Mobility in modern cities: looking for physical laws

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We discuss an exponential law for the trajectories distribution in a urban road network, that has been found analyzing a large set of GPS data on private cars dynamics. This law appears to be common for very different cities, from the size of a town to that of a metropolis. We propose to explain this behavior as an effect of the activities sprawling in modern cities, associated with the existence of a cognitive proximity concept that determines the citizens mobility requests. We discuss the relevance of these results for a complex systems approach to urban mobility.

1 Introduction

The application of dynamical systems models to modern metropolis is becoming an important research field for Complexity Science[3]. Recent observations of urban planners[4] and sociologists[9] have pointed out as only a complex systems approach can take into account the multiple interactions among different urban systems. According to this point of view, the City dynamics is an emergent state of citizens microscopic movements in a evolving environment, and the complex models are useful instruments for the city governance based on a common knowledge among different field experts[1]. The urban mobility modeling is a paradigmatic problem due to the implications for the life quality. Various models[10, 6, 8] have been proposed pointing the attention to the different types of mobility. A common feature is the crucial role of citizens microdynamics (microscopic level) that introduces physical and information-based interactions. The citizens propensities[12] are the real causes of mobility, whereas the transportation networks (mesoscopic level) allow to realize the mobility request. The origin-destination (O-D) models[5] have been recognized to give a too restrictive framework to describe the different kinds of mobility observed in modern metropolis[11]. A new representation of the City[2] has been proposed based on the *chronotopos* concept: the chronotopos is the primigenial entity, whose time dependent activity determines the

citizens request of mobility. Example of chronotopoi are historical centers, commercial areas, university campus, ect... We remark that there are well localized chronotopoi, but there are also chronotopoi sprawled in the city area. The introduction of the chronotopoi provides the citizens with a cognitive map (macroscopic level), that is used to define the agenda[13]. When a citizens decides to visit a chronotopos, this means that he has the demand to perform some activities. A complex dynamical model of urban mobility has to consider the dynamics of the three levels (microscopic, mesoscopic and macroscopic), that have different evolution time scale, and their mutual interactions. In fact, we have a bottom-up action that is typical of physical systems, but also a top-down action due to the cognitive behavior of citizens-agents: for example the individual decisions that depend on the chronotopic city map and on the dynamical state of the transportation networks, have a strong influence on the citizens microdynamics. The construction of such a model is a challenge for the complex systems community, and require large sets of microscopic dynamical data on urban mobility.
In this paper we present some results on data analysis of individual car trajectories in urban spaces, collected for insurance reason, using a GPS systems installed in a sample of $\simeq 1\%$ of the total vehicles population. We have considered the cases of a small town (Senigallia, $\simeq 5 \times 10^4$ inhabitants, near the Adriatic see), a medium city (Bologna, $\simeq 4 \times 10^5$ inhabitants) and a great metropolis (Rome, $\simeq 3 \times 10^6$ inhabitants). The three cities have a historical center that can be considered a great chronotopic area, connected with a residential periphery, but the space topology is very different. We show the existence of an universal behavior in the distribution of the trajectories lengths which is independent on the microscopic structure of the road network[15]. This could be related with the existence of an universal *proximity* concept and with the activities sprawling phenonmenon in modern cities. We discuss also some features of the travel time distribution, using a Ornstein-Uhlenbeck process to model the velocity dynamics in the road network.

2 The experimental data

The collected data give the position, speed, direction and quality of each monitored vehicle. The data recording begins when the engine is switched on and it ends when the engine is switched off. The data are registered when the vehicle has covered a distance of $\simeq 2$ km. The data are associated to a car id, so that we can follow the trajectories of the same driver at different times and days. In the three considered cases (Senigallia, Bologna and Roma), we have selected the inner trajectories: i.e. the trajectories fully contained in a defined area. To have trajectories of comparable length we have chosen three areas of comparable dimension: 130 Km^2 for Senigalla, 160 Km^2 for Bologna and 400 Km^2 for Roma. In the Senigallia case the area is much larger the city itself that is contained in a strip of 2×5 Km^2, whereas in the Bologna case the considered area include the whole municipality. Finally the studied region in Roma is an area containing the main ring (”Grande Raccordo Anulare ”) around the historical center. In fig. 1 we have show the Bologna and Senigallia areas together with the experimental data measured during the whole month of June 2006. The GPS data are sufficiently precise to localize each vehicle in

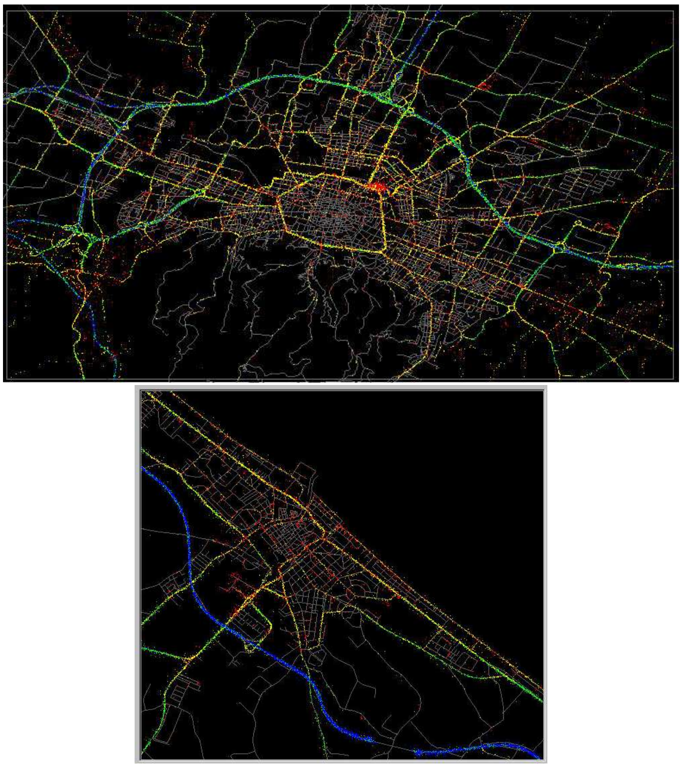

Fig. 1. GPS recorded data in the Bologna(top) and Senigallia(bottom) urban areas during the month June 2006. The colors correspond to the different velocities v: red for $v \leq 30$ Km/h, yellow for $v \leq 60$ Km/h, green for $v \leq 90$ Km/h and blue for $v > 90$ km/h. The areas of the considered regions are comparable: $15 \times 11 \quad Km^2$ for Bologna and $12 \times 11 \quad Km^2$ for Senigallia.

a defined road. We remark that the experimental points are not ergodically spread on the road network, but they are concentrated on a subnetwork ,that contains the main connection streets between the historical center and the periphery. There may be several reasons for this behavior:

I the traffic rules prevent people to use all the possible paths in the road network;

II there is a psychological repulsion to drive along tortuous paths with narrows streets;
III the citizens have a cognitive map representation of the road network and they use the small scale streets only nearby the origin or destination areas.

The third item is consistent with a chronotopic description of the urban space, since the mobility in the chronotopic areas is described as a random walk by the urban planners (*asystematic mobility*). On the contrary outside the chronotopoi, we have an O-D type mobility, and the citizens tends to follow optimal paths.
After a data preprocessing to compensate the GPS position errors, we have computed the path length using metrics:

I the arc-length of the paths on the road network;
II the polygonal metric that sums the Euclidean distances among nearby recorded points;

In both cases the qualitative results on the path-length distribution are the same indicating that our analysis is independent from the geometric details of the road network. In fig. 2 we plot the trajectories length distribution in the city of Roma during February 2007 and in the cities of Bologna and Senigallia during June 2006. Even if the statistical sample is very different (there are almost 2 orders of magnitude between Senigallia and Roma), the behavior is very similar in the three cases, indicating a possible universal character.
Using a semi-log scale it is clear that we have an exponential decay as the length s increases. We also observe an inflection point at a length $\simeq 1.5$ Km, that points out a difference behavior for the short length trajectories. We have checked that this result is scale-invariant with respect to the region dimension, where the inner trajectories are computed. The exponential law is also independent from the considered period: in fig. 3 we show the trajectories length distribution in the Bologna area, recorded during the months of June, September, October 2006.
The independence on the road network topology could be a consequence of the polygonal metric, that does not take into account the small scale structures. Then in the case of Roma, we have computed the lengths distribution using the arc-length metric for the trajectories. The results are shown in the figure 4 (squares) and we get a similar behavior as in fig. 2, even if the slope of the exponential decay is slightly different and the length distribution is spread on a larger interval than in the previous case. This is a consequence of the topological properties of the road network. Finally in order to point out the dynamical aspect of urban mobility, we have computed the travel-time distribution. The result for Roma is plotted in the figure 5 (squares); the presence of an exponential decay for long time trips indicates the existence of an asymptotic average velocity, but still the short time trips show a different behavior.

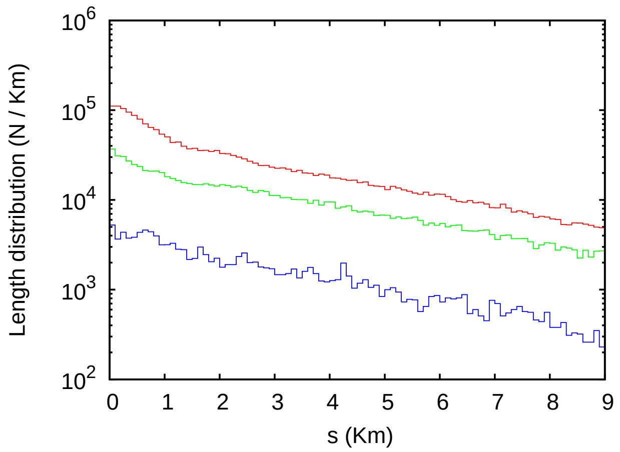

Fig. 2. Length distribution of inner trajectories for the city of Rome (top) during February 2007) and for the cities of Bologna (center) and Senigallia (bottom) during June 2006. Vertical scale is logarithmic and we have used the polygonal metric. The total number of trajectories is 1,176,763 for Roma, 79,154 for Bologna and 13,838 for Senigallia.

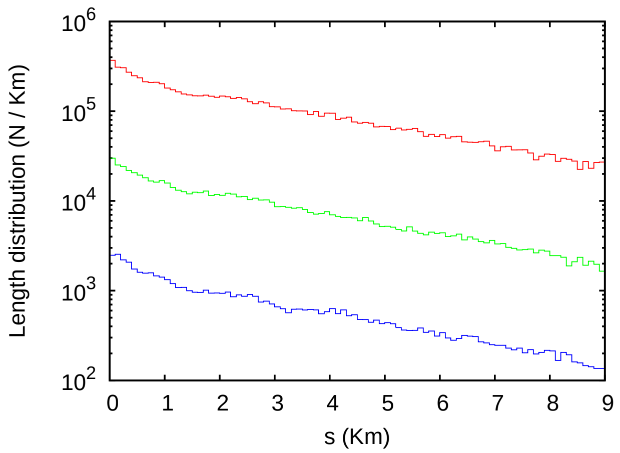

Fig. 3. Length distribution of inner trajectories for the city of Bologna during the months of June (top), September (middle) and October 2006 (bottom) using a logarithmic Vertical. The paths lengths are measured using the polygonal metric. The total number of trajectories plotted is 100,596 for June, 79,154 for September and 66,104 for October. The June (resp. October) data sets has been artificially shifted to allow a clearer comparison by multiplying the distribution by a factor 10 (resp. 1/10).

3 Relevance for complex mobility models

The experimental law for the lengths distribution, could be interpreted by assuming the existence of a *proximity concept* in the individual urban space representation that uses an utility function [16, 7] $U(s)$ to weight the possible local destinations in the individual agenda according to a perceived distance s. The idea of proximity can also introduce long terms effects that determine, for example, the citizens choice of the house location and the urban structure of the city itself[14]. Using a physical point of view, the utility function could be associated to a *proximity force* $F = -dU/ds$ that attracts individuals to stay nearby their living or working areas. The travel lengths distribution is the result of a statistical equilibrium according to the Gibbs distribution

$$\rho(s) = A \exp\left(-\frac{U(s)}{T}\right) \tag{1}$$

where A is a normalizing constant and T is interpreted as a *mobility temperature*: i.e. the population attitude to mobility. However it is really remarkable that the ratio $U(s)/T$ turns out to be the same for the three considered cities. For this reason, in our opinion, the length distribution is the result of a cognitive behavior that is common to a human population represented by our sample. An analysis of the results in fig. 2 points out the citizens mainly use two linear utility functions for the short and for the long lengths trajectories. The observed lengths distribution is an overlap of two exponential distributions

$$\rho(s) = A\left(\exp(-\alpha_l s) + c\exp(-\alpha_s s)\right) \tag{2}$$

In the fig. 4 we show an interpolation(continuous curve) of the experimental data by using the distribution 2, in the case of Roma. The interpolation gives the following parameters: $\alpha_l = .15 \quad \text{km}^{-1}$, $\alpha_s = .6 \quad \text{km}^{-1}$ and $c = .7$. The distribution 2 means that the mobility requests can be divided in two main classes. In the first class there are activities that require long and unfrequent travels with a characteristic scale $1/\alpha_l \simeq 6.5$ km (like for example to reach the working place), and could be related to a O-D mobility. In the second class there are activities that require short and frequent travels with a characteristic scale $1/\alpha_s \simeq 1.6$ km, and create the asystematic mobility on the small scale structure of the road network. In both cases the exponential decay indicates the existence of an incoherent (without memory) process in the mobility requests, that could be related both to the activities sprawling in the City and to the human free will in deciding the daily agenda. The c parameter value .7 gives the relative weight between the two mobilities, that turn out to be comparable.

The existence of two regimes can be also seen in the travel time distribution (see fig. 5), where the relation $s = \overline{v}t$ holds only for long distance travels and cannot explain the short distance travel times. Our explanation is that in the long distance travels there is enough time for the dynamics to reduce the

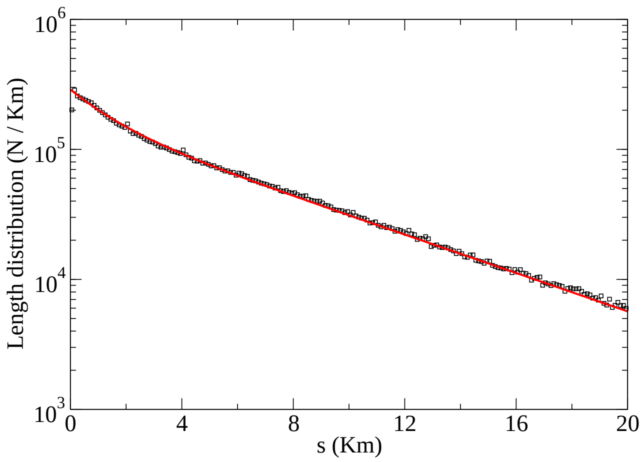

Fig. 4. Interpolation of the experimental data (squares) with the analytical distribution 2 (continuous curve) for the inner trajectories lengths in Roma (February 2006) using the arc-legth metric. The interpolated formula uses the following parameters: $\alpha_l = .15 \quad \mathrm{km}^{-1}$, $\alpha_s = .6 \quad \mathrm{km}^{-1}$, and $c = .7$

effects of fluctuations and to well define an average velocity. On the contrary the short paths dynamics is dominated by stochastic accelerations due to the microscopic interactions and the microscopic structure of road network. Then we assume an effective Ornstein-Uhlenbeck equation for the vehicle velocity

$$\begin{aligned} \dot{v} &= -\gamma(v - \overline{v}) + \sigma\xi(t) \\ \dot{s} &= |v| \end{aligned} \tag{3}$$

where $\xi(t)$ is a white noise ($<\xi(t)>= 0$ and $<\xi(t)\xi(s)>= \delta(t-s)$). In a short time scale ($t \ll 1/\gamma$), v behaves a Wiener process and $<|v|>\simeq \sigma\sqrt{t}$, whereas in a long time scale ($t \gg 1/\gamma$) $<v>$ relaxes towards the average value $\overline{v}$. According to these hypotheses, the travel time distribution $\rho(t)$ is approximated by

$$\rho(t) \propto \begin{cases} \sqrt{t}\exp(-\alpha_s\sigma t^{3/2}) & t \ll 1/\gamma \\ \exp(-\alpha_l\overline{v}t) & t \gg 1/\gamma \end{cases} \tag{4}$$

A comparison of the experimental data with the distribution 4 provided by eq. 3 is shown in fig. 5(continuous curve), where we have used the path length distribution 2. The interpolation with experimental data provides an "average stochastic acceleration" $\sigma \simeq .3$ m/sec$^{3/2}$ and a average velocity $\overline{v} \simeq 32$ Km/h for the long paths. The characteristic time scale for the short trajectories regime, is $1/\gamma \simeq 10 \quad$ min that corresponds to a length of $\simeq 5$ Km using the average velocity. This is consistent with the position of the inflection point in the length distribution 4.

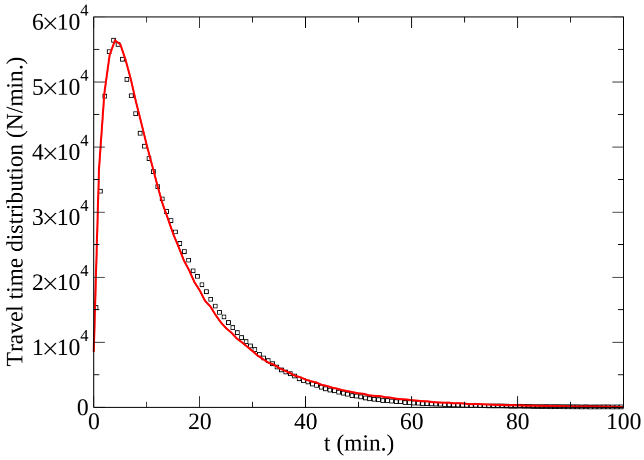

Fig. 5. Comparison of the experimental data (squares) with the travel time distribution(continuous curve) computed by using eq. 3 and the path length distribution 2 in the case of Roma (February 2006). The parameters values are: $\sigma = .3 \quad \mathrm{m/sec}^{3/2}$, $\overline{v} = 32 \quad \mathrm{km/h}$ and $\gamma = .1 \quad \mathrm{min}^{-1}$.

4 Conclusions

In this paper we present some results on a new data set on single vehicle urban dynamics, based on a GPS system to localize the vehicles and to measure their velocity. By studying three cities (Senigallia, Bologna and Roma), we have shown the existence of a general distribution law for the trajectories lengths which is independent from the topological properties of the road network, the scale of the city and the considered period. The distribution law is characterized by two exponential decays for short and long distance travels. We think that this empirical result is an indication for the existence of underlying statistical laws valid for a gas composed by particles able to process information (automata gas). We propose an explanation based on the cognitive concept of proximity and on the presence of the activities sprawling in modern cities. The proximity concept allows to define an utility function for the possible urban paths, that generates the observed distribution as a statistical equilibrium, where the choice among the activities introduce a uncorrelated process in the urban mobility.

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