

WYKŁAD ①

Operatory:

$$\vec{\nabla} = \vec{i}_{e_x} \frac{\partial}{\partial x} + \vec{j}_{e_y} \frac{\partial}{\partial y} + \vec{k}_{e_z} \frac{\partial}{\partial z}$$

$$\vec{E} = [E_x, E_y, E_z]$$

Gradient

$$\text{grad } \varphi = \vec{\nabla} \varphi = \vec{i} \frac{\partial \varphi}{\partial x} + \vec{j} \frac{\partial \varphi}{\partial y} + \vec{k} \frac{\partial \varphi}{\partial z}$$

Mnożenie wektora przez wektor (dywergencja - rozbieżność wektora)

$$\text{div } \vec{E} = \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

(notacja - winowość wektora)

$$\text{rot } \vec{E} = \vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \vec{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_y & E_z \end{vmatrix} - \vec{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ E_x & E_z \end{vmatrix} + \vec{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ E_x & E_y \end{vmatrix}$$

Wykorzystując własność: $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}$

RÓWNANIA MAXWELLA (w wolnej przestrzeni):

$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$
$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$
$\vec{\nabla} \cdot \vec{D} = 0$
$\vec{\nabla} \cdot \vec{B} = 0$

Prawo Biota-Savarda (Ampere'a) $\oint_C \vec{H} d\vec{l} = \iint_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$

Prawo Faraday'a $\oint_C \vec{E} d\vec{l} = - \iint_S \frac{\partial \vec{B}}{\partial t} d\vec{S}$

Prawa Gaussa $\iint \vec{D} d\vec{S} = q$ $\vec{\nabla} \cdot \vec{j} = \frac{\partial \rho}{\partial t}$

$\iint \vec{B} d\vec{S} = 0$

Równania materiałowe

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\epsilon_0 = 8,85 \cdot 10^{-12} \frac{F}{m} = \frac{1}{36\pi} \cdot 10^{-9} \frac{F}{m}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$$

$$Z = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120 \pi \Omega \approx 376,6 \Omega \quad \text{Impedancja próżni}$$

$$i \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \vec{E}$$

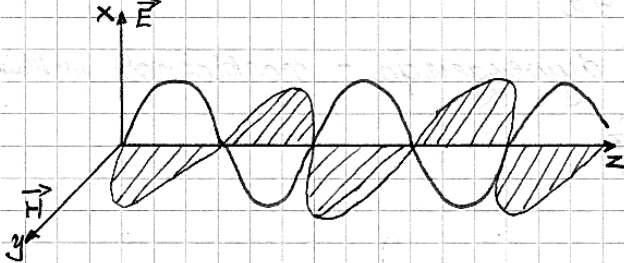
$$\begin{cases} \nabla^2 \vec{E} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2 \vec{H} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \end{cases} \text{ RÓWNANIA FALOWE}$$

Rozwiązanie równania falowego:

$$\vec{E} = \vec{E}_0 e^{i\omega t - \gamma z}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\gamma = i\omega \sqrt{\epsilon_0 \mu_0'} = \alpha + i\beta$$



$$\vec{H} = \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{e}_z \times \vec{E}$$

$$\Delta \omega \ll \omega$$

Długość koherencji:

$$\frac{\lambda^2}{\Delta \lambda} = L$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \cdot 10^8 \frac{m}{s}$$

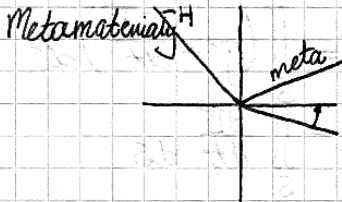
$$n^2 = \frac{c^2}{v^2} = \frac{\epsilon \mu \epsilon_0 \mu_0}{\epsilon_0 \mu_0}$$

Niekiedy piszą: $c^2 = \frac{1}{\epsilon_0 \mu_0}$

W swobodnej przestrzeni \vec{E} i \vec{H} są zawsze ze sobą w fazie.

$$\epsilon^* = \epsilon' - i\epsilon''$$

$$\mu^* = \mu' - i\mu''$$



$$\vec{E} = e^{-\alpha z} e^{i(\omega t - \beta z)}$$

Prędkość faszowa (powierzchnia stałej fazy):

$$v_f = \frac{dz}{dt} = \frac{\omega}{k}$$

Prędkość grupowa (transport energii)

$$v_g = \frac{d\omega}{dk} = \frac{v_f}{1 + \frac{\omega}{v} \frac{dn}{d\omega}}$$

Równania Maxwella

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

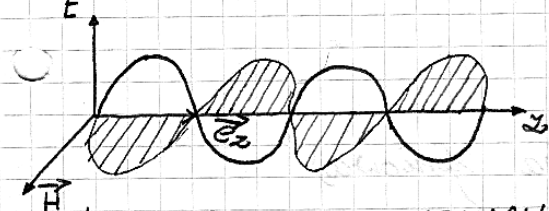
$$\vec{B} = \mu \mu_0 \vec{H}$$

$$\vec{D} = \epsilon \epsilon_0 \vec{E}$$

Rozwiązanie (fala monochromatyczna):

$$\vec{E} = \vec{E}_0 e^{i\omega t - \gamma z}$$

$$\gamma = i\omega \sqrt{\epsilon \mu} = \alpha + i\beta$$



$$\vec{H} = \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{e}_z \times \vec{E}$$

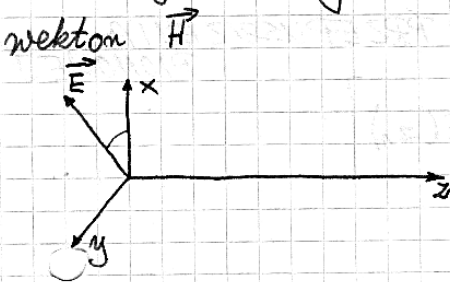
$$\epsilon^* = \epsilon' - i\epsilon''$$

$$n = \pm \sqrt{\epsilon^* \mu^*}$$

$$\mu^* = \mu' - i\mu''$$

OPIS ŚWIATKA SPOLARYZOWANEGO

Plaszczyzna drgań wektora $\vec{E} \Leftrightarrow$ polaryzacja przenoszona przez wektor \vec{H}



$$\vec{E}_x = E_{0x} e^{i(\omega t - kz + \delta_x)}$$

$$\vec{E}_y = E_{0y} e^{i(\omega t - kz + \delta_y)}$$

δ_y - przesunięcie fazy

$$\Delta \delta = \delta_y - \delta_x$$

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - \frac{2E_x E_y}{E_{0x} E_{0y}} \cos \Delta \delta = \sin^2 \Delta \delta$$

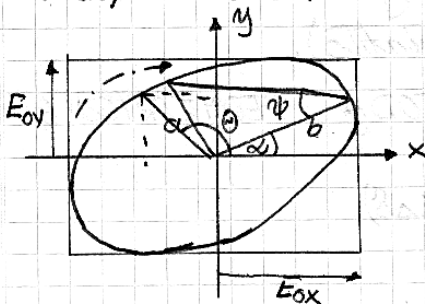
RÓWNANIE ELIPSY

$$a^2 + b^2 = E_{0x}^2 + E_{0y}^2$$

$$\psi = \arctg(b/a)$$

$0 < \Delta \delta < \pi$ POLARYZACJA PRAWOSKRĘTNA

$\pi < \Delta \delta < 2\pi$ POLARYZACJA LEWOSKRĘTNA



WEKTOR JONASA

$$\begin{bmatrix} \vec{E}_x \\ \vec{E}_y \end{bmatrix} = \begin{bmatrix} \vec{E}_{0x} e^{i(\omega t - kz + \delta_x)} \\ \vec{E}_{0y} e^{i(\omega t - kz + \delta_y)} \end{bmatrix} \Rightarrow E(z_0) = \begin{bmatrix} E_{0x} \exp(i\delta_x) \\ E_{0y} \exp(i\delta_x) \end{bmatrix} = \begin{bmatrix} E_{0x} \\ E_{0y} \exp(i\Delta \delta) \end{bmatrix}$$

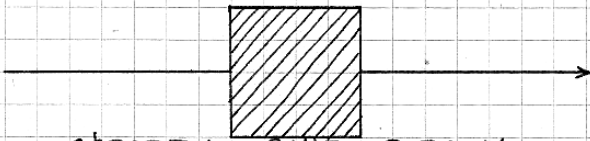
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

POLARYZACJA

POZIOMA

$$\begin{bmatrix} 1 \\ i \end{bmatrix}, \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

MACIERZ JONESA



OŚRODEK ANIZOTROPOWY

$$\begin{bmatrix} E(z_1) \\ E(z_2) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} \begin{bmatrix} E(z_1) \\ E(z_2) \end{bmatrix}$$

$$\begin{bmatrix} E_x(z_1) \\ E_y(z_2) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} E_x(z_1) \\ E_y(z_2) \end{bmatrix}$$

MACIERZ JONESA

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

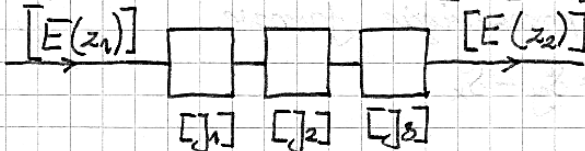
PKYTKI FAZOWE (np.: Emienćfaldmka liniowa)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\delta/4} & 0 \\ 0 & e^{-i\delta/4} \end{bmatrix}$$

POLARYZATOR
LINIOWY
0 OSI
PRZEPUSZCZANIA
POZIOMEJ

POLARYZATOR LINIOWY 0 OSI PRZEPUSZCZANIA
PIONOWEJ

$$\begin{bmatrix} E(z_2) \end{bmatrix} = \begin{bmatrix} J_3 \\ J_2 \\ J_1 \end{bmatrix} \begin{bmatrix} E(z_1) \end{bmatrix}$$



Metoda Jonesa sprawdza się przy opisie zjawisk i doświadczeniach z laserem He-Ne (monochromatyczne światło).

POWIĄZANIE METODY JONESA Z METODĄ GEOMETRYCZNĄ

$$\tan 2\alpha = \frac{2\text{Re}(\chi)}{1 - |\chi|^2}$$

$$\chi = \frac{E_{0y}}{E_{0x}} \exp(i\delta)$$

$$\sin 2\psi = \frac{2\text{Im}(\chi)}{1 - |\chi|^2}$$

Dla $\Delta\omega \ll \omega$ metoda Jonesa jest spełniona.

MACIERZ KOHERENCJI

$$[K] = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix}$$

$$J_{xy} = J_{yx}^*$$

Skłócenie światła opisywane przez koherencje (macierz):

$$I(\theta, \Delta\delta) = \sqrt{\frac{\epsilon_0}{\mu_0}} \langle E(t, \theta, \Delta\delta) E^*(t, \theta, \Delta\delta) \rangle =$$

$$= + 2 \sqrt{J_{xx} J_{yy}} \cos \theta |\mu_{xy}| \cos(\beta_{xy} - \Delta\delta) + J_{yy} \sin^2 \theta + J_{xx} \cos^2 \theta$$

$$\mu_{xy} = |\mu_{xy}| \exp(i\beta_{xy}) = \frac{J_{xy}}{\sqrt{J_{xx}} \sqrt{J_{yy}}}$$

$$P = \frac{I_{\text{SPOL}}}{I_{\text{CAŁ}}} = \frac{I_{\text{MAX}} - I_{\text{MIN}}}{I_{\text{MAX}} + I_{\text{MIN}}} = \sqrt{1 - \frac{4 \det [K]}{[\text{Tr} [K]]^2}} = P \leq 1$$

$$a = \frac{1}{2} \arctg \left(\frac{J_{xy} + J_{yx}}{J_{xx} - J_{yy}} \right)$$

$$\frac{b}{a} = \left| \text{tg} \left\{ \frac{1}{2} \arcsin \left[\frac{i(J_{yx} - J_{xy})}{(J_{xx} - J_{yy})P} \right] \right\} \right|$$

Skłócenie światła po przejściu przez analizator i ćwiertkfabrykę:

$$J_{xx} = \sqrt{\frac{\mu_0}{\epsilon_0}} I(0, 0^\circ) \quad J_{xy} = J_{yx}^*$$

$$J_{yy} = \sqrt{\frac{\mu_0}{\epsilon_0}} I(0, 90^\circ)$$

$$J_{xy} = \sqrt{\frac{\mu_0}{\epsilon_0}} \left\{ \frac{1}{2} [I_{0,45} - I_{0,135}] + \frac{1}{2} i [I_{\pi/4,45} - I_{\pi/4,135}] \right\} =$$

$$= + 2 \sqrt{J_{xx} J_{yy}} \cos \theta |\mu_{xy}| \cos(\beta_{xy} - \Delta\delta) + J_{yy} \sin^2 \theta + J_{xx} \cos^2 \theta$$

Transformacja macierzy koherencji przy przejściu przez otwór

$$[K^{WY}] = [J][K^{WE}][J^*]$$

WYKŁAD ③

7.03.2007

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\frac{\sqrt{2}}{2} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

POLARYZACJE
KOŁOWE

$$\begin{bmatrix} E_{xWY} \\ E_{yWY} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} E_{xWE} \\ E_{yWE} \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$[K] = \langle [E] \otimes [E]^T \rangle$$

$$E^T = [E_x^*, E_y^*]$$

$$[W] = \begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{yx} \\ J_{yy} \end{bmatrix} = \langle [E] \otimes [E^*] \rangle$$

WEKTOR STOCKESA

$$[S] = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} J_{xx} + J_{yy} \\ J_{xx} - J_{yy} \\ J_{xy} + J_{yx} \\ i(J_{yx} - J_{xy}) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{bmatrix} \begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{yx} \\ J_{yy} \end{bmatrix}$$

$$S_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{\text{całk}}$$

$$P = \frac{I_{\text{sp}}}{I_{\text{całk}}} = \sqrt{1 - \frac{4 \det [K]}{(J_{xx} + J_{yy})^2}} = \sqrt{\frac{S_1^2 + S_2^2 + S_3^2}{S_0^2}}$$

$$\alpha = \frac{1}{2} \arctg \left(\frac{S_2}{S_1} \right)$$

$$\frac{b}{a} = \left(\operatorname{tg} \left\{ \frac{1}{2} \arcsin \left(\frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}} \right) \right\} \right)$$

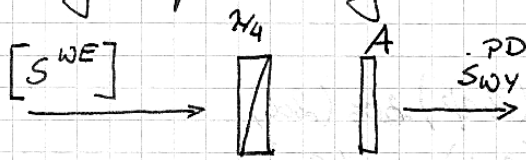
$$[S] = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \sqrt{\frac{\mu_0}{\epsilon_0}} \begin{bmatrix} I(0,0^\circ) + I(0,90^\circ) \\ I(0,0^\circ) - I(0,90^\circ) \\ I(0,45^\circ) - I(0,135^\circ) \\ I(2,4,45^\circ) - I(2,4,135^\circ) \end{bmatrix} = \sqrt{\frac{\mu_0}{\epsilon_0}} \begin{bmatrix} I(0,0^\circ) + I(0,90^\circ) \\ I(0,0^\circ) - I(0,90^\circ) \\ 2I(0,45^\circ) - S_0 \sqrt{\frac{\epsilon_0}{\mu_0}} \\ I(0,0^\circ) + I(0,90^\circ) - 2I(2,4,45^\circ) \end{bmatrix}$$

METODA STOCKESA - MÜLLERA

$$[S_{WY}] = [M] [S_{WE}]$$

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ & & & \\ & & & \\ & & & M_{44} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

W praktyce pomiarowej:



$$[S_{WV}^{PD}]_1 = [M_A][M_{\lambda/4}][S^{WE}]$$

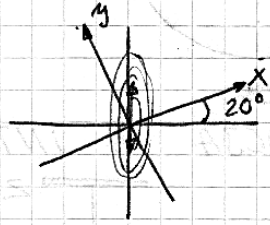
$$[S^{PD}]_2 = [M_{90}][S]$$

$$[S^{PD}]_3 = [M_{45}][S]$$

$$[S^{PD}]_4 = [M_{135}][S]$$

$$[S^{PD}]_5 = [M_{45}][M_{\lambda/4}][S]$$

$$[S^{PD}]_6 = [M_{135}][M_{\lambda/4}][S]$$



$$S_0^{PD}|_1 = \frac{1}{2}(S_0 + S_1) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{0,0^\circ}$$

$$S_0^{PD}|_2 = \frac{1}{2}(S_0 - S_1) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{0,90^\circ}$$

$$S_0^{PD}|_3 = \frac{1}{2}(S_0 + S_2) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{0,45^\circ}$$

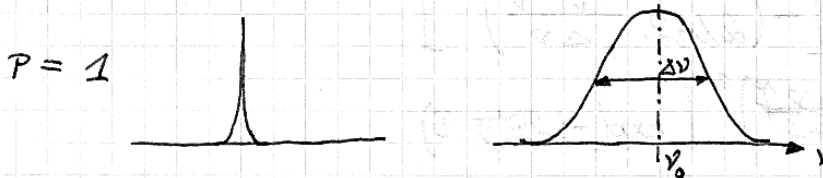
$$S_0^{PD}|_4 = \frac{1}{2}(S_0 - S_2) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{0,135^\circ}$$

$$S_0^{PD}|_5 = \frac{1}{2}(S_0 + S_3) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{\lambda/4,45^\circ}$$

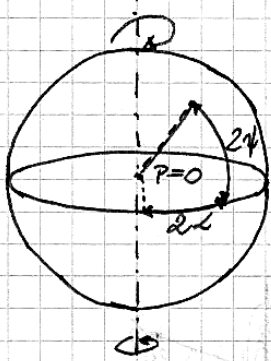
$$S_0^{PD}|_6 = \frac{1}{2}(S_0 - S_3) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{\lambda/4,135^\circ}$$

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = [S^{PD}] = \begin{bmatrix} S_0^{PD} \\ S_1^{PD} \\ S_2^{PD} \\ S_3^{PD} \end{bmatrix} = \sqrt{\frac{\mu_0}{\epsilon_0}} \begin{bmatrix} I_{(0,0^\circ)} + I_{(0,90^\circ)} \\ I_{(0,0^\circ)} - I_{(0,90^\circ)} \\ I_{(0,45^\circ)} - I_{(0,135^\circ)} \\ I_{(\lambda/4,45^\circ)} - I_{(\lambda/4,135^\circ)} \end{bmatrix} = \begin{bmatrix} S_0^{PD}|_1 + S_0^{PD}|_2 \\ S_0^{PD}|_1 - S_0^{PD}|_2 \\ S_0^{PD}|_3 - S_0^{PD}|_4 \\ S_0^{PD}|_5 - S_0^{PD}|_6 \end{bmatrix}$$

ROZCIĄGŁOŚĆ WIDMA ŚWIĄTKA



SFERA POINCARÉ' A



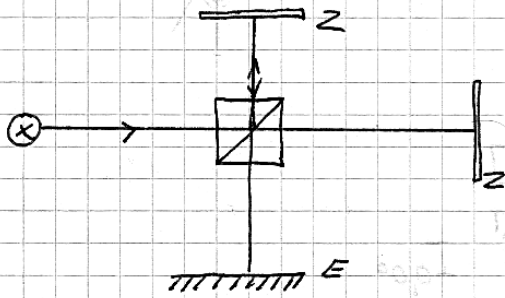
$$\psi = \arctg \frac{b}{a}$$

$$S_1 = S_0 \cos(2\psi) \cos(2\alpha)$$

$$S_2 = S_0 \cos(2\psi) \sin(2\alpha)$$

$$S_3 = S_0 \sin 2\psi$$

POLARYZACJA A INTERFERENCJA



$$E(t) = E_1(t) + E_2(t) = E_1(t) + E_1(t + \tau)$$

$$\tau = \frac{2\Delta L}{c}$$

$$I = \sqrt{\frac{\epsilon_0}{\mu_0}} \langle EE^* \rangle = \sqrt{\frac{\epsilon_0}{\mu_0}} \langle (E_1 + E_2)(E_1 + E_2)^* \rangle = I_1 + I_2 + 2\sqrt{\frac{\epsilon_0}{\mu_0}} \text{Re} \{ \langle E_1^* E_2 \rangle \}$$

$$\Gamma(\tau) = \langle E_1^*(t) E_1(t + \tau) \rangle = \lim_{T_m \rightarrow \infty} \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} E_1^*(t) E_1(t + \tau) dt$$

$$\gamma(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)} = |\gamma(\tau)| e^{i\beta_{12}\tau}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} |\gamma(\tau)| \cos(\beta_{12}\tau - \Delta\delta)$$

$$\Gamma(\tau) = \int_0^\infty 4G(\nu) \exp(-i2\pi\nu\tau) d\nu$$

$$\hat{G}(\nu) = \frac{G(\nu)}{\int_0^\infty G(\nu) d\nu}$$

$$\hat{G}(\nu) = 0 \quad \nu \leq 0$$

$$\gamma(\tau) = \int_0^\infty \hat{G}(\nu) \exp(-i2\pi\nu\tau) d\nu$$

Funkcje widmowej gęstości mocy:

- Rozkład Gaussowski

$$\hat{G}(\nu) = \frac{2\sqrt{\ln 2}}{\sqrt{\pi} \Delta\nu} \exp\left[-\left(2\sqrt{\ln 2} \frac{\nu - \bar{\nu}}{\Delta\nu}\right)^2\right]$$

$$\gamma_G(\tau) = \exp\left[-\left(\frac{\pi\Delta\nu\tau}{2\sqrt{\ln 2}}\right)^2\right] \exp(-i2\pi\bar{\nu}\tau)$$

$$\hat{G}(\nu) = \frac{2\sqrt{\ln 2} (\Delta\nu)^{-1}}{1 + 2\left(\frac{\nu - \bar{\nu}}{\Delta\nu}\right)^2}$$

- Rozkład Lorentzowski

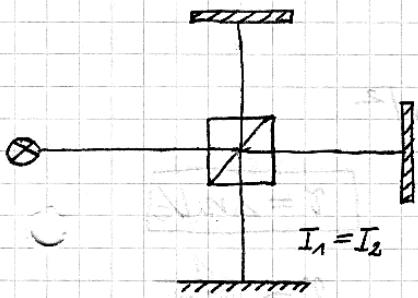
$$\gamma_L(\tau) = \exp(-\pi\nu_0|\tau|) \exp(-i2\pi\bar{\nu}\tau)$$

$$I(\theta, \Delta\delta) = \frac{1}{\mu_0} \langle E(t, \theta, \Delta\delta) E^*(t, \theta, \Delta\delta) \rangle =$$

$$= J_{xx} \cos^2 \theta + J_{yy} \sin^2 \theta + 2 \sqrt{J_{xx} J_{yy}} \cos \theta |\mu_{xy}| \cos(\beta_{xy} + \Delta\delta)$$

$$\mu_{xy} = |\mu_{xy}| \exp(i\beta_{xy}) = \frac{J_{xy}}{\sqrt{J_{xx} J_{yy}}}$$

$$P = \frac{I_{SPOL}}{I_{CAŁK}} = \sqrt{1 - \frac{4 \det[K]}{(\text{Tr}[K])^2}}$$



$$K(\nu) = \frac{I_{\max}(\nu_1) - I_{\min}(\nu_2)}{I_{\max}(\nu_1) + I_{\min}(\nu_2)}$$

$\Delta\omega \ll \omega$ - dla fali quasimonochromat.

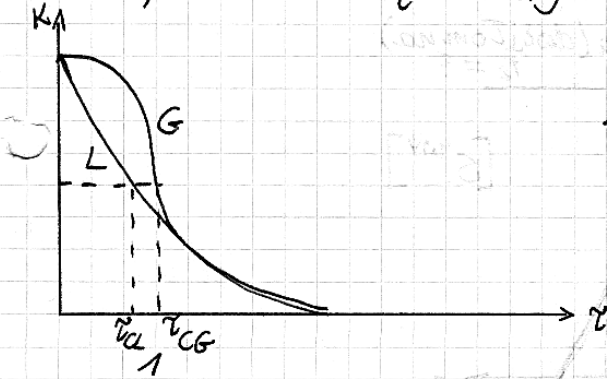
$$\text{Re}\{\Gamma(\nu_1)\} = |\Gamma(\nu)|$$

$$\text{Re}\{\Gamma(\nu_2)\} = |\Gamma(\nu)|$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} |\chi(\nu)| \cos(\beta_{12} - \Delta\delta)$$

$$I_{\max}(\nu_1) = 2I + 2\text{Re}\{|\Gamma(\nu_1)|\} = 2I_1 + 2|\Gamma(\nu)|$$

$$I_{\min}(\nu_2) = 2I - 2\text{Re}\{|\Gamma(\nu_2)|\} = 2I_1 - 2|\Gamma(\nu)|$$



$$\Delta L = c \nu_c = \frac{c^2}{\Delta\lambda} = \frac{\lambda^2}{\Delta\lambda}$$

$$\nu_{CL} = \frac{1}{\delta \Delta\nu}$$

$$\nu_{CG} = \frac{2.2/\ln 2}{\delta \Delta\nu}$$

Nie interferują fale spolaryzowane prostopadle !!!

$\lambda[\mu\text{m}]$	$\Delta\lambda[\mu\text{m}]$	ΔL	
550	750	$\sim 1\mu\text{m}$	Światło nat.
850	$30 \div 50\mu\text{m}$	$24 \div 15\mu\text{m}$	LED
1300	$50 \div 100\mu\text{m}$	$34 \div 17\mu\text{m}$	
850	150	$5\mu\text{m}$	SLD
1300	60	$28\mu\text{m}$	
670	$0.2 \div 2$	$0.2 \div 3\text{mm}$	LD
1300	< 2	$1\text{mm} \div 50\text{m}$	
1550	< 2		
632.8	$\ll 0.1$	$0.1 - 10\text{m}$	He-Ne

Zespolony stopień koherencji w zjawisku polaryzacji jest tożsamy z zespolonym stopniem koherencji w zjawisku interferencji:

$$|\mu_{xy}| = |\gamma(\nu)|$$

$$\gamma_L = \exp(-\pi \Delta\nu \nu)$$

Wtedy można napisać:

$$P = \frac{I_{SPOL}}{I_{CAEK}} = \sqrt{1 - \frac{4 \det[K]}{(\text{Tr}[K])^2}} = \sqrt{1 - \frac{4(J_{xx}J_{yy} - J_{xy}J_{yx})}{(J_{xx} + J_{yy})^2}}$$

Macierz hermitowska:

$$J_{yx} = J_{xy}^*$$

$$J_{xy} J_{yx} = |J_{xy}|^2$$

$$\mu_{xy} = |\mu_{xy}| \exp(i\beta_{xy}) = \frac{J_{xy}}{\sqrt{J_{xx}} \sqrt{J_{yy}}}$$

$$\nu = \Delta n / c$$

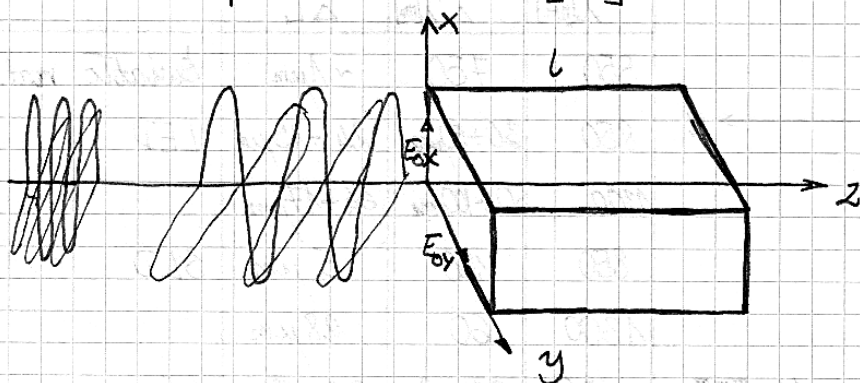
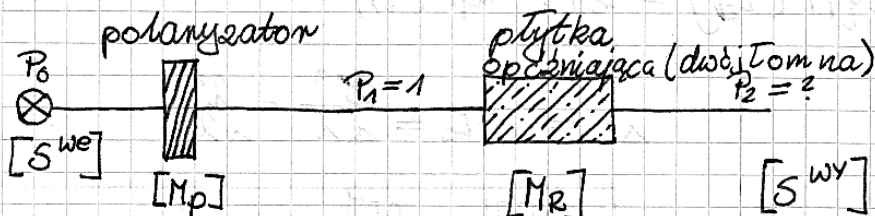
$$\nu_c = \frac{1}{\pi \Delta\nu}$$

$$\nu_c = \Delta L / c$$

$$P = \sqrt{1 - \frac{4(1 - |\mu_{xy}|^2)}{\left(\frac{E_{ox}}{E_{oy}} + \frac{E_{oy}}{E_{ox}}\right)^2}} \Rightarrow$$

l - droga optyczna w osmiodku

$$P_G = \sqrt{1 - \frac{4 \left[1 - \exp\left(-\frac{2\Delta n l}{\Delta L}\right) \right]}{\left(\frac{E_{ox}}{E_{oy}} + \frac{E_{oy}}{E_{ox}}\right)^2}} = \exp\left(-\frac{\Delta n l}{\Delta L}\right)$$



$$[S^{WY}] = [M_R][M_P][S^{WE}]$$

[D] macierz depolaryzacji

$$\frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$[S^{wy}] = [D][M_R][M_P][S^{we}]$$

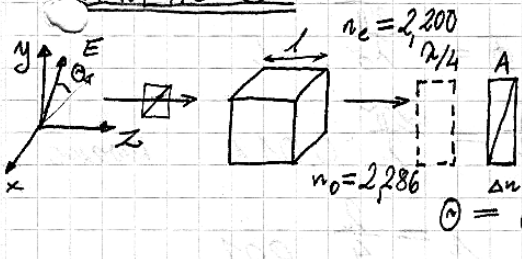
POSTAC MACIERZY DEPOLARYZACJI

$$[D] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c & 0 \\ 0 & 0 & 0 & P_c \end{bmatrix}$$

$$P_c = \sqrt{1 - 4 \dots \frac{\Delta n L}{\Delta L}}$$

WYKŁAD 5

21.03.2007



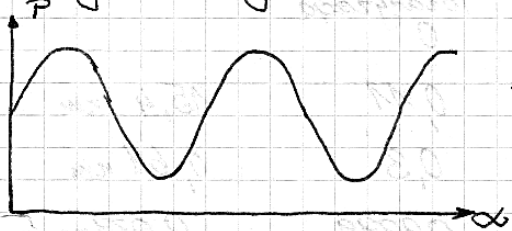
$$P_L = \sqrt{1 - [1 - \exp(-\frac{2\Delta n L}{\Delta L})] \sin^2 2\theta}$$

$$\theta = \arctg\left(\frac{E_{oy}}{E_{ox}}\right)$$

$$P_{Lmin} = \exp\left(-\frac{\Delta n L}{\Delta L}\right)$$

Obracając kryształem otrzymujemy:

$$\Delta L = \frac{\lambda^2}{\Delta n}$$



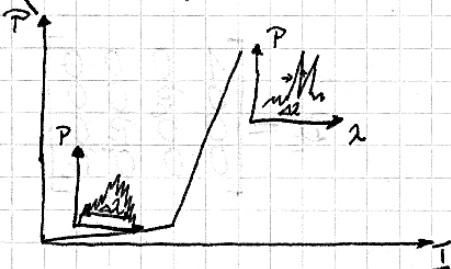
$$P_{Gmin} = \exp\left[-2\left(\frac{\Delta n L}{2\lambda \ln 2 \Delta L}\right)^2\right]$$

$$P_{Lmin} = P_{Gmin}$$

$$\frac{\Delta n L}{4 \ln 2 \Delta L} = 1$$

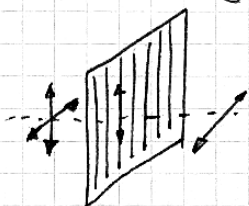
$$\frac{\Delta n L}{\Delta L} = 4 \ln 2 = 2,77$$

$$\left(\frac{E_{ox} + E_{oy}}{E_{ox}} + \frac{E_{oy}}{E_{ox}}\right)^2 = \frac{4}{\sin^2 2\theta}$$



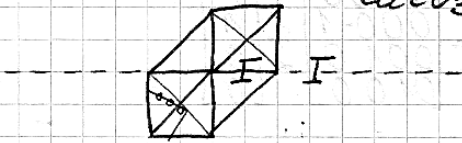
Czy analizator depolaryzuje nam światło?

- Folie dichroiczne

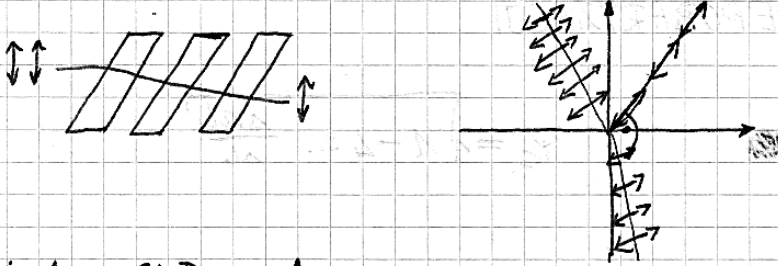


- Polaryzator prostokątny (najlepsze)

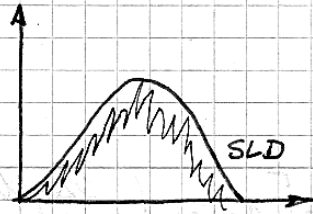
CaCO_3



- Zespół płytek nmiesszonych pod kątem Brewstera



Diody SLD



$$\Phi = \frac{2\pi}{\lambda_0} \Delta n L$$

$$\frac{\Phi}{2} = \frac{2\pi}{\lambda} 0,011 \text{ kwang}$$

$$n = \frac{2}{4} 0,01$$

Dla Lorentzowskiego:

$$\Delta n = -\frac{n^2}{\Delta n L} \ln P_{\text{min}L}$$

Lampa halogenowa $\Delta \lambda = 750 \text{ nm}$

SLD

$$\Delta \lambda = 17,6$$

0,11

15,4 nm

LD

$$\Delta \lambda = 0,44$$

0,3

0,61 nm

Laser He-Ne

$$\Delta \lambda = 0,04$$

0,9999

0,039 nm

$[S] \begin{matrix} \lambda/4 \\ A \end{matrix} [S^{PD}] \neq$

Polaryzacja
0

$$[S^{PD}]_5 = [M_{45}] [D_c] [M_{\lambda/4}] [S]$$

$$[S^{PD}]_6 = [M_{135}] [D_c] [M_{\lambda/4}] [S]$$

$$S_0^{PD}|_5 = \frac{1}{2} (S_0 + P_c S_3) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{\lambda/4, 45^\circ}$$

$$S_0^{PD}|_6 = \frac{1}{2} (S_0 - P_c S_3) = \sqrt{\frac{\mu_0}{\epsilon_0}} I_{\lambda/4, 135^\circ}$$

$$D_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c & 0 \\ 0 & 0 & 0 & P_c \end{bmatrix}$$

$$[S^{PD}] = \begin{bmatrix} S_0^{PD} \\ S_1^{PD} \\ S_2^{PD} \\ S_3^{PD} \end{bmatrix} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ P_c S_3 \end{bmatrix}$$

$$P = \sqrt{\frac{S_1^2 + S_2^2 + P_c^2 S_3^2}{S_0^2}}$$

$$B = \frac{1}{2} \arctg \frac{S_2}{S_1} \quad \beta - \text{azymut}$$

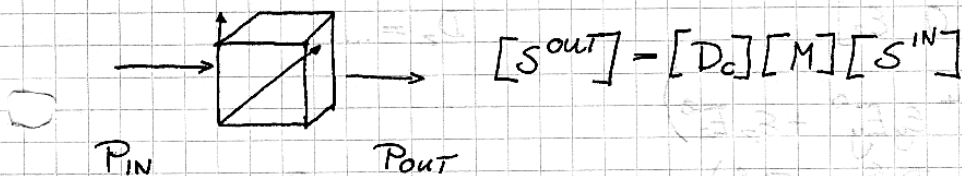
$$\left(\frac{b}{a}\right)_{\text{amiera}} = \left| \operatorname{tg} \left\{ \frac{1}{2} \arcsin \left(\frac{P_c S_3}{\sqrt{S_1^2 + S_2^2 + P_c^2 S_3^2}} \right) \right\} \right| \neq \left(\frac{b}{a}\right)_{\text{reca}}$$

$\stackrel{= P S_0}{\text{}}$

$\Delta L = \frac{\lambda^2}{\Delta \lambda}$	$L_S = \Delta n l$ 1 mm $\Delta n = 0,01$	P_c
Halogen $\sim 1 \mu\text{m}$	$10 \mu\text{m}$	~ 0
LED $\sim 20 \mu\text{m}$	$10 \mu\text{m}$	0,91
SLD $5 \div 10 \mu\text{m}$	— —	$0,24 \div 0,40$
LD $1 \text{mm} \div 100 \mu\text{m}$	— —	$0,99 \div 1$
He-Ne $100 \text{mm} \div 100 \text{m}$	— —	~ 1

WYKŁAD 6

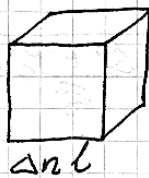
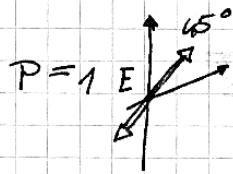
28.03.2007



$$\begin{bmatrix} S_0^{\text{out}} \\ S_1^{\text{out}} \\ S_2^{\text{out}} \\ S_3^{\text{out}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c & 0 \\ 0 & 0 & 0 & P_c \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \frac{2\pi \Delta n l}{\lambda} & \sin \frac{2\pi \Delta n l}{\lambda} \\ 0 & 0 & -\sin \frac{2\pi \Delta n l}{\lambda} & \cos \frac{2\pi \Delta n l}{\lambda} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{\text{out}} = \sqrt{\frac{P_c^2}{1}} \begin{bmatrix} 1 \\ P_c \\ 0 \\ 0 \end{bmatrix} \quad \text{dla} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

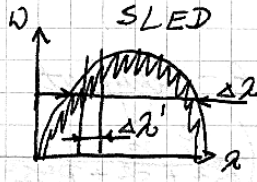
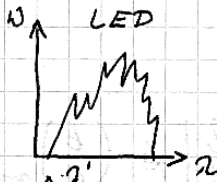
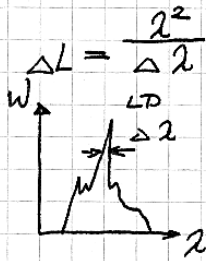
$$P_{\text{out}} = P_c \begin{bmatrix} 1 \\ 0 \\ P_c \cos \\ -P_c \sin \end{bmatrix} \quad \text{dla} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ \cos \\ -\sin \end{bmatrix}$$



$$P_c < P$$

$$P_{c\min L} = \exp\left(-\frac{\Delta n L}{\Delta L}\right)$$

$$P_{c\min G} = \exp\left[-\left(\frac{\Delta n L}{2n/m^2 \Delta L}\right)^2\right]$$



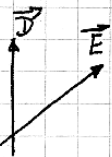
SLD
Lampa halogenowa



Aby obliczyć szerokość widma:

$$\Delta \lambda_L = -\frac{\lambda}{\Delta n L} \ln P_{out\min}$$

$$\Delta \lambda_G = \frac{\lambda^2 2 \Delta n^2}{\Delta n L} \sqrt{\ln P_{out\min}}$$



$$\vec{D} = \hat{\epsilon} \epsilon_0 \vec{E}$$

$$\epsilon_{ij} = \epsilon_{ji}$$

$$D_x = \epsilon_0 (\epsilon_{xx} E_x + \epsilon_{xy} E_y + \epsilon_{xz} E_z)$$

$$D_y = \dots$$

$$D_z = \dots$$

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

$$W = \frac{1}{2} \epsilon_0 (\epsilon_x E_x^2 + \epsilon_y E_y^2 + \epsilon_z E_z^2)$$

$$W = \frac{1}{2 \epsilon_0} \left(\frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z} \right)$$

$$x = \frac{D_x}{\sqrt{2 \epsilon W}}$$

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

$$\epsilon = n^2$$

$$n_x = n_y = n_z$$

ośmiodki bezpostaciowe (szkło) + kryształy regularne

$$n_x = n_y = n_o$$

$$n_z = n_e$$

$$n_o > n_e$$

Heksagonalny
Trójkątny
Tetragonalny

kryształy jednoosiowe ujemne: kalcyt (CaCO3),
niobian litu (LiNbO3),
korund (Al2O3),
ADP, KDP

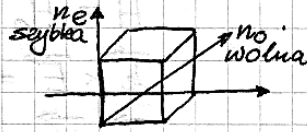
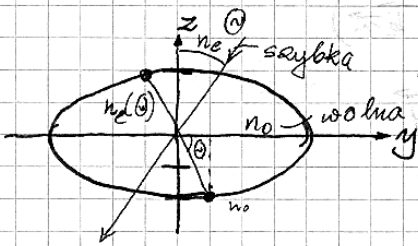
$$n_o < n_e$$

$$n_x \neq n_y \neq n_z$$

SiO₂, TiO₂, H₂O, MgF, ZnO₂SiO₂

BaSO₄, CaSO₄, Topaz

Trojkatny (trójosiowy), jednoskowy, rombowy



$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_o^2} = 1$$

Równanie płaszczyzny prostej k

$$x = \operatorname{tg}\left(\frac{\pi}{2} + \frac{\pi}{2} \cdot \theta\right) y$$

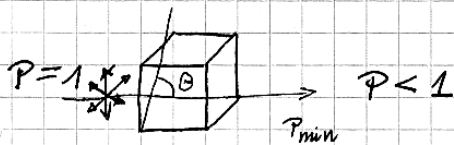
$$x = \operatorname{tg}\left(\frac{\pi}{2} - \theta\right) y$$

$$\begin{cases} \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \\ x = -\operatorname{tg} \theta y \end{cases}$$

$$\frac{y^2}{n_o^2} + \frac{\sin^2 \theta}{\cos^2 \theta} = 1$$

$$n_e^2(\theta) = \frac{y^2}{\cos^2 \theta}$$

$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2 \theta}{n_e^2} + \frac{\cos^2 \theta}{n_o^2}$$



$$P_c = f(\Delta n')$$

$$\Delta n' = n_o - n_e(\theta)$$

Znajdowanie osi optycznej za pomocą pomiaru polaryzacji w trzech różnych płaszczyznach:

$$P_{x \min L} = \exp\left(-\frac{\Delta n'_x l}{\Delta L}\right)$$

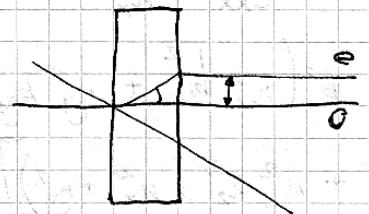
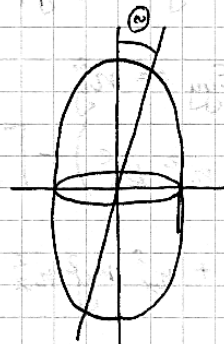
$$P_{y \min L} = \exp\left(-\frac{\Delta n'_y l}{\Delta L}\right)$$

$$P_{z \min L} = \exp\left(-\frac{\Delta n'_z l}{\Delta L}\right)$$

$$\Delta n_i = \frac{\Delta L}{l} \ln P_{i \min}$$

$$\Delta n = n_o - n_e(\theta_i)$$

$$\theta_i = \arcsin \left[\frac{n_e}{n_e(\theta_i)} \sqrt{\frac{n_o^2 - n_e^2}{n_o^2 - n_e^2}} \right]$$



$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

$$\vec{E} = \vec{E}_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

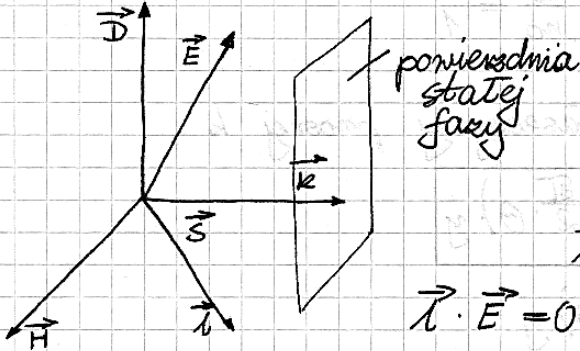
$$\vec{H} = \vec{H}_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

$$\vec{k} \times \vec{H} = -\omega \vec{D}$$

$$\vec{k} \times \vec{E} = \mu_0 \omega \vec{H}$$

$$\vec{k} \cdot \vec{D} = 0$$

$$\vec{k} \cdot \vec{H} = 0$$



$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{l} \cdot \vec{k} = 1$$

$$\vec{l} \times (\vec{k} \times \vec{H}) = -\omega (\vec{l} \times \vec{D})$$

$$\vec{l} \times (\vec{l} \times \vec{E}) = \mu_0 \omega (\vec{l} \times \vec{H})$$

$$\vec{l} \cdot \vec{E} = 0$$

$$\vec{l} \cdot \vec{H} = 0$$

$$\vec{l} \cdot \vec{k} = 1$$

Własności optyczne ośmiodka anizotropowego:

$$\vec{H} = \omega (\vec{l} \times \vec{D})$$

$$\vec{E} = -\omega \mu_0 (\vec{l} \times \vec{H})$$

$$n \neq f(E)$$

$$\vec{k} \times (\vec{k} \times \vec{E}) = \mu_0 \omega (\vec{k} \times \vec{H}) = -\mu_0 \omega^2 \vec{D}$$

$$(\vec{k} \cdot \vec{E}) \vec{k} - k^2 \vec{E} = -\mu_0 \omega^2 \vec{D}$$

$$k^2 E_i - (k_j E_j) k_i = \mu_0 \omega^2 \epsilon_0 \epsilon_{ij} E_j$$

$$E_i = \delta_{ij} E_j$$

$$\epsilon_{ij} = \delta_{ij} \epsilon_{ii}$$

$$\mu_0 \omega^2 \epsilon_0 = \frac{\omega^2}{c^2} = k_0^2$$

$$(k^2 \delta_{ij} - k_i k_j - k_0^2 \delta_{ij} \epsilon_{ii}) E_j = 0$$

$$\det | (k^2 - k_0^2 \epsilon_{ii}) \delta_{ij} - k_i k_j | = 0$$

$$\epsilon_{xx} = n_x^2$$

$$\epsilon_{yy} = n_y^2$$

$$\epsilon_{zz} = n_z^2$$

$$k^2 (n_x^2 k_x^2 + n_y^2 k_y^2 + n_z^2 k_z^2) - [n_x^2 k_x^2 (n_y^2 + n_z^2) + n_y^2 k_y^2 (n_x^2 + n_z^2) + n_z^2 k_z^2 (n_x^2 + n_y^2)] k_0^2 + n_x^2 n_y^2 n_z^2 k_0^4 = 0$$

$n_x \ n_y \ n_z$

płaszczyzna wektora falowego

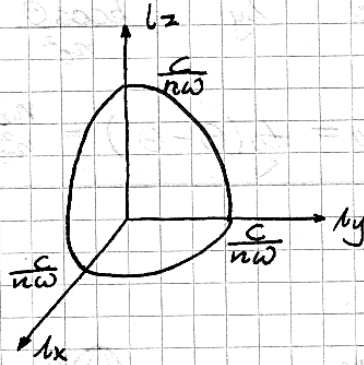
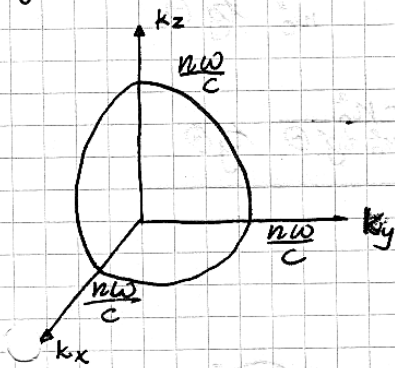
$$\left(\epsilon_{ij} \delta_{ij} - \epsilon_{ij} \epsilon_{ij} - \frac{\delta_{ij}}{\epsilon_{ii}} \epsilon_0^2 \right) D_j = 0$$

$$\epsilon_0 = \frac{c}{\omega}$$

$$\det \left| \epsilon_{ij} \delta_{ij} - \epsilon_{ij} \epsilon_{ij} - \frac{\delta_{ij}}{\epsilon_{ii}} \epsilon_0^2 \right| = 0$$

$$\epsilon_0^2 (\epsilon_{yy}^2 \epsilon_{zz}^2 \epsilon_x^2 + \epsilon_{xx}^2 \epsilon_{zz}^2 \epsilon_y^2 + \epsilon_{xx}^2 \epsilon_{yy}^2 \epsilon_z^2) - [\epsilon_x^2 (\epsilon_{yy}^2 + \epsilon_{zz}^2) + \epsilon_y^2 (\epsilon_{xx}^2 + \epsilon_{zz}^2) + \epsilon_z^2 (\epsilon_{xx}^2 + \epsilon_{yy}^2)] \epsilon_0^2$$

$$+ \epsilon_0^4 = 0$$



$$k^2 = n_0^2 k_0^2 = n^2 \left(\frac{\omega}{c} \right)^2$$

$$\epsilon_0^2 = \frac{1}{n^2} \epsilon_0^2 = \frac{1}{n^2} \left(\frac{c}{\omega} \right)^2$$

Przypadki szczególne:

- ośnówki jednoosiowe

$$n_x = n_y = n_o$$

$$n_z = n_e \neq n_o$$

$$\left(k^2 - k_0^2 n_o^2 \right) \left[n_o^2 (k_x^2 + k_y^2) + n_e^2 k_z^2 - n_o^2 n_e^2 k_0^2 \right] = 0$$

$$\left(\epsilon_0^2 - n_o^2 \epsilon_0^2 \right) \left[n_o^2 \epsilon_z^2 + n_e^2 (\epsilon_x^2 + \epsilon_y^2) - \epsilon_0^2 \right] = 0$$

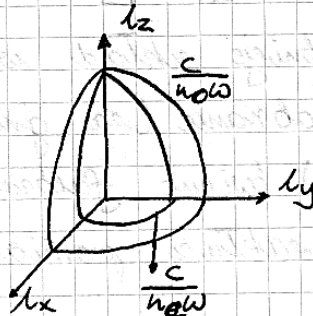
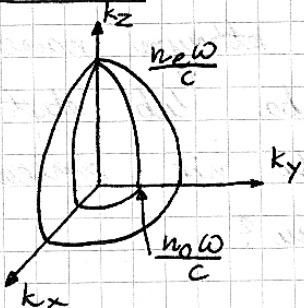
$$k^2 = n_o^2 k_0^2$$

$$\frac{k_x^2 + k_y^2}{n_e^2} + \frac{k_z^2}{n_o^2} = k_0^2$$

$$\frac{\epsilon_x^2 + \epsilon_y^2}{\left(\frac{1}{n_e} \right)^2} + \frac{\epsilon_z^2}{\left(\frac{1}{n_o} \right)^2} = 1$$

$$\epsilon_0^2 = \frac{1}{n_o^2} \epsilon_0^2$$

$n_e > n_o$



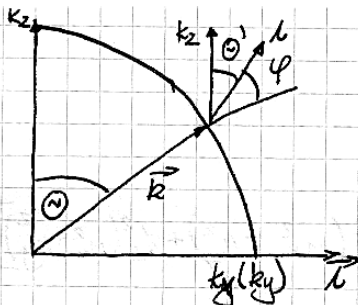
$$k_x^2 + k_y^2 = k^2 \sin^2 \theta$$

$$k_z^2 = k^2 \cos^2 \theta$$

$$\frac{\sin^2 \theta}{n_e^2} + \frac{\cos^2 \theta}{n_o^2} = \frac{k_0^2}{k^2(\theta)} = \frac{1}{n^2(\theta)}$$

$$v_g(\theta) = \frac{c}{n(\theta)} = c \frac{\sqrt{n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta}}{n_o n_e}$$

$$v_g(\theta) = \frac{c}{\sqrt{n_e^2 \sin^2 \theta + n_o^2 \cos^2 \theta}}$$

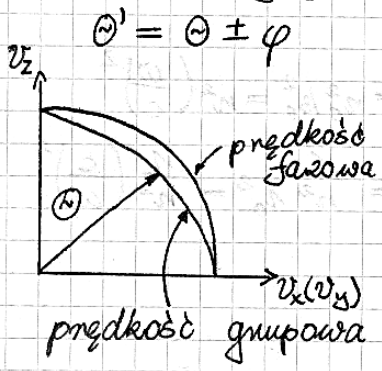


$$\vec{k} \cdot \Delta \vec{l} = 0$$

$$\vec{k} \cdot \vec{l} = 1$$

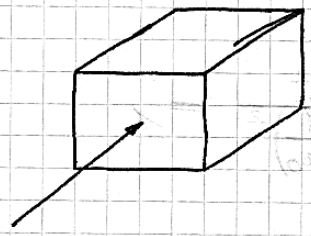
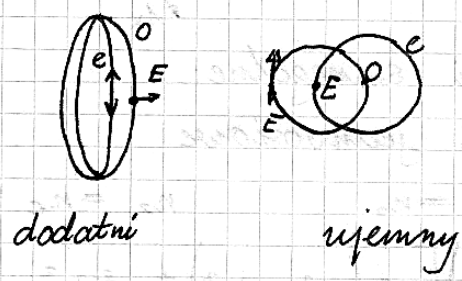
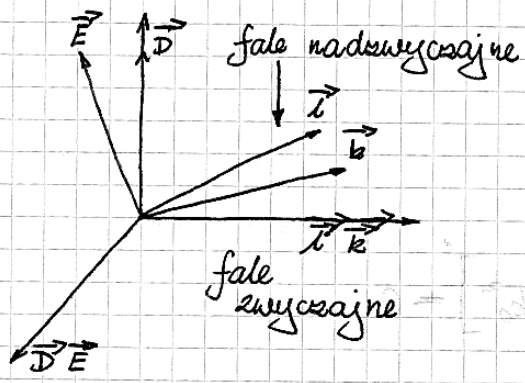
$$\vec{k} \cdot \Delta \vec{l} + \Delta \vec{k} \cdot \vec{l} = 0$$

$$\Delta \vec{k} \cdot \vec{l} = 0$$

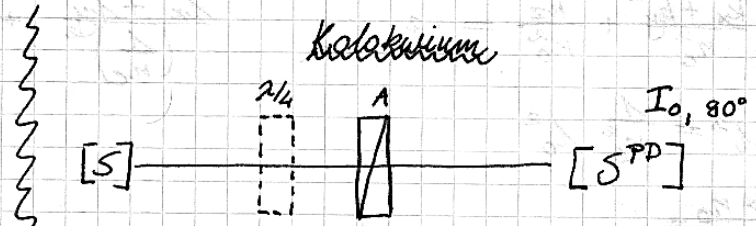


$$\text{tg } \theta' = \frac{l_x}{l_y} = \frac{k \sin \theta}{k \cos \theta} = \frac{n_e^2}{n_o^2} \text{tg } \theta$$

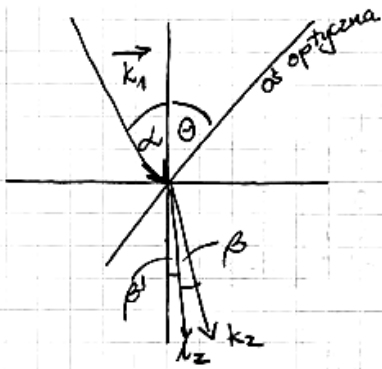
$$\text{tg } \varphi = \text{tg}(\theta - \theta') = \frac{n_e^2 - n_o^2}{n_e^2 + n_o^2 \text{tg}^2 \theta} \text{tg } \theta$$



$$\Delta \varphi = |k_e - k_o| d = \frac{2\pi}{\lambda} |n_e - n_o| d$$



Czy istnieje układ w którym nawet
 że dobrowanie (za grubą lub zbyt
 cienką) orientacja falowa nie zmieni
 lub zakłuci pomiaru?



$$\sin \alpha = n(\beta) \sin \beta$$

$$\frac{\sin^2 \beta}{\sin^2 \alpha} = \frac{\sin^2(\theta + \beta)}{n_c^2} + \frac{\cos^2(\theta + \beta)}{n_o^2}$$

$$\beta' = \arctg \left(\frac{n_o^2}{n_c^2} \operatorname{tg} \beta \right)$$

$$\beta = \arctg \left[\frac{\frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_c^2}}{\left(\frac{1}{n_o^2} - \frac{1}{n_c^2} \right) \sin 2\theta} \sqrt{\frac{\sin^2 \theta}{n_c^2 \sin^2 \alpha} + \frac{\cos^2 \theta}{n_o^2 \sin^2 \alpha} - \frac{1}{n_c^2 n_o^2}} \right]$$

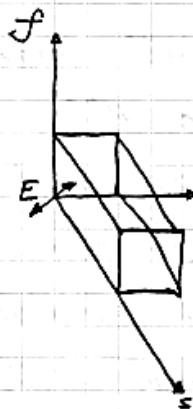
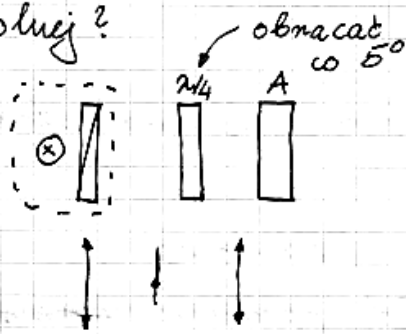
2 maja 4h wykładów.

WYKŁAD 8

25.04.2007.

Zad. I.

1. Jak znaleźć doświadczalnie osie płytki falowej dysponując liniowo spolaryzowanym źródłem światła i analizatorem? Czy możemy rozróżnić oś szybką od wolnej?



2. Jak odróżnić ćwiecfolówkę od innych płytek?

Odchylić ją o 45° względem osi y.

3. Jaki błąd popełnimy, jeżeli zamiast półćwiecfolówki wstawimy ćwiecfolówkę przy znajdowaniu elementów wektora Stockesa?

$$S_3 = I_{(\lambda/4, 45^\circ)} - I_{(\lambda/4, 135^\circ)}$$

$$? = I_{(\lambda/2, 45^\circ)} - I_{(\lambda/2, 135^\circ)} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

$$[S] \otimes \begin{matrix} \text{---} \\ \lambda/4 \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ A \\ \text{---} \end{matrix} [S^{OUT}]$$

$$\frac{1}{2} \begin{bmatrix} S_0 + S_3 \\ S_0 - S_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} S_0 + S_3 \\ S_0 - S_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \Rightarrow \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ -S_2 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Odwrozczenie półfalówki - odchylając o δ polaryzacja zmienia się o $\lambda/2$ zd.

$$\otimes \updownarrow \begin{matrix} \text{---} \\ \lambda/2 \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ A \\ \text{---} \end{matrix} [S^{OUT}]$$

Przekraczając półfalówkę o 45° maksimum na analizatorze otrzymamy obracając go o 90°

4. Jaka może być największa grubość kwarcowej ćwierćfalówki bez kompensacji aby błąd pomiaru depolaryzacji eliptyczności mierzony wiązki światła wywołanej depolaryzacją nie przekroczył 15%.

$$\left(\frac{b}{a} \right)_{zm} = \left| \operatorname{tg} \left\{ \frac{1}{2} \arcsin \left(\frac{P_C S_3}{\sqrt{S_1^2 + S_2^2 + (P_C S_3)^2}} \right) \right\} \right| = \frac{b}{a} \operatorname{arcc} 2$$

$$P_C = \operatorname{arcc} \left(\frac{-\sin \Delta \lambda}{\lambda^2} \right)$$

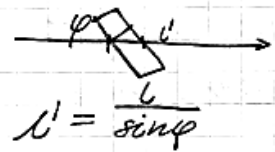
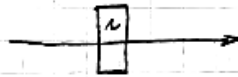
$$l_{max} = ? \Leftrightarrow \frac{\left| \left(\frac{b}{a} \right)_{zm} - \left(\frac{b}{a} \right) \right|}{\frac{b}{a}} \leq 0,15$$

5. Jaki błąd popełniamy, jeżeli wstawimy ćwierćfalówkę o złe dobranej długości fali: np 8λ w stosunku do nominalnej podanej przez producenta? Czy istnieje prosty sposób kompensacji tego błędu?

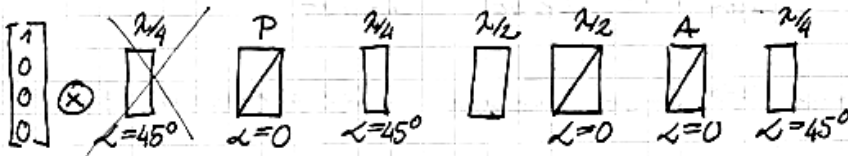
$$\frac{2\pi}{\lambda} \Delta n l = \Phi$$

$$\frac{2\pi}{\lambda + \delta\lambda} \Delta n l = \underbrace{\Phi}_{\frac{\pi}{2}} + \delta\Phi$$

$$\delta\Phi = \frac{2\pi \Delta n l}{\lambda} \frac{(-\delta\lambda)}{\lambda + \delta\lambda}$$



Zad. 5.



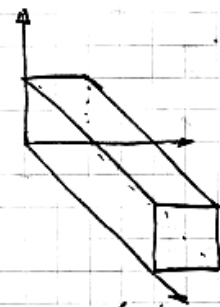
Zad. 2.

1) Wykonaj wykres (przynajmniej 5 punktów) funkcji $P_{min} = f(\theta)$, gdzie θ oznacza kąt pomiędzy kierunkiem propagacji a osią optyczną dla kryształu nobianu litu w postaci sześciangu o boku 10 mm ($n_o = 2.286$, $n_e = 2.200$).

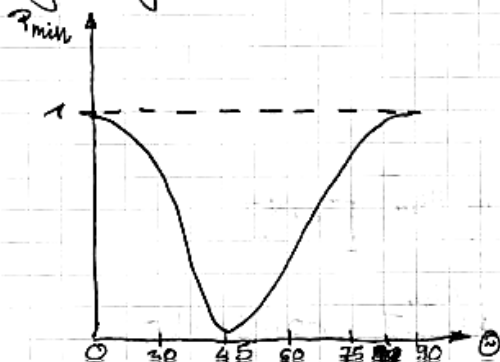
2) Jak będzie wyglądał ten wykres dla lasera gazowego o podstawowych parametrach takich samych jak dla diody laserowej ($\lambda, \Delta\lambda$).



$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2 \theta}{n_e^2} + \frac{\cos^2 \theta}{n_o^2}$$



3) Czy istnieje kąt θ przy którym można uzyskać takie samo P_{min} bez względu na charakter widma (lorentzowskie czy gaussowskie)? Jaka musiałaby być dwójtomność osnówka aby było to możliwe?



$$1) P_{minL} = \exp\left(-\frac{[\Delta n_i l \Delta\lambda]}{\lambda^2}\right)$$

$$2) P_{minG} = \exp\left(-\left[\frac{[\Delta n_i l \Delta\lambda]}{2 l n^2 \lambda^2}\right]^2\right)$$

$$3) \Delta n_i = n_o - n_e(\theta)$$

$$P_{minL} = P_{minG}$$

$$\frac{\Delta n_i \Delta z}{2 \cos^2 \theta} = 1 \Rightarrow n_i = \dots, \theta = \dots$$

Ład 3. Sprawdzić czy metoda pomiarów czterech natężeń światła jest tożsama z metodą sześciu pomiarów przy znajdowaniu wektora Stockesa

$$S_2 = 2I_{0,45} - I_{(0,0)} - I_{90} \Leftrightarrow I_{0,45} - I_{0,135}$$

$$I_{0,0} + I_{0,90} \Leftrightarrow I_{0,45} + I_{0,135} \quad \text{Równoważne !!!}$$

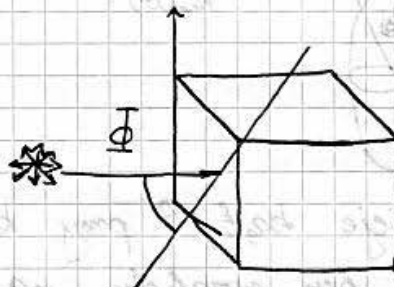
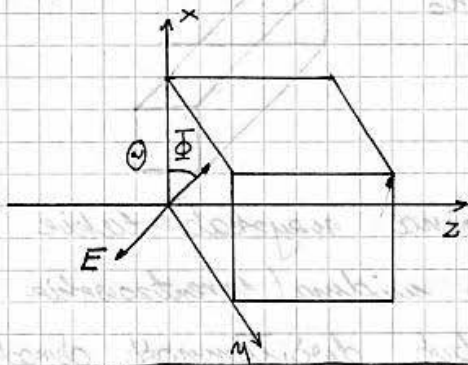
$$S_3 = I_{\pi/4,45} - I_{\pi/4,135} \Leftrightarrow I_{0,0} + I_{0,90} - 2I_{\pi/4,45}$$

$$[S] \Rightarrow \begin{matrix} \begin{matrix} I_{\pi/4,45} \\ I_{\pi/4,135} \end{matrix} \\ \begin{matrix} A_{45} \\ A_{135} \end{matrix} \end{matrix} \quad \frac{1}{2}S_0 + \frac{1}{2}S_3 - \frac{1}{2}S_0 + \frac{1}{2}S_3 = S_3 \Leftrightarrow S_0 - 2 \left[\frac{1}{2}(S_0 + S_3) \right]$$

$$\frac{1}{2} \begin{bmatrix} S_0 + S_3 \\ I_{\pi/4,45} \\ I_{\pi/4,135} \end{bmatrix} \quad \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

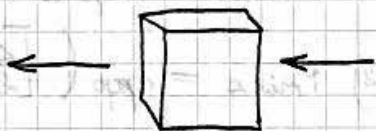
WYKŁAD ③

16. V 2007



$$P = \sqrt{1 - [1 - \exp(-\frac{2 \sin k}{\Delta L})] \sin^2 2\Phi}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$[S^{OUT}] = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ \vdots & \dots & \dots & \dots \\ m_{41} & \dots & \dots & m_{44} \end{bmatrix} [S^{IN}]$$

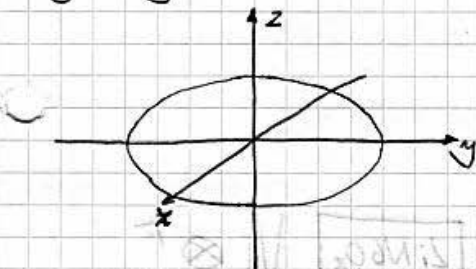
16 możliwych pomiarów

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} \Rightarrow m_{11} + m_{12} = a_1$$

$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow m_{11} + m_{12} = a_2$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Indykatorysa



$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

$$\Delta\left(\frac{1}{n^2}\right) = rE + gE^2 + \dots$$

Pod wpływem pola elektrycznego nastąpi deformacja elipsoidy.

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + n_e + 2a_{23}yz + 2a_{31}zx + 2a_{12}xy = 1$$

$$\begin{bmatrix} a_{11} - \frac{1}{n_x^2} \\ a_{22} - \frac{1}{n_y^2} \\ a_{33} - \frac{1}{n_z^2} \\ a_{23} \\ a_{31} \\ a_{12} \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & \dots & \dots \\ \vdots & \dots & \dots \\ n_{61} & n_{62} & n_{63} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

MACIERZ WSPÓŁCZYNNIKÓW
ELEKTROOPTYCZNYCH

LiNbO₃ 3m

$$\begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$$

$$r_{13} = 8,6 \cdot 10^{-12} \frac{m}{V}$$

$$r_{33} = 30,8 \cdot 10^{-12} \frac{m}{V}$$

$$r_{22} = 3,4 \cdot 10^{-12} \frac{m}{V}$$

$$r_{51} = 2,8 \cdot 10^{-8} \frac{m}{V}$$

Dla kierunku propagacji Y i pola zewnętrznego E_z :

Oś optyczna zawsze z .

$$n_x = n_y = n_o \quad n_z = n_e \quad E_x = E_y = 0$$

$$\left(n_o^{-2} + n_{12} E_z\right) x^2 + \left(n_o^{-2} + n_{13} E_z\right) y^2 + \left(n_e^{-2} + n_{33} E_z\right) z^2 = 1$$

$$n_x' = n_o - \frac{1}{2} n_{12} n_o^3 E_z$$

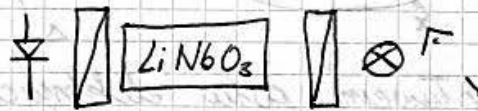
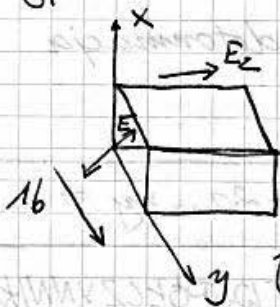
$$n_z' = n_e - \frac{1}{2} n_{33} n_e^3 E_z$$

NAPIĘCIE POKFALOWE

$$\varphi = \frac{2\pi}{\lambda} \Delta n l \quad \sim 10^5 \frac{V}{m}$$

$$\Delta\varphi = \frac{2\pi}{\lambda_o} (n_x - n_o) = \frac{\pi}{\lambda_o} n_o^3 n_{12} E l$$

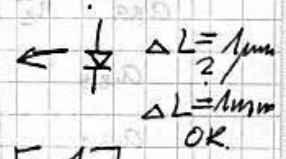
Przypadek 1 (1b)



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \frac{2\pi}{\lambda} \Delta n l & -\sin \frac{2\pi}{\lambda} \Delta n l \\ 0 & 0 & \sin \frac{2\pi}{\lambda} \Delta n l & \cos \frac{2\pi}{\lambda} \Delta n l \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$S^{out} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Upraszczając na przypadek fali padającej pod kątem na kryształ optyczny:

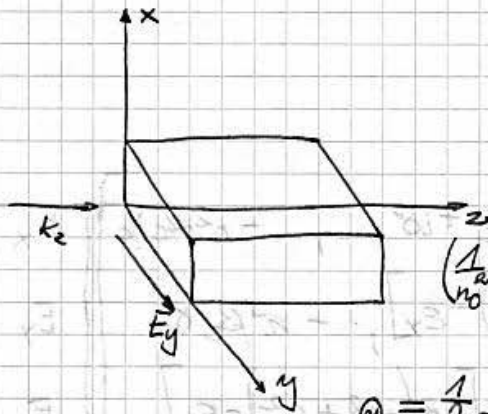


$$S^{out} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos \frac{2\pi}{\lambda} \Delta n l \\ 0 & 0 & \sin \frac{2\pi}{\lambda} \Delta n l \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$S^{out} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \cos \frac{2\pi}{\lambda} \Delta n l \\ \sin \frac{2\pi}{\lambda} \Delta n l \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ P_c \cos \frac{2\pi}{\lambda} \Delta n l \\ P_c \sin \frac{2\pi}{\lambda} \Delta n l \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 + P_c \cos \frac{2\pi}{\lambda} \Delta n l \\ 1 + P_c \sin \frac{2\pi}{\lambda} \Delta n l \end{bmatrix}$$

$$\Delta n_b = \left| (n_e - n_o) + \frac{1}{2} (n_o^3 r_{13} - n_e^3 r_{33}) E \right|$$



$$E_x = E_z = 0$$

$$\left(\frac{1}{n_o^2} - r_{22} E \right) x^2 + \left(\frac{1}{n_o^2} + r_{22} E \right) y^2 + \frac{1}{n_e^2} z^2 + 2r_{51} y z E = 1$$

$$\left(\frac{1}{n_o^2} - r_{22} E \right) x^2 + \left(\frac{1}{n_o^2} + r_{22} E \right) y^2 + \frac{1}{n_e^2} z^2 + 2r_{51} y z E = 1$$

$$\Theta_1 = \frac{1}{2} \arctg \frac{2r_{51} E}{\frac{1}{n_o^2} - \frac{1}{n_e^2} + r_{22} E}$$

$$\left(\frac{1}{n_o^2} - r_{22} E \right) x'^2 + \left(\frac{1}{n_o^2} + r_{22} E + \frac{r_{22}^2 E^2}{\frac{1}{n_o^2} - \frac{1}{n_e^2} + r_{22} E} \right) y'^2 + \left(\frac{1}{n_e^2} - \frac{r_{51}^2 E^2}{\frac{1}{n_o^2} - \frac{1}{n_e^2} + r_{22} E} \right) z'^2 = 1$$

$$\Delta \varphi = \frac{2\pi}{\lambda_0} r_{22} n_o^3 E l$$

$$\text{dla } r_{42} E \ll r_{22} E$$

$$\Delta n = ?$$

WYKŁAD 10

23.05.2007

MAGNETOOPTYKA

$$\text{rot } \vec{H} = \epsilon_0 \frac{\partial \hat{\epsilon} \vec{E}}{\partial t}$$

$$\text{div } \hat{\epsilon} \vec{E} = 0$$

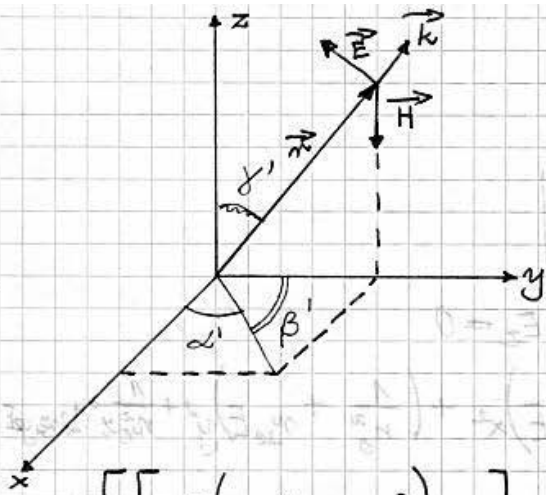
$$\text{rot } \vec{E} = -\mu_0 \frac{\partial \hat{\mu} \vec{H}}{\partial t}$$

$$\text{div } \hat{\mu} \vec{H} = 0$$

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{11(\omega)} - i\delta & 0 \\ i\delta & \epsilon_{22(\omega)} \\ 0 & 0 & \epsilon_{33(\omega)} \end{bmatrix}$$

$$\mu = 1$$

	Dwójtomność liniowa	Dwójtomność katowa aktywność optyczna
Kryształy ośrodków dwójtomne		
Elektrooptyka	Pockels	
Magnetooptyka	Cotton-Mouton	rotator Faradaya



$$\Delta \vec{E} - \nabla (\nabla \cdot \vec{E}) - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{k} \cdot \vec{r} = n k_0 (\alpha c x + \beta c y + \gamma c z)$$

$$\begin{bmatrix} [n^2(\beta_c^2 + \gamma_c^2) - \epsilon_x] & -n^2 \alpha c \beta_c + i \delta & -n^2 \alpha c \gamma_c \\ -n^2 \alpha c \beta_c - i \delta & [n^2(\alpha_c^2 + \gamma_c^2) - \epsilon_x] & -n^2 \beta_c \gamma_c \\ -n^2 \alpha c \gamma_c & -n^2 \beta_c \gamma_c & [n^2(\alpha_c^2 + \beta_c^2) - \epsilon_z] \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

Aby otrzymać rozwiązanie wyznacznik macierzy musi być równy 0:

$$n^4 (\epsilon_x \alpha_c^2 + \epsilon_y \beta_c^2 + \epsilon_z \gamma_c^2) + n^2 [(\epsilon_x \epsilon_y - \delta^2)(\alpha_c^2 + \beta_c^2) + \epsilon_z (\epsilon_x \alpha_c^2 - \epsilon_y \beta_c^2) + \epsilon_z (\epsilon_x + \epsilon_y) \gamma_c^2] +$$

$$+ \epsilon_z (\epsilon_x \epsilon_y - \delta^2) = 0$$

$$\vec{k} = [0, 0, k_z]$$

$$\vec{H} = [0, 0, H]$$

$$\alpha_c = \beta_c = 0 \quad \gamma_c = 1$$

$\epsilon_x = \epsilon_y = \epsilon_z = \epsilon_F$ - ciało bezpostaciowe

$$n^4 - 2n^2 \epsilon_F + \epsilon_F^2 - \delta^2 = 0$$

$$n_{\pm}^2 = \epsilon_F \pm \delta^2 \quad \text{- dwójtomność kolowa}$$

$$\begin{bmatrix} \pm i \delta & -\delta & 0 \\ \delta & \pm \delta & 0 \\ 0 & 0 & -i \epsilon_z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_F & -i \delta & 0 \\ i \delta & \epsilon_F & 0 \\ 0 & 0 & \epsilon_F \end{bmatrix}$$

$$E_y = \pm i E_x$$

$$\vec{E} = \vec{E}_0 \exp(i(\omega t - n \vec{k}_0 \cdot \vec{r}))$$

$$E_x^{\pm} = E_0^{\pm} \exp[i(\omega t - k_0 n_{\pm} z)]$$

$$E_y^{\pm} = \pm i E_0^{\pm} \exp[i(\omega t - k_0 n_{\pm} z)]$$

$$\Theta_F = \pi(n_+ - n_-) \frac{l}{\lambda_0} = \frac{\pi}{\lambda} \Delta n_{CB} l$$

Θ_F - kat notaqi Faradayda

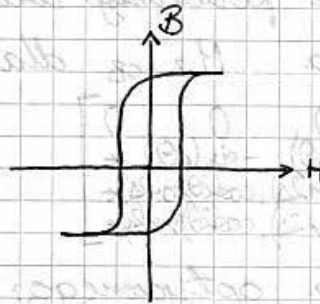
$$\Theta_F = \sqrt{H_0} l$$

$$V = \frac{\omega \epsilon_0 \mu_0 \pi}{N \epsilon N_F} (\epsilon_F - 1)^2 \mu(H_0)$$

$$\lambda_0 = 0,633 \mu\text{m}$$

$$V_{SiO_2} = 0,0476 \cdot 10^{-4} \left[\frac{\text{rad}}{\text{A}} \right]$$

$$V_{FR-B} = 0,88 \cdot 10^{-4} \left[\frac{\text{rad}}{\text{A}} \right]$$



$$\Theta_F = \Phi \cdot l$$

BIG (granat bismutowy, $\lambda = 1,550 \text{ nm}$) = 3150 $\left[\frac{\text{rad}}{\text{m}} \right]$

$Y_3 Fe_5 O_{12}$

Dla propagacji $E_x = E_y = E_{CM}$

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{CM} & -i\delta & 0 \\ i\delta & \epsilon_{CM} & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

$$\gamma_c = 0 \quad \beta_c + \alpha_c = 1$$

$$n^4 + n^2 \left[\epsilon_{CM} - \frac{\delta^2}{\epsilon_{CM}} + \epsilon_z \right] + \epsilon_z \left(\epsilon_{CM} - \frac{\delta^2}{\epsilon_{CM}} \right) = 0$$

$$n_{||}^2 = \epsilon_z \quad n_{\perp}^2 = \epsilon_{CM} \left(1 - \frac{\delta^2}{\epsilon_{CM}^2} \right)$$

$n = n_{||} \quad E_x = E_y = 0$

$$E_{||} = E_z = E_{0z} \exp[i(\omega t - n_{||} k_0 s)]$$

$n = n_{\perp} \quad E_z = 0$

$$E_{\perp} = \sqrt{E_x^2 + E_y^2} = E_{0\perp} \exp[i(\omega t - n_{\perp} k_0 s)]$$

$$\Delta = k_0(n_{||} - n_{\perp})s = k_0 n_{\perp} L B s$$

$$\Delta = f(H^2)$$

Dla układu

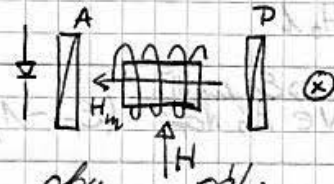


$$\begin{bmatrix} S_{0out}^{DET} \\ S_{1out}^{DET} \\ S_{2out}^{DET} \\ S_3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$I_{DET} = \sqrt{\frac{\epsilon_0}{\mu_0}} S_0^{DET} = \frac{1}{4} \left(\right)$$

Dla zjawiska Cottona - Muttona macierz osrodka magnetoptycznego MO zmieni się

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\Delta) & -\sin(2\Delta) \\ 0 & 0 & \sin(2\Delta) & \cos(2\Delta) \end{bmatrix}$$

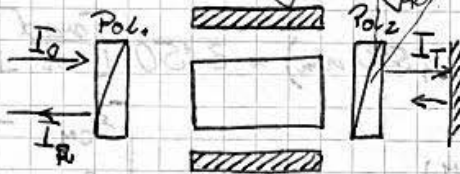


Macierz osrodka poddanego działaniu obu pól:

Macierz Mullera M_{LB+CB} dla liniowej i kołowej dwojtonności

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & * \\ 0 & \sin(2\theta)\cos(2\Delta) & \cos(2\theta)\cos(2\Delta) & * \\ 0 & \sin(2\theta)\sin(2\Delta) & \cos(2\theta)\sin(2\Delta) & * \end{bmatrix}$$

Dla izolatora optycznego:



$$[S^R] = [M^{P1}] [M^{ROT}] [M^{P2}] [M^{ROT}] [M^{P2}] [M^{ROT}] [M^{P1}] [S^P]$$

$$[S^T] = [M^{P2}] [M^{ROT}] [M^{P1}] [S^P]$$

$$L = -10 \lg \left(\frac{I_B}{I_T} \right)$$

WYKŁAD 10

30.05.2007

ANIZOTROPIA OPTYCZNA W ŚWIATKOWODACH



Dla teorii słabego pola możemy mówić o modach LP:

$$\vec{E}(x, y, z) = u_1(z) \vec{E}_x(x, y) + u_2(z) \vec{E}_y(x, y)$$

$$[J] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$L_b = \frac{2\pi}{\Delta\beta} = \frac{\lambda}{\Delta N_{eff}}$$

$$[S] = \begin{bmatrix} \langle U_1 U_1^* \rangle + \langle U_2 U_2^* \rangle \\ \langle U_1 U_1^* \rangle - \langle U_2 U_2^* \rangle \\ \langle U_1 U_2^* \rangle + \langle U_1^* U_2 \rangle \\ -i(\langle U_1, U_2 \rangle - \langle U_2, U_1^* \rangle) \end{bmatrix}$$

Dla przykładu:

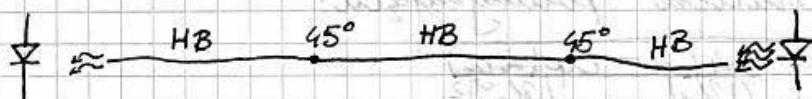
1) światłowod laboratoryjny

$$\Delta N_{\text{eff}} = \frac{\lambda_0}{L_{\text{BO}}} = \frac{0,66 \cdot 10^{-6} \text{ m}}{1,32 \cdot 10^{-3} \text{ m}} = 0,0005$$

2) dla LiNbO_3

$$\Delta N_{\text{eff}} = 0,086$$

OPIS PROPAGACJI ŚWIATKA W ŚWIATKOWODZIE DWÓJŁOŻNYM



$$[S^{\text{out}}] = [M_{L2}] [M_{CZ}] [M_{L1}] [S^{\text{in}}]$$

Dla miennika mocy optycznej:

$$P_{\text{out}} = \oint I^{\text{DET}} ds$$

Dla detektora

$$I^{\text{DET}} = \sqrt{\frac{\epsilon_0}{\mu_0}} S_0^{\text{out}}$$

Macierz Müllera światłowodu:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\Delta\beta L) & -\sin(\Delta\beta L) \\ 0 & 0 & \sin(\Delta\beta L) & \cos(\Delta\beta L) \end{bmatrix}$$

Ze względu na charakter zmoda wprowadzamy macierz depolaryzacji

$$[S^{\text{out}}] = [M_{L2}] [D_P] [M_{CZ}] [M_{L1}] [S^{\text{in}}]$$

gdzie D_P i P_C wynosi:

$$[D_p] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c & 0 \\ 0 & 0 & 0 & P_c \end{bmatrix}$$

$$P_c = \sqrt{1 - \frac{4 \left[1 - \exp(-2\eta_c) \right]}{\left(\frac{|u_1|}{|u_2|} + \frac{|u_2|}{|u_1|} \right)^2}}$$

$$\eta_{SL} = \frac{2\omega L}{\Delta L L_B}$$

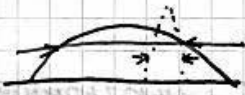
$$\eta_{SG} = \left[\frac{2\omega L}{2(\ln 2)^2 \Delta L L_B} \right]^2$$



$$\Delta L = \frac{\Delta^2}{\Delta^2}$$

Najlepsze rozwiązanie: znaleźć drogę koherencji testując je za pomocą znanego kryształu

Niektóre elementy mogą działać jak filtry nycinając pewną część pasma



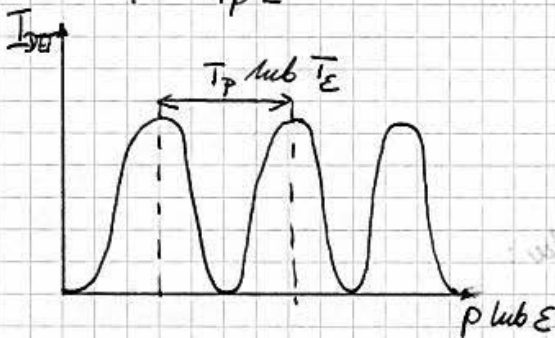
Dla P_c^* - macierz polaryzująca:

$$P_c^* = \sqrt{1 - \frac{4 \left[1 - \exp(-2\eta_c) \right]}{\left(\frac{|u_1|}{|u_2|} + \frac{|u_2|}{|u_1|} \right)^2}}$$

Dla ciśnienia:

$$L_B = \frac{L_{B0}}{1 + \frac{P L_{B0}}{T_P L}}$$

L_{B0} - początkowa droga zdundnień
 P - ciśnienie



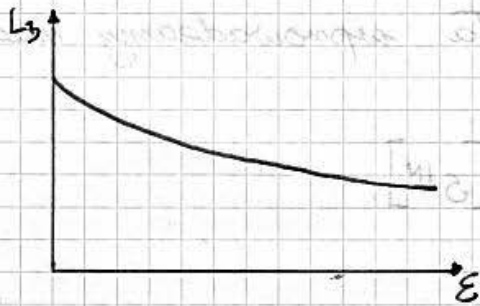
$$T_P L = \text{const}$$

$$\varepsilon = \frac{\delta L}{L}$$

Dla strainu (naprężenia)

$$L_B = \frac{L_{B0}}{1 + \frac{\varepsilon L_{B0}}{T_E L}}$$

$\left(\frac{\delta}{\delta} \right)$ - kompensacja



Dynamika czujnika

$$\eta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$\eta = \sqrt{\frac{2}{\left(\frac{|u_1|}{|u_2|} + \frac{|u_2|}{|u_1|} \right)^2} - 1 + P^2}$$

Dla kąta 45° $u_1 = u_2$ daje $P = \eta$

Wtedy $\begin{bmatrix} S^{OUT} \\ S^{OUT} \\ S^{OUT} \\ S^3 \end{bmatrix} = [M_{L2}] [D_p] [M_{CZ}] [M_{L1}] [S^{IN}]$ $\left. \begin{matrix} \swarrow 45^\circ \\ \searrow 45^\circ \end{matrix} \right\}$
 $[S^{OUT}] = [M_{L2}] [D_p] [M_{CZ}] [S^{IN}]$

$$\begin{bmatrix} S^{DET} \\ S^{OUT} \\ S^{OUT} \\ S^3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P_c & 0 & 0 \\ 0 & 0 & P_c & 0 \\ 0 & 0 & 0 & P_c \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$C = \cos(\Delta\beta L)$ $S = \sin(\Delta\beta L)$

Mając $[S^{OUT}]$ możemy obliczyć stopień polaryzacji:

$$[S^{OUT}] = [M_A] [D_p] [M_{CZ}] [S^{IN}]$$

$$\begin{bmatrix} 1 \\ 0 \\ C \\ S \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ P_c C \\ P_c S \end{bmatrix}$$

$$P = \sqrt{\frac{P_c^2 C^2 + P_c^2 S^2}{1}} = P_c$$

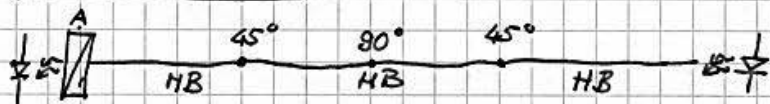
$$P = \frac{\sqrt{(1+P_c C)^2}}{1+P_c C}$$

$$I_{det} = \frac{E_0^2}{\mu_0} \cdot S_0^{DET} = \frac{1}{2} (1 + P_c \cos \Delta\beta L)$$

$$[S^{OUT}] = \frac{1}{2} \begin{bmatrix} 1+P_c C \\ 0 \\ 1+P_c C \\ 0 \end{bmatrix}$$

$$P = \sqrt{\frac{S_1^2 + S_2^2 + S_3^2}{S_0^2}}$$

— stopień polaryzacji



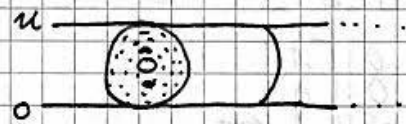
Maciers obrotu o 90 stopni

$$[S^{OUT}] = [M_A] [M_{L\text{odpr}}] [D_{CZ}^*] [T_{-80}] [M_{LCZ}] [T_{80}] [D_{L1}] [M_{L1}] [M_p] [S^{IN}]$$

$$[T_{-80}] = [T_{80}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_{-45}] = [T_{45}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dla światłowodów fotonicznych



Kompensacja dyspersji w telekomunikacji

