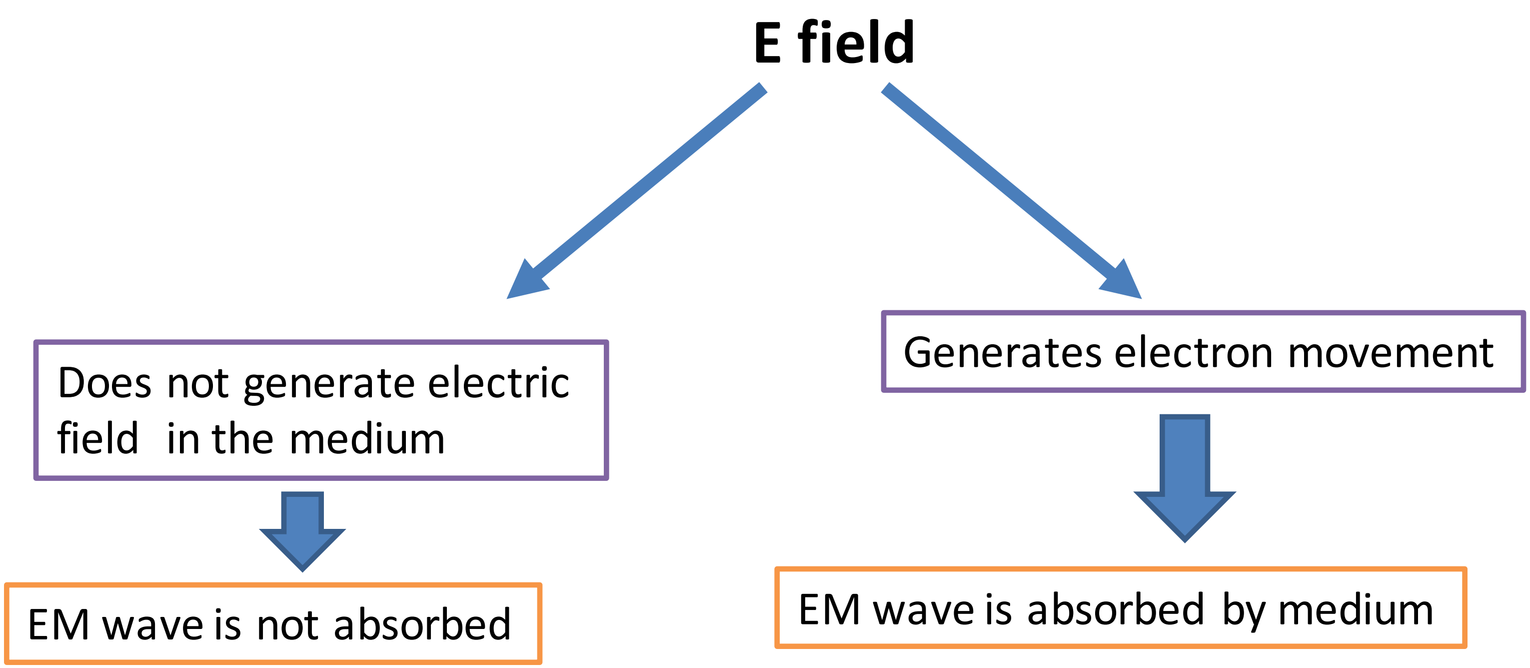
EM wave in the isotropic absorbing medium

- EM wave can generate electric field in the medium,



EM wave in the isotropic absorbing medium

Electrical conductivity s must be included in Maxwell's equations

$$\operatorname{rot}\mathbf{H} - \dot{\mathbf{D}} = \sigma\mathbf{E},$$

$$\dot{D} = \varepsilon\varepsilon_0\dot{E}$$

$$\dot{E} = i\omega E$$

$$\operatorname{rot}\mathbf{H} - \varepsilon_0\left(\varepsilon - \frac{i\sigma}{\varepsilon_0\omega}\right)\dot{\mathbf{E}} = 0.$$

For dielectrics:

$$\operatorname{rot}\mathbf{H} - \varepsilon_0\varepsilon\,\dot{\mathbf{E}} = 0.$$

$$\boxed{\underline{\varepsilon} = \varepsilon - \frac{i\sigma}{\varepsilon_0\omega},}$$

Electrical permitivity of the conductor

EM wave in the isotropic absorbing medium

$$\underline{n} = \sqrt{\underline{\varepsilon}} = n\sqrt{1 - \frac{\mathrm{i}\sigma}{\varepsilon\varepsilon_0\omega}},$$

Refractive index for conductor

… by expanding it to the series:

$$\sqrt{1-x} \approx 1 - \frac{1}{2}x + \cdots$$

$$\boxed{\underline{n} = n(1 - \mathrm{i}\kappa), \quad \text{gdzie} \quad \kappa = (\sigma/2\omega\varepsilon\varepsilon_0).}$$

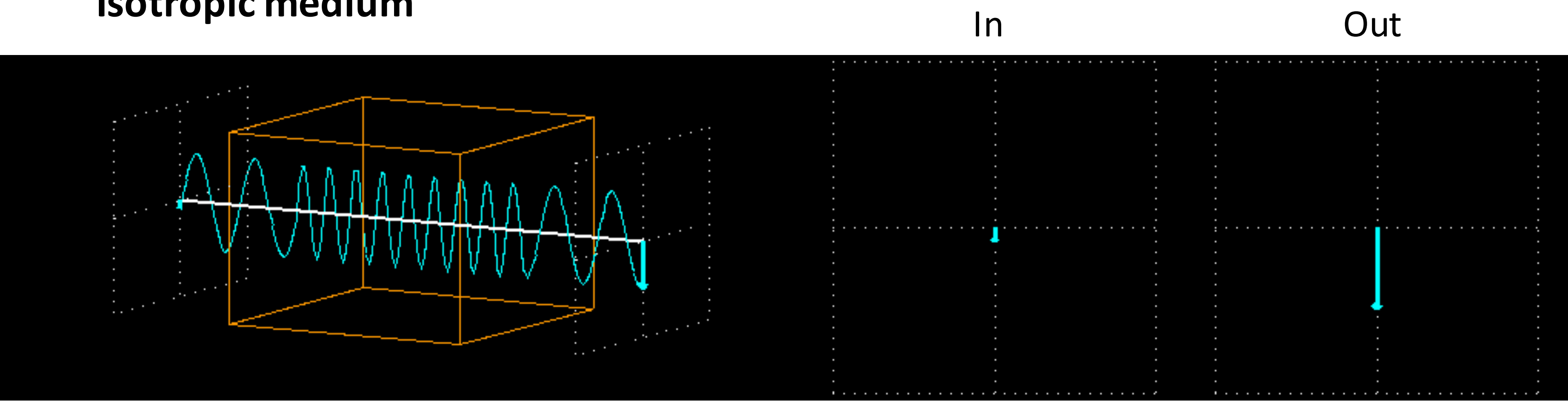
$$\underline{k} = \omega\frac{\underline{n}}{c}$$

Wave number for absorbing medium

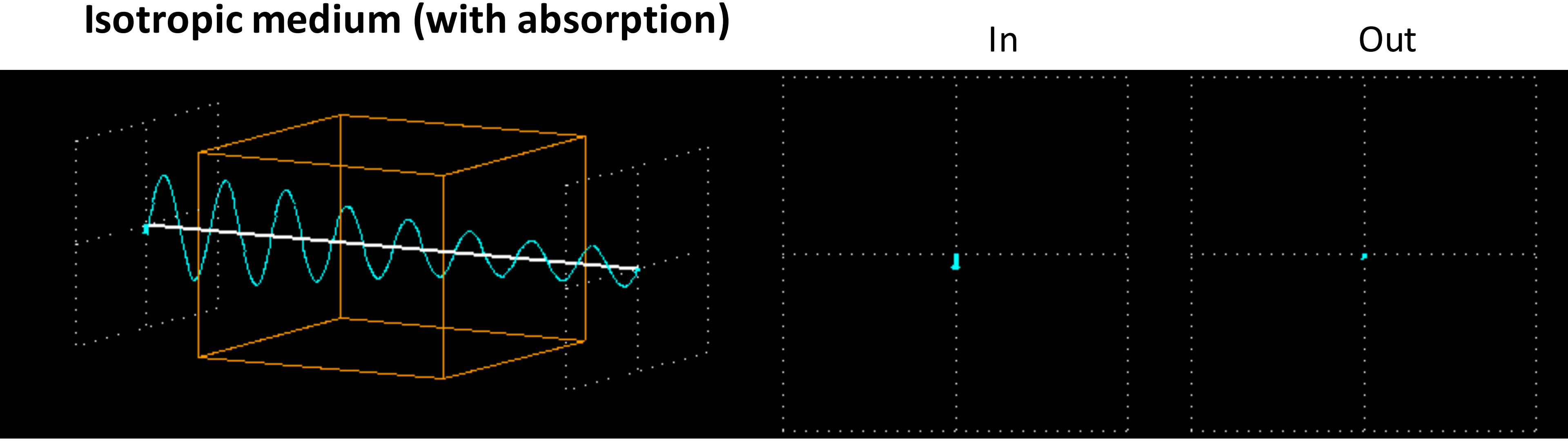
$$\mathbf{E} = \mathbf{E}_0 \exp\left(-\omega\kappa\frac{z}{c}n\right)\exp\left[\mathrm{i}\omega\left(t - \frac{z}{c}n\right)\right].$$

Amplitude of EM wave is decreasing expotentialy during propagation in absorbing medium

Isotropic medium



Isotropic medium (with absorption)



Optical anisotropy

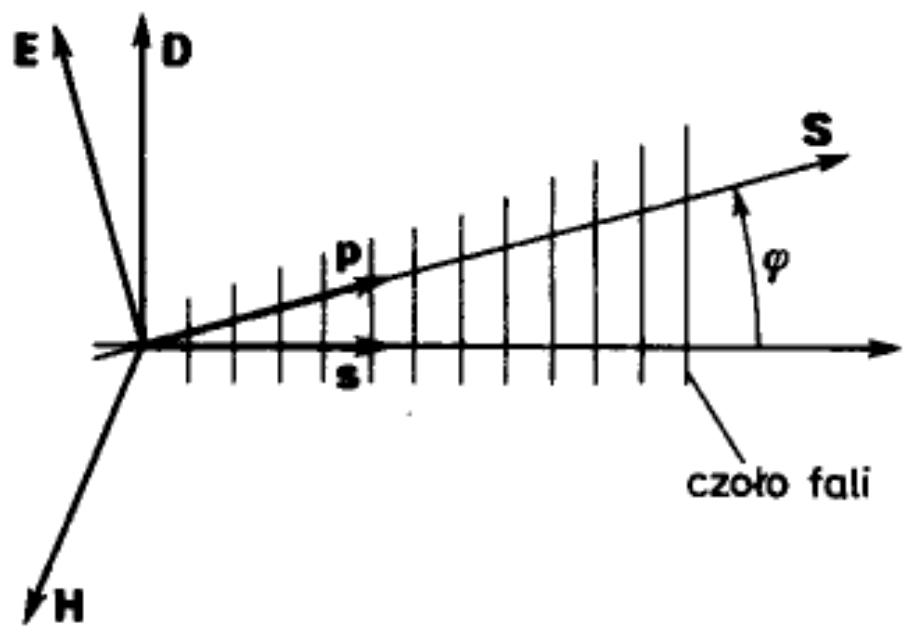
For flat EM wave in linearly anisotropic dielectric:

$$\mathbf{H} \times \mathbf{s} = c_n \mathbf{D}.$$

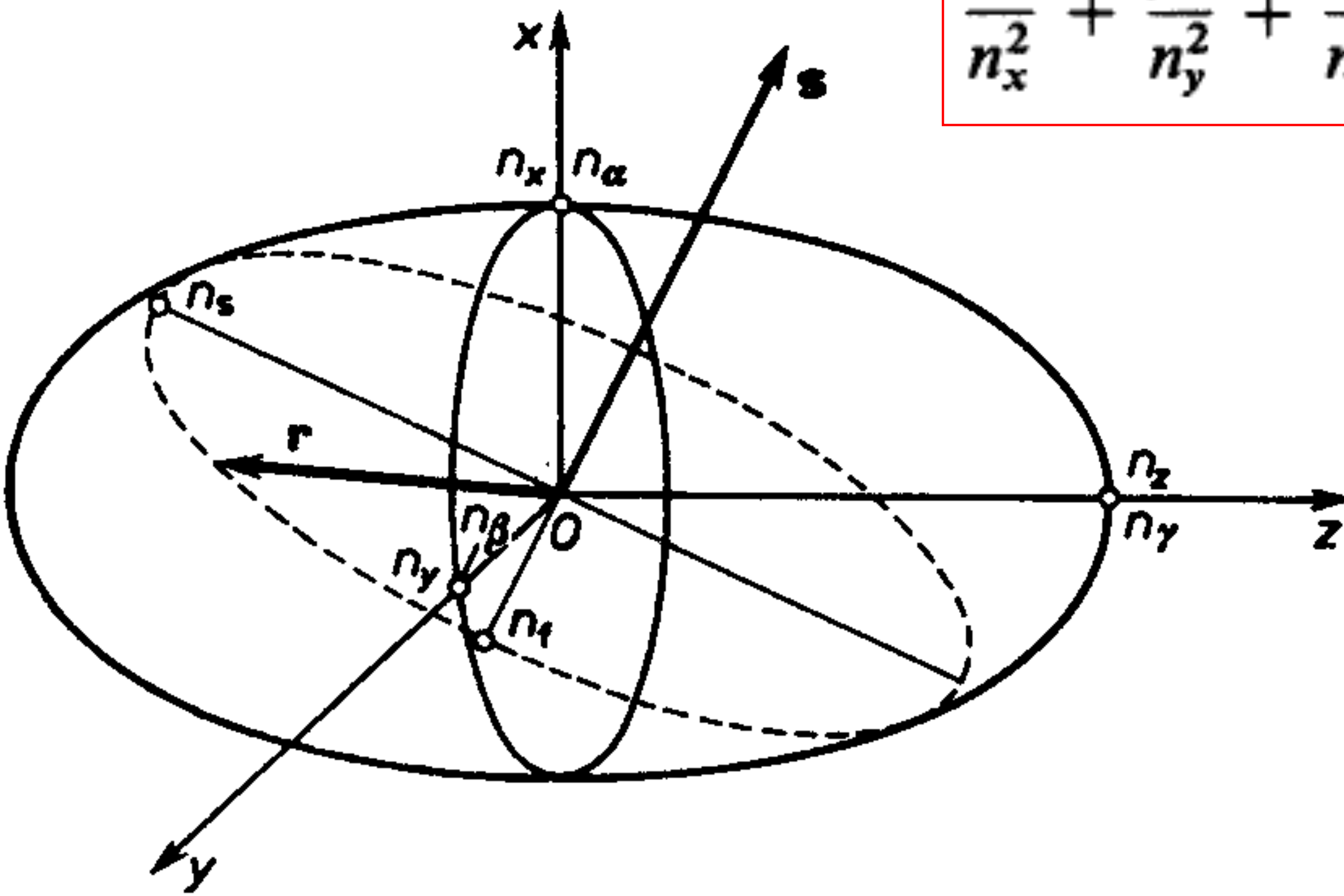
$$\mathbf{s} \times \mathbf{E} = c_n \mu\mu_0 \mathbf{H}.$$

<u>Properties:</u>

1. Vectors ***E,D, S, s*** lays in the same plane perpendicular to vector ***H,***
2. Vector ***E*** oscilates perpendicularly to the direction of vestor **S**,
3. Vector ***D*** is perpendicular to the normal **s** (tangent to the wavefront)



Refractive index indicatrix

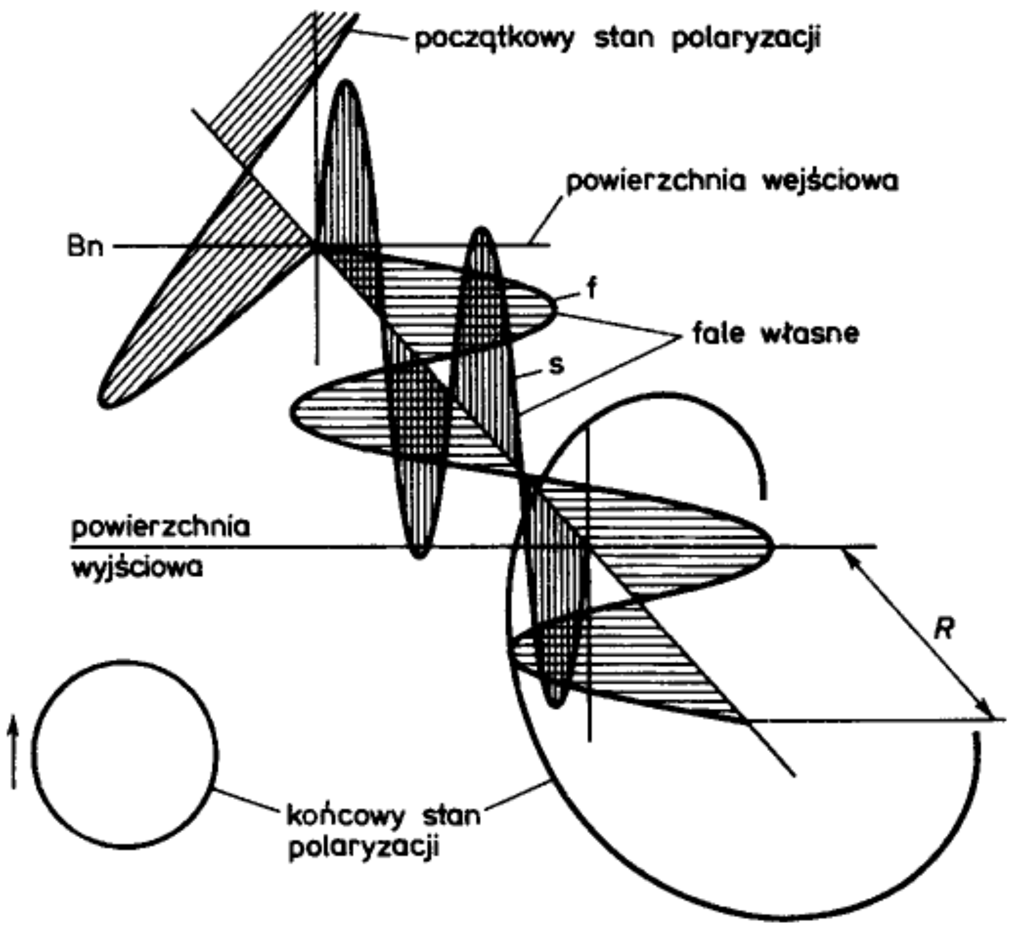


$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1,$$

Indicatrix of refractive indices

1. In the uniaxial medium two waves have the same direction as normal ***s***,
2. Long and short semi-axis of the ellipsoid corresponds to ordinary and extraordinary refractive index,
3. Refractive index n_x corresponds to vector D that oscilates along x axis,
4. z axis is parallel to the oscilation of vector D_z where refractive index is equal to n_z

EM wave in linearly birefringent dielectric

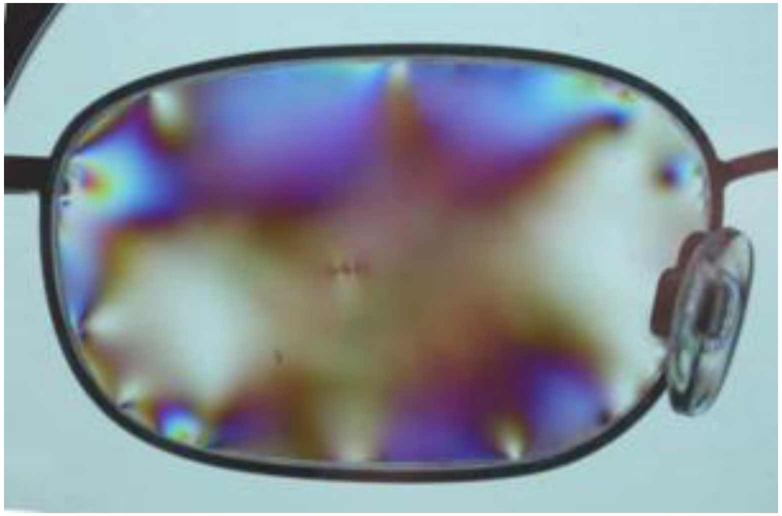


- Birefringent dielectric of thickness d,
- EM wave is being decomposed into two orthogonal linearly polarized components,
- first eigenwave (fast) and second eigenwave (slow) – only these two waves can propagate in the medium,
- Superposition of these eignewaves occurs after passing through the medium,

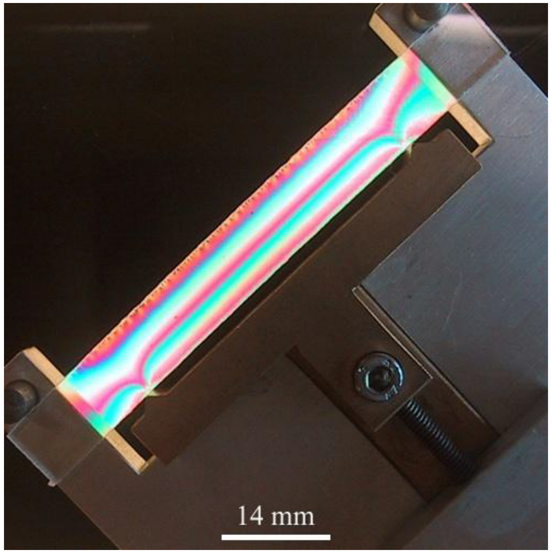
$$R = d\,|n' - n''|,$$

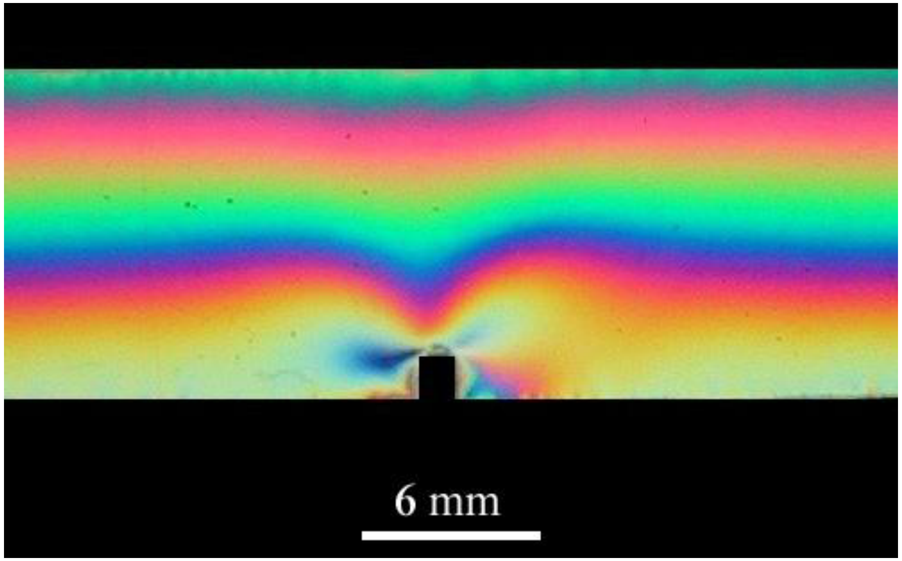
$$\gamma = 2\pi R/\lambda.$$

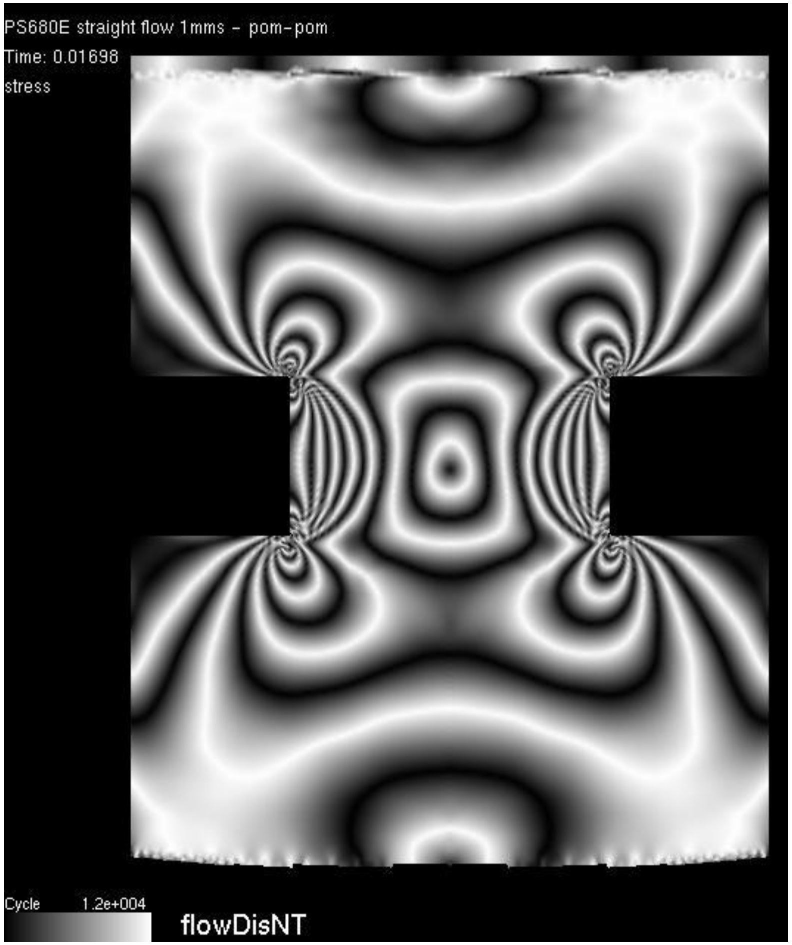
Induced birefringence



Induced birefringence







Induced birefringence

- Refractive indices ellipsoid

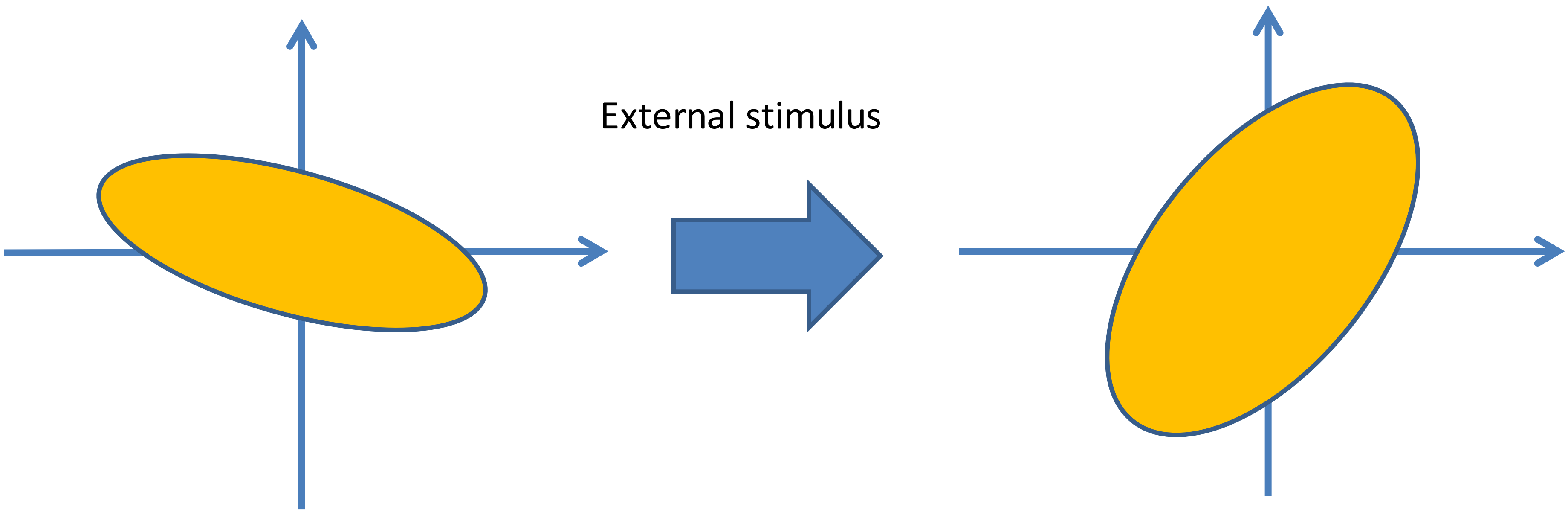
$$B_{xx}x^2 + B_{yy}y^2 + B_{zz}z^2 + B_{xy}xy + B_{yz}yz + B_{xz}xz = 1$$

where: $B_{kl} = \frac{1}{n_{kl}^2}$

External factor (electric field, magnetic field, heat, strain) generates change of the shape and spacial orientation of the ellipsoid (but still it will be an ellipsoid)

$$(B_{xx} + \Delta B_{xx})x^2 + (B_{yy} + \Delta B_{yy})y^2 + (B_{zz} + \Delta B_{zz})z^2 +$$
$$2(B_{xy} + \Delta B_{xy})xy + 2(B_{yz} + \Delta B_{yz})yz + 2(B_{xz} + \Delta B_{xz})xz = 1$$

Induced birefringence



Tensor of refractive indices can be simplified due to its symmetry

$$\begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \rightarrow \begin{bmatrix} B_1 & B_6 & B_5 \\ B_6 & B_2 & B_4 \\ B_5 & B_4 & B_3 \end{bmatrix}$$

Piezo-optic and elasto-optic effects

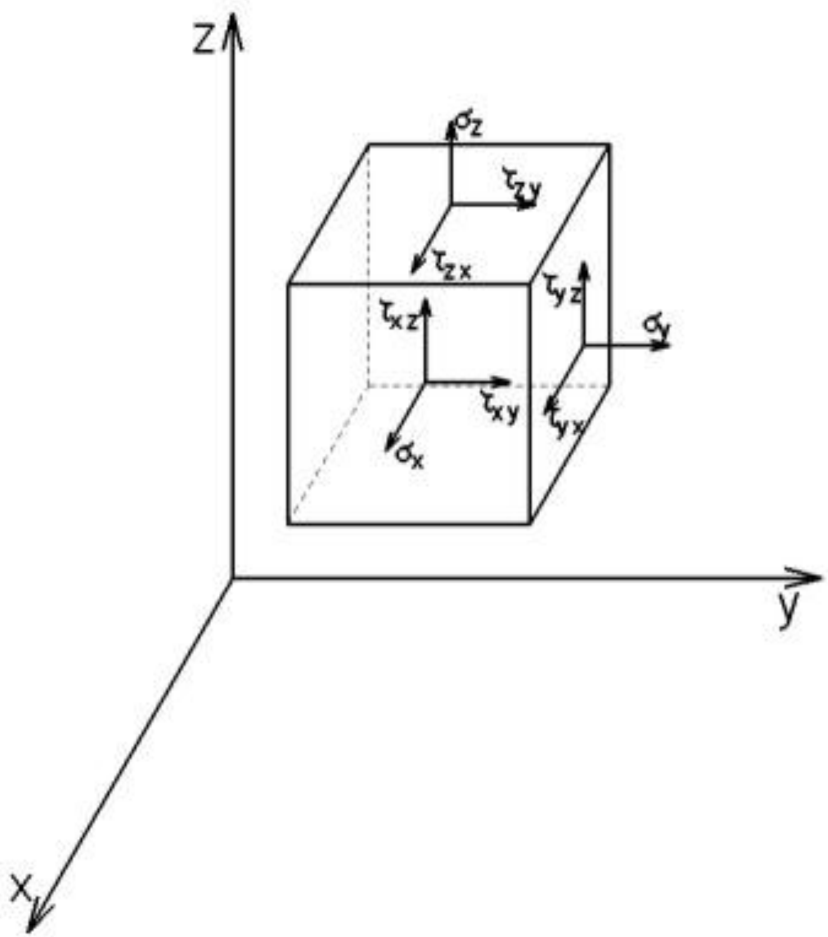
- Photo-elasticity,
- Birefringence generated by tensions or deformation,

Increments can be defined as: $$\Delta B_{ij} = \Pi_{ijkl}\sigma_{kl}$$

where: $(i, j, k, l = x, y, z)$

s – tension tensor

Piezo-optic and elasto-optic effects



$$\sigma_{kl} = \frac{F_k}{S_l} \doteq \sigma_{lk} = \frac{F_l}{S_k}$$

k – index representing direction of force $\boldsymbol{F_k}$,
l – index representing the axis perpendicular to the surface on which force F_k is acting,

Piezo-optic and elasto-optic effects

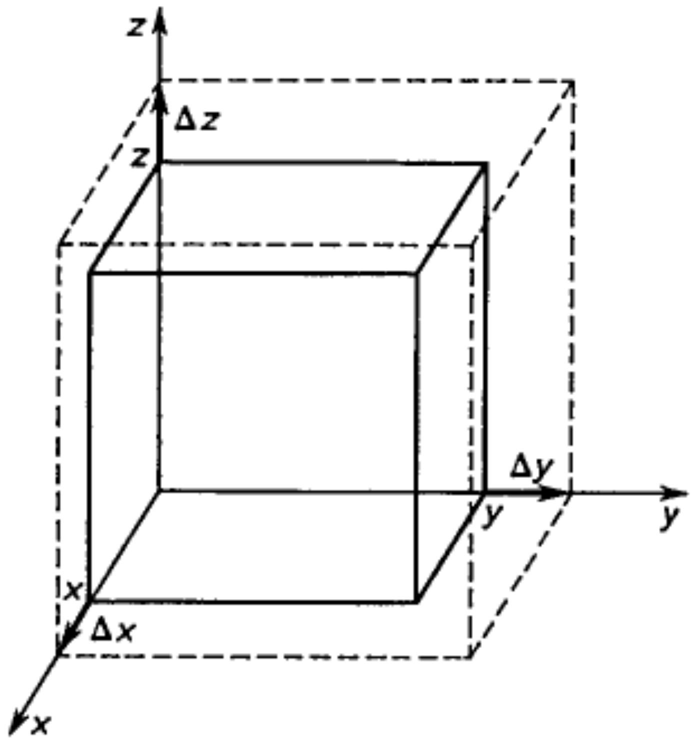
- Elasto-optic effect – birefringence change due to deformation of the medium,

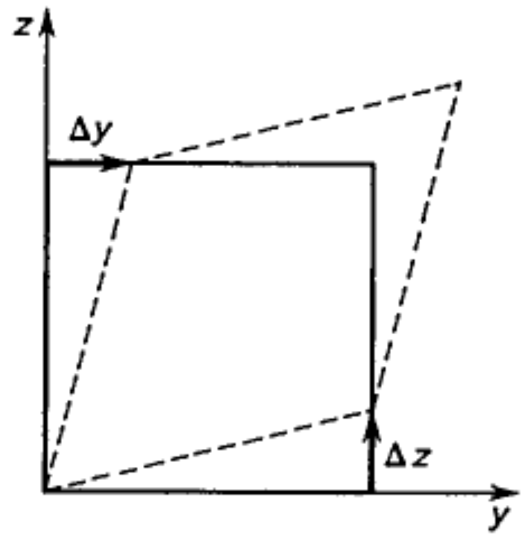
$$\Delta B_{ij} = \mathrm{P}_{ijrs}\gamma_{rs}$$

P_{ijrs} - Components of elasto-optic tensor

γ_{rs} - components of deformation tensor

Piezo-optic and elasto-optic effects





$$\gamma_{rs}=\frac{\Delta r}{s}\ (r\neq s)$$

$$\gamma_{xx}=\frac{\Delta x}{x},\ \gamma_{yy}=\frac{\Delta y}{y},\ \gamma_{zz}=\frac{\Delta z}{z},\ \gamma_{rs}=\frac{\Delta r}{s}\ (r=s)$$

$$\gamma_{yz}=\frac{\Delta y}{z}=\gamma_{zy}=\frac{\Delta z}{y}$$

Both tensors are dependent (according to Hooke's law):

$$\gamma_r = s_{rj}\sigma_j \qquad (r, j = 1, 2, \ldots, 6),$$

Calculating birefringence of the crystal with tension

- Isotropic crystal

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{11} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{12} & \Pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{aligned} \Delta B_1 &= \Pi_{11}\sigma_1 + \Pi_{12}\sigma_2 + \Pi_{12}\sigma_3, & \Delta B_4 &= 0, \\ \Delta B_2 &= \Pi_{12}\sigma_1 + \Pi_{11}\sigma_2 + \Pi_{12}\sigma_3, & \Delta B_5 &= 0, \\ \Delta B_3 &= \Pi_{12}\sigma_1 + \Pi_{12}\sigma_2 + \Pi_{11}\sigma_3, & \Delta B_6 &= 0. \end{aligned}$$

Calculating birefringence of the crystal with tension

Ellipsoid of normals can be described as:

$$(B + \Delta B_{xx})x^2 + \left(B + \Delta B_{yy}\right)y^2 + (B + \Delta B_{zz})z^2 = 1$$

Due to irregularity of the medium: Bxx = Byy = Bzz = B

We assume that EM wave is propagating along z axis:

(so our refractive index ellipsoid is described as)

$$(B + \Delta B_{xx})x^2 + \left(B + \Delta B_{yy}\right)y^2 = 1$$

Material constants

$$\Delta n = n^3 \frac{(\Pi_{11} - \Pi_{22})(\sigma_1 - \sigma_2)}{2}$$

Calculating birefringence of the crystal with tension

By defining piezo-optic constant:

$$\Delta n = c_p(\sigma_1 - \sigma_2)$$

And elasto-optic constant:

$$\Delta n = c_e(\gamma_1 - \gamma_2),$$

Elasto-optic constant

$$c_e = n^3 (p_{11} - p_{12})/2.$$

Electro-optic effect

- External electric field can influence the birefringence of the medium:

$$n' = n_0 + a' E + b' E^2 + c' E^3 + d' E^4 \ldots$$
$$n'' = n_0 + a'' E + b'' E^2 + c'' E^3 + d'' E^4 \ldots$$

Refractive index for E=0

Constants for specific wavelength

$$n' = n'_0 + a'E$$
$$n" = n"_0 + a"E$$

Pokels effect - occurs in crystals without symmetry axis

$$n' = n'_0 + b'E^2$$
$$n" = n"_0 + b"E^2$$

Kerr effect – occurs in isotropic or possessing symmetry point media,

Pokels effect

$$\Delta B_{kl} = r_{klm} E_m \quad (k, l, m = x, y, z),$$

Optical tensor is related to **electro-optic tensor**

ADP Crystal

– uniaxial medium,

- Electric field along z axis,
- EM wave also along z axis (longitudinal Pockels effect)

Pockels effect

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix}$$

Only DB6 is nonzero

$$\Delta B_6 = r_{63} E_z$$

Refractive index ellipsoid：

$$\frac{x^2+y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63} E_z xy = 1.$$

(because the wave is along z axis)

$$\frac{x^2+y^2}{n_o^2} + 2r_{63} E_z xy = 1.$$

Pockels effect

We transform our coordinates to x', y' which is retated on 45° in respect to x, y.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix},$$

$$x'^2\left(\frac{1}{n_0^2} + r_{63} E_z\right) + y'^2\left(\frac{1}{n_0^2} - r_{63} E_z\right) = 1,$$

$$n_{x'} = \frac{n_0}{\sqrt{1+n_0^2 r_{63} E_z}} \approx n_0 - r_{63} n_0^3 E_z/2,$$

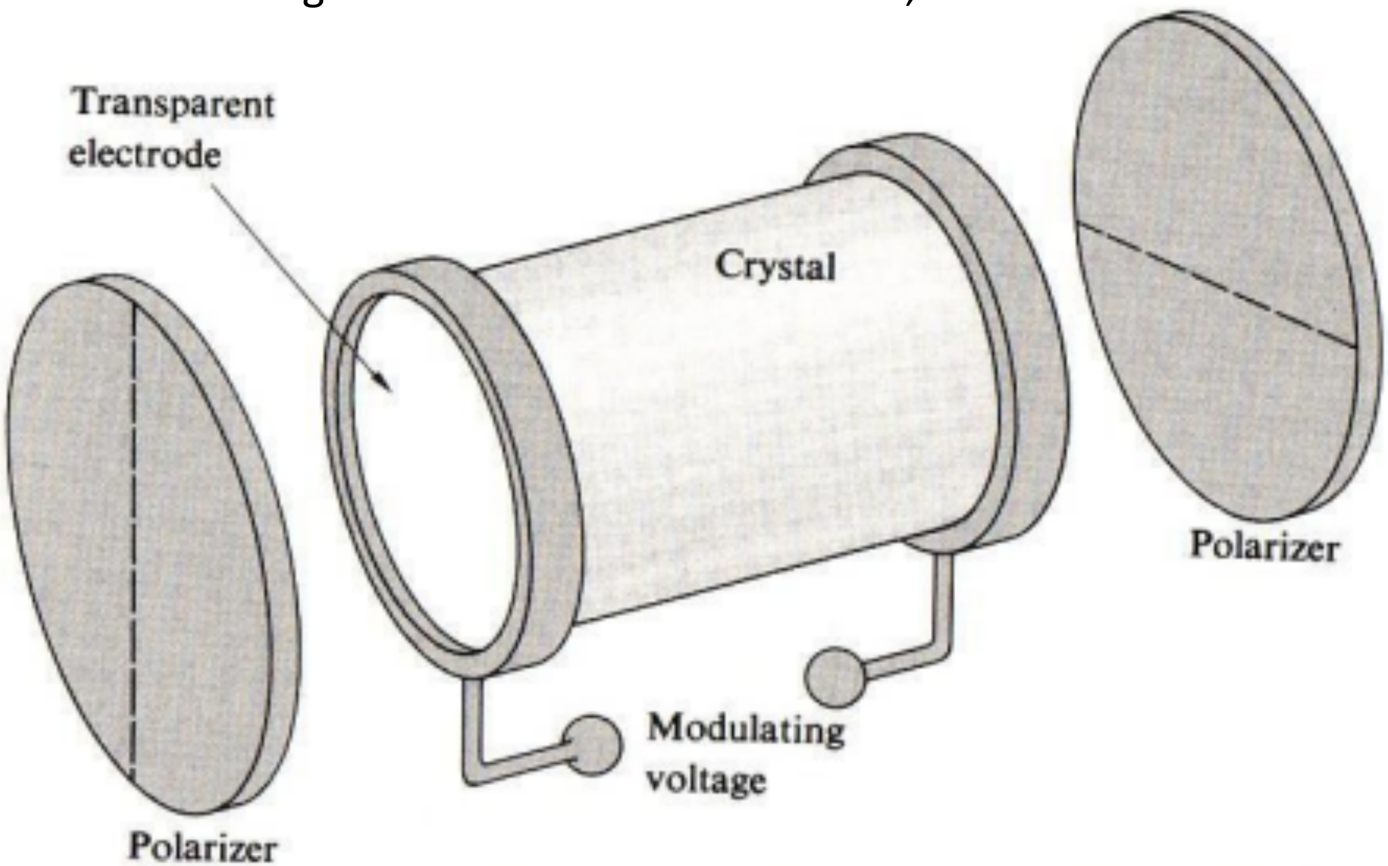
$$n_{y'} = \frac{n_0}{\sqrt{1-n_0^2 r_{63} E_z}} \approx n_0 + r_{63} n_0^3 E_z/2.$$

$$\boxed{\Delta n = r_{63} n_o^3 E_z}$$

Pockels effect

Pockels cell

- Birefringence of the crystal is changing under influence of electric field,
- Allows temporal modulation of phase difference between two eigenwaves,
- Can be observed as longitudinal and transversal effect,



Kerr effect

- In general, components with higher power than 2 can be neglected,

$$\Delta B_i = R_{ikl} E_k E_l, \quad (i = 1, 2, \ldots, 6;\ k, l = x, y, z).$$

- can occur in media with any type of symmetry,
- for isotropic media (glasses, liquids), E field induces birefringence with optical axis along direction of electric field lines,

$$\Delta n \sim E^2$$

- Can be used as switchable retardation plate,

Kerr effect

Calculating square electro-optic effect

- Isotropic crystal or nonbirefringent,
- E field acts along z axis,

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{12} & 0 & 0 & 0 \\ R_{12} & R_{11} & R_{12} & 0 & 0 & 0 \\ R_{12} & R_{12} & R_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ E_y E_z \\ E_x E_z \\ E_x E_y \end{bmatrix}$$

Kerr effect

$$\Delta B_1 = \Delta B_2 = R_{12} E_z^2, \quad \Delta B_3 = R_{11} E_z^2, \quad \Delta B_4 = \Delta B_5 = \Delta B_6 = 0.$$

Ellipsoid of normals will be defined as:

$$x^2\left(\frac{1}{n_0^2} + R_{12} E_z^2\right) + y^2\left(\frac{1}{n_0^2} + R_{12} E_z^2\right) + z^2\left(\frac{1}{n_0^2} + R_{11} E_z^2\right) = 1,$$

We obtain an uniaxial medium with binormal axis along z axis,

$$n_o \approx n_0 - 0{,}5 R_{12} n_0^3 E_z^2, \quad n_e \approx n_0 - 0{,}5 R_{11} n_0^3 E_z^2,$$

$$\boxed{\Delta n = 0{,}5(R_{12} - R_{11}) n_o^3 E_z^2}$$

Kerr effect

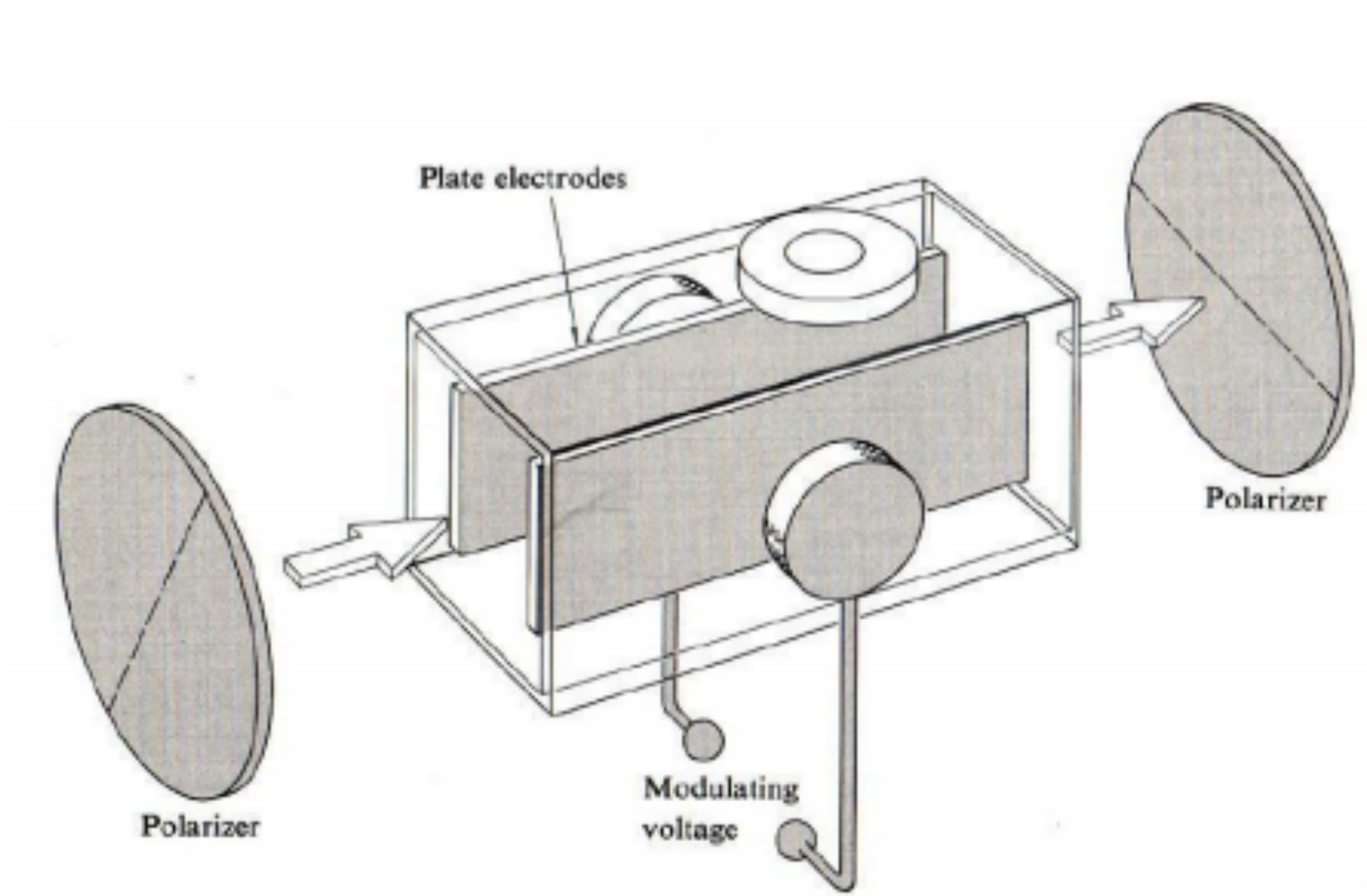


Plate electrodes
Polarizer
Polarizer
Modulating
voltage