Light in different media

A short review

• Reflection of light:

Angle of incidence = Angle of reflection

 $\theta_1 = \theta_1'$

• Refraction of light:

Snell's law

 $n_1\sin(\theta_1) = n_2\sin(\theta_2)$





Refraction of light

• Total internal reflection

For some critical angle light beam will be reflected:

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}$$

For some critical angle light beam will be reflected:

Optical fibers





Optical elements



Summary

- Real image can be projected on a screen
- Virtual image exists only for observer
- **Plane mirror** is a flat reflecting surface Plane Mirror: i = -p
- Convex mirrors make objects smaller
- Concave mirrors make objects larger

Spherical Mirror:
$$f = \frac{1}{2}r$$

Geometrical Optics

• "Geometrical" optics (rough approximation): light rays ("particles") that travel in straight lines.

• "Physical" Classical optics (good approximation): electromagnetic waves which have amplitude and phase that can change.

• Quantum Optics (exact): Light is BOTH a particle (photon) and a wave: wave-particle duality.





Refraction



$$E_{1} = A_{1} \cos \left(\omega t - k_{1x}x - k_{1y}y\right)$$
$$E_{2} = A_{2} \cos \left(\omega t - k_{2x}x - k_{2y}y\right)$$
For $y = 0$ $\frac{E_{1}}{E_{2}}$ the same for all x, t
$$k_{1x} = k_{2x}$$

$$n_1 \sin \alpha = n_2 \sin \beta$$

Refraction

$$E = E_0 \exp(i\omega t - ik_x x - ik_y y) + c.c.$$
$$k_y = \sqrt{k^2 - k_x^2} = \frac{\omega}{c} \sqrt{n_2^2 - n_1^2 \sin^2 \alpha} = \pm i |k_y|$$

$$E(y) = E_0 \exp\left(\pm \left|k_y\right| y\right)$$



Polarization

Polarization By Reflection

Different polarization of light get reflected and refracted with different amplitudes ("birefringence").

At one particular angle, the parallel polarization is NOT reflected at all! This is the "Brewster angle" $\theta_{\rm B}$, and $\theta_{\rm B} + \theta_{\rm r} = 90^{\circ}$. (Absorption)

$$n_1 \sin \theta_{\rm B} = n_2 \sin(90^\circ - \theta_{\rm B}) = n_2 \cos \theta_{\rm B}$$

$$\tan \theta_{\rm B} = \frac{n_2}{n_1}$$



Component perpendicular to page
 Component parallel to page

Polarizing Sunglasses



Polarized Sunglasses













Linear polarization



 $E_{v} = A\sin(2\pi x/\lambda - \omega t)$

Vertically (**y** axis) polarized wave having an amplitude *A*, a wavelength of λ , and an angular velocity (frequency * 2π) of ω , propagating along the **x** axis.

Linear polarization

Vertical

$$E_{y} = A\sin(2\pi x / \lambda - \omega t)$$



Horizontal

$$E_z = A\sin(2\pi x/\lambda - \omega t)$$



Linear polarization



- superposition of two waves that have the same amplitude and wavelength,
- are polarized in two perpendicular planes and oscillate in the same phase.
- Oscillating in the same phase means that the two waves reach their peaks and cross the zero line in the same moments

Circular polarization

Right circular

$$E_y = A\sin(2\pi x/\lambda - \omega t + 90^\circ)$$

$$E_z = A\sin(2\pi x/\lambda - \omega t)$$



Left circular $E_y = A \sin(2\pi x / \lambda - \omega t - 90^\circ)$ $E_z = A \sin(2\pi x / \lambda - \omega t)$



Superposition of two circularly polarized waves





- two circularly polarized waves can meet as well.
- In that case, the fields are added according to the rules of vector addition, just as with plane-polarized waves.
- The superposition of two circularly polarized light beams can result in various outcomes.

Any linearly polarized light wave can be obtained as a superposition of a left circularly polarized and a right circularly polarized light wave, whose amplitude is identical.

Polarization

 Some crystals strongly absorb one of these beams while letting the other beam pass right through These materials are called *polarizers*

 Take a thin sheet of such a material and embed it between two sheets of glass or cellulose, and you have a Polaroid filter



 other types of polarizers: Glan Thompson's, Glan Taylor's, Wollaston's, Brewster Windows The direction of polarization of a linearly polarized electromagnetic wave is the direction of the E- field. A polarizing filter used as an analyzer, the intensity I_{max} of the light transmitted through the analyzer depends on the angle φ between the polarization direction of the incident light and the polarizing axis of the analyzer



Malus's law, polarized light passing through an analyzer



Polarization



(b)

(a)

Light interaction with media

Maxwell eqs for dielectric materials

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$
$$\nabla \times \vec{H} = \frac{\partial}{\partial t}\vec{D}$$
$$\nabla \cdot \vec{D} = 0$$
$$\nabla \cdot \vec{B} = 0$$

$$\begin{split} \vec{P} &= \varepsilon_0 \chi \vec{E} \\ \vec{D} &= \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1 + \chi) \vec{E} = \varepsilon_0 \varepsilon \vec{E} \end{split}$$

Light and Matter

- Opaque
 - absorbs or reflects all light
- Transparent
 - allows light to pass through completely
- Translucent
 - allows some light to pass through



Interaction of light and matter: Absorption

The process by which EM radiant energy is absorbed by a molecule or particle and converted to another form of energy

Material with an extinction coefficient ϵ



In

The light gets weaker (its amplitude drops)

$$E_{y} = Ae^{-\varepsilon x}\sin(2\pi x/\lambda - \omega t)$$



Out

Circularly polarized waves in an absorbing medium



- The intensity of light decreases exponentially inside the shown piece of material.
- After the light exits the medium, its field vector rotates as before but its length is lower than the original value.



Interaction of light and matter: Refraction



Material with an *index of* refraction n

The light slows down inside the material, therefore its wavelength becomes shorter and its phase gets shifted



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In

Out

Circular dichroism

Material having different extinction coefficients for right and left circularly polarized lights: ε_R and ε_L



Plane-polarized light becomes elliptically polar



 $E_{y} = Ae^{-\varepsilon_{R}x} \sin(2\pi x/\lambda - \omega t + 90^{\circ}) + Ae^{-\varepsilon_{L}x} \sin(2\pi x/\lambda - \omega t - 90^{\circ})$ $E_{z} = Ae^{-\varepsilon_{R}x} \sin(2\pi x/\lambda - \omega t) + Ae^{-\varepsilon_{L}x} \sin(2\pi x/\lambda - \omega t)$

Circular bi-refringence

Material having different refraction indices for right and left circularly polarized lights: $n_{\rm R}$ and $n_{\rm L}$



The plane of polarization of planepolarized light gets rotated



 $E_{y} = A\sin(2\pi n_{\rm R}x/\lambda - \omega t + 90^{\circ}) + A\sin(2\pi n_{\rm L}x/\lambda - \omega t - 90^{\circ})$ $E_{z} = A\sin(2\pi n_{\rm R}x/\lambda - \omega t) + A\sin(2\pi n_{\rm L}x/\lambda - \omega t)$

Circular dichroism AND bi-refringence

Material having different extincion coefficients AND refraction indices for right and left circularly polarized lights: $\varepsilon_{\rm R}$ and $\varepsilon_{\rm L}$ AND $n_{\rm R}$ and $n_{\rm L}$



Plane polarized light gets elliptically polar, with the great axis of the ellipse being rotated relative to the original plane of polarization



 $E_{y} = Ae^{-\varepsilon_{R}x} \sin(2\pi n_{R}x/\lambda - \omega t + 90^{\circ}) + Ae^{-\varepsilon_{L}x} \sin(2\pi n_{L}x/\lambda - \omega t - 90^{\circ})$ $E_{z} = Ae^{-\varepsilon_{R}x} \sin(2\pi n_{R}x/\lambda - \omega t) + Ae^{-\varepsilon_{L}x} \sin(2\pi n_{L}x/\lambda - \omega t)$

Light scattering

Types of Scattering in the Atmosphere

The process whereby EM radiation is absorbed and immediately re-emitted by a particle or molecule – energy can be emitted in multiple-directions

- Rayleigh
- Mie
- Geometric

The type of scattering is controlled by the size of the wavelength relative to the size of the particle

Blue Sky & Red Sunsets

- Molecules in atmosphere scatter light rays.
- Shorter wavelengths (blue, violet) are scattered more easily.



NOON

- less atmosphere
- less scattering
- blue sky, yellow sun

SUNSET

- more atmosphere
- more scattering
- orange-red sky & sun

Rayleigh Scattering in Visible Spectrum

- Occurs when the wavelength >> the particle size, Rayleigh scattering occurs at a molecular level,
- Blue sky
- Red Sunset
- Blue water (from underwater)

Rayleigh Scattering



- Wavelength of light is much larger than scattering particles,
- Blue light ~4000 Angstroms, scattering particles ~1 Angstrom (1A=10⁻¹⁰ m)

Intensity of Rayleigh Scattering

$$I_{\lambda} = I_{0\lambda} e^{-\beta_s x}$$
$$\beta_s = \lambda^{-4}$$

SO

$$I_{\lambda} = I_{0\lambda} e^{-\frac{x}{\lambda^4}}$$

The degree of Rayleigh scattering is inversely proportional to the fourth power of the EM wavelength

Angular Intensity Factor

$$I(\phi) = I_0(1 + \cos^2 \phi)$$

• Where *f* is the angle between the incident beam and the scattered light so for non-polarized light

$$I_{\lambda} = I_{0\lambda} e^{-\frac{x}{\lambda^4}} (1 + \cos^2 \phi)$$

Rayleigh Scattering

- Can be explained in terms of oscillations of dipole atoms when excited by an EM wave
- Derivation explains polarization as well
- This is why the sky is blue!
- The higher frequencies are scattered more by the molecules in the atmosphere
- Reds and oranges just pass through, so the sky appears blue

Scattered radiation Incident beam Atom

Most Rayleigh scattering occurs in the upper 10 km of the stratosphere

Rayleigh Scattering

A 5 mW green laser pointer is visible at night due to Rayleigh scattering and airborne dust.





Rayleigh Scattering from Particles

1. When scattering from a particle of size *d* with light of wavelength λ , the Rayleigh scattering is found to be:

$$I = I_o \frac{1 + \cos^2 \theta}{2R^2} \left(\frac{2\pi}{\lambda}\right)^4 \left(\frac{n^2 - 1}{n^2 + 2}\right)^2 \left(\frac{d}{2}\right)^6$$

2. where R is the distance to the particle, n is the index of refraction, and θ is the scattering angle.

Degree of Polarization

- 1. In general, Rayleigh scattering is for randomly polarized incident flux and the scattered flux will be polarized.
- 2. The degree of polarization induced by scattering from a small particle exposed to randomly polarized flux is:

$$DOP = \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta}$$

Bohren and Huffman (1998)

Rayleigh Scattering from Particles

The clear sky appears blue because Rayleigh scattering high in the atmosphere influence short wavelength (blue) radiation the most

Note UV radiation is not scattered by the upper atmosphere because it is absorbed by the OZONE Layer



Mie Scattering

- Begin with Maxwell's equations
- Derive a wave equation in spherical polar coordinates
- Provide boundary conditions at surface of a sphere
- Solve the partial differential wave equation for dependence on r, θ , ϕ .

Mie Scattering

- Occurs when the wavelength \cong particle size,
- Explains scattering around larger droplets such as Corona around the sun or moon, Glory and similar phenomena.
- Occurs with particles that are actually 0.1 to 10 times the size of the wavelength
- Primary Mie scatterers are dust particles, soot from smoke
- Mie scatterers are found lower in the Troposphere



Non-selective Scattering



Mie Scattering cont.

$$I = \pi^2 r^4 \left[1 - \frac{1}{2} \frac{m^2}{1} + \frac{1}{3} \frac{m^4}{(1 \cdot 2)^2} - \frac{1}{4} \frac{m^6}{(1 \cdot 2 \cdot 3)^2} + \cdots \right]^2$$

 The bracket has the form of a Bessel function of the first order and

$$m = \frac{\pi r \sin \rho}{\lambda}$$

