Why Two Variants of Signal May be not the Best Solution Especially in the Kauffman Networks Used for Real Adaptive Systems

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Abstract

The main topic of this paper is the introduction of more than two equally probable variants of signal to the Kauffman networks which can then become different from the Boolean networks. An intuitive premise and interpretative condition of such assumption for real adaptive systems are shortly shown but we focus on its effects which differ from the effects of the commonly used assumption of only two variants. For more than two variants the level of damage equilibrium is much higher and the dynamics of different network types, especially scale-free network also differs significantly. These differences show when simplification to two variants may lead to incorrect effects. Along the way we introduce a simple and intuitive coefficient of damage propagation which describes well the first crucial period of damage spreading and serves as a simple indicator of when the system should become chaotic. We also propose a simple algorithm for statistical simulation of damage spreading which only calculates damaged nodes instead of two full systems - the damaged one and the undisturbed one.

Keywords: Kauffman network; Boolean network; damage spreading; chaos; adaptive system.

1 Introduction

The main idea of this paper is to introduce more than two equally probable signal variants (s¿2) especially to the Kauffman networks and to show that such

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a solution can be better and lead to more adequate description and predictions. We collect arguments of different types. Some of them have subjective bases or interpretative character - such are presented shortly in the Introduction. Interpretation is the base of the proposed assumption $s > 2$, but we also show that using it we obtain significantly different results (ch.3 and 4).

The most adequate network type to describe a living object or a system designed by human, whose main assessed properties are effects of its function, is Kauffman network [23, 25]. However, in the Kauffman model there are two assumptions, which we do not share in our view especially of the typical living and human-designed systems. The first of these assumptions is an interpretative estimation that spontaneous order typically has a large significance in such systems and the second one - that the number two of signal variants is adequate to describe function and behaviour of such systems especially when the assumption of equal probability of these two variants is added.

We estimate that the typical living object or a system designed by human are chaotic - for such systems we observe a large set of really random changes (not predicted in designing process) which with high probability can cause a large avalanche of damage. These observations, however, are subjective, we will not prove them. There is a certain strange exception the gene regulatory network described by the Kauffman model [24, 37, 31, 32] using Boolean networks, however there are also other, less unusual examples [28, 8].

We use the term 'chaos' as Kauffman[25] does. It differs from the more common definition used for continuous arguments of a function. We have a finite number N of nodes, each one with a small natural number s of possible states. Such chaotic systems differ from the ordered ones in damage spreading. Damage is the difference between two identical functioning systems which appears as an effect of some disturbance in one of these systems [22]. Typically a very small change is investigated, which initiates damage, e.g. a change of state of one element of the system. For chaotic systems a small initiation of damage typically causes a large avalanche of damage which spreads onto a big part of the system, however, it ends at an equilibrium level. The existence of this equilibrium level as the limit of damage growth is the main difference between this 'chaos' and the more commonly used definition. High stability of ordered systems does not allow for damage to evolve into an avalanche.

Let k be the number of node outputs (outgoing links) and K - the number of node inputs. We assume constant K i.e . equal for all nodes of the network. We neglect the strange case of $K = 1$ therefore K has to be greater or equal 2. We consider an autonomous network therefore $< k >= K$ and $< k >=> 2$ but for a particular node $k = 1$ and $k = 0$ can happen. For completeness we repeat: s is the number of equally probable variants of signal. Note, that using s we know that they are equally probable.

We introduce (ch.3) a simple intuitive indicator of ability of damage to explode. This is $w = k(1 - s)/s$ - 'coefficient of damage propagation' (or 'damage multiplication' on one element of system if only one input signal is changed) which indicates how many output signals of a node will be changed on the average if one input signal is changed (for the random function used by nodes). (We assume minimal P - internal homogeneity[25] for this all paper.) An average w is interesting for the whole network or for its part. For the whole network the case $w = 1$ occurs only for $s = 2$ and $k = 2$. For all other cases it is $w > 1$ and the damage should statistically explode onto a large part of the system, which means that the system is chaotic.

The case $s = 2$ is extreme - there is no smaller sensible value for s - but it is even more extreme as we have seen. If we simultaneously use $k = 2$ in the same way, then we obtain an especially extreme case - order instead of chaos. Such extreme values of parameters assumed for the model should have special known causes, other than useful simplicity of the model, especially if we need chaos as we estimate above. Generally a safer approach is to use not so extreme values for an unknown parameter.

Typically while describing adaptive systems we encounter two alternatives but they usually have very different probabilities. This is also a subjective observation but we have a certain explanation of it: One of the typical ways leading to two alternatives is our concentration on one particular event and collecting all the remaining events as the second alternative. This second alternative is NOT the first one, and we obtain two alternatives. There are lots of alternatives in such a case in the reality and this is important for the mechanism but we are only interested in one of them. Typically it is 'the proper event' when we are concerned with systems which adapt. Note, for system which adapt the notions: 'proper' and 'correct' are defined using fitness. This is the main, however, simple and important cause of introduction more than two alternatives. Note, we estimate that this cause of observation two alternatives is typical.

Using more than two alternatives for the description of mechanism of such a case seems much more adequate. E.g. when we are going to describe the long process leading from genes to some properties directly assessed using fitness we should remark that there are 4 nucleotides, 20 amino acids and other unclear spectra of alternatives. In this set of spectra really two alternatives seem to be an exception. Sousa in [34] considers the scale-free network and more than two different opinions and he obtains a vote distribution in better agreement with reality. Similarly Stauffer et al. [35, 21, 36] consider Q opinion states. Luque and Ballesteros [27] also have a doubt about the adequateness of two signal variants when they also similarly introduce more than two variants. We found a suggestion only in [30] that in Kauffman model the number of variants can be different from two.

Using equal probability of these alternatives is probably the only way to define probability needed for prediction and calculation. In such a way we obtain more than two equally probable signal variants ($s > 2$) which we are going to introduce in this paper into the Kauffman networks used for general description of real adaptive systems.

The parameter p was used as the probability of one of two alternatives. Parameter s describes another new aspect and mechanism. For $s > 2$ damage always statistically should grow (whenever it has room to grow) which our 'coefficient of damage propagation' shows, and we always should obtain chaos. For extreme p and small $K > 2$, however, we obtain order [10, 3].

Number s of equally probable variants of signals is the next main parameter of system, like Kauffman's K - number of inputs per element and P - the internal homogeneity in Boolean functions or the above mentioned p. These parameters define a system as chaotic or ordered.

Next we examine the differences of effects obtained using our new approach and the old one. We find two important differences. We find the first of them by expanding the Kauffman method of calculation of equilibrium level of damage (ch.3) to cases of $s > 2$. The levels for $s > 2$ are very different from the case of $s = 2$ (see fig.1). The parameter s has a much stronger influence on these levels (moving up their upper limit two times) than the K parameter used up till now for exploration of chaotic regime, therefore $s > 2$ cannot be substituted by $K > 2$.

The result of our simulation shows the second important difference (ch.4). The parameter s, especially for lower values, has a significant influence on the behaviour of different network types (see fig.5), especially scale-free networks, in the first crucial period of damage growth.

Both these differences in effects of the assumptions $s = 2$ and $s > 2$ (neglecting the possibility of phase transition from chaos to order) enter the range of qualitative differences. This confirms the importance of this choice.

The assumptions of two variants and their equal probability are also used in a wide range of similar models like e.g. cellular automata, Ising model or spin glasses [22]. They are typically applied as safe, useful simplifications which should be used for preliminary recognition. But just like in the case of Boolean networks these assumptions may not be so safe and should be checked carefully. In the original application of Ising model and spin glasses to physical spin they are obviously correct, but these models are nowadays applied to a wide range of problems, from social (e.g. opinion formation [19]) to biological ones, where such assumptions are typically simplifications.

We use our special simplified algorithm for this simulation (described in ch.4). It is dedicated for statistical investigation of damage spreading in Kauffman and similar networks in synchronous mode. Only damaged nodes are calculated here. The process stops when damage fades out or achieves the equilibrium level. In the classic method two full systems are calculated - the disturbed and the undisturbed one and at each time step they are compared to measure the damage.

2 Agitation, Intuition and Interpretation - Estimation of Parameters for the General Model of Real Adaptive Systems

The most adequate network type to describe a living object or a system designed by human, whose main assessed properties are effects of its function, is Kauffman network [23, 25]. However, in the Kauffman model there are two assumptions, which we do not share in our view especially of the typical living and human-designed systems. The first of these assumptions is an interpretative estimation that spontaneous order typically has a large significance in such systems and the second one - that the number two of signal variants is adequate to describe function and behaviour of such systems especially when the assumption of equal probability of these two variants is added. These discrepancies are not independent in the case of useful equality of probability of the variants - 'order' appears only for two variants (when internal homogeneity P of function is minimal which we assumed for the whole of this paper). However, there is one strange exception from our view - the main application of Kauffman model. Gene regulatory networks [24] are described successfully [37, 31, 32] using Boolean networks, but this is an exception which proves the statistical rule.

The terms: 'Kauffman networks' and 'Boolean networks' were synonymous. For more than two variants of signal which we are going to introduce in this paper such networks cannot be 'Boolean' anymore but they can and should remain 'Kauffman networks'. Systems created or maintained by humans and living systems grow or are designed under adaptive conditions. We name them 'adaptive systems'. We distinguish this set of systems because of the aspect of purposefulness which is typically present when such systems are described. Summarizing our observations of these systems we have collected a few remarks or intuitive estimations. We will not prove them but we will agitate. These remarks will build an intuitive base of assumption we will introduce, which should have some base.

2.1 Typical living object or a system designed by human are chaotic

If a really random change happens, as opposed to a predicted event to which the system is adapted, then with high probability we can expect a large avalanche of damage which typically leads to a critical malfunction of the system. This is chaotic behaviour in the meaning of Kauffman model [25]. Note, we exclude here the large set of random changes which are 'predicted' by the adaptive process of designing of these systems because the system's reactions to such changes are not random but special. We know which changes are not random in the human-designed systems but in a living system we only know that it is an effect of the Darwinian mechanism and we expect that lots of reactions to random-like changes are not random. The existence of a large set of changes which create damage avalanches is enough to treat such systems as chaotic. This important estimation will be a part of the basis of adequateness of our assumption of more than two equally probable signal variants for the considered set of systems. Typically we find mechanism of such a not random reaction which is based on negative feedbacks. It is homeostatic in the typical meaning of this term and it is 'ultrastable' using Kauffman's term [25]. Kauffman uses the 'homeostatic stability' term for spontaneous resistance of system to disturbance which may also result from adaptation but it is an effect of system type founded by adaptation which is 'ordered'. Damage avalanches on great scale are impossible in ordered systems, they should fade out quickly.

Examples of such really random changes occur when control parameters rise above their critical levels [28] which are the boundary of tested (i.e. predicted) range of these parameters or in other cases which are so strange [8] (i.e. have especially small probability or are especially complicated which also leads to small probability) that they did not occur often enough or due to complication - not solved yet or possible mechanisms are too expensive. For living objects one mechanism is always standing by - it is reproduction, it may be cheaper.

Do you believe us that you are chaotic? If not, then imagine, please, that you are a patient. Can a surgeon expect during a medical operation on your body that his mistakes will be neutralized by your 'homeostatic stability' (in Kauffman sense)?

2.2 Case 'two variants of signal' is extreme, may lead to another phase

A methodological and philosophical reason for more than two variants of signal is that the case of 'two variants' is an extreme one - there is no smaller sensible value for number of variants, only higher values. When we model a real system and we do not know the value of a parameter we should take a middle value, i.e. probably near the average value. This is a known, safe method leading to more adequate models. It assumes that the more average value is typically the more probable one which is suggested by the typical Gaussian distribution. Discontinuities can appear anywhere and for more probable values of a parameter we have no way to avoid it but for extreme values, which therefore are special and more naturally lead to special effects, we can and should avoid them due to the expectation of their low probability.

This is philosophy, however, the extreme case $s = 2$ (two equally probable signal variants) together with the similar extreme case $K = 2$ (two inputs per node) really leads [10] to an especially extreme case - crossing of phase transition from chaos (which occurs for all others s and K) to order (which occurs only for such a combination). Then this philosophy works. For $s = 2$ and $K > 2$ there are no such special cases and $s = 2$ seems safe in this area but 'working philosophy' suggests not to use such an extreme value, but a more probable higher value instead. Note that using such a suggestion we are coherent with the above estimation that modelled adaptive systems should be chaotic.

2.3 Alternatives are typically not equally probable

Typically in the description of adaptive systems we encounter two alternatives. However, the assumption of typical occurrence of equal probability of such alternatives seems a great simplification. We do not like to use description notions such as: 'correct', 'proper' or 'special' for an alternative, (such terms are defined using fitness in the set of systems which adapt) but everybody agrees that the 'correct' alternative is typically much less probable. Only novice gamblers do not agree but they will agree. This expected inequality of two variants of signal was described using probability p for one of the alternatives [10, 3]. We will propose another solution.

2.4 Introduction of more than two equally probable signal variants

Why do we use two alternatives which is an extreme case? What about more than two alternatives which is more safe? One of the typical ways leading to two alternatives is our concentration on one particular specific event and collecting all the remaining events as the second alternative. This occurs especially in adaptive systems because of the aspect of purposefulness of considered alternatives. One of them is 'proper', 'correct' or 'special' as we mention above. There are lots of alternatives in such a case in the reality but we are only interested in one of them. We use this 'interesting one' and NOT this 'interesting one', and we obtain two alternatives.

Using more than two alternatives for the description of mechanism of such a case seems much more adequate. E.g. when we are going to describe the long process leading from genes to some properties directly assessed using fitness we should remark that there are 4 nucleotides, 20 amino acids and other unclear spectra of alternatives. In this set of spectra really two alternatives seem to be an exception. Sousa in [34] considers the scale-free network and more than two different opinions and he obtains a vote distribution in better agreement with reality. Similarly Stauffer et al. [35, 21, 36] consider Q opinion states. Luque and Ballesteros [27] also have a doubt about the adequateness of two signal variants when they also similarly introduce more than two variants. We found a suggestion only in [30] that in Kauffman model the number of variants can be different from two.

We should not expect that in a real large network the alternatives coding different meanings for each node always have exactly equal probabilities and that the numbers of them are the same. Using equal probability of these alternatives is the typical simplification, however, it is useful and maybe it is the only way to define the probability needed for prediction and calculation. We know that all nucleotides and amino acids have probabilities not exactly equal but similar and such a simplification can be assumed for more general qualitative models.

We denote the number of equally probable signal variants by s. Note, such a description contains the assumption of equal probability.

Case $s > 2$ differs from the one described by p in the statistical mechanism and its result. For extreme p and small $K > 2$ order is expected [10, 3] but for $s > 2$ chaos is always expected - damage should grow up to an equilibrium which our simple coefficient of damage propagation introduced in the next chapter shows easily.

Another parameter P, named 'internal homogeneity of Boolean function'[25] is also used for certain problems connected to the inequality of probability of signal variants. It also describes a different aspect of this idealisation. Parameters s and P work in opposite direction when they differ from their typical value - the smallest one. Higher s causes chaos but higher P allows to avoid it.

Summarizing, we propose to use $s > 2$ (more than two equally probable signal variants) and we hope that this is much more adequate for description of the typical adaptive system. Such an assumption leads to chaotic systems which we expect from our interpretation and observation, and it explains the observed inequality of probability of the two alternatives. It is a different assumption and mechanism than the one leading to known p or P parameters.

Opposite $s = 2$ is typically used in Kauffman model [25]. Its basic application is the genetic regulatory network [24, 33] where 1 is interpreted as an active gene and 0 as an inactive one. This strange case seems adequate and gives results close to the experimental data [37, 31, 32] although it describes an adaptive system.

3 Coefficient w of Damage Propagation

3.1 Definition and meaning

Now, after above two agitation chapters, let us define a concrete coefficient w of damage propagation: $w = k * (s - 1)/s$ where k is the number of node outputs. Kauffman considers constant (for a particular system) number K of node inputs, and considers an autonomous network which has no external inputs or outputs. For autonomous networks $< k >= K$: the average k in the network is equal to the fixed K. The coefficient w shows how many output signals of a node are changed on the average if one (or more) of its input signals is changed. In the case of 'one changed input signal' this coefficient can be named 'coefficient of damage multiplication on one node'. If it is greater than one ($w > 1$) then damage should statistically grow and create an avalanche which spreads onto a large part of a system. It is similar to the coefficient of neutron multiplication in a nuclear chain reaction - if it is less than one then we have a nuclear power station, if it is greater than one then an atomic bomb explodes.

The coefficient of damage multiplication depends on the functions draw - if functions are not properly random then the coefficient w may be greater or less than the above. The coefficient of damage multiplication is a simple and intuitive indicator of the possibility of damage avalanche and therefore of the system's place on the chaos-order axis but it is only the first approximation as we will show later. Note that case other than $w > 1$ appears only for case $k = 2$ and $s = 2$ and both these parameters are here in their smallest values. The case of $k < 2$ is sensible for one particular node but not as the average value in the whole, typical, randomly build network describing reality; however we can find the case $K = 1$ in the literature [25, 37]. For all other cases where $s > 2$ or $k > 2$ we have $w > 1$ and in such a case damage statistically should explode onto a large part of the system.

The number s of equally probable variants of signals is the next among the main parameters of a system which define the system as chaotic or ordered. It is similar to Kauffman's K - number of node inputs and P - the internal homogeneity in Boolean functions or probability p for one of two alternatives. In the first theoretical approximation the coefficient w can substitute two of them (s and K) in this role but other important features of a system depend on the parameters s and K individually and differ although the coefficient w is the same. One of such features is the level of damage equilibrium for chaotic networks which differs much stronger in dependency on the parameter s than on the parameter K. The second one occurs when we investigate various types of networks, differing mainly in the distribution of node degree k: we obtain different results in 'real fade out' especially for low s and scale-free network than for higher s. Such conclusion is an effect of simulation described at the end of this paper.

3.2 w^t Describes first critical period of damage spreading

When the avalanche is still small and the range of interaction is a whole and big system (large number N of elements of system) then the probability of more than one changed input signal is also small and damage is well described by w as $d(t) = d_0 w^t$ which is shown in fig.1.2. This is a critical period of time t, when damage is still so small that probability of its fade out is not to be neglected. Later it practically cannot fade out but the cases of more than one changed input signal occur more and more often and the real multiplication of damage becomes smaller and smaller up to the moment of achieving a stable level of damage (fig.1.1 and fig.1.2). These figures are calculated in a theoretical way described in Kauffman book [25], expanded to case $s > 2$: If a denotes a part of system B with the same states of nodes as an undisturbed system A, then a^K is the probability that the node has all its K inputs with the same signals in both systems. Such nodes will have the same state in the next time point $t+1$. The remaining $1-a^K$ part of nodes will have a random state, which will be the same as in the second system A with probability $1/s$. The part of system which does not differ in $t+1$ is therefore $a^K + (1-a^K)/s$. The damage $d = 1-a$. For $K = 2$ we obtain $d_2 = d_1 * w - d_1^2/2 * w$ where for small d_1 we can neglect the second element.

3.3 Aggregate of automata - the simplest case of network for 'w'

In the Kauffman networks all k outputs of a node transmit the same signal - it is the state of node, the value of its function (fig.2.1). To understand the coefficient w of damage multiplication we must average by lots of nodes. It is much simpler and more intuitive (which is important for introducing such a method into biology) if each output link of a node has its own signal to transmit, which need not be the same as on other outputs in the same node. In such a case the function's argument and value are a K- and k-dimensional vectors (fig.2.2). Due to function uniformity it is useful to fix $K = k$. I have introduced

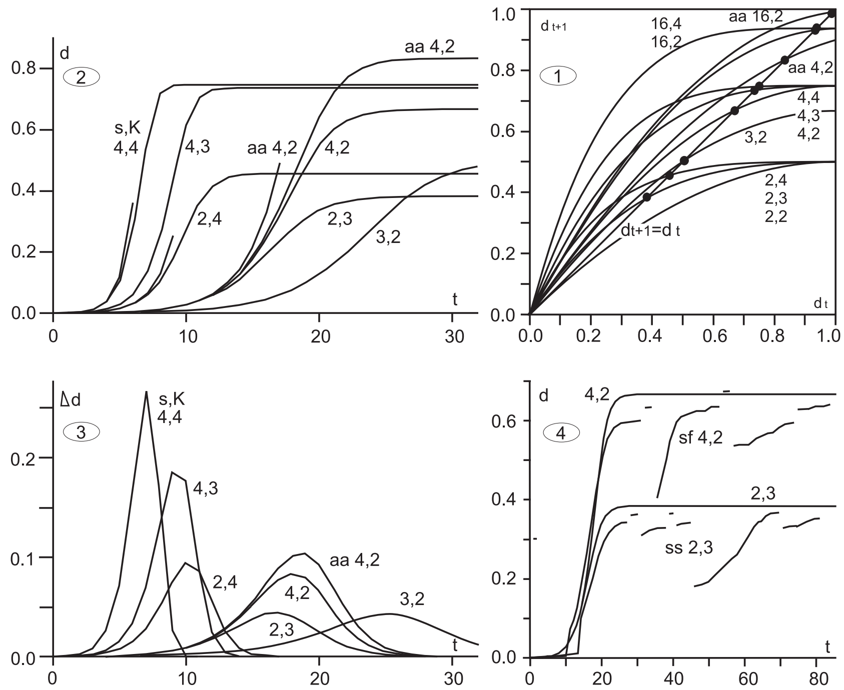

Figure 1: Theoretical damage spreading calculated using Kauffman method [25] p.199. (1) Damage change at one time step in synchronous calculation of system. It is fig.5.8 in [25] extended for the case $s > 2$ and for aa network type. The crossing of curves $d_{t+1}(d_t)$ with line $d_{t+1} = d_t$ shows equilibrium levels dmx up to which damage can grow. These levels are reached in (2) on the left which shows damage size in time dependency. A simplified expectation $d(t) = d_0 w^t$ using coefficient w is shown (Three short curves to the left of the longer reaching equilibrium). This approximation is good for the first critical period when d is still small. (4) shows examples of experimental curves in comparison to their theoretical expectations (see table 1). In (3) the increase of damage in consecutive time steps is shown. Experimental curves are similar but wider, the small difference at the point of maximum is shown in table 1.

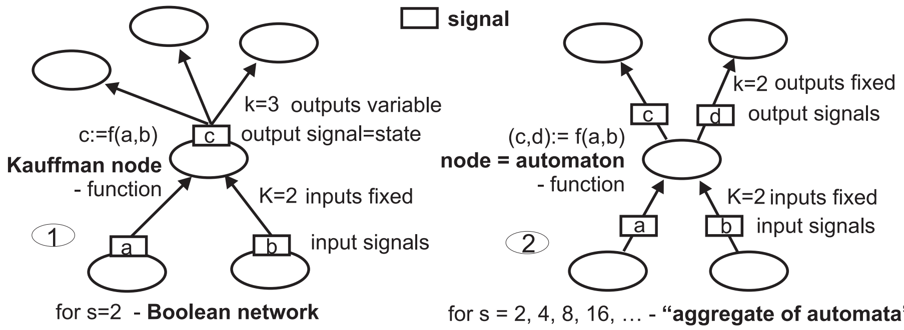

Figure 2: The basic elements of Kauffman network (1) and aggregate of automata - *aa* network type (2). Nodes - ovals, signals - rectangles, links - arrows. Each node transforms incoming (input) signals into output signals using a function, these signals are transmitted through links to the next nodes as their input signals. K - number of input signals (or links) of a particular node. k - number of output links of a particular node (node degree). For a particular node of Kauffman network (case 1 on the left) there is one output signal (state of node) which is sent by k output links. s - number of equally probable variants of signal values (in Boolean network $s = 2$, i.e. true and false). In the case (2) of the *aa* network k is fixed and each output link has its own signal, possibly different from others.

such a network in [15, 16] where I have named it 'aggregate of automata'. For this network if $K = 2$ then $d_2 = d_1 * w - d_1^2 * (s-1)^2/(s+1)/s$ which is obtained similarly as above. Note, that for small d_1 we can neglect element with d_1^2. Theoretical curves for aggregate of automata for case $s = 4$ and $K = k = 2$ are also included on fig.1.1-3. These figures show that the level of damage equilibrium for aggregate of automata is much higher than for Kauffman networks.

4 Are New Network Types and $s > 2$ Similar to Area Investigated by Kauffman?

4.1 Project formulation

The Kauffman formula gives useful ability to differentiate k within the network and to investigate different types of networks which differ in the distribution of node degree $P(k)$ like Erdős-Rényi random networks, on which Kauffman had worked, or e.g. nowadays famous Barabási-Albert scale-free networks. This is because the definition of function does not change if k changes. Due to this reason K is fixed and for a directed network only the k parameter is typically

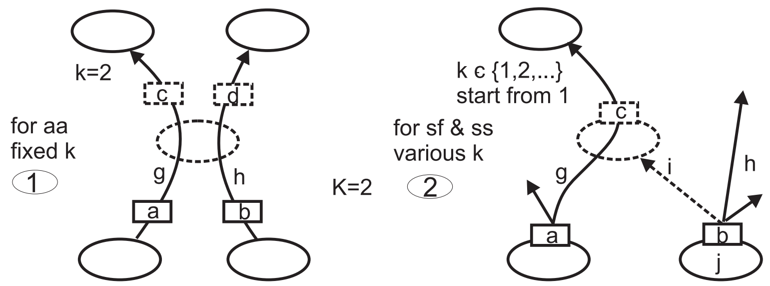

Figure 3: Additions patterns for aggregate of automata aa (1) and Kauffman networks ss and sf (2). ($K = 2$) Links g and h (and function) of node are drawn. Node j is drawn directly instead of link h for ss. For $K > 2$ additional inputs are constructed like the right ones (h or j). The ak network is maintained as aa but there is only one output signal c (d=c).

used as the degree of a node. Scale-free networks occur to be more adequate to describe reality [7, 6, 18, 19, 9, 12] than old Erdős-Rényi random networks.

Now two interesting questions appear: 1- are there any significant differences in damage spreading in different network types?; 2- what is the effect of higher s introduced above. We have investigated these questions in a simulation using our simplified algorithm dedicated for statistical analysis of damage spreading. The consistency of its result with Kauffman's expectation makes this simplification trustworthy. This algorithm will be used later for some simple and intuitive definition of complexity threshold, useful at the end of this path to investigation (also using this algorithm) of structural tendencies in adaptive evolution of a complex system. These tendencies describe some known and interesting phenomena in ontogeny development or human activity which however have not been explained until now.

In this paper we compare damage spreading in five types of autonomous networks: 'er' - random (Erdős-Rényi [11]), 'sf' - scale-free (Barabási-Albert [7, 6]), 'ss' - single-scale [1], 'aa' - aggregate of automata and 'ak' - a network similar to aa with fixed $K = k$, but using Kauffman formula where one state of a node is transmitted by all its outputs. Construction of the network simulation has two stages: construction of the network and damage investigation in the constant network. Construction of the network depends on the chosen network type. Except for the type 'er' - random networks, all networks have a rule of growth. Aggregate of automata 'aa' and 'ak' needs to draw K links in order to add a new node. These links are broken and their beginning parts become inputs to the new node and their ending parts become its outputs (fig.3.1.).

For 'ss' - single-scale network the new node is connected to the node present in the network with equal probability for each existing node. For 'sf' - scale-free network the new node is connected with another node with probability proportional to its node degree, i.e. to the k of this existing node. For both types we draw one link first and we break it like for aa and ak to define one output and its destination node and the first input. For sf type at least one such output is necessary to participate in further network growth. Later we draw the remaining inputs according to the rules described above - for ss by drawing the node directly, for sf by drawing a link and using its source node (fig.3.2.). If $K = 2$ then only one input follows the rules, but it is enough to obtain the correct $P(k)$ distribution characteristic for these network types.

Damage can take various forms, e.g. in complex computational networks [29] but typically damage spreading in scale free networks describes: epidemic spreading [18], opinion formation [12, 19, 34] and attack or error effects [9, 13]. However, these networks typically are not directed networks and their important aspect is the spatial description which uses a particular lattice shape. They also are constructed in a different way, not only using preferential attachment [18, 19]. A partially directed scale-free network was used in [36] preceded by [35, 21]. These networks describe opinion agreement process. In this approach the direction of links is used for construction of a social network and consequently - initiative to contact; however, during opinion exchange information flows in both directions and each of the talking nodes randomly takes the opinion of its partner. This second aspect is more similar to signals flow in Kauffman network, but here it is undirected and therefore this approach is not similar to Kauffman networks. The dynamics of Boolean networks with scale free topology were studied by Aldana [3] and Kauffman [26], now Iguchi et al. [20]. They look for the difference between the dynamics of er (here called: RBN) and the scale-free random Boolean network (SFRBN). All they use s=2, flexible k and K, therefore their networks differ from our sf.

4.2 Simplified algorithm of damage spreading

The classic method of damage observation uses two processes which are compared: A for an unchanged system and B for system with damage initiation[22]. We observe one process - only damage spreading, but this process is only statistically correct in a concrete range of situations. The main assumption is: We consider chaotic system where damage can fade out only when it is still small but when it is large, then it grows up to an equilibrium level where in our algorithm it also stops (pseudo fade out). These two cases are mixed, however they have different interpretations. Cases which stop between them with a middle damage d have no interpretation and can be permissible only in negligible frequency. Such cases occur only for $s = 2$ (sf 3,2; 2,4 and ss 2,3 in small but visible level and especially for sf 2,3 network (fig.4.1) in high level) which confirms that $s = 2$ is extreme. The case $s, K = 2, 2$ is for every network type out of range of permissible levels of middle damage and we cannot use our algorithm for its investigation - it consists only of real fade out cases (fig.1.1) or very low damage equilibrium levels but its long tail for higher d is strongly incorrect (too short) in the simulation.

We calculate only the nodes with one or more changed input signals[16]. We do not care what remaining input signals are. They can be changed before or after the calculation of this particular node, e.g. as effect of feedbacks loop. If a node is affected by damage, which means that at least one input signal is changed, then the node function is calculated using 'old' remaining input signals, but only once. In this paper we also do not use concrete functions for nodes. If the input state is changed, then the output state is random. This calculation gives an answer, whether output signals of this node have undergone any changes. If its input signals change later then it will not be calculated next time - for statistically correct damaged area it is not needed. Any initiation of a particular node in a particular network should statistically lead to the same damaged area but in each particular case it may be different. We, however, are not interested in a particular case but in a statistical result. Such an algorithm works fast and gives correct statistical effects.

An intuition behind this algorithm can be found when we consider a network without feedbacks, where each signal on the node output is equal to the value of the function of current signals on the node inputs. It is not a typical system state - in the next time step in the synchronous mode nothing will be changed. In such a case for calculation of node with a changed input signal (as a result of damage) we can use the old signals on the remaining inputs. After a finite number of time steps the process will stop but it never happens that a node obtains a damaged input state for a second time. The damaged part will become a tree, and all the node states will be equal to the function value of current node inputs (excluding the first initiated node) as was the case at the beginning. In the case with feedbacks sometimes an already calculated node gets a damaged input signal for a second time. For measuring the statistical effect only it is not necessary to examine its initiation for the second time, however, if such second initiation will be processed, then the process may never stop (but still reach an equilibrium level of damage).

In this paper we investigate the damage in a system of a particular size. When a network achieves the assumed number N of nodes we stop the growth and we start to initiate damage: we change the output state of each node using all remaining possibilities as damage initiation. It is the smallest initiation and in the first few steps the damage can fade out. It is a real fade out of damage. In this short way damage can meet an already damaged node which is not calculated for the second time (which helps damage to fade out), however, such an event has a very small probability. We assume that if damage fades out when it is small, then it is not due to meeting an already calculated node. This is a simplification of our algorithm. In this case the number of damaged nodes is interpreted as the number of damaged nodes during the whole process from initiation to real fade out. If coefficient $w > 1$ then on average the damage grows. If damage is great, i.e. the number of nodes with changed output state is large (the number of calculated nodes due to their input state was changed is also large), then it practically cannot fade out (probability of such events is very low, we neglect them), but during this damage growth there are less and less nodes which are not reached by damage yet. Therefore the avalanche of damage must slow down and stop (the growth). It looks like a fadeout, but it is equivalent to the achievement of the stable level by the damage which appears at the end of curves in fig.1.2 or on cross of curves with line '$d_{t+1} = d_t$' in fig.1.1 This level is an equilibrium state, as fig.1.1 shows. In our simplification the process stops at this level due to the 'pseudo-fade out' on already damaged nodes. Now the number of damaged nodes is interpreted as the number when equilibrium occurs and it describes the statistical state of the system at one specific time step (i.e. at any moment after the equilibrium is reached).

The number of nodes with changed output state (i.e. number of damaged nodes) divided by N is equivalent to the damage size d, despite the fact that they are damaged during the whole process (using our algorithm), not only in the last time step. Note, if damage fades out, regardless of the way it happens (pseudo or real fade-out), then in the last time step no node is really damaged and damage size looks like zero at this time t. However, such a view is incorrect in the case of pseudo fade out, it does not take into consideration the fact that we do not recalculate a damaged node when its input signal changes for a second time. Such a false suggestion appears due to the simplification of our algorithm.

We calculate the damage using a fi-fo queue for nodes with changed input signals waiting for calculation. The queue length in time t dependency is very similar to the one shown in fig.1.3. Small differences at the point of maximum are shown in table 1. The time step number t is defined observing this queue but for control of the process it is not necessary.

4.3 Simulation effects and comparison to theoretical expectations

The computer program realizing the above algorithm is prepared for $s = 2$, 3, 4, 8, 16, 32, 64 and K =2, 3, 4. We have investigated the whole of this area but the most interesting part is the area near the phase transition from chaos to order where differences are larger. For comparison of dependency in main parameters s and K we choose five cases described as s, K: 2,3; 2,4; 3,2; 4,2; 4,3 for five network types described above: er, ss, sf, ak, aa. In this set there are $K = 3$ and 4 for $s = 2$, $s = 3$ and 4 for $K = 2$. Similarly for $K = 3$ and $s = 4$ the second parameter has two variants. Cases 2,3 and 4,2 have the same $w = 1.5$. The coefficient w is the smallest for case 3,2 (1.33) and the largest in the shown set for 4,3 (2.25). The simulation results are shown in fig.4 and fig.5, also in table 1 but only in the main fig.5 the whole set of the above enumerated cases is shown. For the most interesting networks sf and er the full set of combination s and K in range of values 2, 3, 4 is show in fig.5.3. Each simulation consists of 600 000 damage initiations - e.g. for $s = 4$ (excluding aa) 100 different networks grow randomly up to $N = 2000$ and $N = 3000$ nodes and later each node has its output state changed 3 times. For aa and $s = 4$ we use 20 networks and the output state is changed 15 times.

As the first effect of simulation we are going to show a comparison to the expectation presented in fig.1.3 and fig.1.2. The obtained distributions of in-

Table 1: Some simulation results and their comparison to the theoretical expectations showed in fig.1. d1 - dmx (equilibrium level of damage size, see fig.1) taken from maximum position of the right peak in $P(d)$ distribution where d is the damage size when damage fades out (in the sense of our algorithm) d2 - dmx taken from the first stable maximum (plateau) of d in $d(t)$ (fig.1.2 and fig.1.4) t1 - position of maximum in increasing $d(t)$ (fig.1.3). Typically lower in simulation. t2 - visible range of the right peak in increasing $d(t)$ (fig.1.3). Typically higher in simulation. t3 - position of maximum of the right peak in $P(t)$ distribution where t is the time step number when damage fades out (in the sense of our algorithm)

s,K	network	d1	d2	t1	t2	t3
2,3	aa	1338	1319	16	35	28
	ak	772	690	14	34	25
	er	770	880	14	34	25
	ss	774	654	13	40	24
	sf	818	877	12	45	21
	theor.	764		17	26	
4,2	aa	1668	1672	17	32	27
	ak	1335	1307	16	31	26
	er	1335	1336	15	34	24
	ss	1335	1202	16	36	25
	sf	1344	1217	16	43	26
	theor.	1667aa	1333	18	28	
4,3	aa	1938	1908	10	16	15
	ak	1473	1515	10	15	14
	er	1473	1441	10	17	14
	ss	1476	1495	10	19	14
	sf	1476	1485	10	26	15
	theor.	1472		10	14	

crease of damage in consecutive time steps in synchronous mode of calculation are very similar to the ones shown in fig.1.3. (A small difference at the point of maximum is shown in table 1). However this similarity conceals some statistical diversity of speed of damage spreading process. This diversity appears when we try to compare obtained results especially for networks sf and ss (e.g. fig.1.4) to fig.1.2. In fig.1.4 a case for ss 2,3 is presented and a similar case sf 4,2; they are both less rough than sf 2,3 and sf 3,2. We can identify a few independent processes with different speeds of damage growth. This diagram is plotted for each particular case of damage initiation replacing the old one. When damage reaches the level of equilibrium it stops (in our simplified algorithm) and there is no data for later time steps. In this later area, therefore, we can observe other processes which are earlier and slower. For network types scale-free and single-scale this speed is strongly connected with the time of reaching the hubs by damage. For network types er and obviously ak and aa there are no hubs and the obtained curves are more uniform and similar to the theoretical ones shown in fig.1.2.

Fig.4.4-7 (right column) shows the distribution of time of damage fadeout in both real and 'pseudo' cases. There are two peaks in this distribution: one for real fadeout in the first steps (early fadeout) and the second one for 'pseudo-fadeout' when damage reaches the equilibrium level at the last time steps. For the network cases with wide range of node degrees like sf and ss with a large fraction of $k = 1$ nodes the probability of early fadeout is much greater especially for small $s = 2$. If $K = 3$ then 60% nodes for sf and 33% for ss have $k = 1$ but there are 11% and 20% nodes of $k > 4$ which have 55% and 46% outgoing links. If $K = 2$ then respectively for sf and ss networks, 67% nodes and 50% nodes have $k = 1$, there are 7% and 6% nodes of $k > 4$ which have 34% (sf) and 19% (ss)outgoing links . For $s = 2$ nodes with $k = 1$ have $w = 1/2$ and early fadeout is more probable than for $s = 4$ where $w = 3/4$. Hubs are present in this case. The biggest hub ($k = 955$) appears in sf when $K = 4$, for $K = 3$ it reaches $k = 520$. This single hub takes 12% (the second 9%) of all the outgoing links. Hubs decrease the average k and in effect also the average w for remaining nodes, which helps damage to fade out before the first hub is achieved. For er network even $k = 0$ occurred but nodes with $k < 2$ constitute less than 1/4 of all the nodes. If s is small, e.g. $s = 2$, then the coefficient w is locally especially low. Note that we have used local coefficient w for the explanation. On the opposite end (only of Kauffman mode) the case of ak 4,3 lies where $k < 2$ and hubs are absent and the coefficient w of damage propagation is high and equal for all the nodes. In such a case the early fadeout is very small and most of the damage grows until the equilibrium level is reached.

The right column in fig.4 consist of four different distributions. They are all plotted for networks of $N = 2000$ nodes, but this sequence looks like a sequence of distributions of stage of e.g. ak 4,3 growth. (We investigate this process in another paper in much more detail. It can be a base for the definition of complexity threshold.) This means that sf 2,3 in left (fig.4.4) looks like small, not yet mature networks ak or er. In an sf network 2,3 in the most cases (80%) the damage fades out (real fade out) without reaching hubs. Such damage

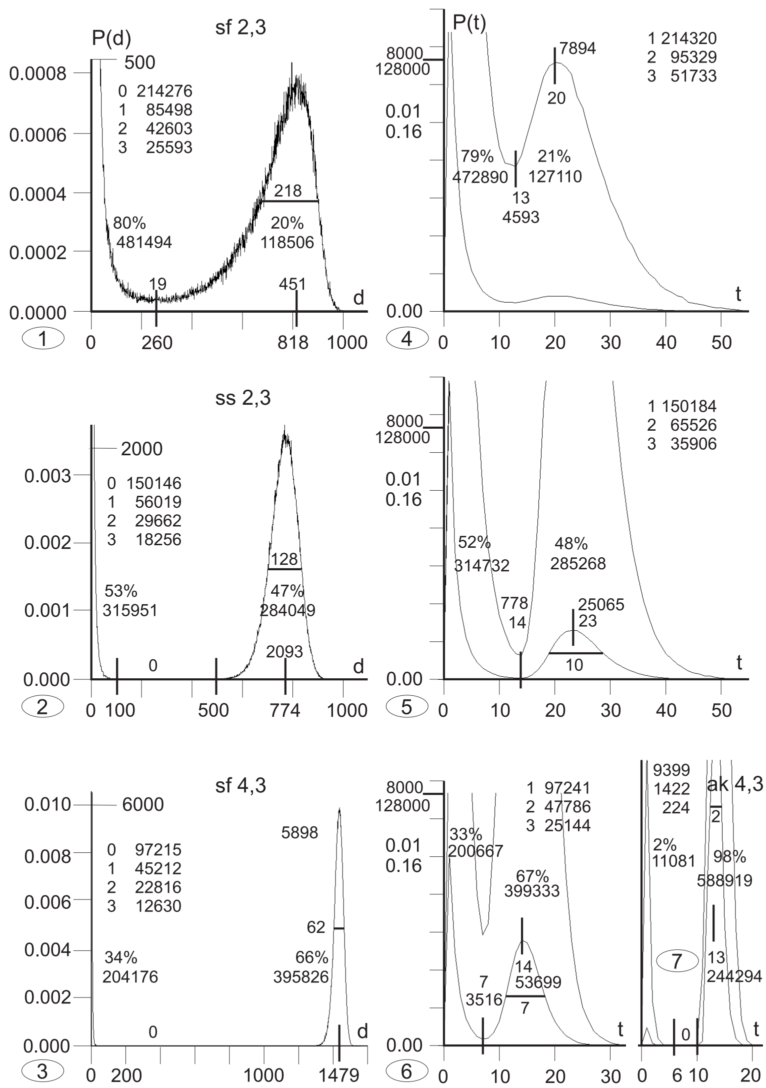

Figure 4: Distribution of damage size (left column) and time (right column) when damage really or 'pseudo' (via stabilization at the equilibrium level) fades out. In the lowest row the typical chaotic form of this distribution is shown, but the higher rows, for sf 2.3 (minimum of chaos) and ss 2,3 networks are not so chaotic. All networks contain 2000 nodes. All distributions are obtained from 600000 events of damage initiation. The positions and values of minima between peaks and the right maximum are shown. The width of the right peaks at half of their height is also shown.. For the left peaks a few of the first values are shown. The number of events in both peaks and the percent of all the 600000 events in each peak are shown - this important information is hard to estimate only from the shown figures. E.g. in (3) a left peak exists, it contains 34% of the events but it is hard to see.

behaviour is more typical of 'ordered' networks rather than 'chaotic' ones. For ss 2,3 and sf 4,2 both the peaks have a similar area (fig.4.5) but the left peak is much higher. Later, (fig.4.6) for sf 4,3 and ak 2,3 the left peak has a smaller area but is still higher and the right peak which describes chaotic behaviour contains most of the cases.

The phenomenon of different speed of damage spreading described above is the reason behind the large width of these peaks and lack of sharp boundary between them here (fig.4.4-6). This remark suggests that the variable t - time of damage fadeout is not the best choice. However, the variable t is interesting in practice and therefore often used [22]. A similar distribution of damage fadeout in the variable: damage size d, shown in fig.4.1-3 (left column) appears much better suited for the description and understanding of the underlying mechanisms. Using such a variable we also obtain the same two peaks but this time they are very narrow and a big segment of exact zero frequency lies between them typically. The only exception from this rule is the extreme case of sf 2,3 (fig.4.1) but we have discussed the causes of this exception above. The case of sf 3,2 and ss 2,3 follows the rule (fig.4.2.) but the second peak is not very narrow and we can find some single cases between the peaks. (These cases have no interpretation in our algorithm.) All the remaining cases are similar to the last sf 4,3 shown in fig.4.3, small differences concern proportion of both peaks and the peaks' width.

Position of maximum of the second peak is exactly equal to the theoretical point of equilibrium of damage size (dmx), it is obviously much more exact than the values of maximum which can be read from the distribution like in fig.1.4 which are for one particular process. The comparison of this 3 values for different cases is shown in table 1.

As it was discussed above, the new network types, especially the scale-free networks, due to concentration of a great part of links in a few hubs, exhibit significant differences in behaviour of damage spreading. These differences appear especially near the boundary of chaos and order and are more intensive for $s = 2$. To summarize these differences we show fig.5.1 where we compare average (for all initiations) damage size d for each simulated case of network type and s, K. This average allows to distinguish between real and pseudo fadeout because d has different interpretation for each of them and these cases should not be mixed. We used $d = 0$ for real fadeout cases which are separated using threshold on $d = 250$. However this correction in the biggest case for special sf 23 is still relatively small, it is 3.58% only. For sf 2,4 and 3,2 it is 1.1% and for all other cases it is less than 0.5% and cannot be visible in the figure. Depicted data have 3 decimal digits of precision, therefore the shown differences are not statistical fluctuations. As it can be seen, using higher $s = 4$ for $K = 2$ causes different behaviour of damage spreading than for $s = 2$ and $K = 3$, especially for er network type, despite the same value of coefficient $w = 1.5$, therefore these both parameters cannot substitute each other, i.e. we cannot limit ourselves to one of them or to the coefficient w.

Fig.5.1 contains two different causes which differ results. One of them was already described at the end of ch.3.1. It is the different level of damage equi-

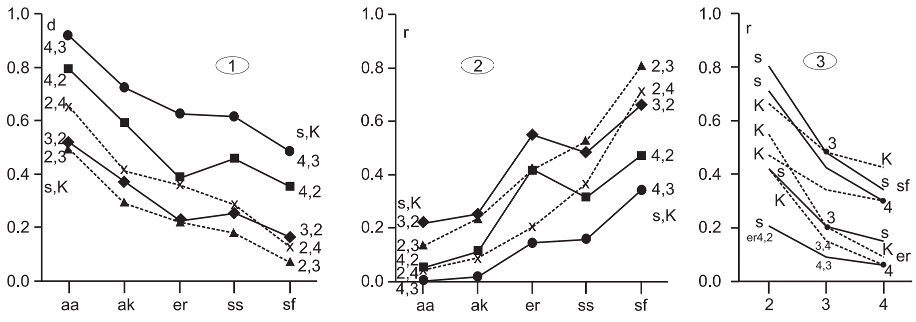

Figure 5: Average (for all initiations) damage size d at the process end (1) and real (early) fadeout as part r of all initiated processes (2) for five different network types and small values of parameters s and K. Comparison of slopes of dependencies r on s and K for sf and er networks (3). The points have 3 decimal digits of precision. Note, d considers also early (real) fadeout (for which $d = 0$ is taken), not only the equilibrium level for large damage, which for er, ss and sf is probably equal to the value for ak as fig.1 expects. Cases of parameters s and K are selected for easy comparison of dependency on them.

librium visible in the theoretically obtained (ch.3.2) fig.1. If we remove this aspect from fig.5.1, then what remains is the second cause which is connected to the early (real) fadeout. This cause depends mainly on the network type and the s parameter which is depicted in fig.5.2. This figure shows how big is the part of initiations which end in real fadeout. Compare the point for sf 2,3 to the fig.4.1. This aspect contains the mechanism of er distinctness which results from the events $k = 0$.

These investigations using simulations of different network types were designed and included in this paper to show that the parameter s is important and we cannot limit ourselves to the parameter K only. The dependency on s is about as strong as the dependency on K but it also differs from dependency on K for different network types. In the aggregate of automata the state of a node has s^K variants and this network type has obviously stronger (and different) dependency on these parameters than Kauffman networks. The ss and ak networks exhibit symmetrical dependency in s and K but for the most interesting sf and er network types there is no symmetry, which is depicted in the fig.5.3. For sf the dependency on s is stronger but for er - weaker than the dependency on K. For each network type two sets of two 'lines' are shown. Each 'line' consists of three cases for the same value of parameter s or K and all three values (2, 3, 4) of the second parameter. E.g. we compare the line consisting of (s,K) cases 3,2 3,3 3,4 and the line of 2,3 3,3 4,3 cases. These two lines have one common case 3,3 indicated in the figure. We are interested in the slopes of both compared lines. The relative slopes appear to be approximately constant for both parts of each lines. For both network types shown the slopes for parameters s and K are significantly different which already for two steps leads to significantly different values. These differences are not big but may be important.

The significantly lower damage size for sf network which can be seen in fig. 5 is known [13, 9] as the higher tolerance of a scale-free network of attack.

5 Conclusion

The first conclusion of this paper is that the parameter s (number of equally probable variants of signal) is an important one and cannot be neglected or substituted by the parameter K (number of node inputs) and others like P (internal homogeneity) and p (the probability of one of two signals), in the investigation of damage spreading behaviour, especially for scale-free networks. Next we collect arguments of various types that especially for adaptive systems describing real living or human-designed systems, the s parameter should generally be greater than two which placed these systems in the chaotic area. (This conclusion, however, has one famous exception - the gene regulatory network.) Such an assumption expands the notion of 'Kauffman network' which up till this proposition was the synonym of 'Boolean network'. We believe that the typically observed case of two alternatives with different probability has typically bases whose description using p is incorrect and s should be used instead.

Along the way of investigation of properties of the parameter s using simulation, we have found that the contemporary first BA scale-free network has significantly different behaviour in damage spreading than the Erdős-Rényi random network which was used in Kauffman's path and these differences are largest for $s = 2$. Generally, the networks with higher frequency of $k < 2$ nodes (k is node degree - the number of node outputs, for autonomous systems $K = k$ in the average) have higher chances of damage fade out in the critical beginning period. If hubs are present then this chance also increases because they decrease the average k for remaining nodes which helps the damage to fade out before the first hub is reached.

For this simulation we design and describe here a special simplified algorithm which we also use for the next investigation, of 'complexity threshold' and 'structural tendencies'. This algorithm uses only calculation of damage spreading up to reaching the equilibrium level instead of calculation and comparison of two systems - damaged and undisturbed one as in the classic method.

The coefficient of damage propagation, which we introduce in this paper, connects two main parameters K and s and describes the first, critical period of damage spreading (in the first theoretical approximation). It is simple and intuitive and it easily shows when damage should explode: damage should explode always if $k > 2$ or $s > 2$. The ability to produce an explosion of damage is one of the definitions of a chaos which we used following Kauffman for chaotic systems. The above mentioned influence of the hubs suggests that such a coefficient should be considered more locally, e.g. in the area of damage spreading.

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