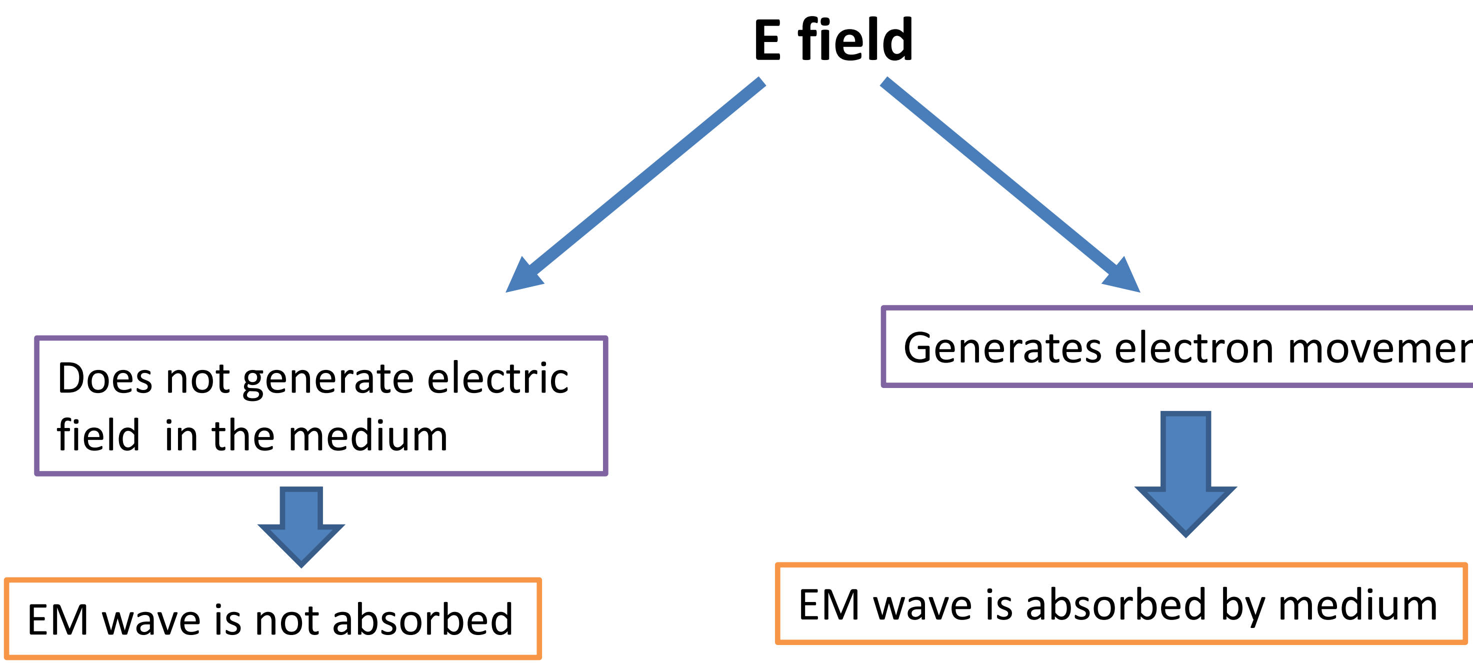
EM wave in the isotropic absorbing medium

- EM wave can generate electric field in the medium,



EM wave in the isotropic absorbing medium

Electrical conductivity σ must be included in Maxwell's equations

$$\mathrm{rot}\,\mathbf{H} - \dot{\mathbf{D}} = \sigma\,\mathbf{E},$$

$$\dot{D} = \varepsilon\varepsilon_0\dot{E}$$

$$\dot{E} = i\omega E$$

$$\mathrm{rot}\,\mathbf{H} - \varepsilon_0\left(\varepsilon - \frac{\mathrm{i}\sigma}{\varepsilon_0\omega}\right)\dot{\mathbf{E}} = 0.$$

For dielectrics:

$$\mathrm{rot}\,\mathbf{H} - \varepsilon_0\varepsilon\,\dot{\mathbf{E}} = 0.$$

$$\underline{\varepsilon} = \varepsilon - \frac{\mathrm{i}\sigma}{\varepsilon_0\omega},$$

Electrical permitivity of the conductor

EM wave in the isotropic absorbing medium

$$\underline{n} = \sqrt{\underline{\varepsilon}} = n\sqrt{1 - \frac{i\sigma}{\varepsilon\varepsilon_0\omega}},$$

Refractive index for conductor

… by expanding it to the series:

$$\sqrt{1-x} \approx 1 - \frac{1}{2}x + \cdots$$

$$\underline{n} = n(1 - i\kappa), \quad \text{gdzie} \quad \kappa = (\sigma/2\omega\varepsilon\varepsilon_0).$$

$$\underline{k} = \omega\frac{\underline{n}}{c}$$

Wave number for absorbing medium

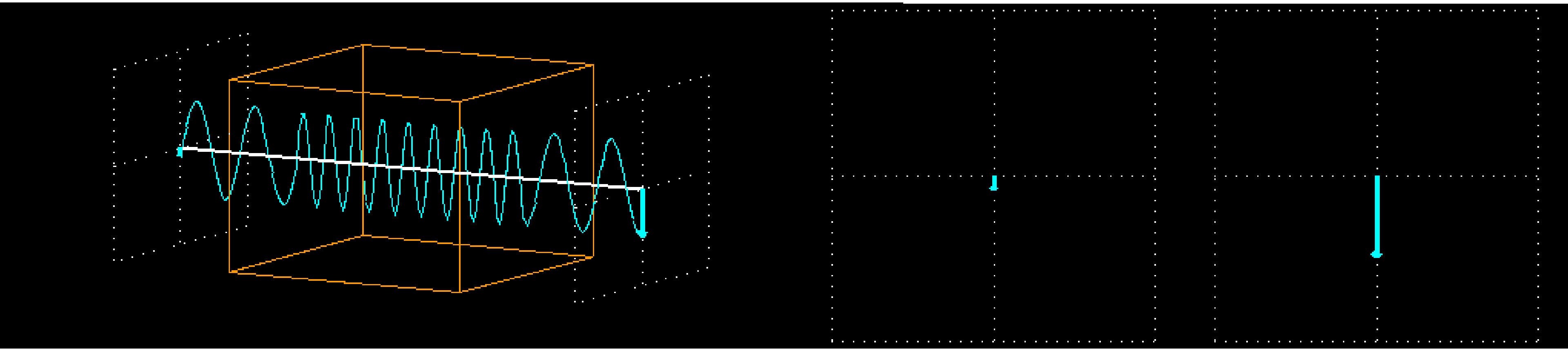
$$\mathbf{E} = \mathbf{E}_0 \exp\left(-\omega\kappa\frac{z}{c}n\right)\exp\left[i\omega\left(t - \frac{z}{c}n\right)\right].$$

Amplitude of EM wave is decreasing expotentialy during propagation in absorbing medium

Isotropic medium

In

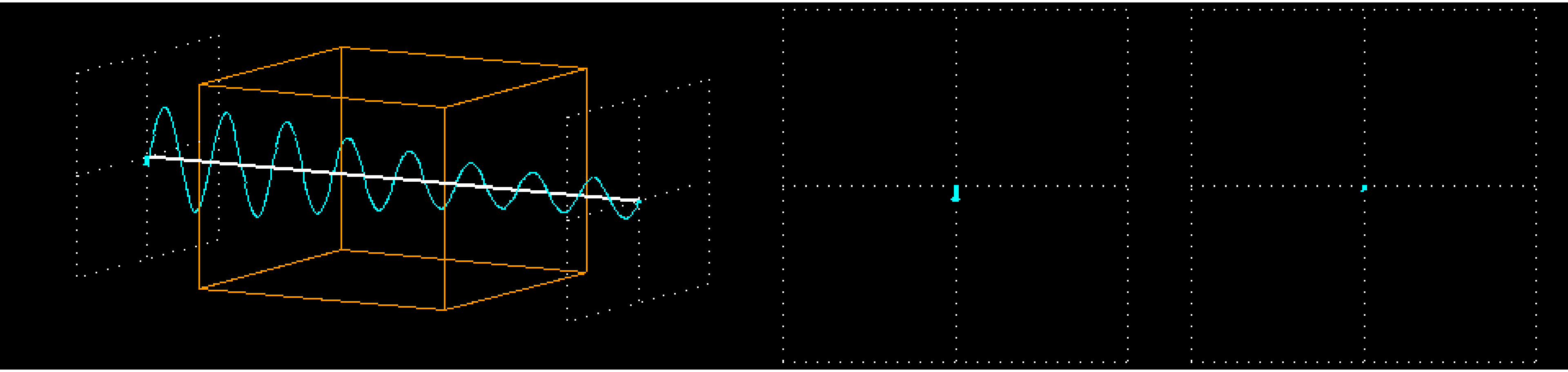
Out



Isotropic medium (with absorption)

In

Out



Optical anisotropy

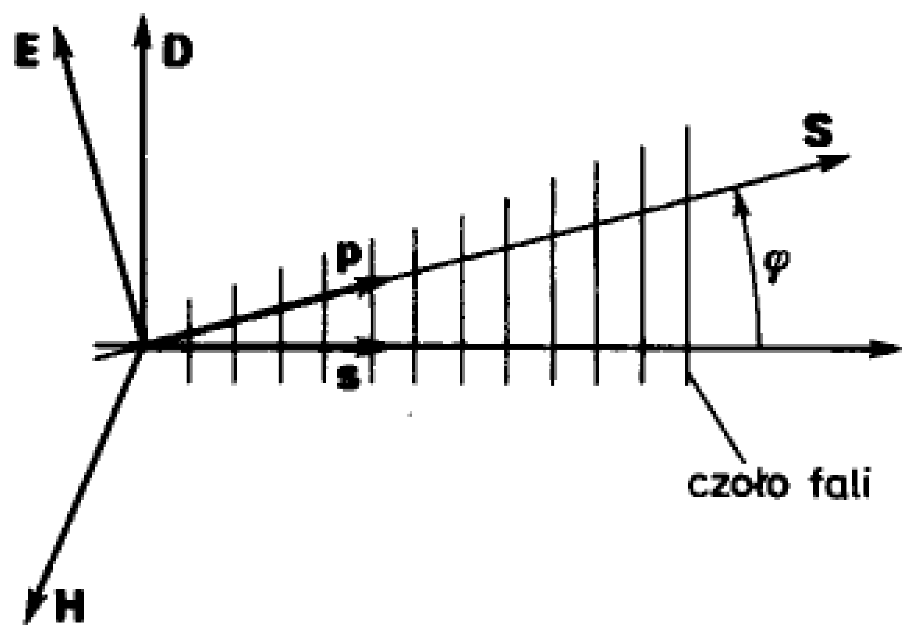
For flat EM wave in linearly anisotropic dielectric:

$$\mathbf{H} \times \mathbf{s} = c_n \mathbf{D}.$$

$$\mathbf{s} \times \mathbf{E} = c_n \mu\mu_0 \mathbf{H}.$$

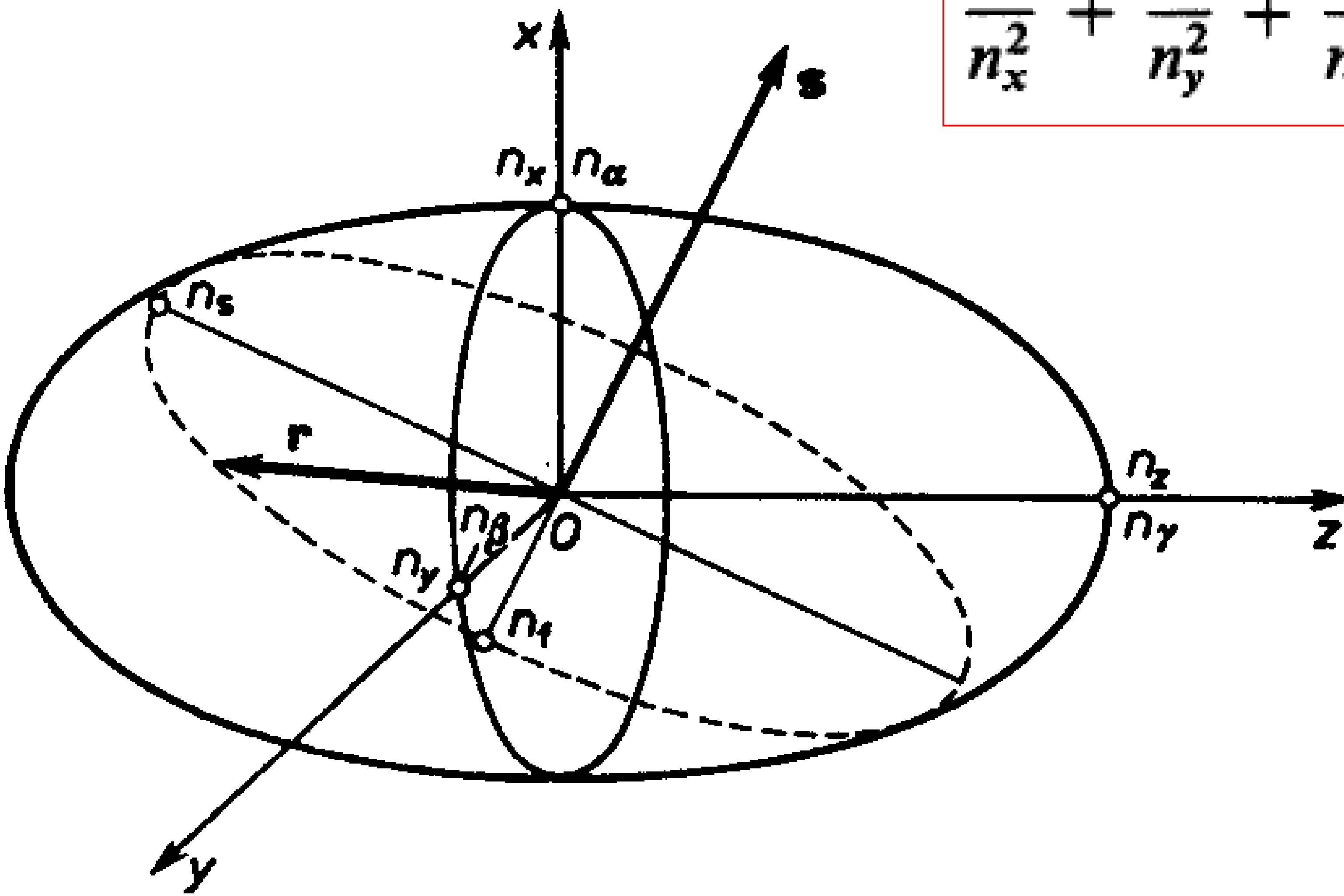
Properties:

1. Vectors ***E,D, S, s*** lays in the same plane perpendicular to vector ***H,***
2. Vector ***E*** oscilates perpendicularly to the direction of vestor **S**,
3. Vector ***D*** is perpendicular to the normal **s** (tangent to the wavefront)



Refractive index indicatrix

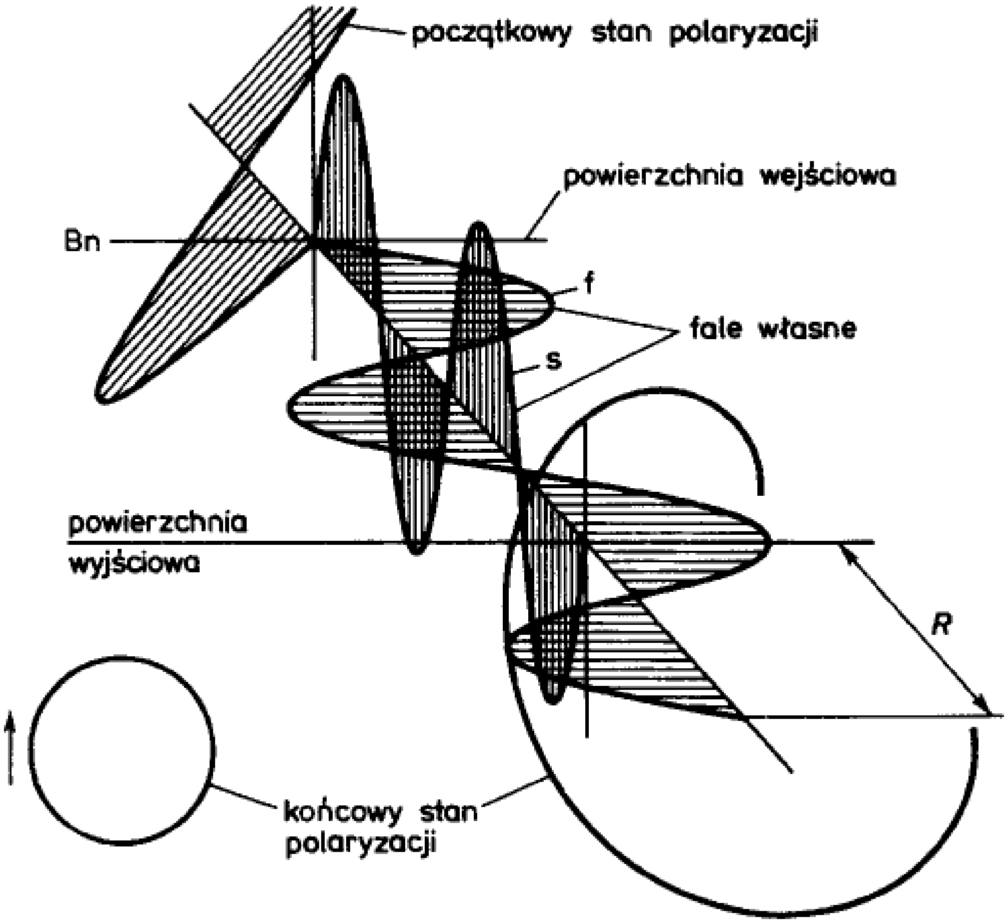
$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1,$$



Indicatrix of refractive indices

1. In the uniaxial medium two waves have the same direction as normal ***s***,
2. Long and short semi-axis of the ellipsoid corresponds to ordinary and extraordinary refractive index,
3. Refractive index n_x corresponds to vector D that oscilates along x axis,
4. z axis is parallel to the oscilation of vector D_z where refractive index is equal to n_z

EM wave in linearly birefringent dielectric

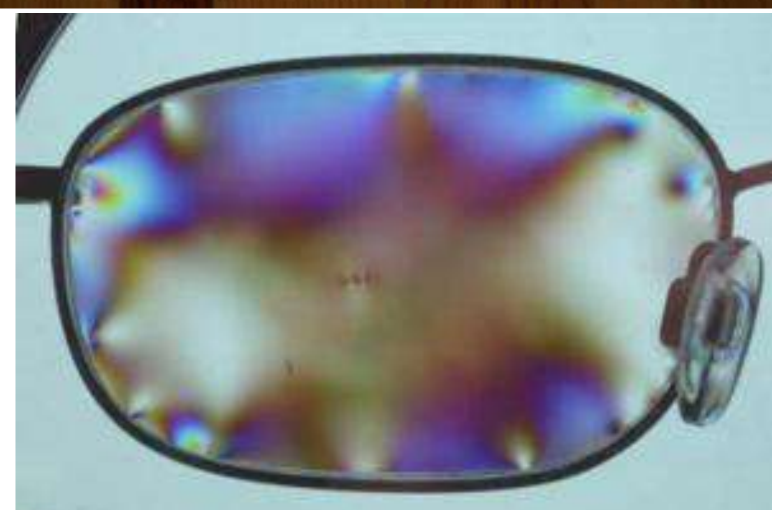


• Birefringent dielectric of thickness d,
• EM wave is being decomposed into two orthogonal linearly polarized components,
• first eigenwave (fast) and second eigenwave (slow) – only these two waves can propagate in the medium,
•Superposition of these eignewaves occurs after passing through the medium,

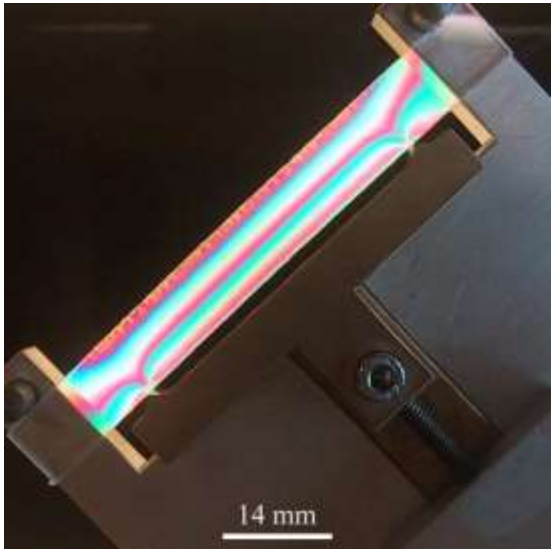
$$R = d\,|n' - n''|,$$

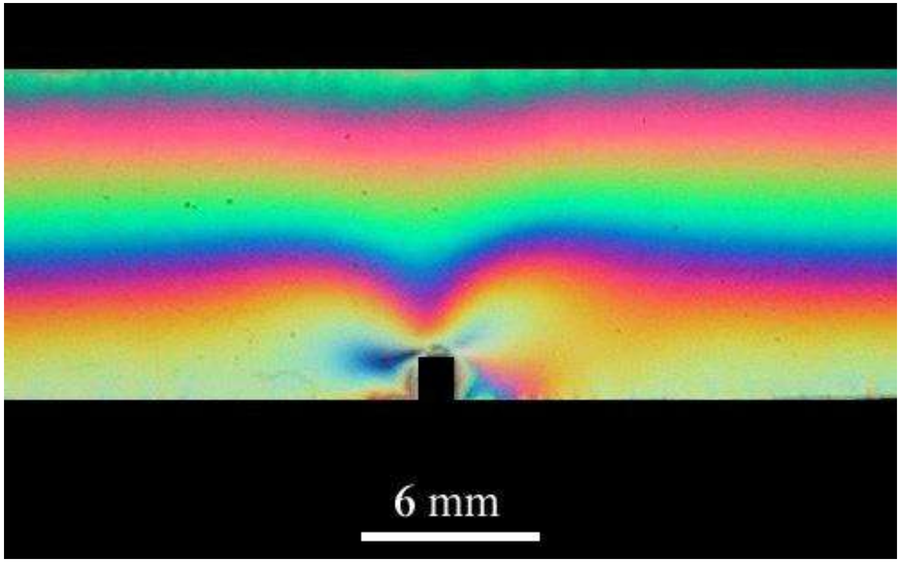
$$\gamma = 2\pi R/\lambda.$$

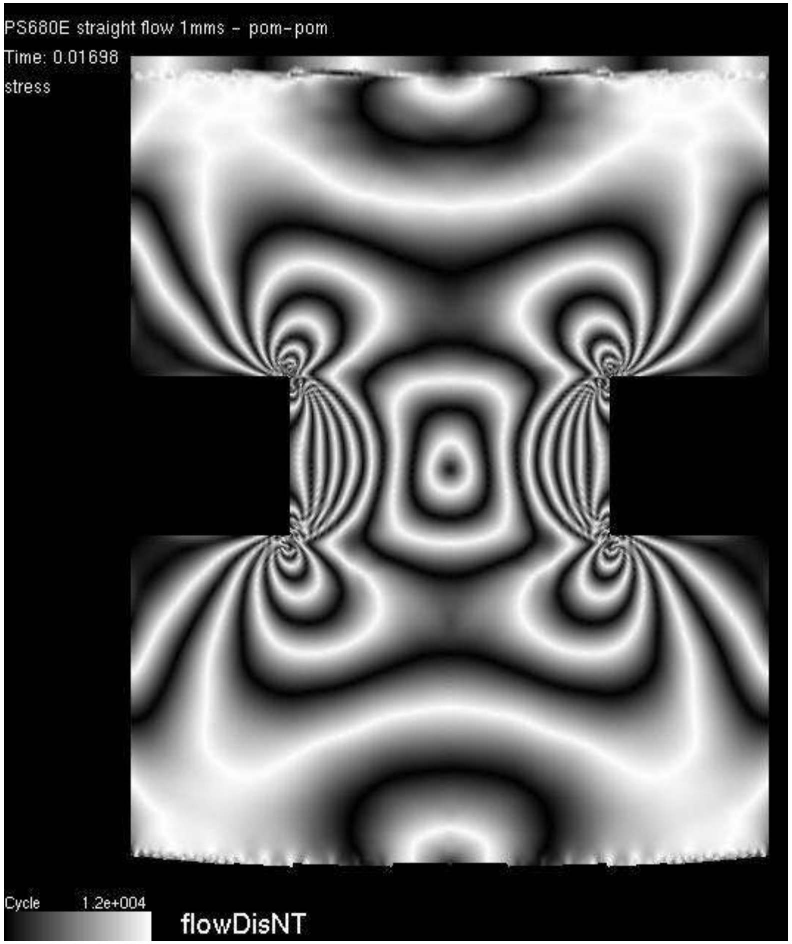
Induced birefringence



Induced birefringence







Induced birefringence

- Refractive indices ellipsoid

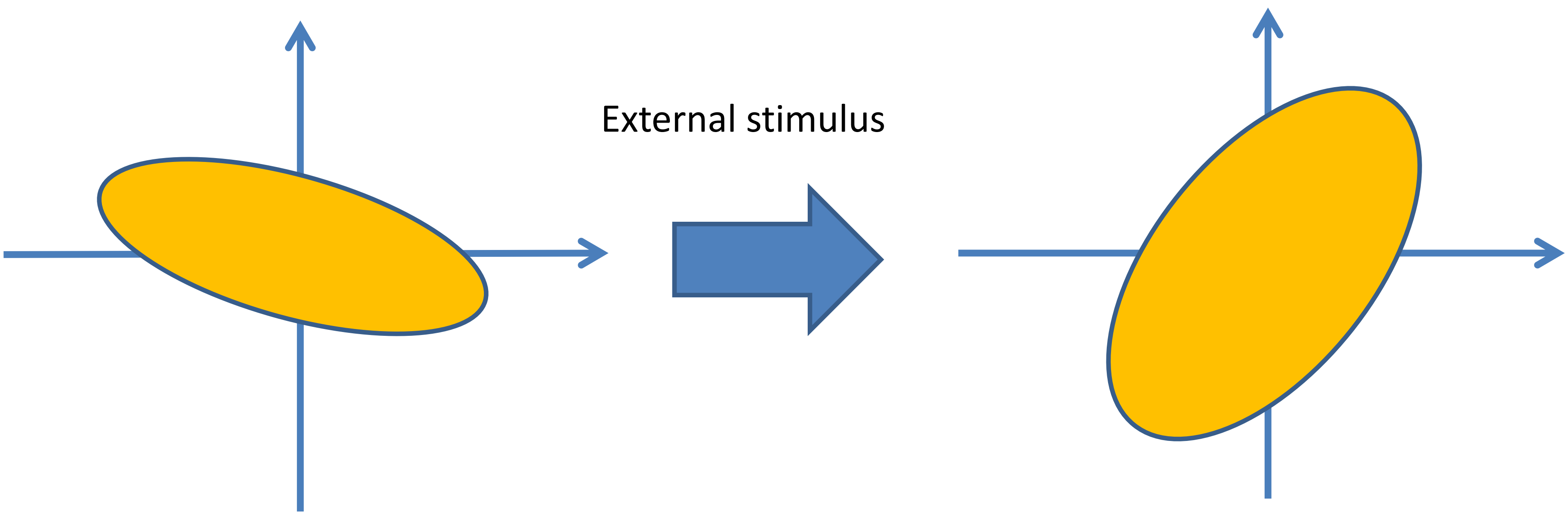
$$B_{xx}x^2 + B_{yy}y^2 + B_{zz}z^2 + B_{xy}xy + B_{yz}yz + B_{xz}xz = 1$$

where: $B_{kl} = \frac{1}{n_{kl}^2}$

External factor (electric field, magnetic field, heat, strain) generates change of the shape and spacial orientation of the ellipsoid (but still it will be an ellipsoid)

$$(B_{xx} + \Delta B_{xx})x^2 + (B_{yy} + \Delta B_{yy})y^2 + (B_{zz} + \Delta B_{zz})z^2 +$$
$$2(B_{xy} + \Delta B_{xy})xy + 2(B_{yz} + \Delta B_{yz})yz + 2(B_{xz} + \Delta B_{xz})xz = 1$$

Induced birefringence



Tensor of refractive indices can be simplified due to its symmetry

$$\begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \rightarrow \begin{bmatrix} B_1 & B_6 & B_5 \\ B_6 & B_2 & B_4 \\ B_5 & B_4 & B_3 \end{bmatrix}$$

Piezo-optic and elasto-optic effects

- Photo-elasticity,
- Birefringence generated by tensions or deformation,

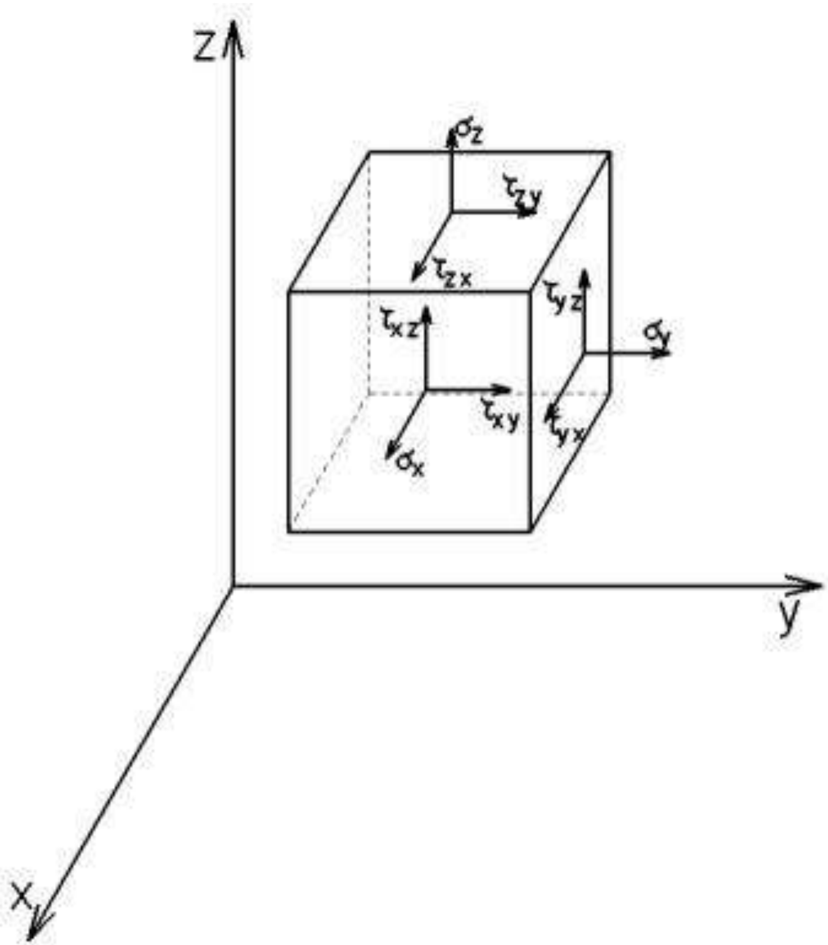
Increments can be defined as:

$$\Delta B_{ij} = \Pi_{ijkl}\sigma_{kl}$$

where: $(i, j, k ,l = x, y, z)$

σ – tension tensor

Piezo-optic and elasto-optic effects



$$\sigma_{kl} = \frac{F_k}{S_l} \doteq \sigma_{lk} = \frac{F_l}{S_k}$$

k – index representing direction of force ***F_k***,
l – index representing the axis perpendicular to the surface on which force F_k is acting,

Piezo-optic and elasto-optic effects

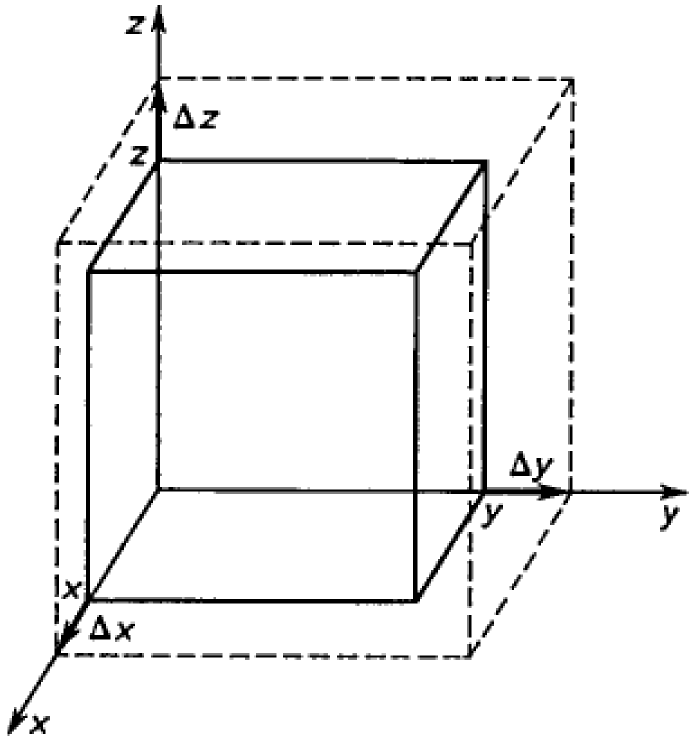
- Elasto-optic effect – birefringence change due to deformation of the medium,

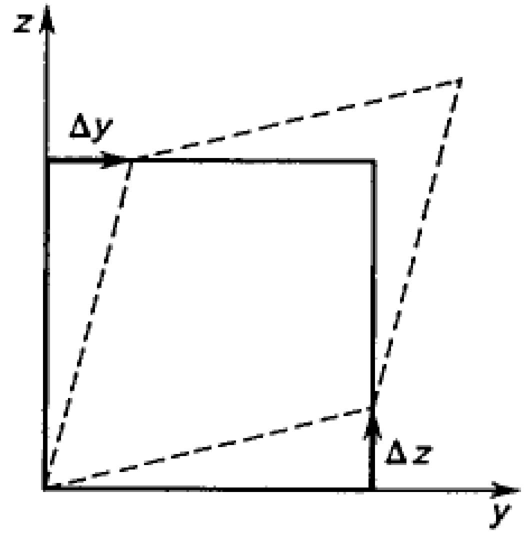
$$\Delta B_{ij} = \mathrm{P}_{ijrs}\gamma_{rs}$$

P_{ijrs} - Components of elasto-optic tensor

γ_{rs} - components of deformation tensor

Piezo-optic and elasto-optic effects





$$\gamma_{rs}=\frac{\Delta r}{s}\ (r\neq s)$$

$$\gamma_{xx}=\frac{\Delta x}{x},\ \gamma_{yy}=\frac{\Delta y}{y},\ \gamma_{zz}=\frac{\Delta z}{z},\ \gamma_{rs}=\frac{\Delta r}{s}\ (r=s)$$

$$\gamma_{yz}=\frac{\Delta y}{z}=\gamma_{zy}=\frac{\Delta z}{y}$$

Both tensors are dependent (according to Hooke's law):

$$\gamma_r = s_{rj}\sigma_j \qquad (r, j = 1, 2,\ldots, 6),$$

Calculating birefringence of the crystal with tension

- Isotropic crystal

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{11} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{12} & \Pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{aligned} \Delta B_1 &= \Pi_{11}\sigma_1 + \Pi_{12}\sigma_2 + \Pi_{12}\sigma_3, & \Delta B_4 &= 0, \\ \Delta B_2 &= \Pi_{12}\sigma_1 + \Pi_{11}\sigma_2 + \Pi_{12}\sigma_3, & \Delta B_5 &= 0, \\ \Delta B_3 &= \Pi_{12}\sigma_1 + \Pi_{12}\sigma_2 + \Pi_{11}\sigma_3, & \Delta B_6 &= 0. \end{aligned}$$

Calculating birefringence of the crystal with tension

Ellipsoid of normals can be described as:

$$(B + \Delta B_{xx})x^2 + \left(B + \Delta B_{yy}\right)y^2 + (B + \Delta B_{zz})z^2 = 1$$

Due to irregularity of the medium: Bxx = Byy = Bzz = B

We assume that EM wave is propagating along z axis:

(so our refractive index ellipsoid is described as)

$$(B + \Delta B_{xx})x^2 + \left(B + \Delta B_{yy}\right)y^2 = 1$$

Material constants

$$\Delta n = n^3 \frac{(\Pi_{11} - \Pi_{22})(\sigma_1 - \sigma_2)}{2}$$

Calculating birefringence of the crystal with tension

By defining piezo-optic constant:

$$\Delta n = c_p(\sigma_1 - \sigma_2)$$

And elasto-optic constant:

$$\Delta n = c_e(\gamma_1 - \gamma_2),$$

Elasto-optic constant

$$c_e = n^3 (p_{11} - p_{12})/2.$$

Electro-optic effect

- External electric field can influence the birefringence of the medium:

$$n' = n_0 + a' E + b' E^2 + c' E^3 + d' E^4 \ldots$$
$$n'' = n_0 + a'' E + b'' E^2 + c'' E^3 + d'' E^4 \ldots$$

Refractive index for E=0

Constants for specific wavelength

$$n' = n'_0 + a'E$$
$$n'' = n''_0 + a''E$$

Pokels effect - occurs in crystals without symmetry axis

$$n' = n'_0 + b'E^2$$
$$n'' = n''_0 + b''E^2$$

Kerr effect – occurs in isotropic or possessing symmetry point media,

Pokels effect

$$\Delta B_{kl} = r_{klm} E_m \quad (k, l, m = x, y, z),$$

Optical tensor is related to **electro-optic tensor**

ADP Crystal

– uniaxial medium,

- Electric field along z axis,
- EM wave also along z axis (longitudinal Pockels effect)

Pockels effect

$$
\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix}
$$

Only ΔB_6 is nonzero

$$\Delta B_6 = r_{63} E_z$$

Refractive index ellipsoid :

$$\frac{x^2+y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63} E_z xy = 1.$$

(because the wave is along z axis)

$$\frac{x^2+y^2}{n_o^2} + 2r_{63} E_z xy = 1.$$

Pockels effect

We transform our coordinates to x’, y’ which is retated on 45° in respect to x, y.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix},$$

$$x'^2\left(\frac{1}{n_0^2} + r_{63}E_z\right) + y'^2\left(\frac{1}{n_0^2} - r_{63}E_z\right) = 1,$$

$$n_{x'} = \frac{n_0}{\sqrt{1+n_0^2 r_{63}E_z}} \approx n_0 - r_{63}n_0^3 E_z/2,$$

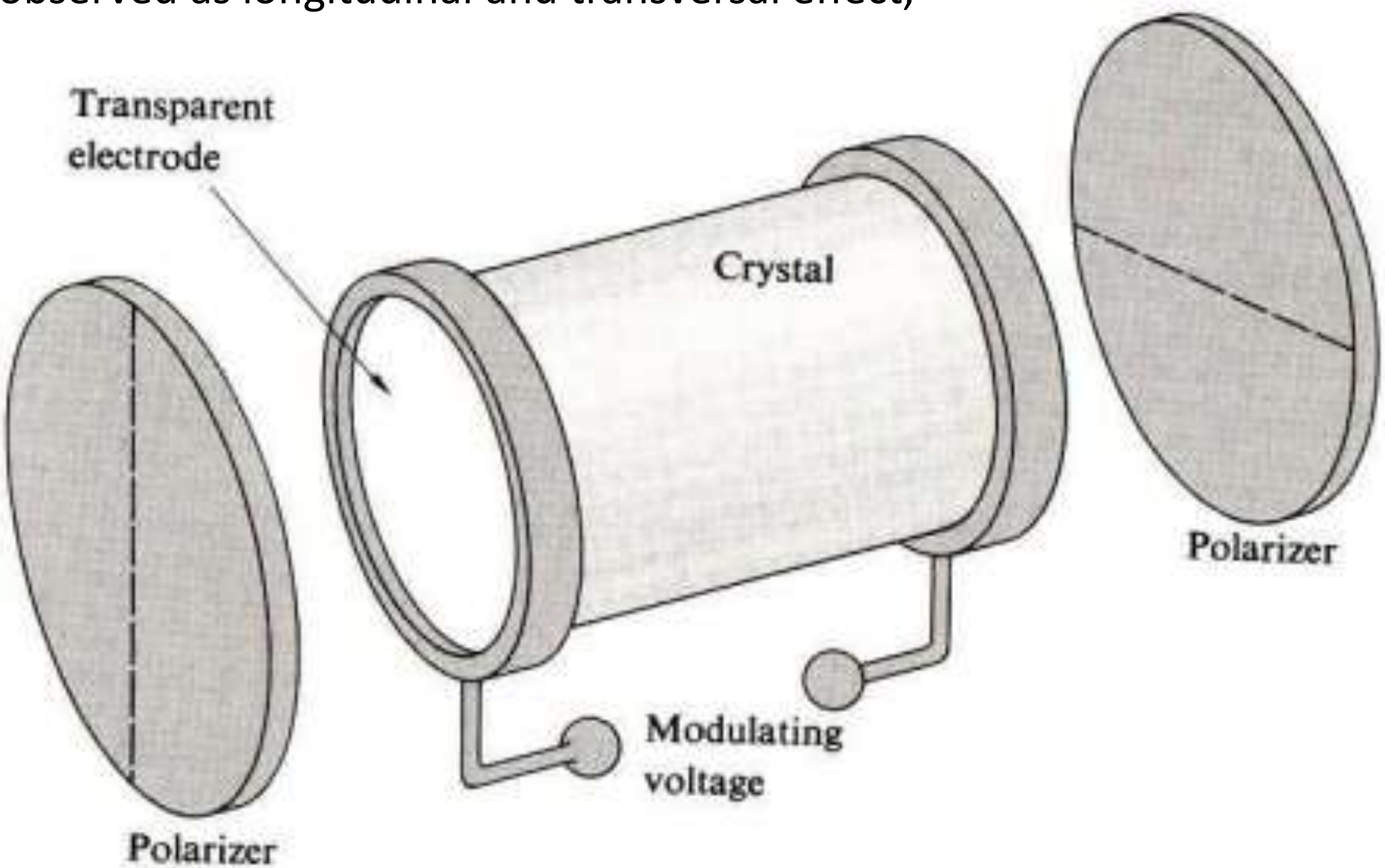
$$n_{y'} = \frac{n_0}{\sqrt{1-n_0^2 r_{63}E_z}} \approx n_0 + r_{63}n_0^3 E_z/2.$$

$$\Delta n = r_{63} n_o^3 E_z$$

Pockels effect

Pockels cell

- Birefringence of the crystal is chenging under influence of electric field,
- Allows temporal modulation of phase difference between two eigenwaves,
- Can be observed as longitudinal and transversal effect,



Kerr effect

- In general, components with higher power than 2 can be neglected,

$$\Delta B_i = R_{ikl} E_k E_l, \quad (i = 1, 2, \ldots, 6;\ k, l = x, y, z).$$

- can occur in media with any type of symmetry,
- for isotropic media (glasses, liquids), E field induces birefringence with optical axis along direction of electric field lines,

$$\Delta n \sim E^2$$

- Can be used as switchable retardation plate,

Kerr effect

Calculating square electro-optic effect

- Isotropic crystal or nonbirefringent,
- E field acts along z axis,

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \\ \Delta B_4 \\ \Delta B_5 \\ \Delta B_6 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{12} & 0 & 0 & 0 \\ R_{12} & R_{11} & R_{12} & 0 & 0 & 0 \\ R_{12} & R_{12} & R_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ E_yE_z \\ E_xE_z \\ E_xE_y \end{bmatrix}$$

Kerr effect

$$\Delta B_1 = \Delta B_2 = R_{12} E_z^2, \quad \Delta B_3 = R_{11} E_z^2, \quad \Delta B_4 = \Delta B_5 = \Delta B_6 = 0.$$

Ellipsoid of normals will be defined as:

$$x^2\left(\frac{1}{n_0^2} + R_{12} E_z^2\right) + y^2\left(\frac{1}{n_0^2} + R_{12} E_z^2\right) + z^2\left(\frac{1}{n_0^2} + R_{11} E_z^2\right) = 1,$$

We obtain an uniaxial medium with binormal axis along z axis,

$$n_o \approx n_0 - 0{,}5 R_{12} n_0^3 E_z^2, \quad n_e \approx n_0 - 0{,}5 R_{11} n_0^3 E_z^2,$$

$$\Delta n = 0{,}5(R_{12} - R_{11}) n_o^3 E_z^2$$

Kerr effect

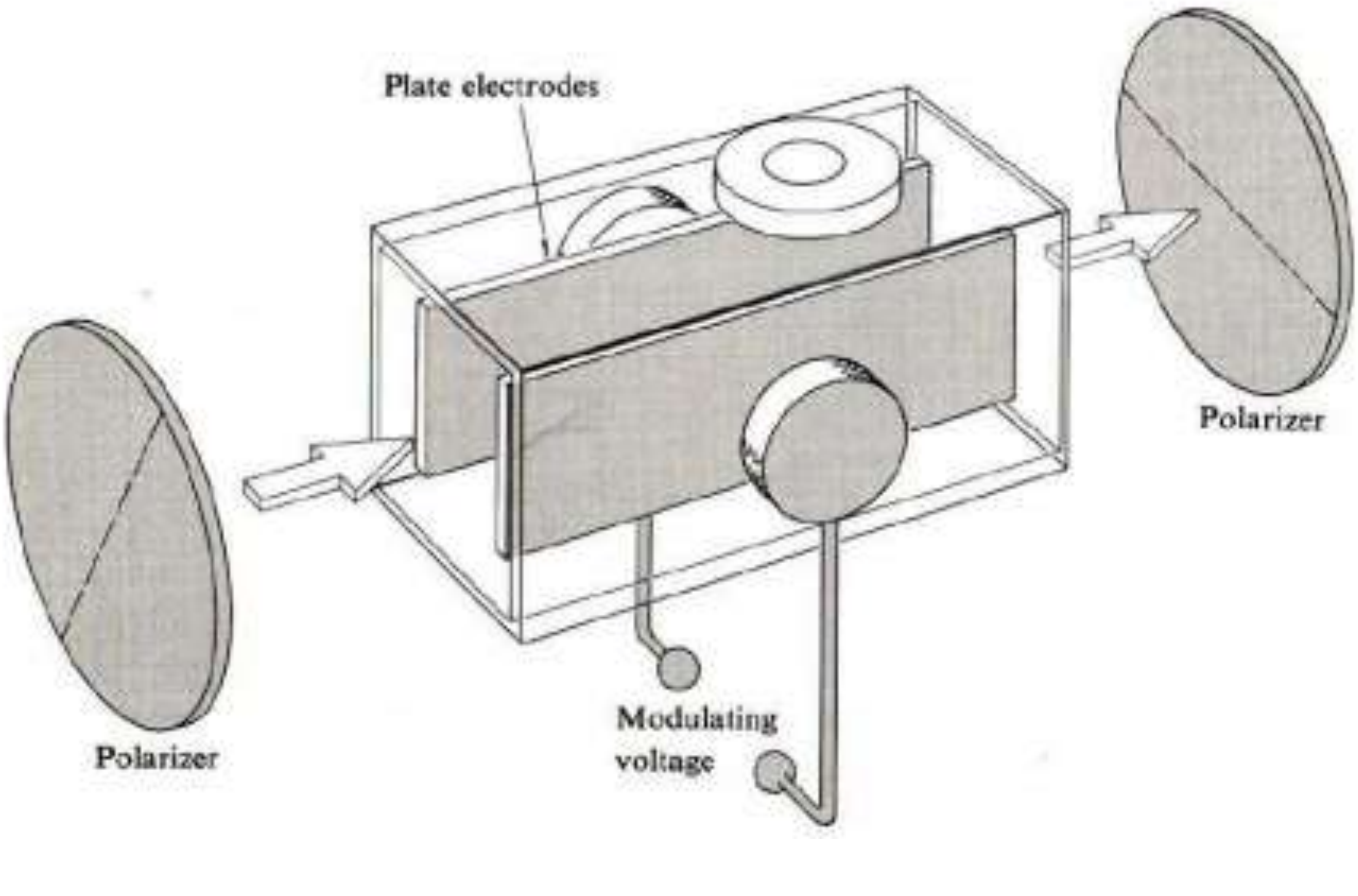


Plate electrodes
Polarizer
Polarizer
Modulating
voltage