

Multifractal Continuous-Time Random Walk on financial markets

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Wydział Fizyki, Politechnika Warszawska**

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Zespoły badawcze

Wydział Fizyki, UW:
Andrzej Kasprzak - doktorant
RK

Dept. de Fisica Fonamental, Univ. Barcelona:
Josep Perelló
Jaume Masoliver

Plan seminarium

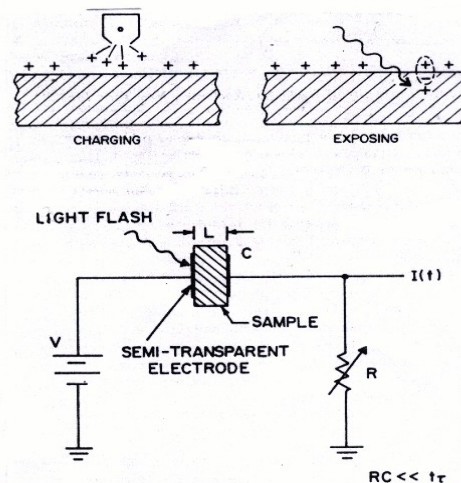
1. Sukcesy tradycyjnego modelu
'Continuous-Time Random Walk' (CTRW)
2. 'Multifractal Continuous-Time Random Walk' (MCTRW)
3. Heurystyczne rozszerzenie modelu MCTRW
4. Od MCTRW do termodynamiki
5. Konkluzje i co dalej?

Transport dyspersyjny w materiałach amorficznych

EXPERIMENTAL BASIS OF ANOMALOUS TRANSPORT AND DIFFUSION IN CONDENSED MATTER

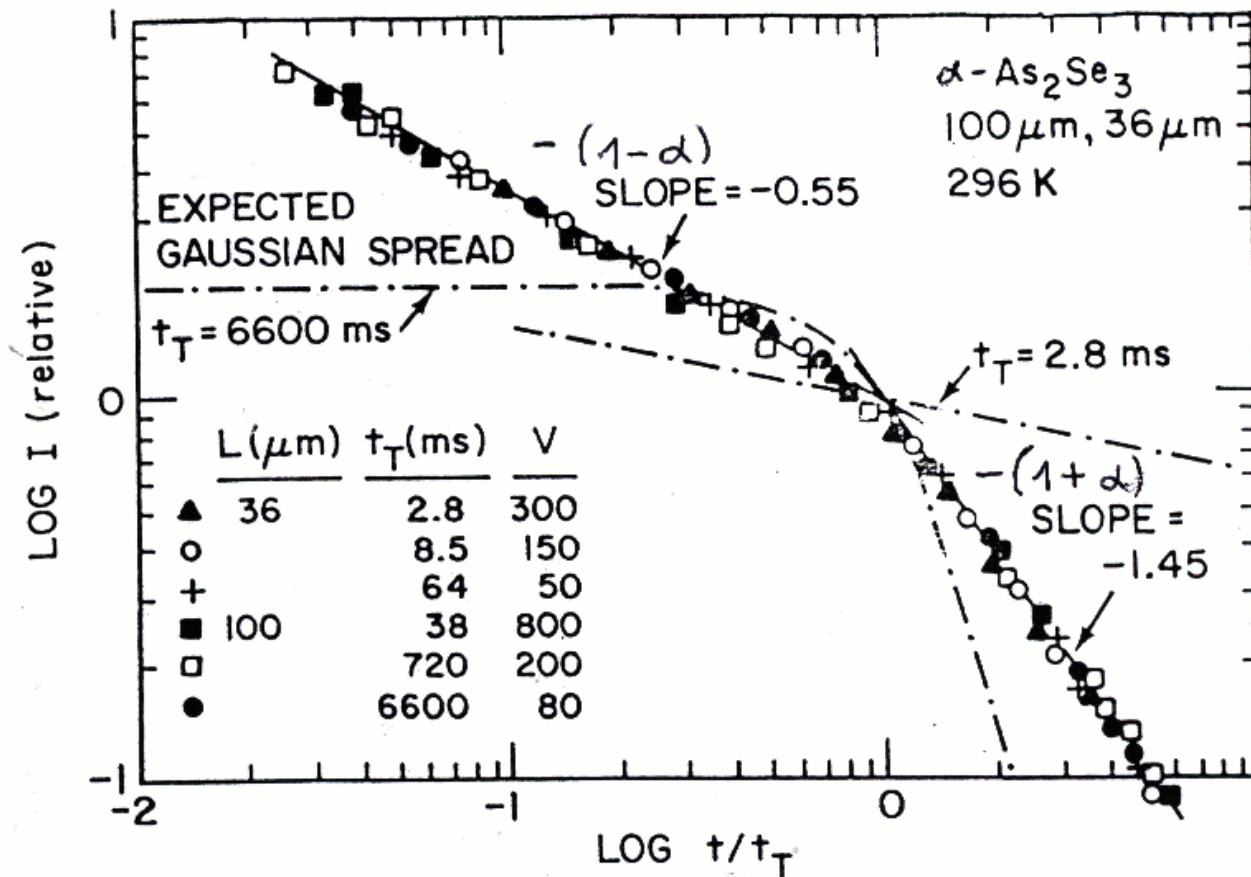
Scharfe: PR B (1970), Pfister: PRL (1974, 1976),
Pfister and Scher: PR B (1977)

**Experimental setup for xerographic copying process:
transient photoconductivity measurement**



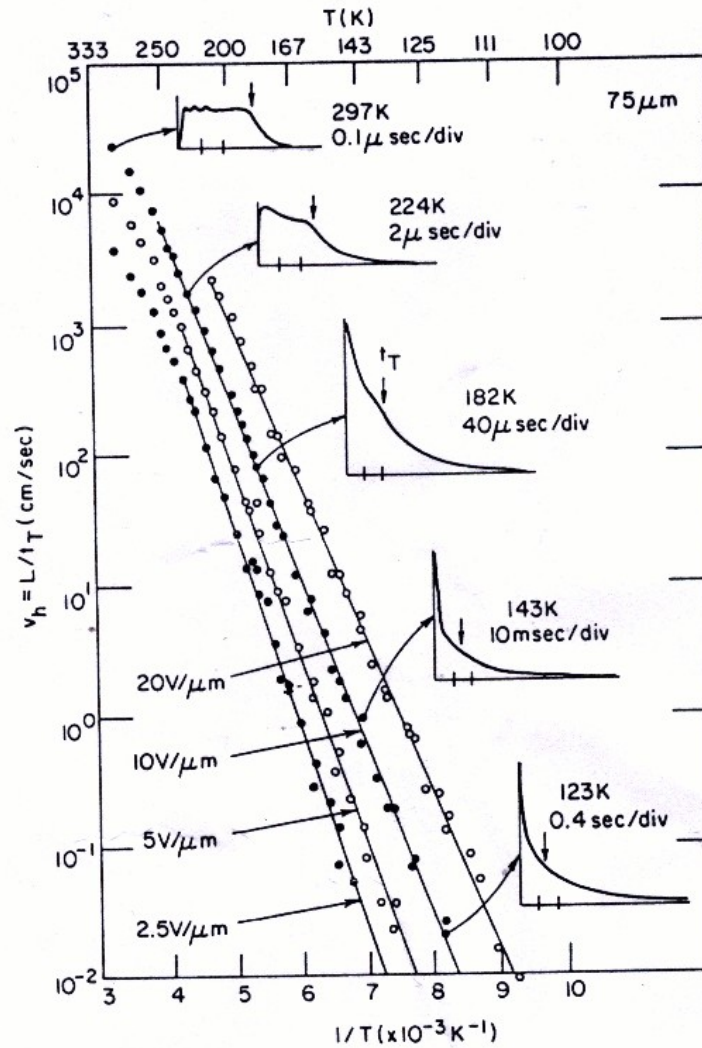
Measurement of time-dependent current $I(t)$
induced by holes moving across the sample

Universality of current shape for amorphous binary chalcogenide $\alpha\text{-As}_2\text{Se}_3$: transient current vs. time



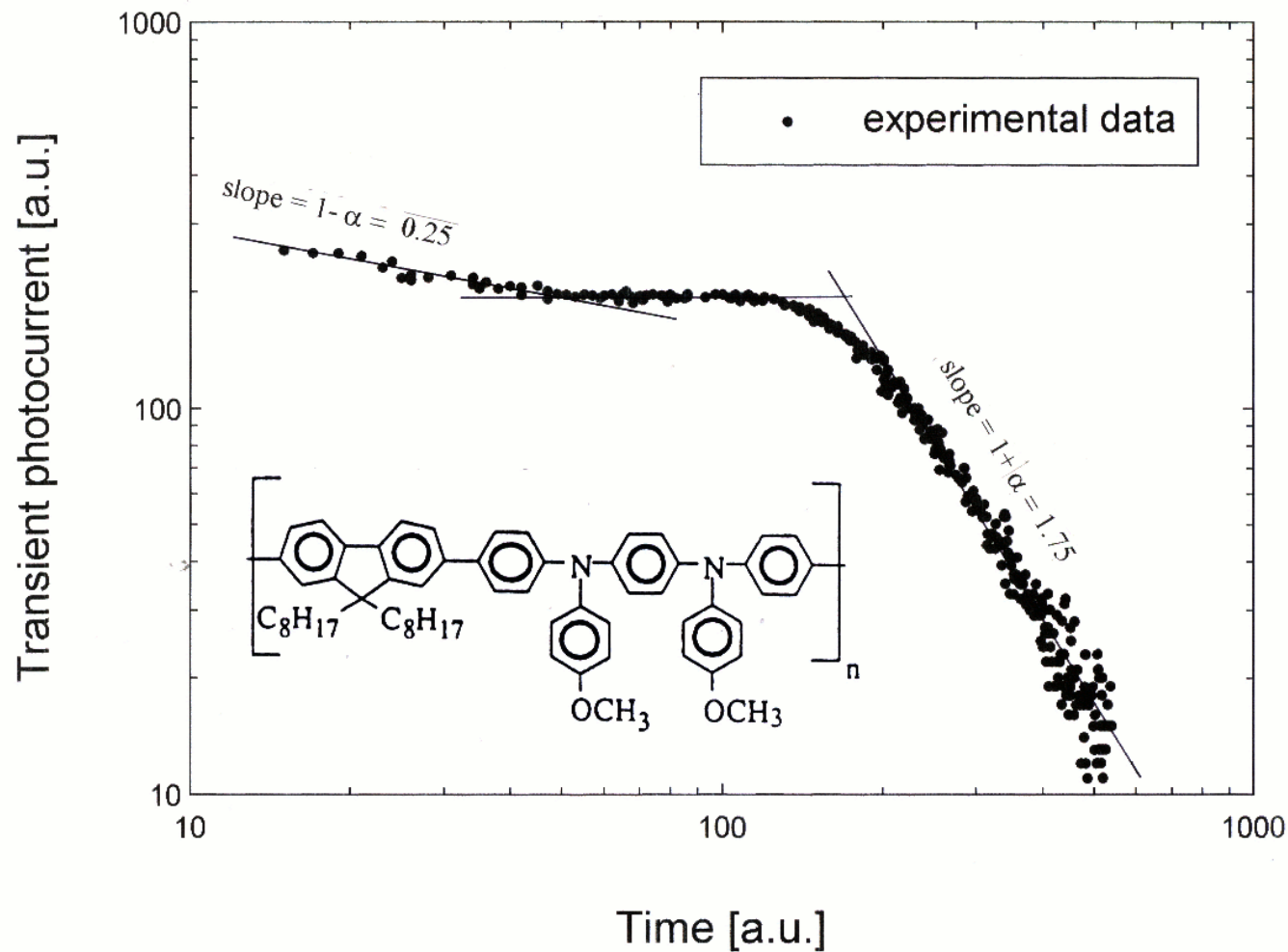
Log-log plots of (relative) hole transient current $I(t)/I(t_T)$ vs. (relative) time t/t_T

Hole drift velocity vs. inverse temperature for amorphous α -As₂Se₃



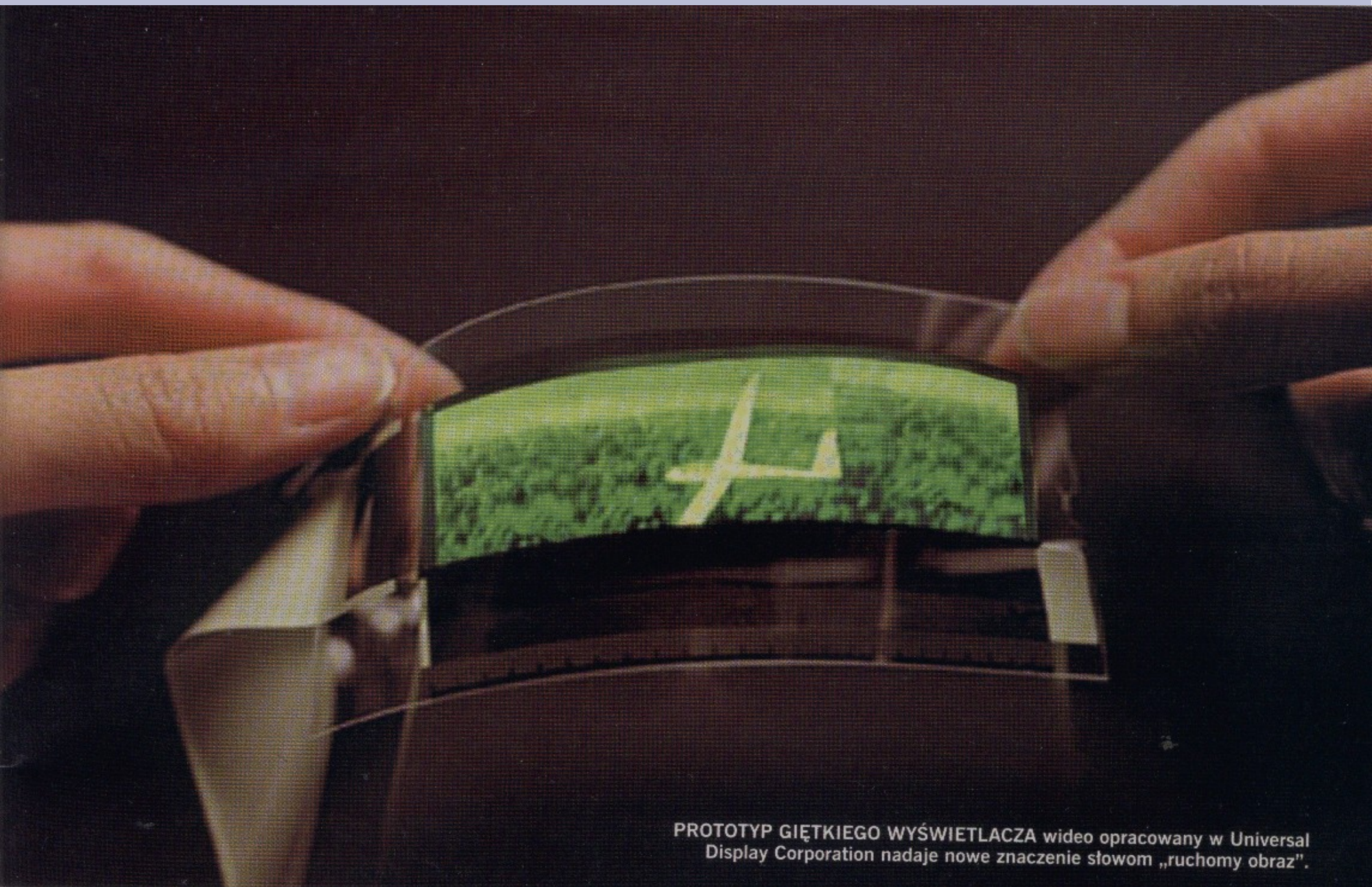
Semilog plot of hole drift velocity v_h vs. inverse temperature $1/T$

Photocurrent in OLED (Organic Light Emitted Diode)



Log-log plot of electron transient current $I(t)$ vs. time t

Giętkie wyświetlacze OLED



PROTOTYP GIĘTKIEGO WYŚWIETLACZA wideo opracowany w Universal Display Corporation nadaje nowe znaczenie słowom „ruchomy obraz”.

Założenia tradycyjnego modelu CTRW

1. Energetic landscape:

$$\rho(\varepsilon) \sim \exp(-\varepsilon / \langle \varepsilon \rangle)$$

2. Poisson, conditional Pausing-Time Distribution (PTD):

$$\psi(t|\varepsilon) \sim \exp(-t/\tau(\varepsilon))$$

3. Arrhenius law:

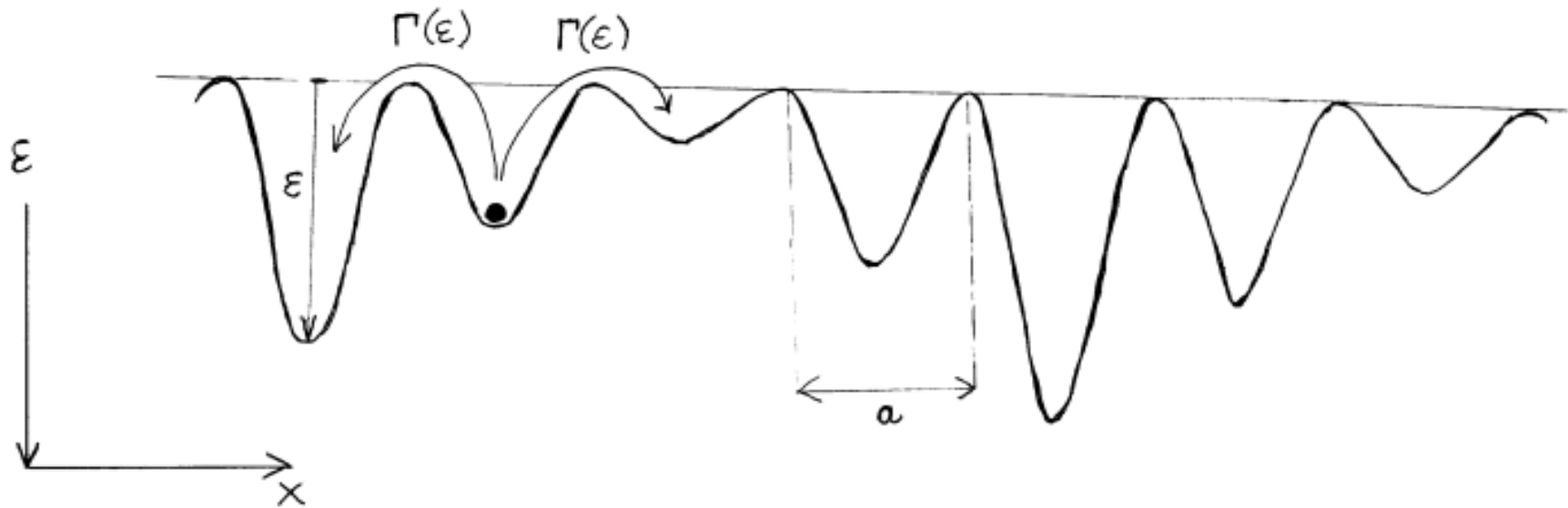
$$\Gamma(\varepsilon) \sim 1/\tau(\varepsilon) \sim \exp(-\varepsilon/kT)$$

Random-trap or random-valley model

M. Kozłowska, R.K., *Anomalous transport and diffusion versus extreme value theory*, Physica A 357 (2005) 282-304

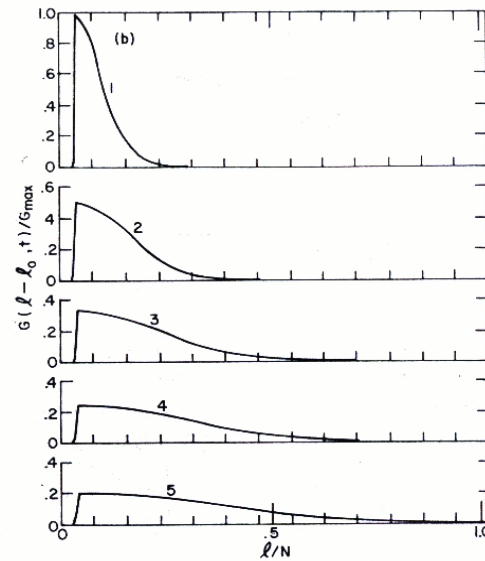
Random-trap or valley model

- $\Gamma(\varepsilon) = \Gamma_0 \cdot \exp(-\varepsilon/k_B T)$

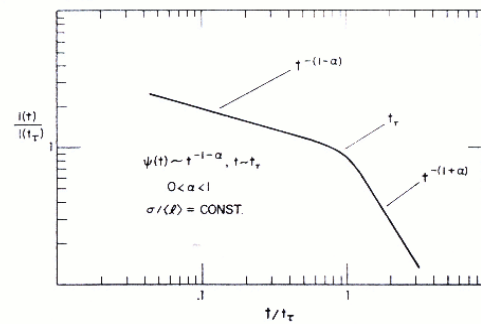


- $g(\varepsilon) = \frac{1}{\langle \varepsilon \rangle} \cdot \exp(-\varepsilon/\langle \varepsilon \rangle)$

Non-Gaussian, broad carrier propagator in an electric field and the corresponding current

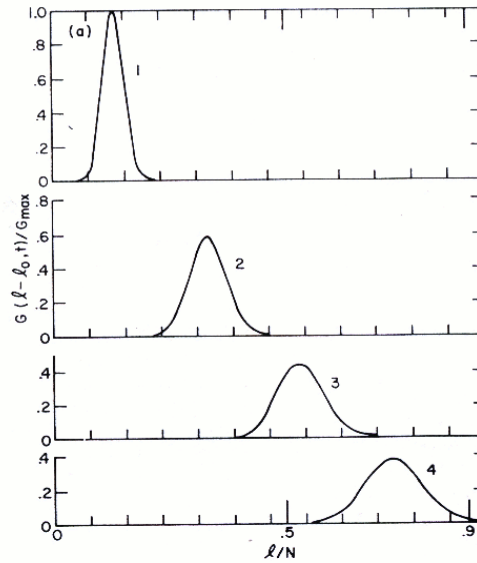


Relative propagator $G(I-I_0, t)/G_{\max}$ for the carrier package vs. relative distance l/N

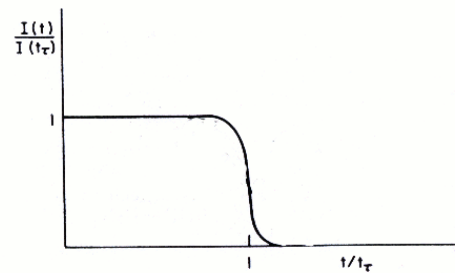


Plot of (relative) transient current $I(t)/I(t_r)$ vs. relative time t/t_r , for a propagating non-Gaussian, broad package

Gaussian carrier propagator in an electric field and the corresponding current



Relative propagator $G(l-l_0, t)/G_{\max}$ for the carrier package vs. relative distance l/N



Plot of (relative) transient current $I(t)/I(t_r)$ vs. relative time t/t_r for a propagating Gaussian package

Pełna PTD oraz prawdopodobieństwa skumulowane

Pełna PTD, dla $t \rightarrow \infty$:

$$\psi(t) = \int_0^{\infty} \psi(t|\varepsilon) \rho(\varepsilon) d\varepsilon \sim 1/t^{1+\alpha}, \quad \alpha = kT/\langle \varepsilon \rangle < 1$$

Prawdopodobieństwa skumulowane:

$$\Psi(t|\varepsilon) = \int_t^{\infty} \psi(t'|\varepsilon) dt'$$

$$\Psi(t) = \int_t^{\infty} \psi(t'|\varepsilon) \rho(\varepsilon) dt' = \int_t^{\infty} \psi(t') dt'$$

Momenty czasowe dowolnego rzędu

$$\langle t^q | \varepsilon \rangle = \int_0^\infty t^q \psi(t | \varepsilon) dt \sim \Gamma(1 + q) \exp\left(q \frac{\varepsilon}{kT}\right)$$

$$\langle t^q \rangle = \int_0^\infty \langle t^q | \varepsilon \rangle \rho(\varepsilon) d\varepsilon \sim \frac{1}{1 - \alpha/q}, \quad q < \alpha,$$

$$\langle t^q \rangle \sim \infty, \quad q \geq \alpha$$

**Pełen sukcesów kierunek badawczy
oparty na **wykładniczej postaci $\rho(\varepsilon)$**
obejmujący:**

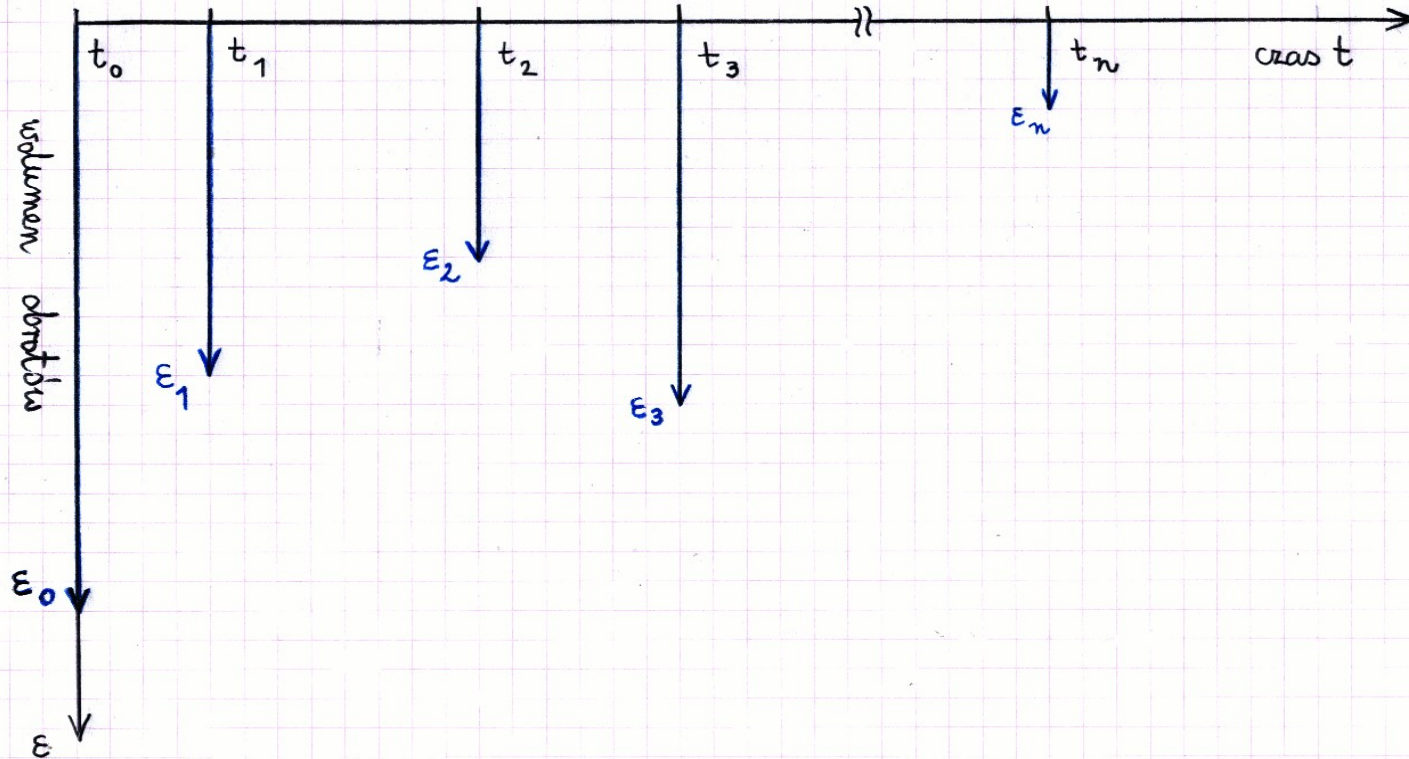
hierarchiczność, samopodobieństwo
i samopowinowactwo,
relacje skalowania, RG, singularność,
bezskałowość, DSI i log-periodyczność,
poszerzone rozkłady (grube ogony),
rzadkie i ekstremalne zdarzenia

...

'Multifractal Continuous-Time Random Walk' podejście oparte na 'stretched exponential' $\rho(\varepsilon)$

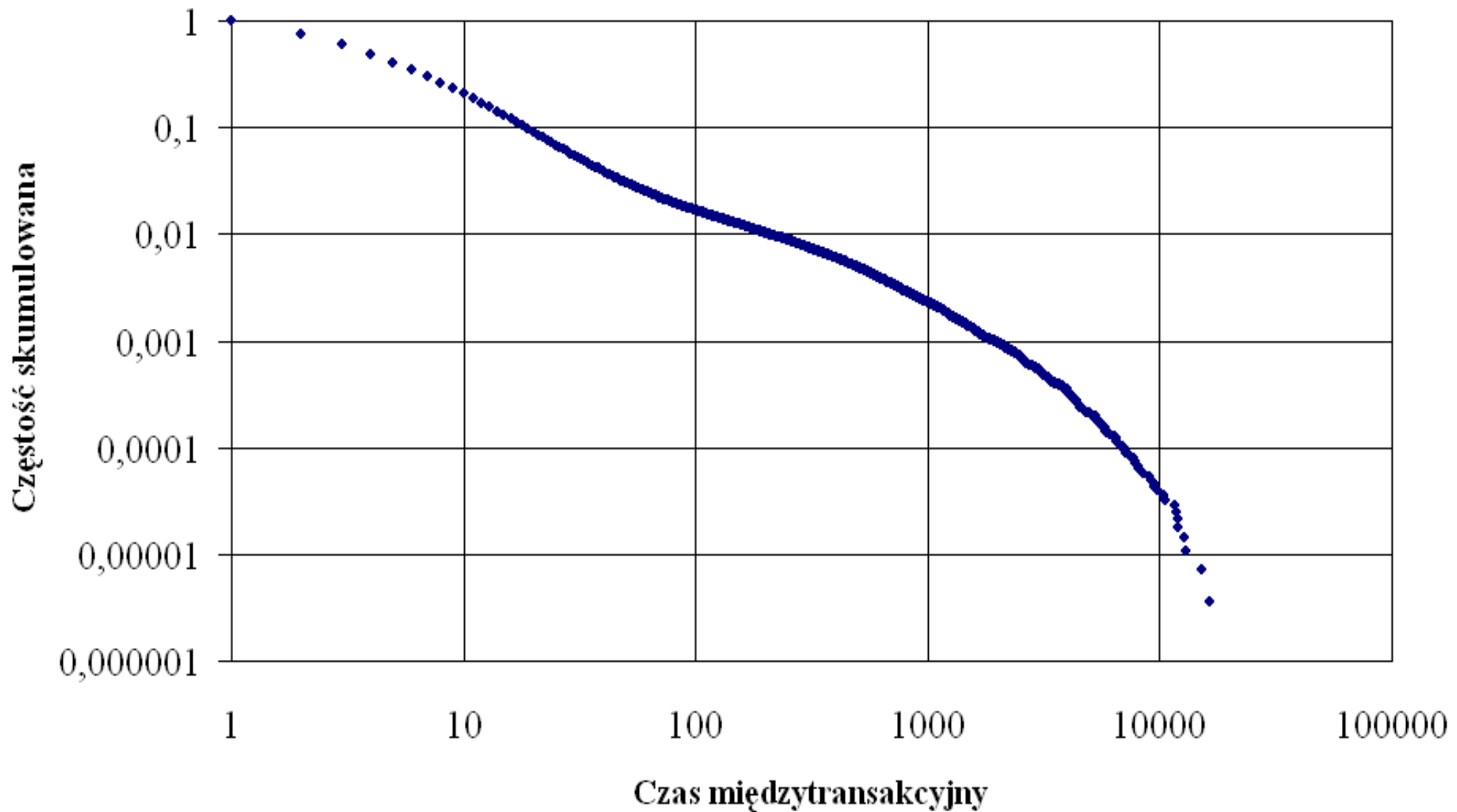
Izomorfizm pomiędzy 'random-trap (random-valley) model'
a strukturą obrotów kontraktami futures

Conditional PTD $\psi(t|\varepsilon)$: gęstość prawdopodobieństwa, że w przedziale czasu t nie ma żadnej transakcji, pod warunkiem iż na początku tego przedziału ich wolumen wyniósł ε .



Prawdopodobieństwo skumulowane: zagadka

F. Mainardi, M. Raberto, R. Gorenflo, E. Scalas, Physica A 287 (2000) 486



**MCTRW: formula based on
'stretched exponential' $\rho(\varepsilon)$**

$$\rho(\varepsilon) \sim \exp\left(-\frac{1}{2} \left(\frac{|\varepsilon - \mu|}{\sigma}\right)^\alpha\right), \alpha > 1$$

Moment rzędu q (using 'saddle-point approximation'(SPA)):

$$\frac{\langle t^q \rangle}{\Gamma(1+q)} \sim L^\lambda q + q^{\alpha/(\alpha-1)}, \alpha > 1, q \geq 0,$$

$$L = \exp(b), b = 2^{\frac{1}{\alpha-1}} (\alpha-1) \left(\frac{\sigma \beta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \lambda = \frac{\mu \beta}{b}$$

L jest naturalną skalą zjawisk ekonomicznych związanych z obrotem kontraktami futures

Heuristic extension of MCTRW

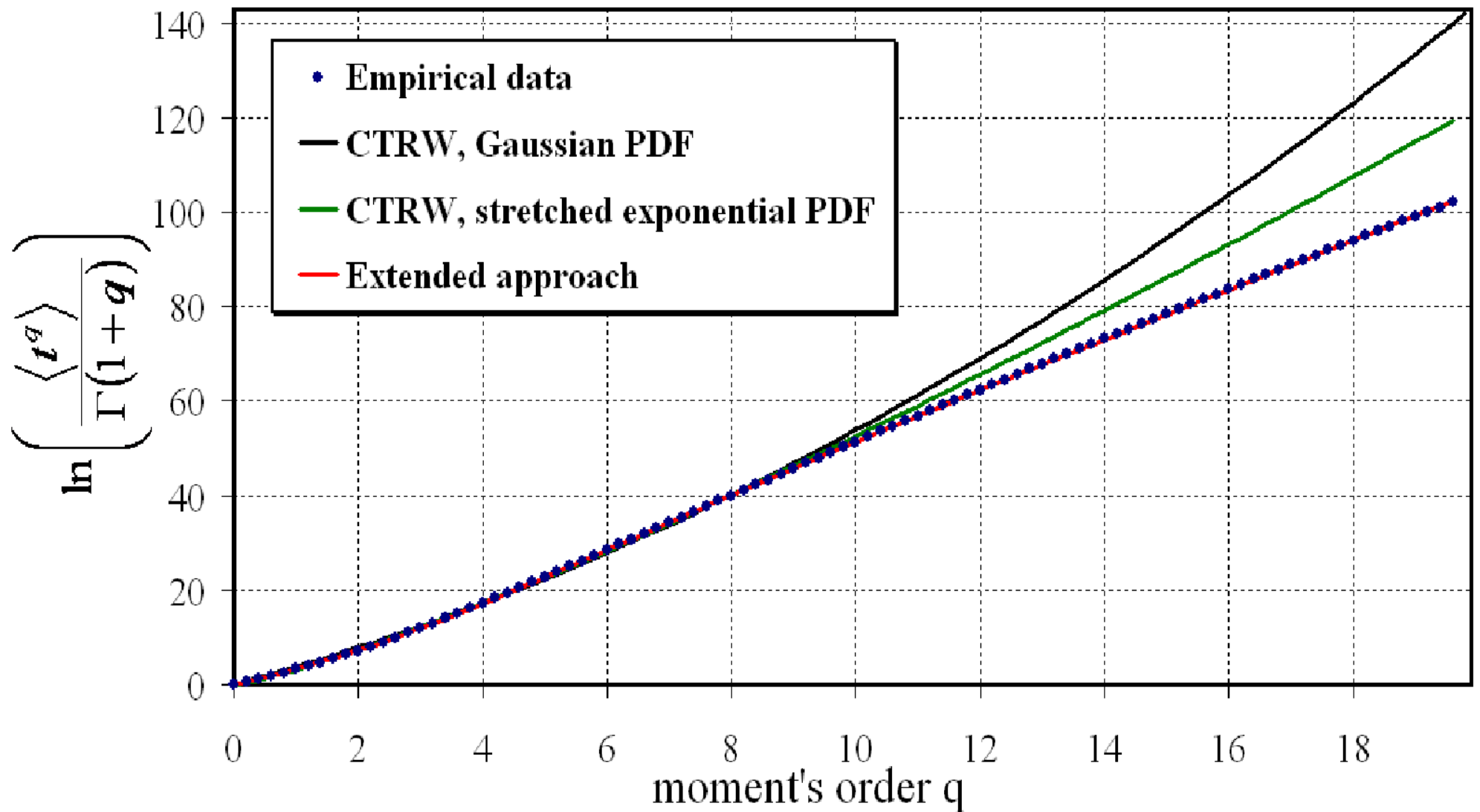
Two requirements:

- (i) for $q \rightarrow 0$ formulas of the MCTRW and its heuristic extension should give the same formulas,
- (ii) for $q \rightarrow \infty$ heuristic extension of the MCTRW should give an unifractal

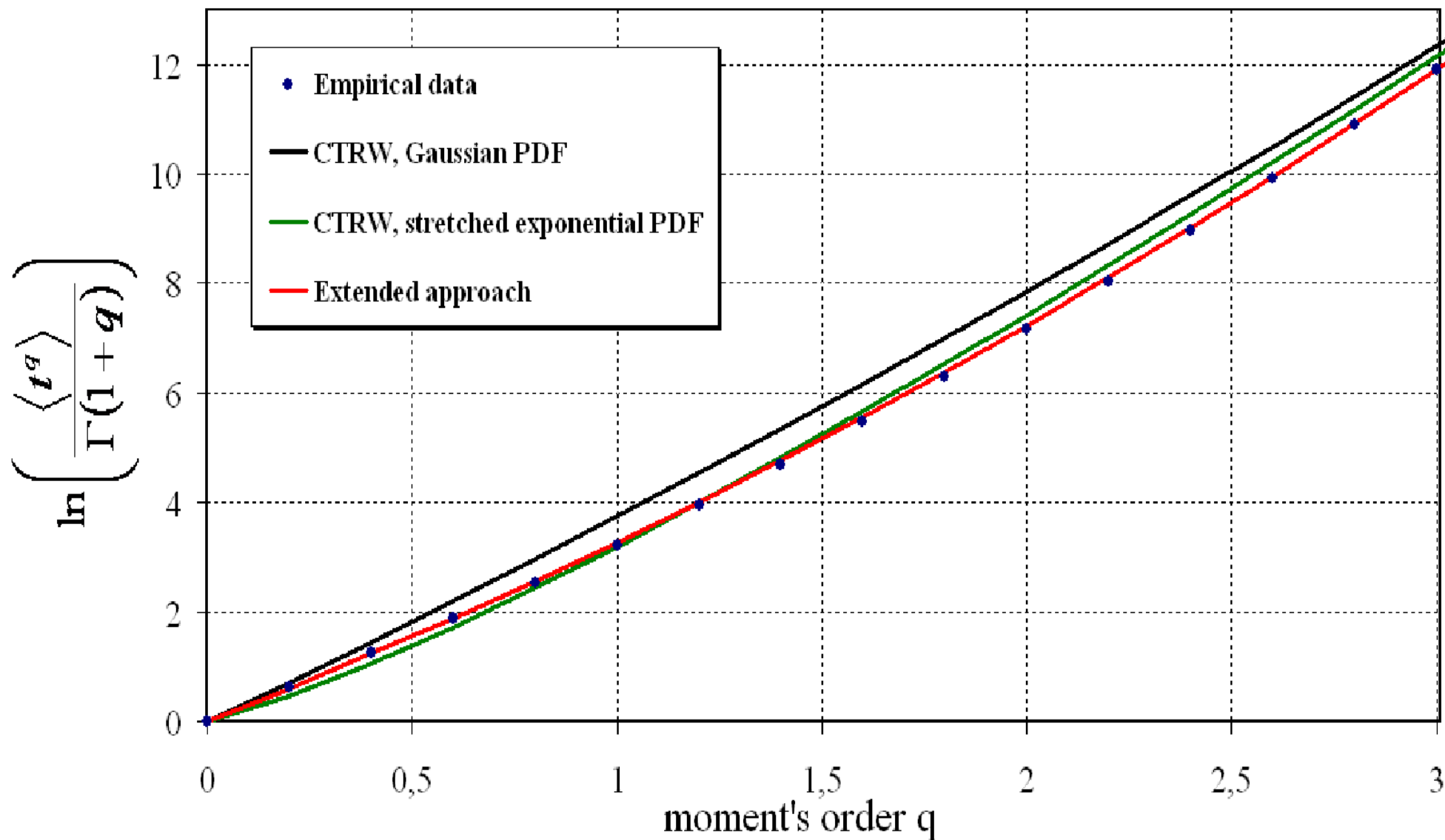
General multifractal formula

$$\frac{\langle t^q \rangle}{\Gamma(1+q)} \sim L^{\left(\frac{\mu\beta}{b} \operatorname{sgn}(q) + \frac{1}{b_1} [1 - \exp(-b_1 |q|^{\alpha/(\alpha-1)})] |q| \right)}$$

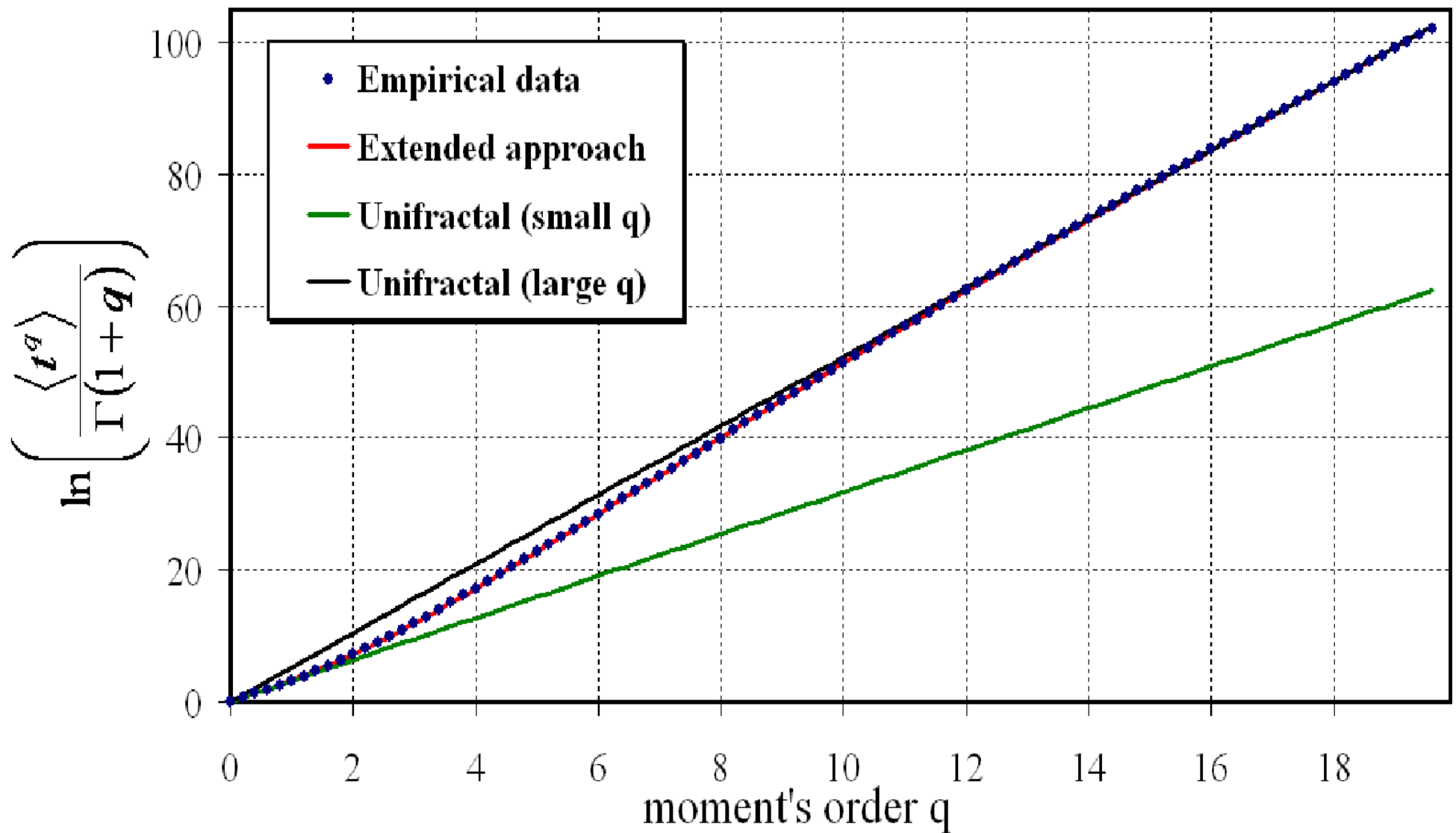
Dane empiryczne: kontrakty na kurs wymiany US/DEM (na Forex)



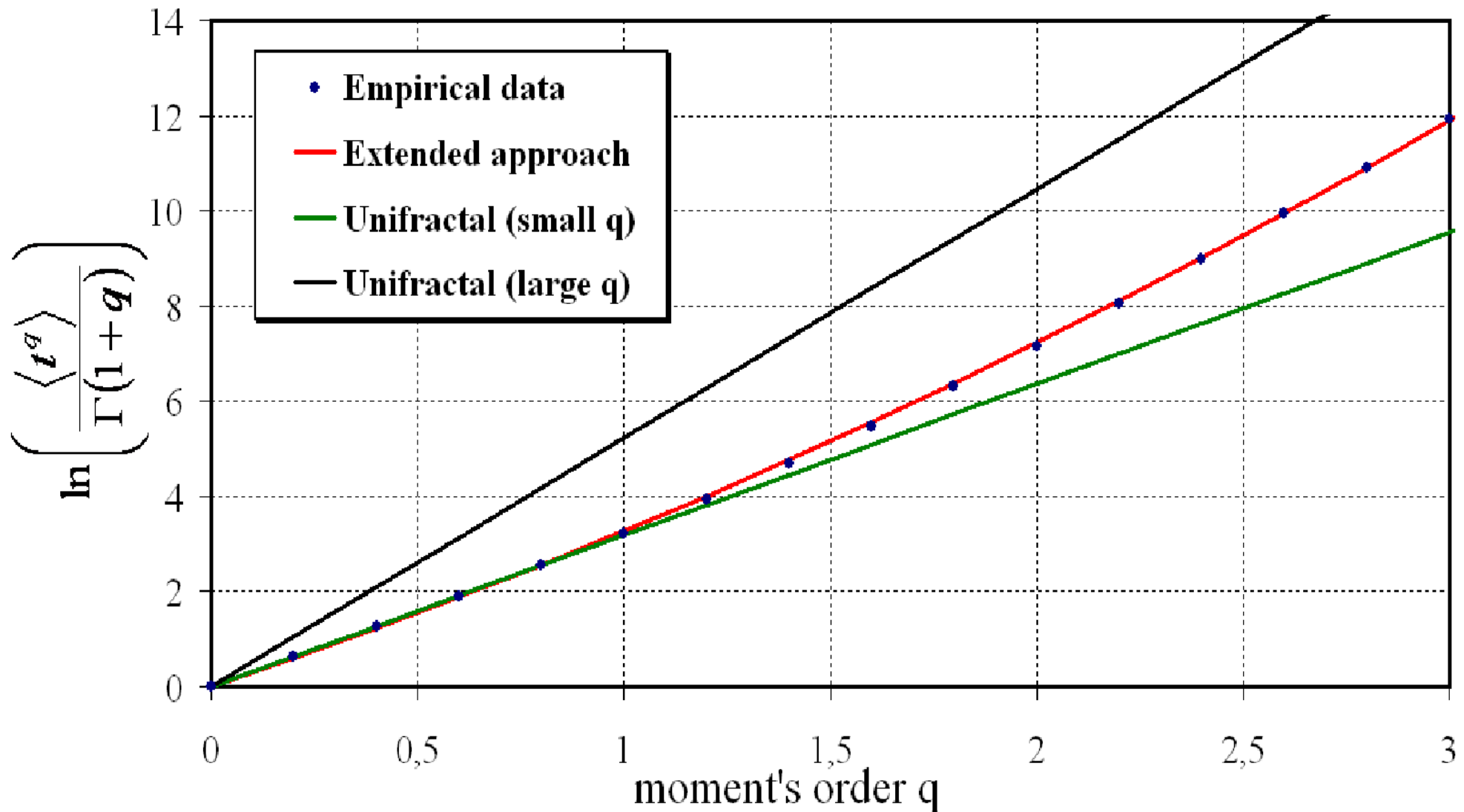
Dane empiryczne: kontrakty na kurs wymiany US/DEM (na Forex), powiększenie



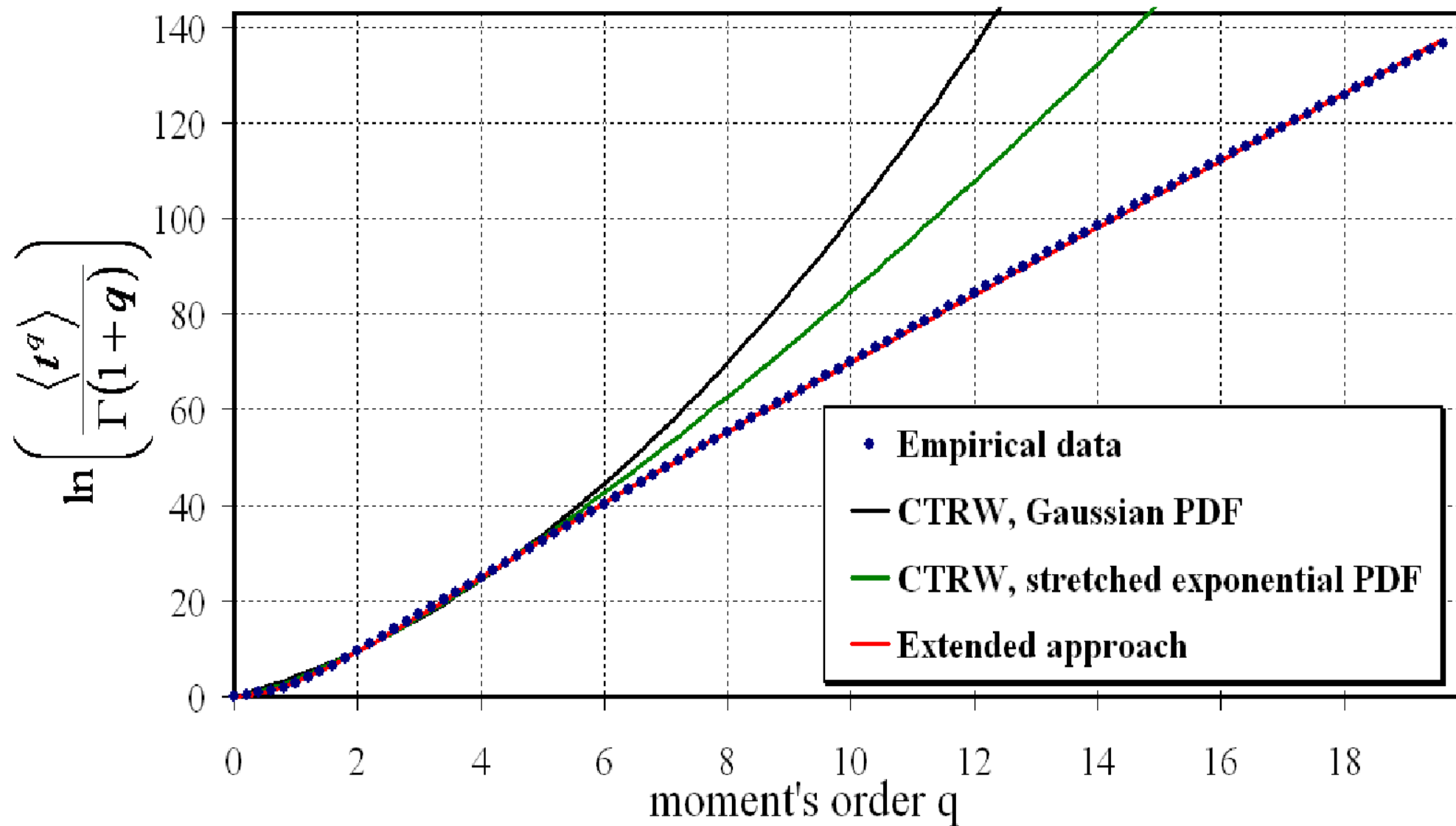
Dane empiryczne: kontrakty na kurs wymiany US/DEM (na Forex), unifraktale



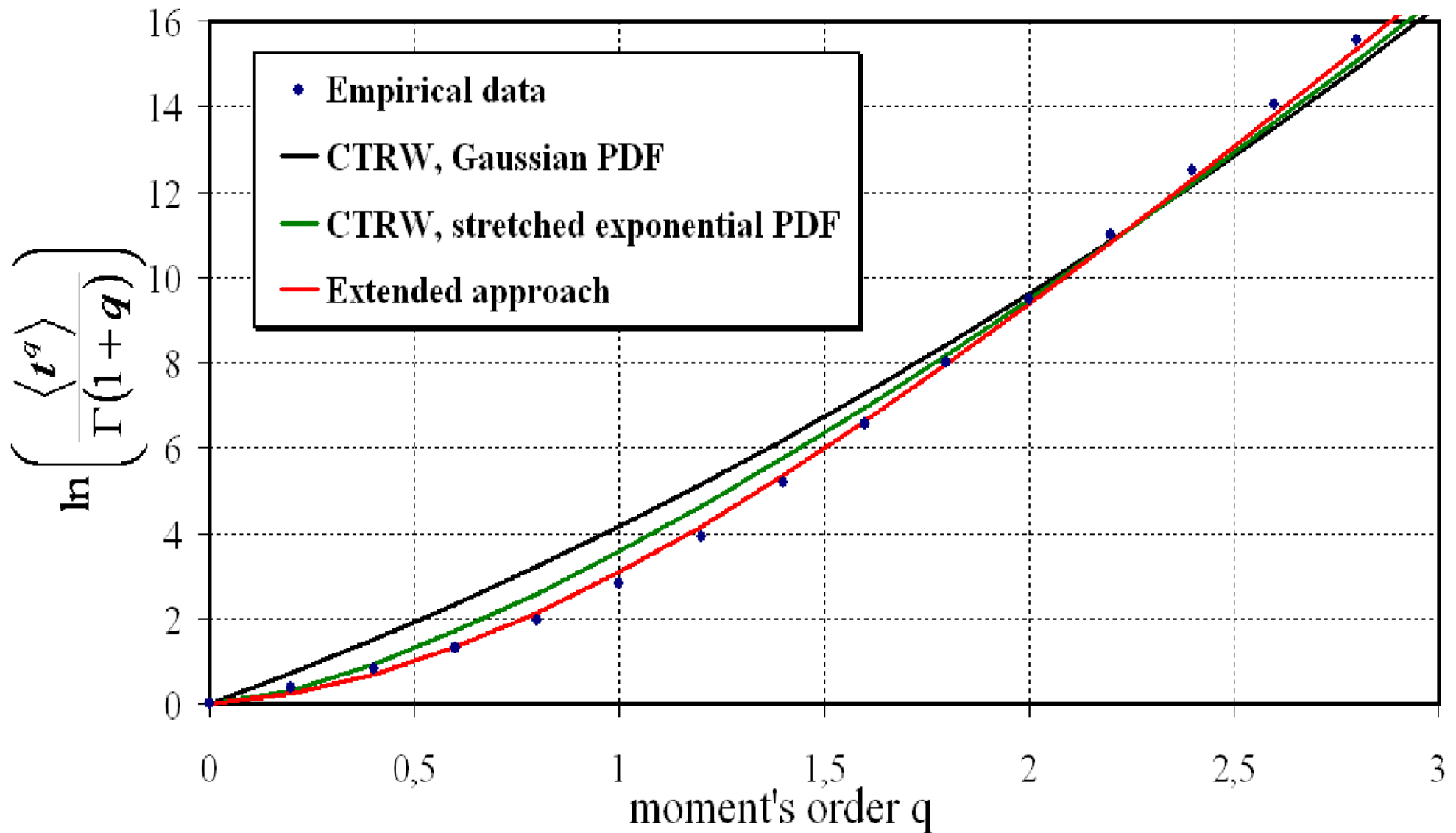
Dane empiryczne: kontrakty na kurs wymiany US/DEM (na Forex) powiększenie i unifraktale



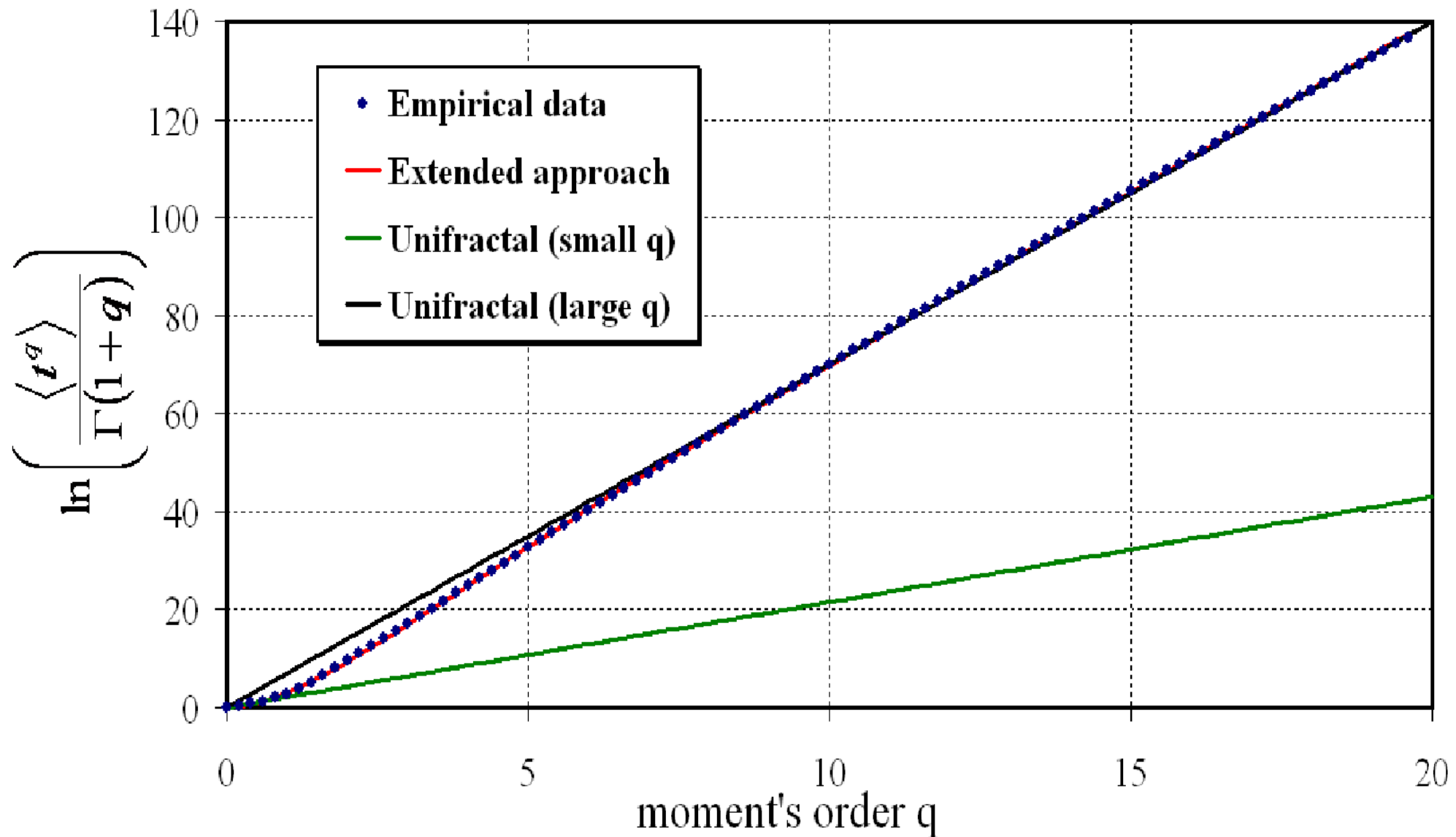
Dane empiryczne: kontrakty na WIG (GPW)



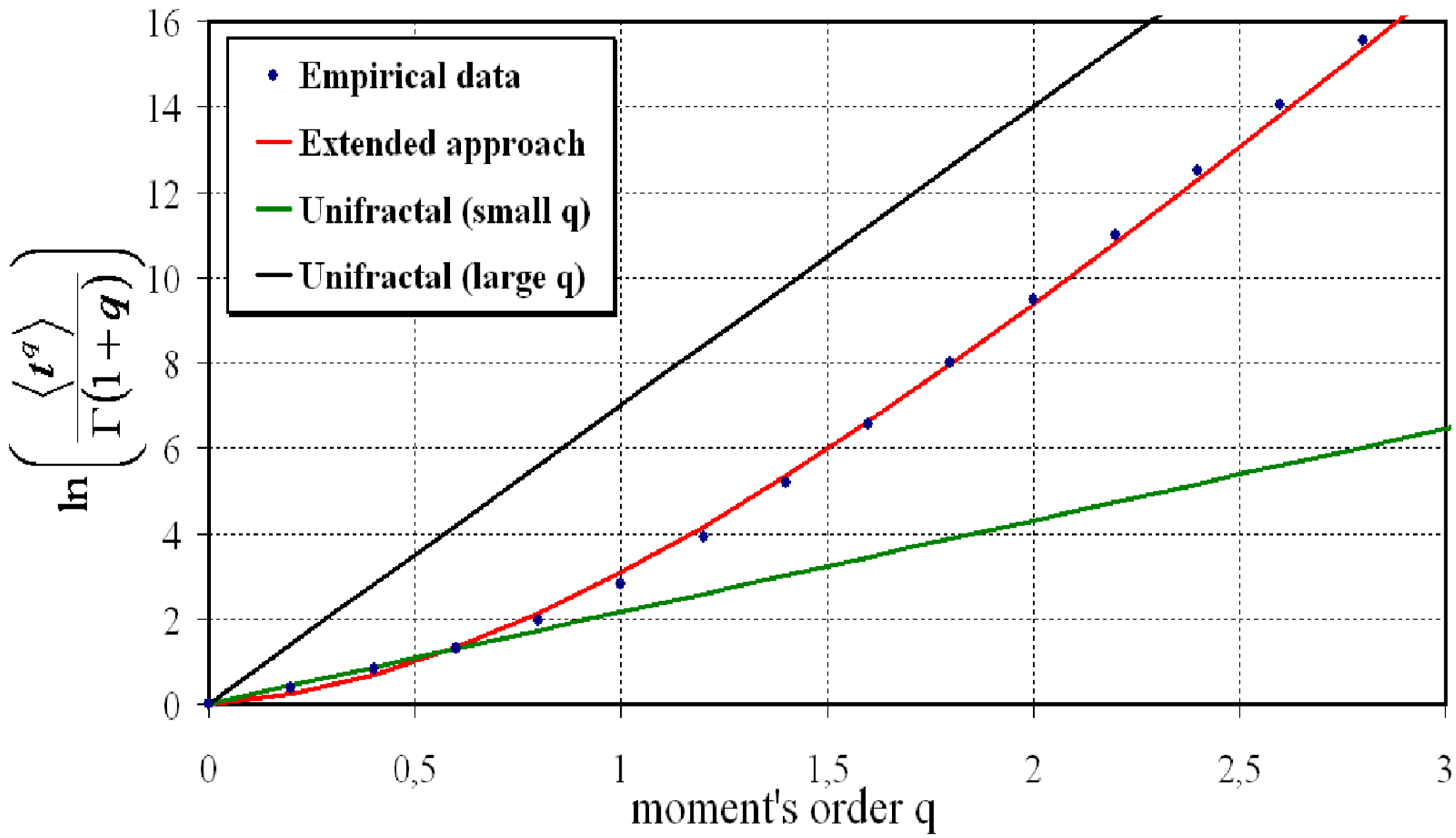
Dane empiryczne: kontrakty na WIG (GPW), powiększenie



Dane empiryczne: kontrakty na WIG (GPW) unifraktale



Dane empiryczne: kontrakty na WIG (GPW) powiększenie, unifraktale



Formal relation MCTRW - thermodynamics

Scaling form of partition function (within the 'SPA'):

$$Z(q) \sim \sum_i p_i^q \sim \int dt \int d\varepsilon [\psi(t|\varepsilon) \rho(\varepsilon)]^q \sim \langle t^Q \rangle \sim L^{-\tau(q)},$$
$$Q = \frac{1-q}{q^{1/\alpha}}$$

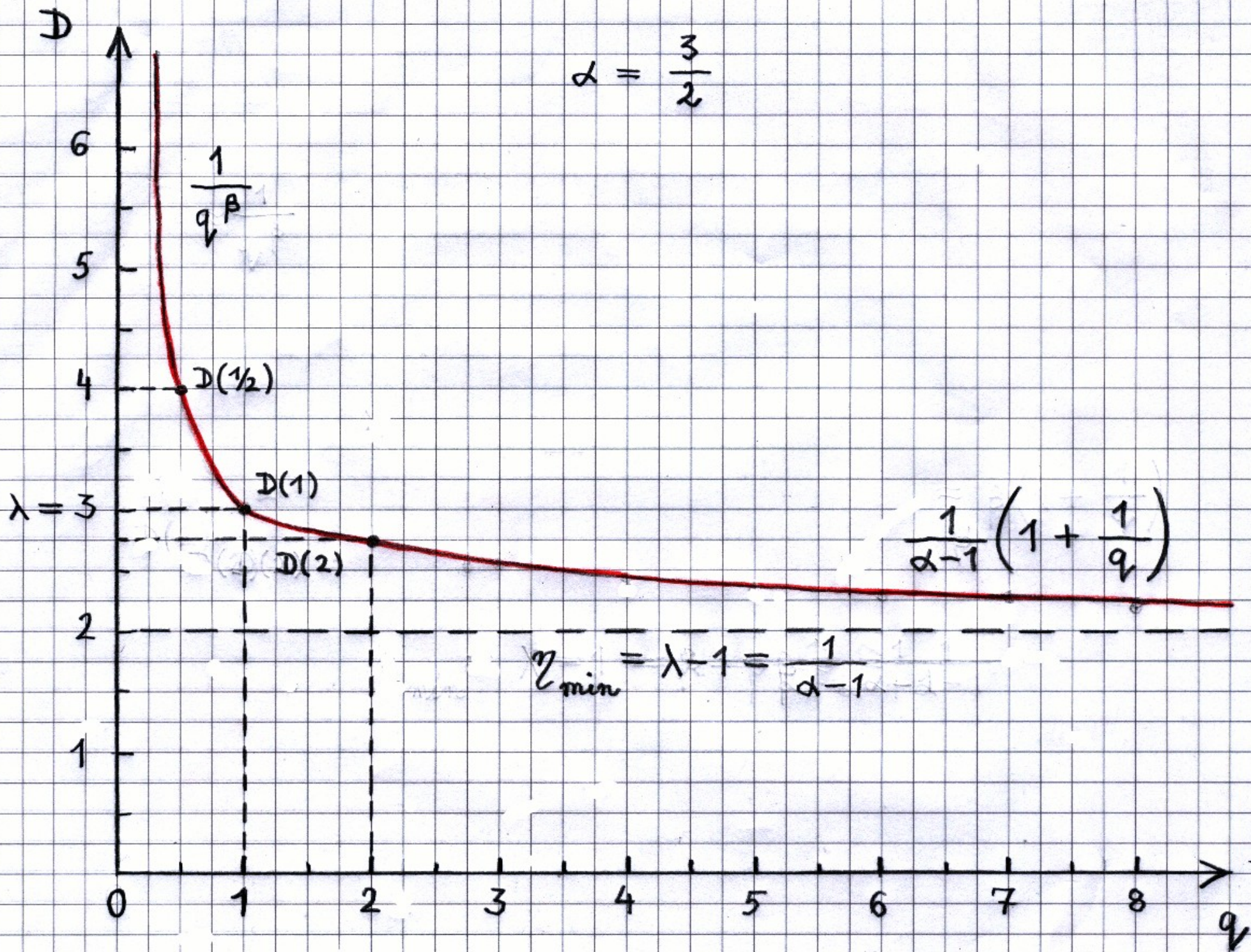
Scaling exponent $\tau(q)$

$$\tau(q) = (q-1) D(q),$$

Formal Rényi dimensions

$$D(q) = \lambda + \left(\frac{|1-q|}{q} \right)^\gamma \text{sgn}(1-q), \quad \gamma = \frac{1}{\alpha-1}$$

$$\alpha = \frac{3}{2}$$



Exists Legendre transformation

$$\tau(q) = q \eta(q) - f(\eta(q)), \quad \eta = \frac{d\tau}{dq}, \quad q = \frac{df}{d\eta}$$

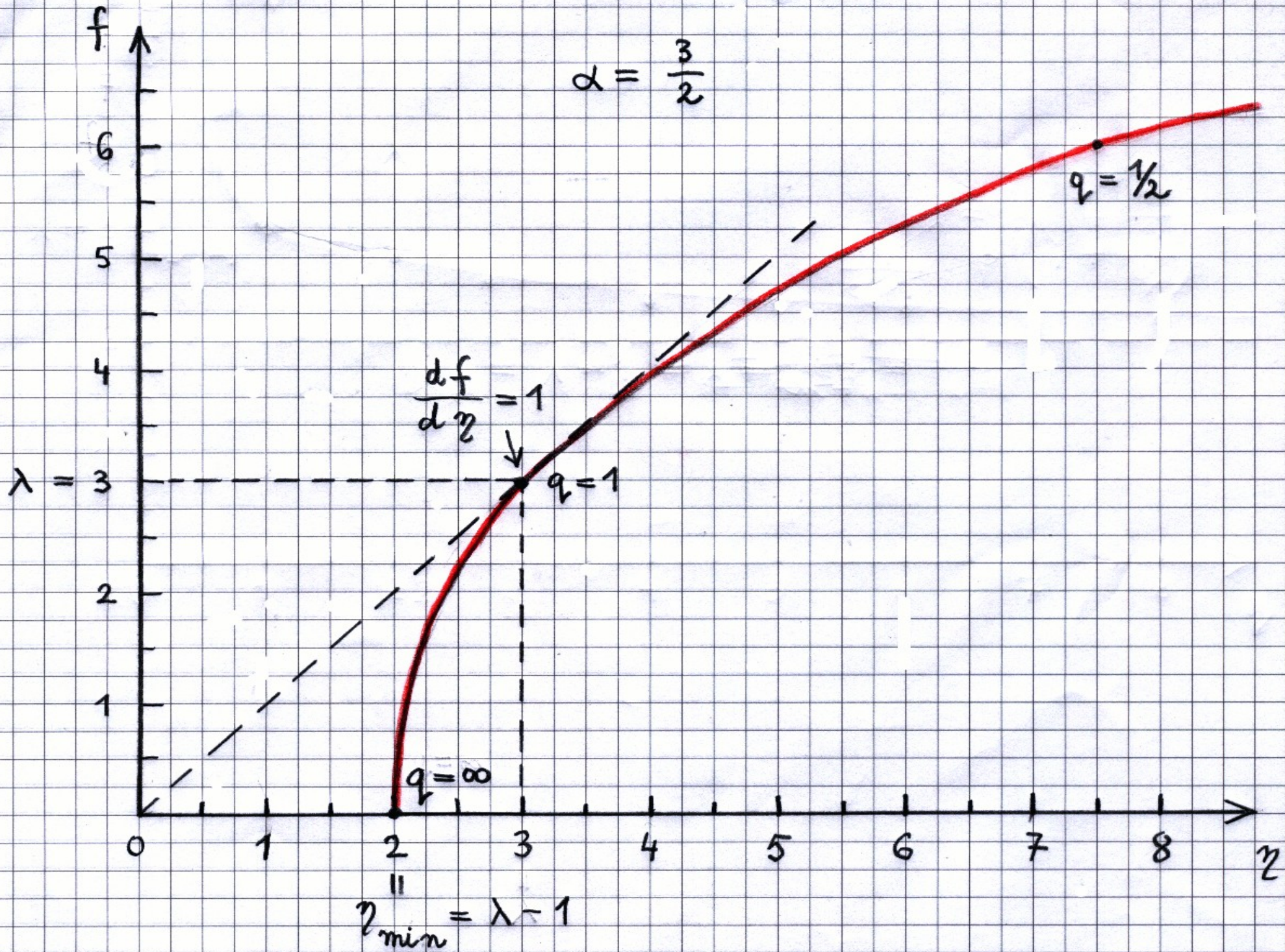
$$\eta(q) = \lambda + \frac{|1-q|^\gamma}{q^{1+\gamma}} (q + \gamma) \operatorname{sgn}(1-q)$$

Spectrum of local exponents (singularities)

$$f(\eta(q)) = \lambda + (1 + \gamma) \left(\frac{|1-q|}{q} \right)^\gamma \operatorname{sgn}(1-q)$$

$$\eta = f(\eta) + \gamma \left(1 - \frac{1}{\alpha} \right)^\alpha |\lambda - f(\eta)|^\alpha$$

We have unrestricted, unlimited multifractal(!)



Specific heat

$$c_V(\beta) = \left(\frac{\partial (E/V)}{\partial T} \right)_V = -\beta^2 \left(\frac{\partial^2 (\beta F/V)}{\partial \beta^2} \right)_V = -q'^2 \frac{d^2 \tau(q')}{dq'^2} = c_L(q'),$$

$$c_L(q') \equiv c(q') = \gamma(\gamma + 1) \frac{|1 - q'|^{\gamma-1}}{q'^{\gamma}}$$

Formal relation between multifractality and thermodynamics

Thermodynamics	Multifractality
β	q'
V	$b(= \ln(L))$
$E(\beta)/V$	$\eta(q')$
$\beta F(\beta)/V$	$\tau(q')$
$S(E)/V$	$f(\eta)$
$c_V(\beta)$	$c_L(q')$

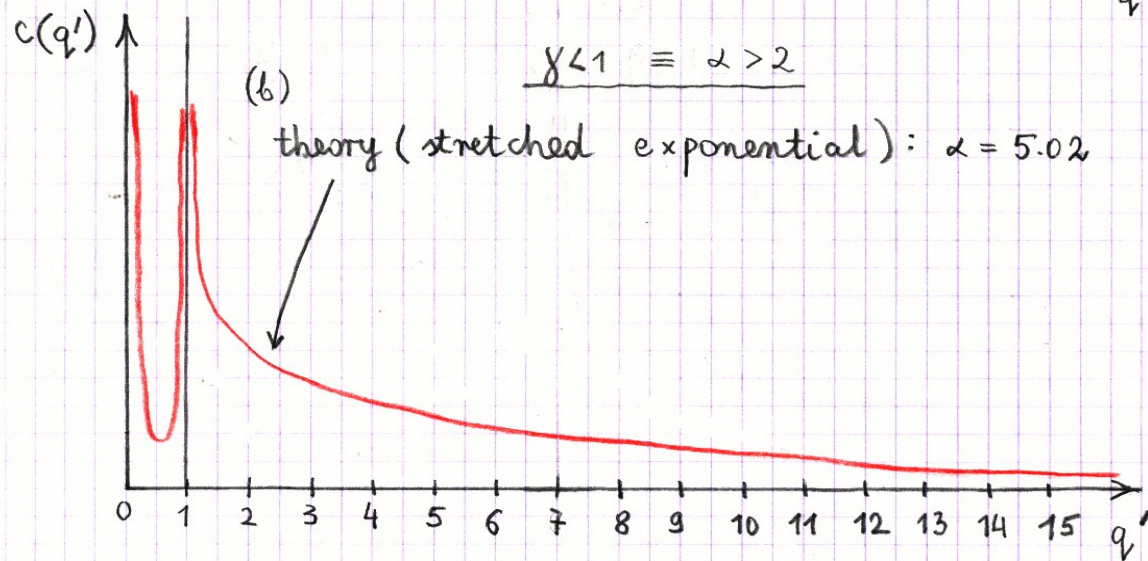
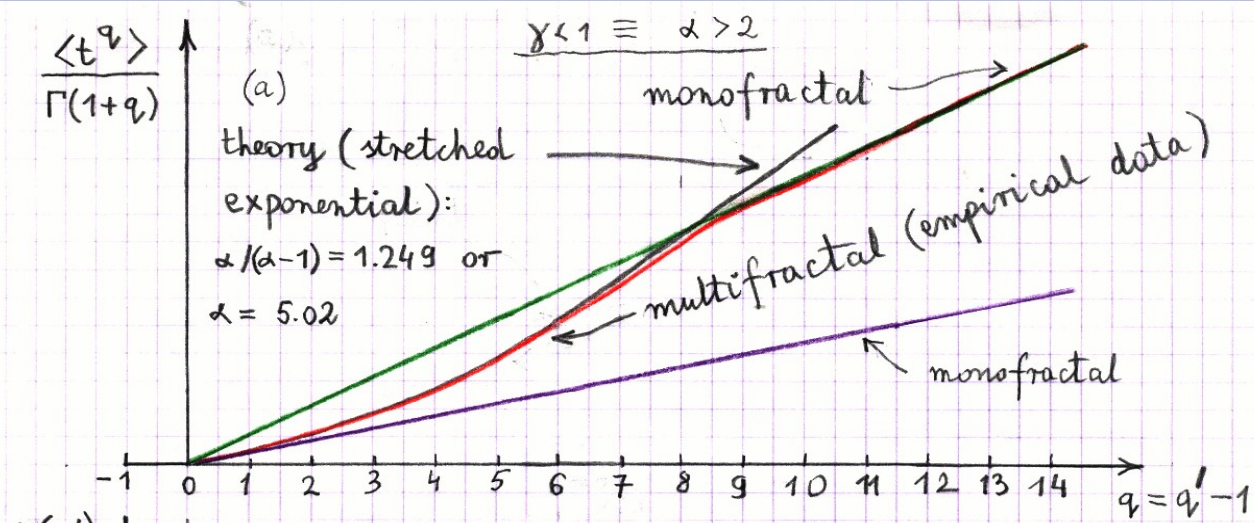
Równości termodynamiczne

Definicja energii swobodnej

$$F(\beta) = E(\beta) - \frac{1}{\beta} S(\beta)$$

Definicja temperatury

$$\beta = \frac{dS(E)}{dE}$$

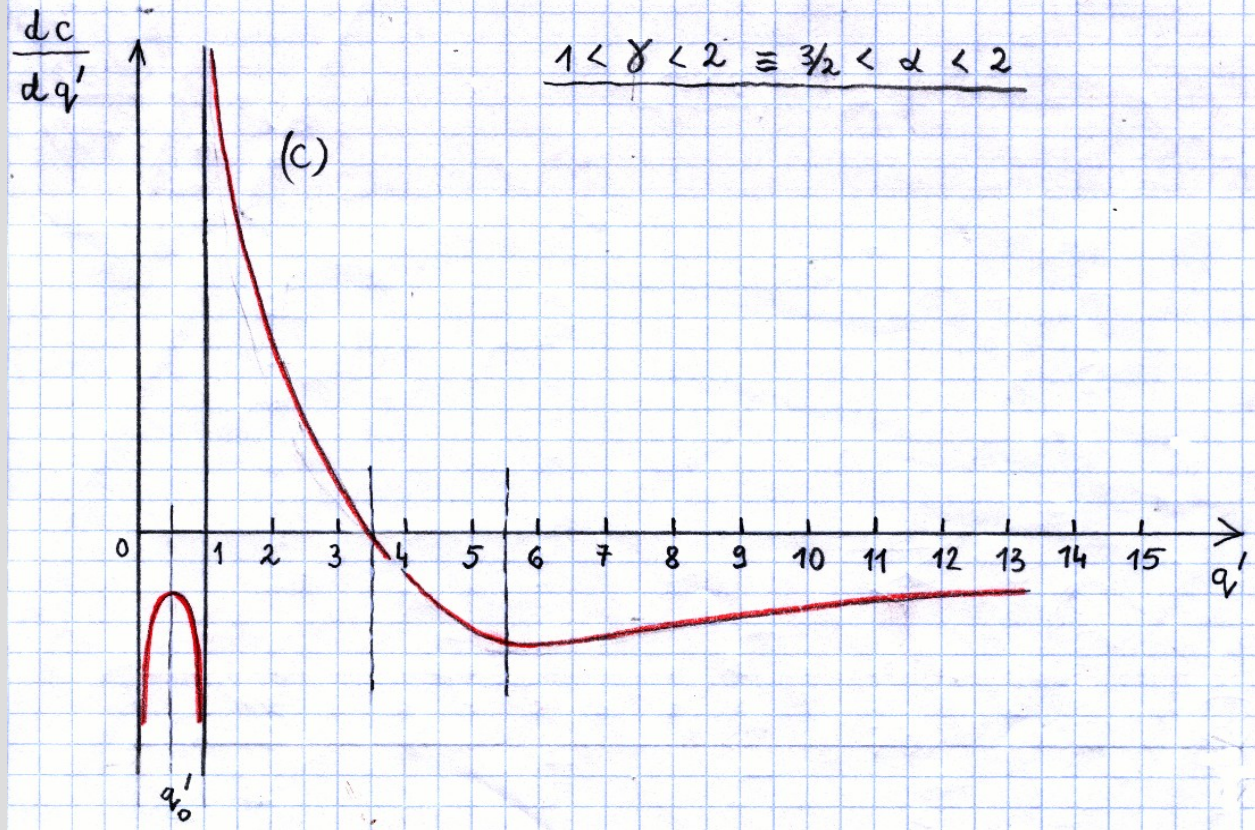
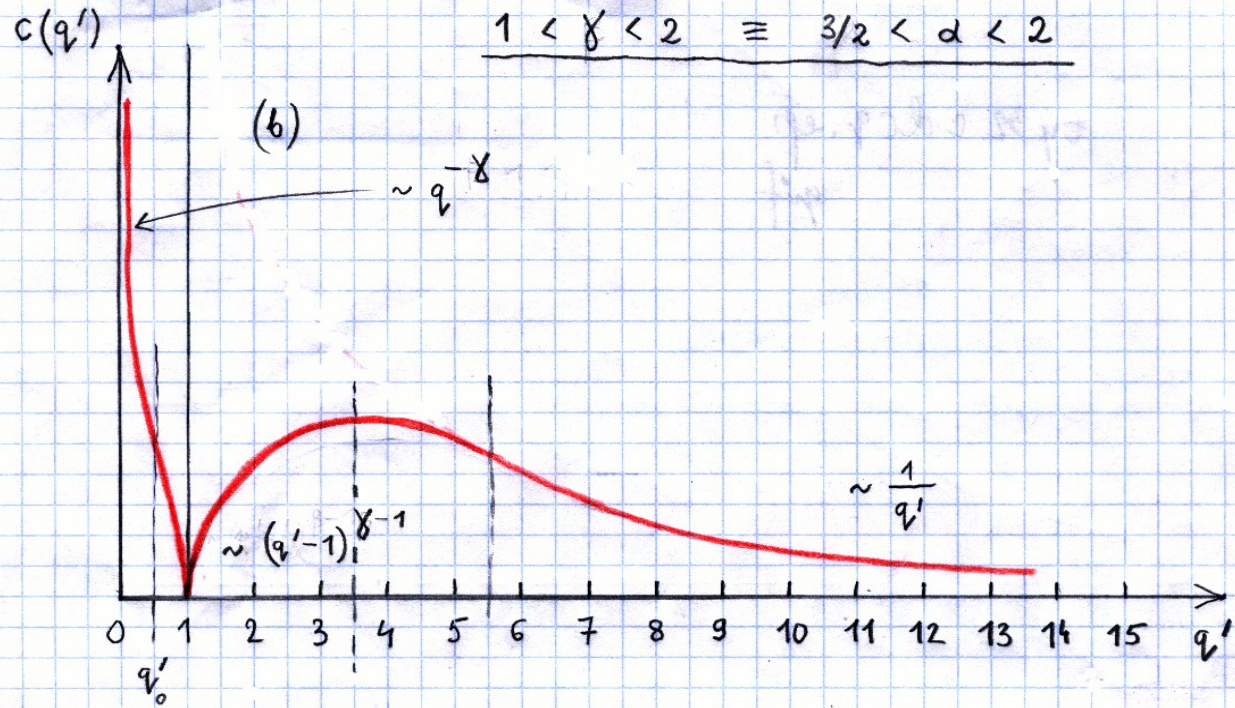


$q' \rightarrow 1$: phase transition from multifractal to monofractal is observed.

Hypothesis

For $q \rightarrow \infty$ we can expect phase transition from multifractal to monofractal.

For $q \rightarrow -1$ we can expect phase transition from monofractal to something (perhaps multifractal).



General conclusions

1. Multifractal version of the CTRW was constructed.
2. Well agreement between empirical data (intertransaction time-intervals for some futures) and predictions of MCTRW was found.
3. Left-sided, unrestricted and unlimited spectrum of local exponents was found.
4. Higher-order phase transition, e.g. between multi- and unifractal was found.

Co dalej? Badanie: statystyk oraz transmisji informacji pomiędzy różnymi skalami.