Magnetooptic effects

•Solution from Maxwell's equations,
•Material parameters must be given,

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \vec{\vec{\varepsilon}} \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \times \vec{E} = \mu_0 \vec{\vec{\mu}} \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\vec{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

- unitar tensor,
- commonly used term in analysis of optical fields
- influence of magnetic field is expressed in terms of off-diagonal elements

$$\vec{\vec{\mu}} = 1$$

$$\Delta \vec{E} - \nabla \cdot \nabla \vec{E} - \vec{\vec{\varepsilon}} \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\Delta\vec{E} = \begin{bmatrix} \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} & 0 & 0 \\ 0 & \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} & 0 \\ 0 & 0 & \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\vec{\nabla}\cdot\vec{\nabla}\,\vec{E} = \begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x \partial z} \\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y \partial z} \\ \frac{\partial^2}{\partial z \partial x} & \frac{\partial^2}{\partial z \partial y} & \frac{\partial^2}{\partial z^2} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

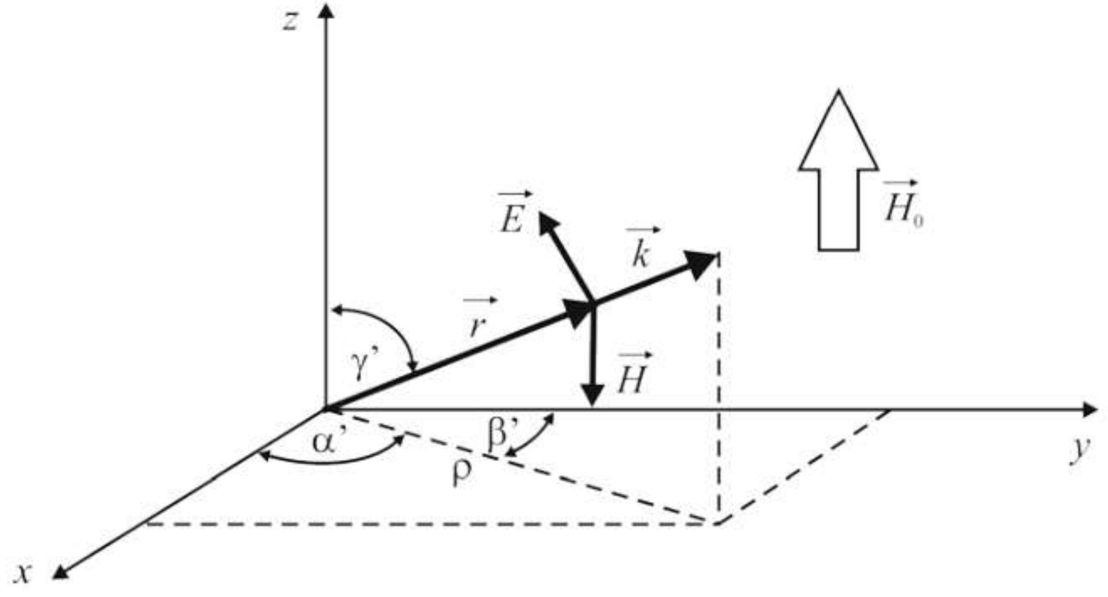
$$\Delta\vec{E} - \nabla \cdot \nabla\vec{E} - \vec{\vec{\varepsilon}}\varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

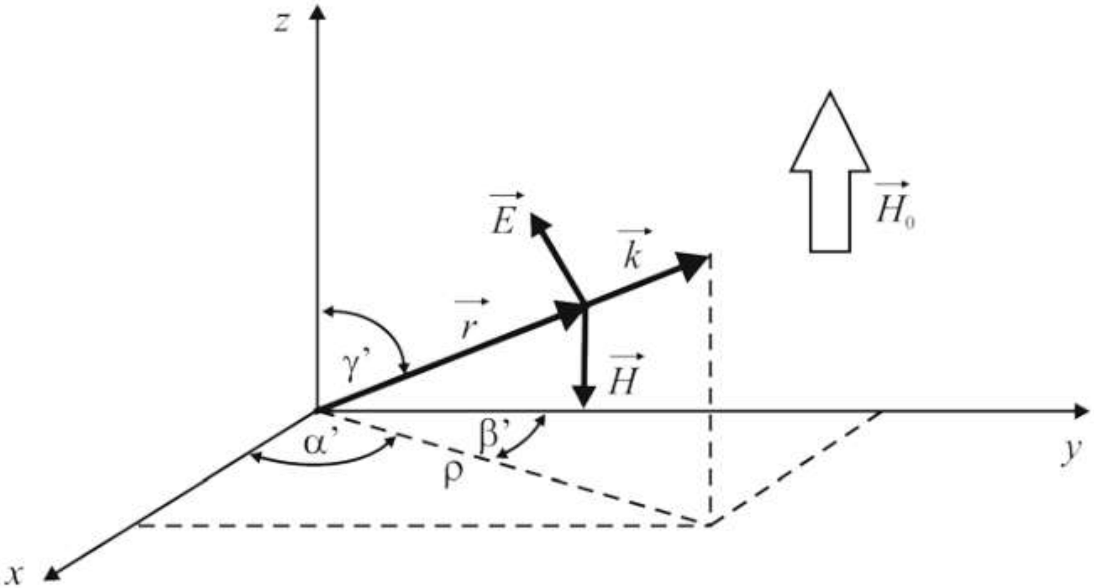
Solution :

$$\vec{E} = \vec{E}_0 \exp\left[\mathrm{i}\left(\omega t - n\vec{k}_0 \vec{r}\right)\right]$$

gdzie: $k_0 = \frac{2\pi}{\lambda_0}$

Wektor falowy w próżni





Oznaczenia:

$$\cos(\alpha') = \alpha_c$$

$$\cos(\beta') = \beta_c$$

$$\cos(\gamma') = \gamma_c$$

Direction Cosinuses of wave vector in respect to coordination axes

Product of wave vector and radios r can be expressed as:

$$\vec{k} \cdot \vec{r} = nk_0 \left(\alpha_c x + \beta_c y + \gamma_c z \right)$$

Tensor $\bar{\bar{\varepsilon}}$ under the influence of external magnetic field *H₀* :

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \varepsilon_x & -i\delta & 0 \\ i\delta & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$\Delta\vec{E} - \nabla\cdot\nabla\vec{E} - \vec{\vec{\varepsilon}}\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\vec{E} = \vec{E}_0 \exp\left[\mathrm{i}\left(\omega t - n\vec{k}_0\vec{r}\right)\right]$$

$$\left(\Delta - \vec{\nabla}\cdot\vec{\nabla} - \bar{\bar{\varepsilon}}\,\varepsilon_0\,\mu_0\omega^2\right)\vec{E} = 0$$

Considering $\bar{\bar{\varepsilon}}$

$$\begin{bmatrix} \left[\mathrm{n}^2\left(\beta_c^2+\gamma_c^2\right)-\varepsilon_x\right] & -\mathrm{n}^2\alpha_c\beta_c+i\delta & -\mathrm{n}^2\alpha_c\gamma_c \\ -\mathrm{n}^2\alpha_c\beta_c-i\delta & \left[\mathrm{n}^2\left(\alpha_c^2+\gamma_c^2\right)-\varepsilon_y\right] & -\mathrm{n}^2\beta_c\gamma_c \\ -\mathrm{n}^2\alpha_c\gamma_c & -\mathrm{n}^2\beta_c\gamma_c & \left[\mathrm{n}^2\left(\alpha_c^2+\beta_c^2\right)-\varepsilon_z\right] \end{bmatrix}\begin{bmatrix} \mathrm{E}_x \\ \mathrm{E}_y \\ \mathrm{E}_z \end{bmatrix} = 0$$

In order to solve this, the characteristic determinant must vanish

Eqiations for possible refractive indices:

$$n^4\left(\varepsilon_x\alpha_c^2+\varepsilon_y\beta_c^2+\varepsilon_z\gamma_c^2\right)-n^2\Big[\left(\varepsilon_x\varepsilon_y-\delta^2\right)\left(\alpha_c^2+\beta_c^2\right)+$$

$$+\varepsilon_z\left(\varepsilon_x\alpha_c^2-\varepsilon_y\beta_c^2\right)+\varepsilon_z\left(\varepsilon_x+\varepsilon_y\right)\gamma_c^2\Big]+\varepsilon_z\left(\varepsilon_x\varepsilon_y-\delta^2\right)=0.$$

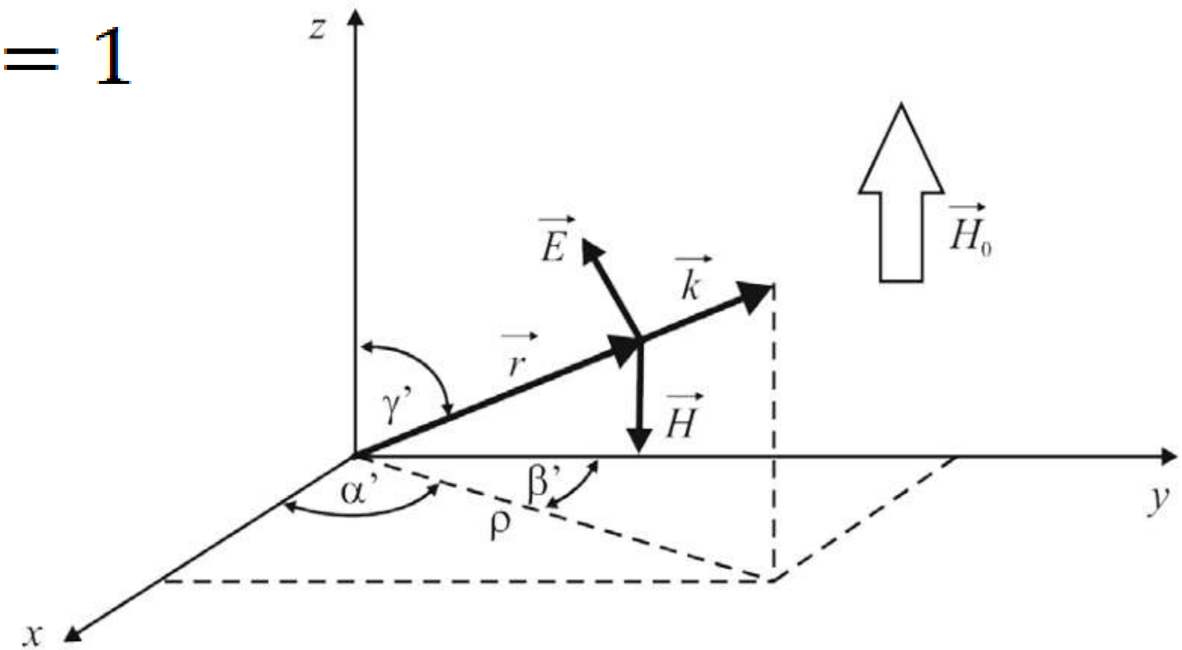
Two cases can be considered:

1). Propagation along Oz axis – along the direction of external magnetic field,
(Faraday’s rotation),

2). Propagation perpendicularly to the magnetic field vector
(Cotton-Mouton effect)

Faraday's effect

$$\alpha_c = \beta_c = 0 \qquad \gamma_c = 1$$



To simplify:

$$\varepsilon_x = \varepsilon_y = \varepsilon_F$$

- no-shape medium,
- crystal with regular structure,
- uniaxial crystal with optical axis along OZ axis

$$n^4 - 2n^2\varepsilon_F + \varepsilon_F^2 - \delta^2 = 0$$

Solution: $$n_\pm^2 = \varepsilon_F \pm \delta$$

Faraday’s effect

Two possible values of refractive index, so two eigenwaves can propagate in the medium,

$$\begin{bmatrix} \pm i\delta & -\delta & 0 \\ \delta & \pm i\delta & 0 \\ 0 & 0 & -i\varepsilon_z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

$$E_x^{\pm} = E_0^{\pm} \exp\left[i\left(\omega t - k_0 n_{\pm} z\right)\right],$$

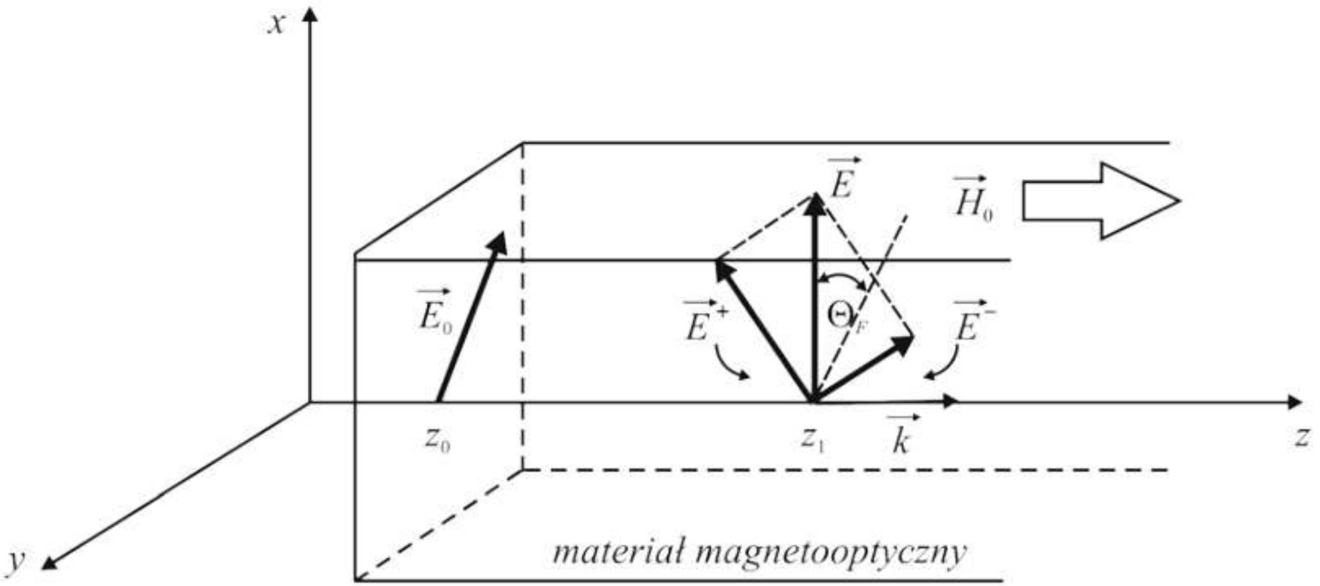
$$E_y^{\pm} = \pm i E_0^{\pm} \exp\left[i\left(\omega t - k_0 n_{\pm} z\right)\right].$$

Two circularly polarized waves (left-handed and right-handed)

Faraday’s rotation angle

$$\Theta_F = V H_0 \Delta z$$

Faraday's effect



$$\Theta_F = \pi(n_+ - n_-)\Delta z/\lambda_0 = \pi \Delta n_{CB} \Delta z/\lambda_0$$

Path length

In paramagnetics and diamagnetics, the rotation angle is proportional to intensity of magnetic field

$$\Theta_F = \mathrm{V}\, H_0 \Delta z$$

Verdett constant

Faraday's effect

For ferromagnetics and ferrimagnetics, rotation angle is larger and is not proportional to the intensity of magnetic field

$$\varphi_{H_0=H_{nas}} = \frac{\Theta_F}{\Delta z}$$

Proper rotation in total magnetic saturation state

It can be proved that nondiagonal elements of tensor $\bar{\bar{\varepsilon}}$ for medium ($\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon_F$)

$$\varepsilon_{12} = -\varepsilon_{21} = -i\,\delta = -i\,\frac{\omega \varepsilon_0 \mu_0}{N\,e}\left(\varepsilon_F - 1\right)^2 \mu\left(H_0\right) H_0$$

where: N – Avogadro number, e – electron charge, ω – frequency of optical wave , $\mu(H_0)$ – magnetic permitivity of the medium,

Faraday’s effect

For ferro- i ferrimagnetics, Faraday’s rotation angle is defines as:

$$\Theta_F = \pi(n_+ - n_-)\frac{\Delta z}{\lambda_0} \approx \frac{\pi\delta\,\Delta z}{\lambda_0\sqrt{\varepsilon_F}} = \frac{\varepsilon_0\mu_0\varepsilon_F\pi}{N\,e\,\lambda_0\sqrt{\varepsilon_F}}(\varepsilon_{F2} - 1)^2\mu(H_0)\,H_0$$

$$V = \frac{\omega\varepsilon_0\mu_0\pi}{N\,e\,\lambda_0\sqrt{\varepsilon_F}}(\varepsilon_F - 1)^2\mu(H_0)$$

Verdett’s constant for ferro- i ferrimagnetics

- For diamagnetics and paramagnetics, V does not depend on external field
- For ferro- and ferrimagnetics, V is changing with magnetic field intensity H up to saturation state,

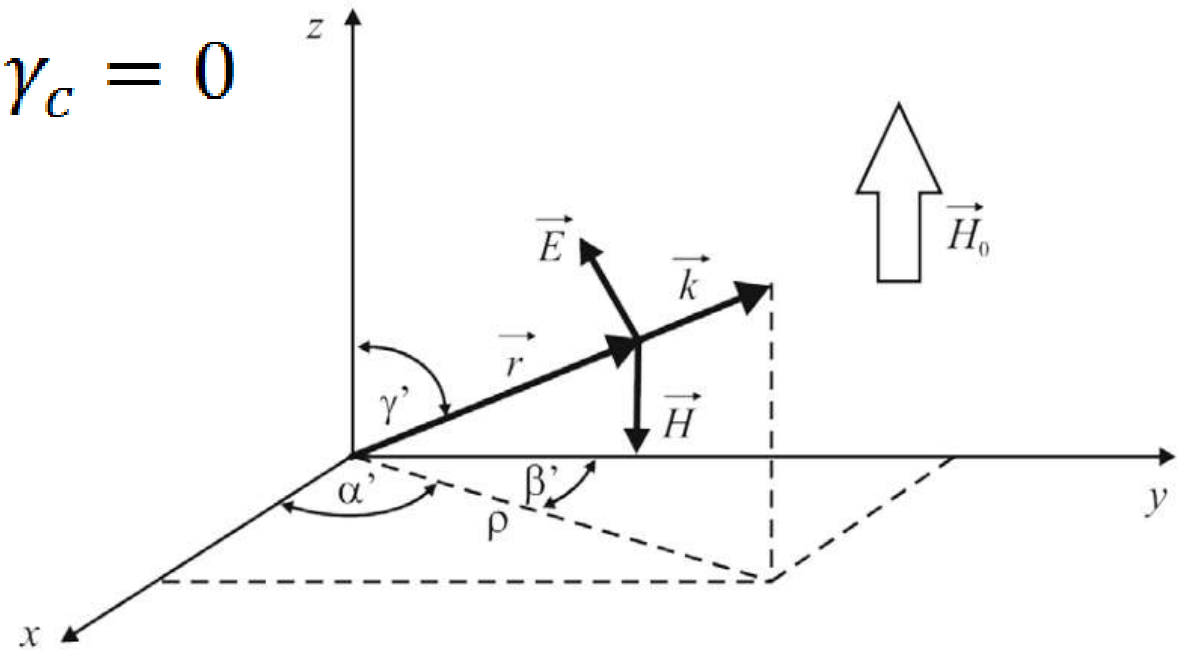
Lp.	Materiał magneto-optyczny	Własności magnetyczne i struktura krystalograficzna	Długość fali λ [μm]	Stała Verdeta V [rad/A]	Tłumienność ξ [1/m]	Dobroć magnetooptyczna V / ξ [rad m/A] (*) φ / ξ [rd]	dla silnych magnetyków (*) $\varphi=V\times H_{max}$ [rad/ m]	źródło informacji
1.	*FR-5*	Szkło paramagnetyczne	0,633	$-0{,}88\cdot10^{-4}$	3	$3{,}11\cdot10^{-5}$	-	[28]
2.	*EuSe*	Kryształ paramagnetyczny	0,633	$-2{,}82\cdot10^{-4}$	$1{,}73\cdot10^{4}$	$1{,}65\cdot10^{-8}$	-	[29]
3.	$FeBr_2$	Heksagonalne kryształy warstwowe ferromagnetyczno – antyferromagnetyczne	0,633	(*)	200	3,5 (*)	700	[30]
4.	$PbMo0_4$	Kryształ paramagnetyczny (akustooptyczny)	0,633	$0{,}66\cdot10^{-4}$	10	$0{,}66\cdot10^{-5}$	-	[31]
5.	*KDP*	Kryształy diamagnetyczne (elektrooptyczne)	0,633	$0{,}046\cdot10^{-4}$	10	$0{,}46\cdot10^{-5}$	-	[32]
6.	*ADP*	Kryształy diamagnetyczne (elektrooptyczne)	0,633	$0{,}05\cdot10^{-4}$	10	$0{,}05\cdot10^{-5}$	-	[32]
7.	$Si0_2$	Diamagnetyczne szkło "światłowodowe"	0,633	$0{,}0476\cdot10^{-4}$	0,1	$4{,}76\cdot10^{-5}$	-	[33]
8.	$Cd_{0,55}Mn_{0,45}Te$	Półprzewodnik "półmagnetyczny"	0,633	$25\cdot10^{-4}$	15	$16{,}8\cdot10^{-5}$	-	[34, 35]
9.	$YFeO_3$	Kryształ ferrimagnetyczny o strukturze ortoferrytów	1,55	(*)	0,5	1050 (*)	525	[36]
10.	$Y_3Fe_50_{12}$	Kryształ ferrimagnetyczny o strukturze granatu	0,633 1,15	(*) (*)	10^{5} 1	0,017 (*) 295 (*)	1700 295	[27] [27]
11.	$(YbTbBi)_3 Fe_50_{12}$	Kryształ ferrimagnetyczny o strukturze granatu	1,3 1,55	(*) (*)	1 1	3150 (*) 2100 (*)	-3150 -2100	[37]

Attenuation (absorption coefficient)

$$I(z) = I exp(-\xi z)$$

Cotton-Mouton effect

$\varepsilon_x = \varepsilon_y = \varepsilon_{CM}$ $\gamma_c = 0$



$$n^4 - n^2\left[\left(\varepsilon_{CM} - \frac{\delta^2}{\varepsilon_{CM}}\right) + \varepsilon_z\right] + \varepsilon_z\left(\varepsilon_{CM} - \frac{\delta^2}{\varepsilon_{CM}}\right) = 0$$

Solutions:

$$n_{II}^2 = \varepsilon_z \qquad n_{\perp}^2 = \varepsilon_{CM}\left(1 - \frac{\delta^2}{\varepsilon_{CM}^2}\right)$$

Cotton-Mouton effect

$$\begin{bmatrix} \left[n^2(\beta_c^2+\gamma_c^2)-\varepsilon_x\right] & -n^2\alpha_c\beta_c+i\delta & -n^2\alpha_c\gamma_c \\ -n^2\alpha_c\beta_c-i\delta & \left[n^2(\alpha_c^2+\gamma_c^2)-\varepsilon_y\right] & -n^2\beta_c\gamma_c \\ -n^2\alpha_c\gamma_c & -n^2\beta_c\gamma_c & \left[n^2(\alpha_c^2+\beta_c^2)-\varepsilon_z\right] \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}=0$$

(Can be simplified)

$$\left(n^2\beta_c^2-\varepsilon_{CM}\right)E_x-\left(n^2\alpha_c\beta_c-i\delta\right)E_y=0,$$

$$\left(n^2\alpha_c^2-\varepsilon_{CM}\right)E_y-\left(n^2\alpha_c\beta_c-i\delta\right)E_x=0,$$

$$\left(n^2-\varepsilon_{CM}\right)E_z=0,$$

Cotton-Mouton effect

Solution for $n = n_{\parallel}$

$$E_x = E_y = 0,$$

$$E_z = E_{0z} \exp\left[\mathrm{i}\left(\omega t - n_{\mathrm{II}} k_0 \rho\right)\right] = E_{\mathrm{II}},$$

$$\rho = \sqrt{x^2 + y^2} = \alpha_c x + \beta_c y$$

Solution for $n = n_{\perp}$

$$E_z = 0,$$

$$E_{\perp} = \sqrt{E_x^2 + E_y^2} = E_{0\perp} \exp\left[\mathrm{i}\left(\omega t - n_{\perp} k_0 \rho\right)\right].$$

- Inside of material, there are two orthogonal linear polarized waves,
- Phase velocities of both waves are different,
- The medium behaves as uniaxial crystal with optical axis parallel to vector H_0

Cotton-Mouton effect

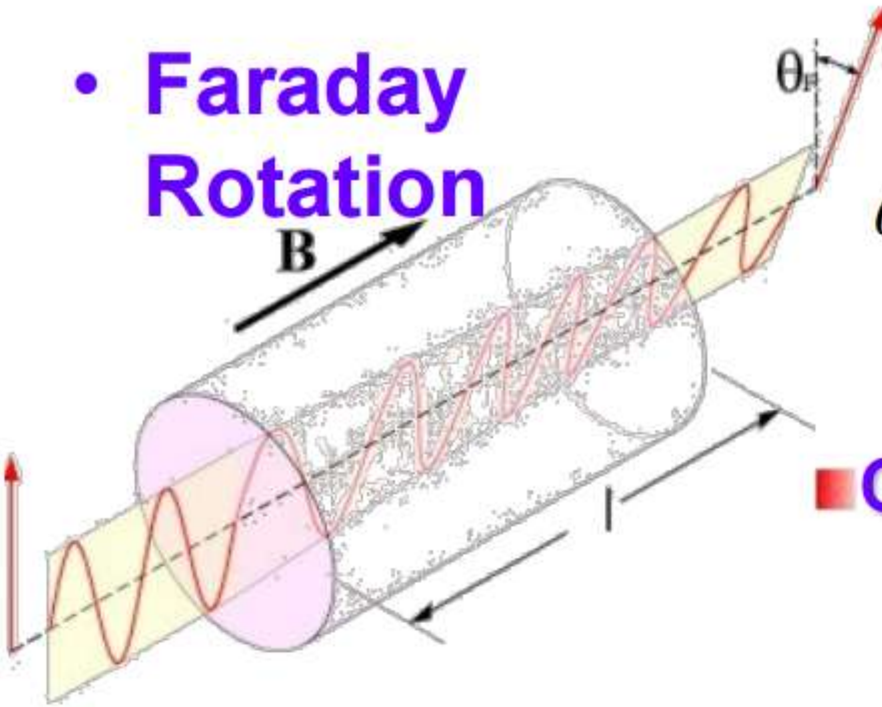
- At every point of the crystal, whose alignment is defined by vector ρ, a elliptically polarized wave as a superposition of two orthogonal waves is observed,
- Magnetic birefringence – Cotton-Mouton effect,
- Phase difference is calculated accordint to:

$$\Delta = k_0 (n_{II} - n_{\perp}) \rho = k_0 \, \Delta n_{LB} \rho \approx k_0 \frac{\sqrt{\varepsilon_{CM}} \delta^2}{\varepsilon_{CM}} \rho$$

$$\Delta = \frac{\pi \omega^2 \varepsilon_0^2 \mu_0^2}{N e \lambda_0 \varepsilon_{CM}^{3/2}} (\varepsilon_{CM} - 1)^4 \mu^2 (H_0) H_0^2 \rho$$

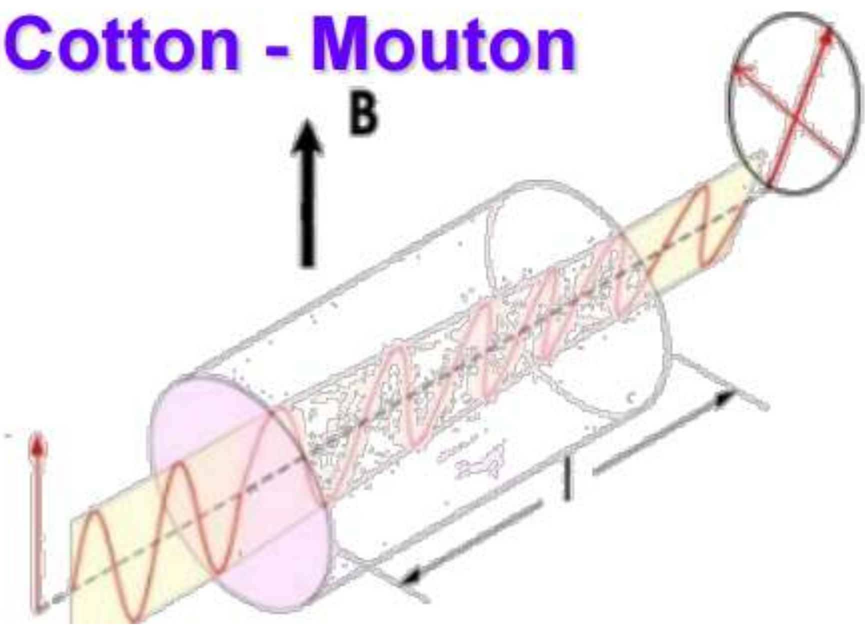
- Cotton-Mouton effect is a second order phenomena, and phase difference is a nonlinear function of H_0

- **Faraday Rotation**



$$\theta_F = \pi \Delta n_{cb} \frac{l}{\lambda_0} = \frac{\pi \varepsilon_0 \mu_0 \varepsilon (\varepsilon - 1)^2 l}{N \varepsilon \lambda_0 \sqrt{\varepsilon}} B$$

- **Cotton - Mouton**



$$\phi_{CM} = k_0 \Delta n_{LB} = \frac{\pi \omega^2 \varepsilon_0^2 \mu_0^2 (\varepsilon - 1)^4 l}{N e \lambda_0 \varepsilon^{3/2}} B^2$$

Other megnetooptic effects

Circular and linear dichroism

- observed together with Faraday's and Cotton-Mouton effects,
- in reality, components of electric permitivity tensor $\bar{\bar{\varepsilon}}$ are complex,
- refractive index will be represented as: $n^* = n' - in''$

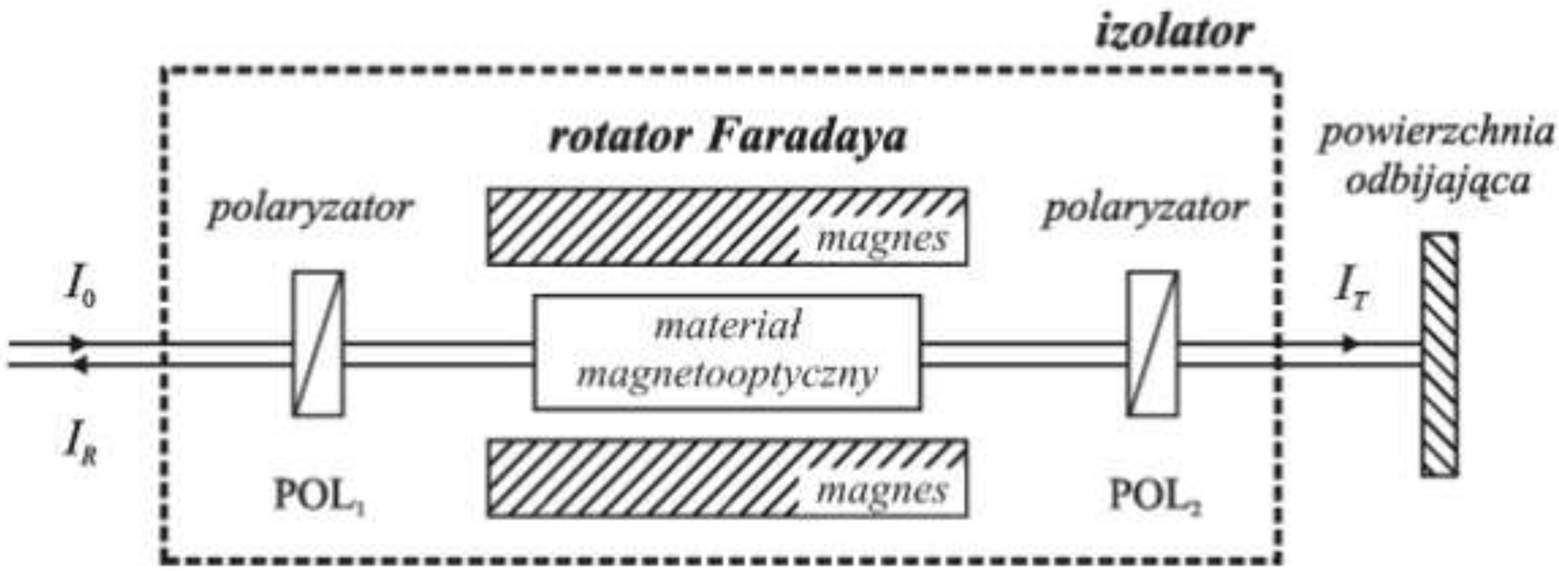
- Faraday's rotation can be defines as:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = E_0 \begin{bmatrix} 1 \\ \pm i \end{bmatrix} \exp\left(-2\pi n''_{\pm} z/\lambda_0\right) \exp\left[i\left(\omega t - 2\pi n'_{\pm} z/\lambda_0\right)\right]$$

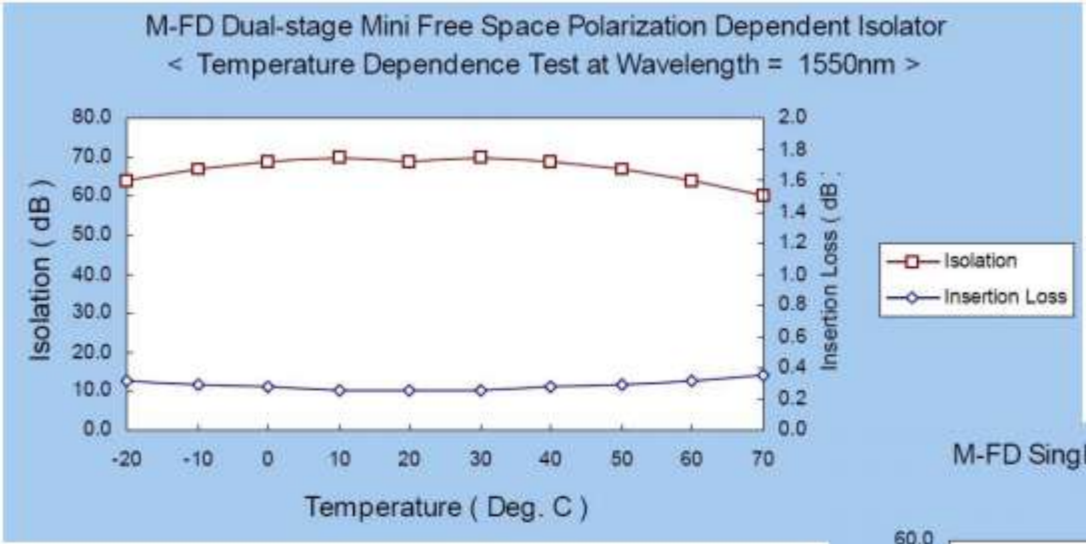
Optical isolator

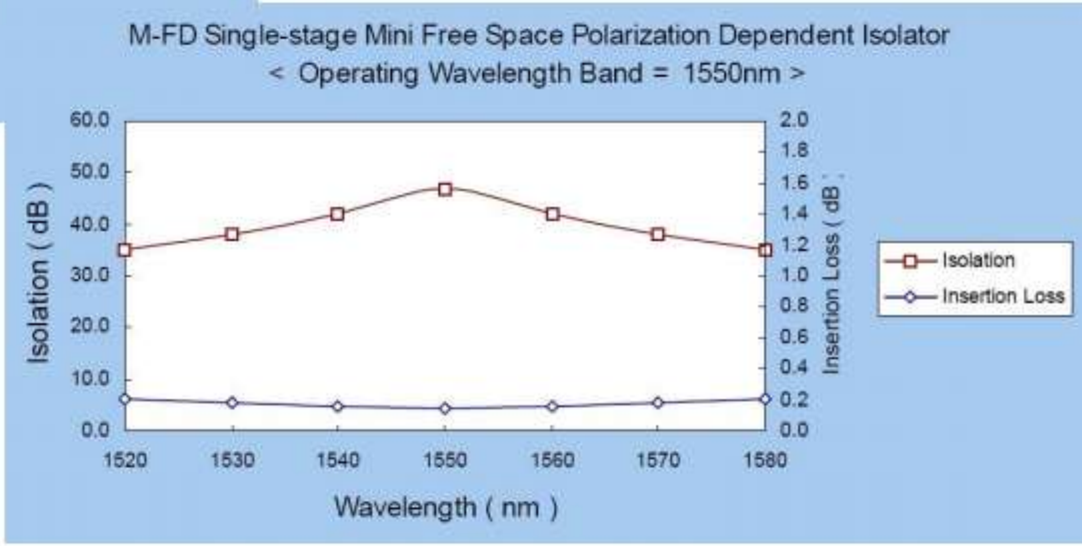
$$Isolation = 10 * \log\left(\frac{I_T}{I_R}\right)$$

$$Insertion \quad loss = -10 * \log\left(\frac{I_T}{I_O}\right)$$

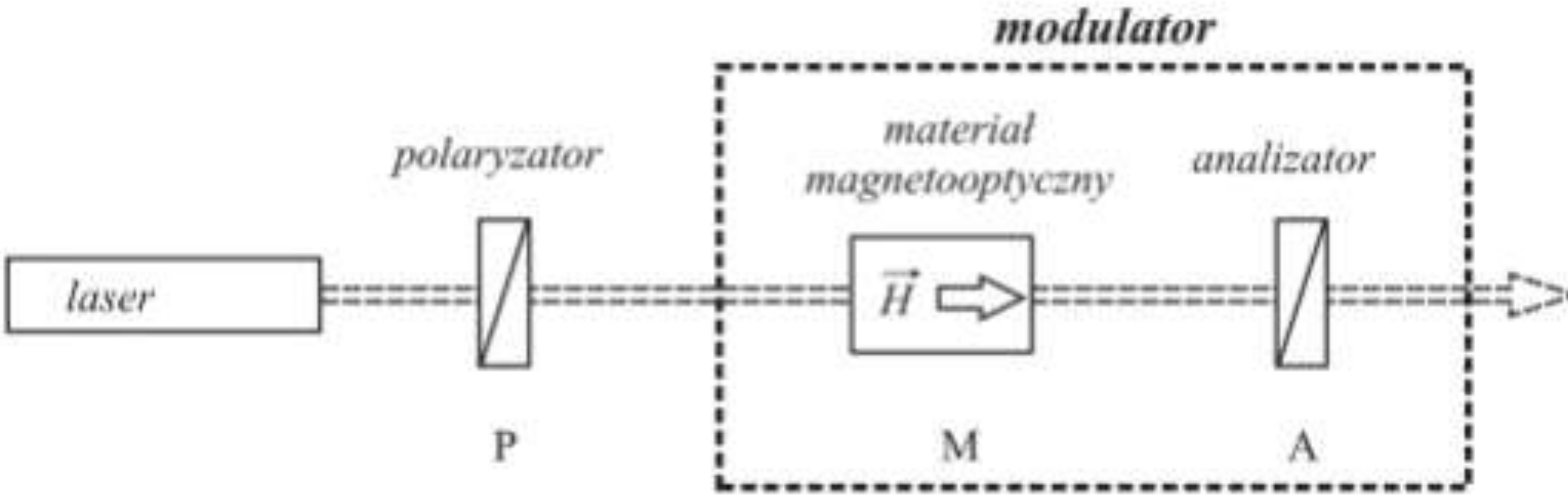


Optical isolator





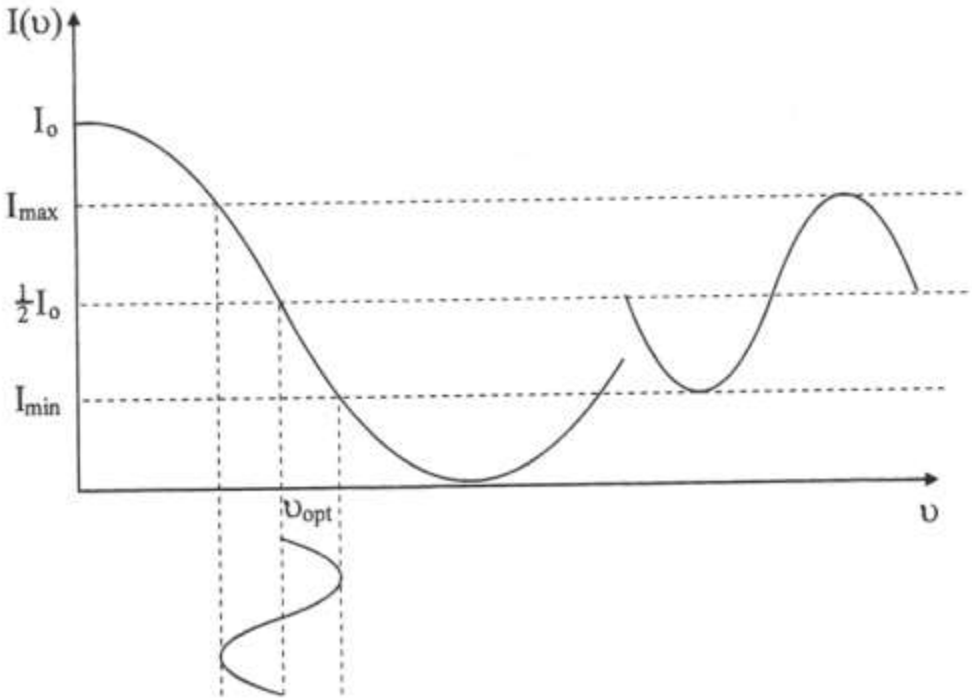
Optical modulator

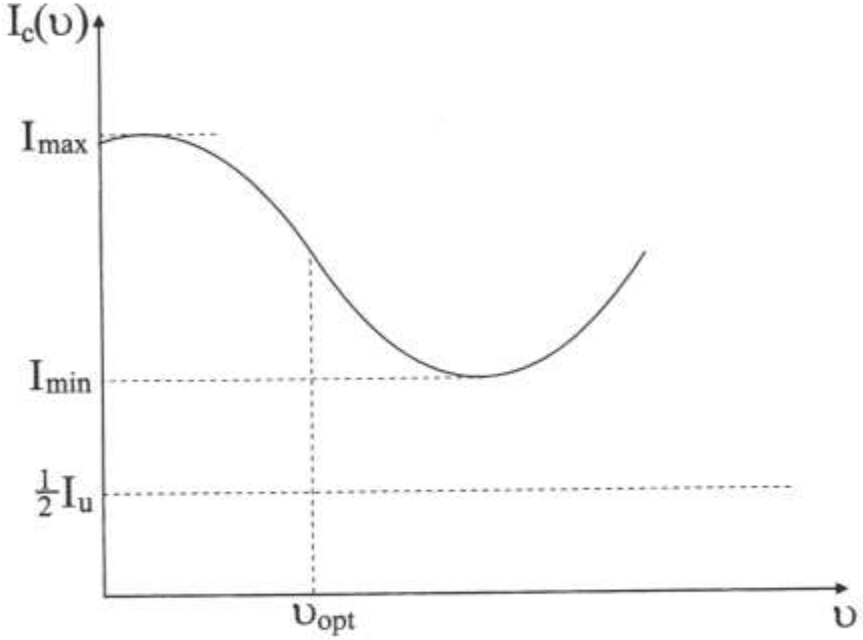


$$I = I_0 \cos^2 \upsilon$$

$$m = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Optical modulator





$$I = I_0 \cos^2 \upsilon$$

$$m = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Optical circulator