.9; they increase with the time window Δt and vary in time depending on observation days. The nontrivial value of α implies that market participants may be affected by both endogenous and exogenous factors, or that they behave with a strong temporal correlation. The dependence α can be explained by the heterogeneity of Hurst exponents that increase with the mean activity of a given currency pair. This dependence also follows from the increasing contribution of common fluctuations for larger Δt and can be related to synchronous and desynchronous states of information transmission in the FX market. The standard deviations of the specific fluctuations scale with $\alpha_{spe} \approx 0.68$ as a function of activity means and this scaling exponent is nearly independent of the time window Δt. The specific fluctuations dominate the dynamics of rare currency pairs, while the common fluctuations are essential for hard currency pairs with large mean activity.

It follows from our analysis that for short time scale $\Delta t \approx 1$ (min), the dynamics of the FX market are substantially driven by the specific processes while for Δt around 2 (h) the common factors start to be important. This observation is consistent with the results of [27] on the stock market fluctuations. It means that in the stock markets and in the FX market, incoming news shows finite time diffusion on the short time scale. On the other hand, since the coherent response can be observed, collective decisions occur at a longer time scale.

There are certain similarities as well as differences between our results for FX market quotation activity fluctuations and observations of the stock market value fluctuations [22,27]. In both cases there is a Taylor-like scaling of fluctuation amplitudes with nonuniversal characteristic exponents α dependent on the time window Δt. Hurst exponents for both systems increase with mean activity $\langle X_{i,\Delta t}\rangle$ of a currency pair or of a mean stock value. This dependence is, however, very

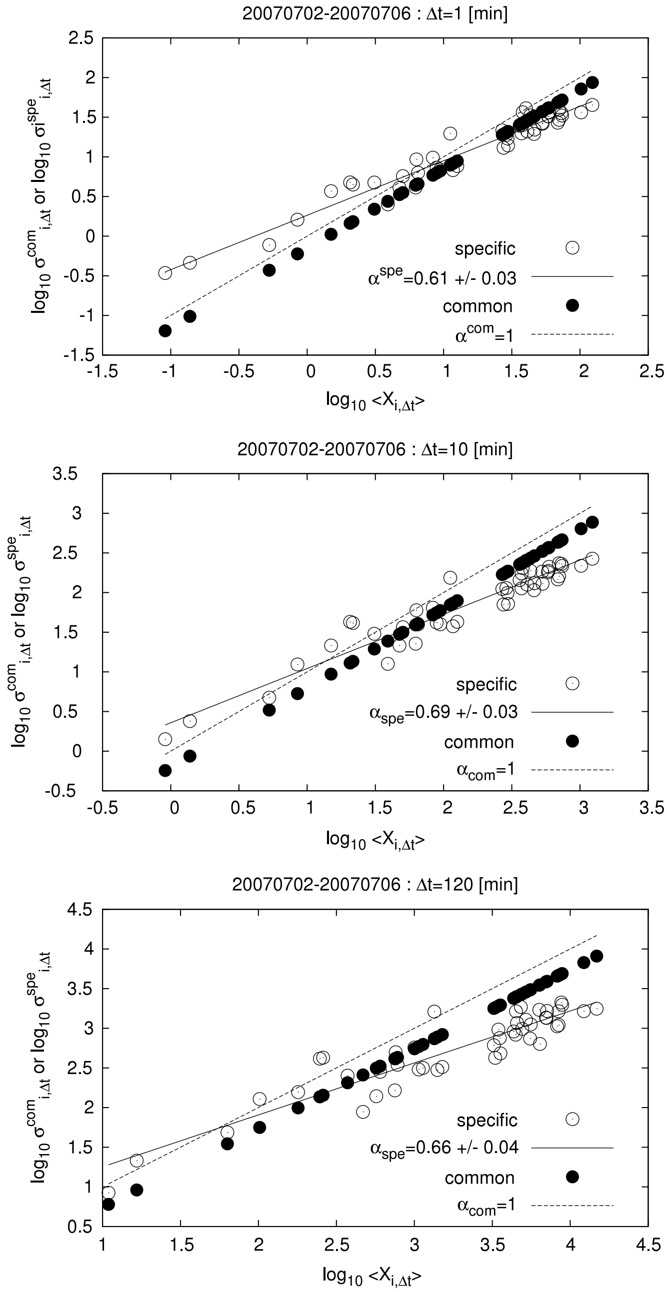

Fig. 10. Log–log plots of a mean of the specific fluctuation (unfilled circle) and of that of the common fluctuation (filled circle) and their standard deviation for each currency pair at $\Delta t = 1$ (min)(a), $\Delta t = 10$ (min) (b), and $\Delta t = 120$ [min] (c) for a period from 2nd to 6th July 2007. The *x*-axis represents a mean value, and the *y*-axis a standard deviation. By using the ordinary least squares, the straight line is estimated as $\log_{10} C_{spe} = 0.26\pm0.03$ and $\alpha_{spe} = 0.69\pm0.03$ at $\Delta t = 1$ [min], $\log_{10} C_{spe} = 0.36\pm0.08$ and $\alpha_{spe} = 0.68\pm0.03$ at $\Delta t = 10$ (min), and $\log_{10} C_{spe} = \pm0.60\pm0.14$ and $\alpha_{spe} = 0.66\pm0.04$ at $\Delta t = 120$ (min). The slope of the dashed line is unity.

clear for the stock market data (in the case of $\Delta > 300$ (min) and it is very noisy for the FX market. The other important difference is the magnitude of common fluctuations $\sigma^{\rm com}_{i,\Delta t}$ that, in the case of the FX market can be larger than the amplitude of specific fluctuations $\sigma^{\rm spe}_{i,\Delta t}$ even for short time scales $\Delta t = 1$ (min) for very active currency pairs with large centrality values (e.g. GBP/CHF or GBP/JPY).

As for future work, more careful analysis, including the relationship between auto-correlation and cross-correlation, is needed. In addition, persistent investigation using exhaustive data for long periods, and mathematical modeling of the FX market from an information transmission point of view, should be conducted. One candidate for an adequate model is that of the stochastic processes [18,20] and another is that of agent-based models in the financial markets [8,33,34].

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