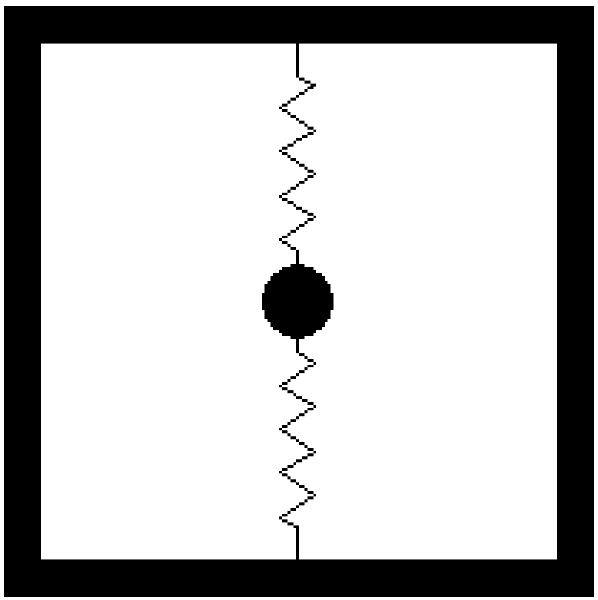
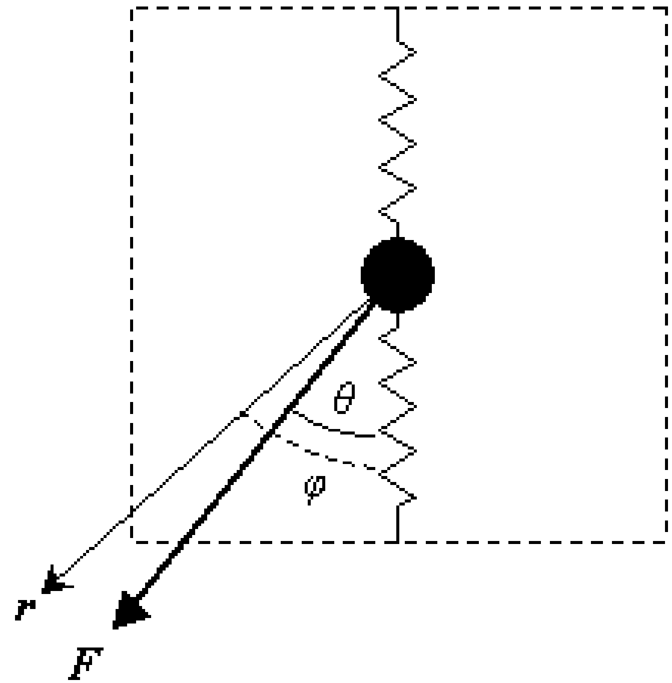
Anisotropy

Mechanical analogy to anisotropic response

A stimulus does not induce a response parallel to it,

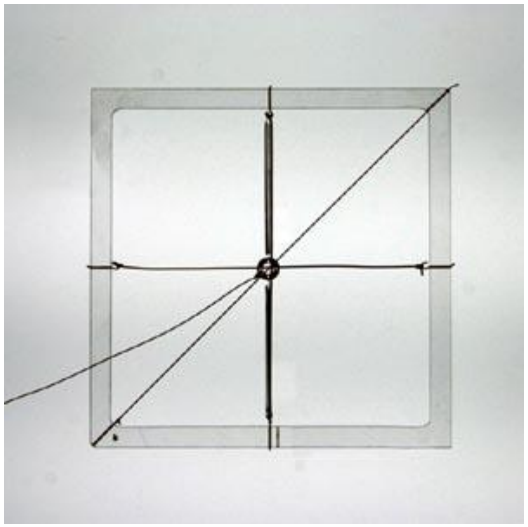


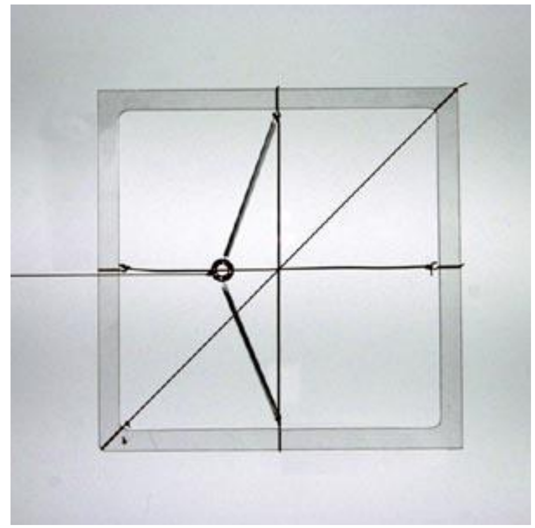


A response is the displacement r at an angle φ

Mechanical analogy to anisotropic response – principal directions

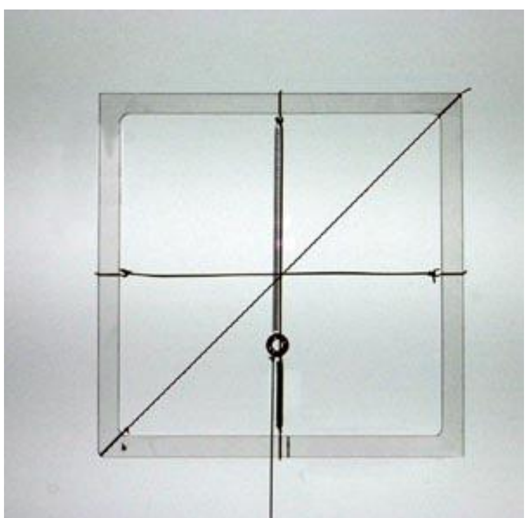
No force applied

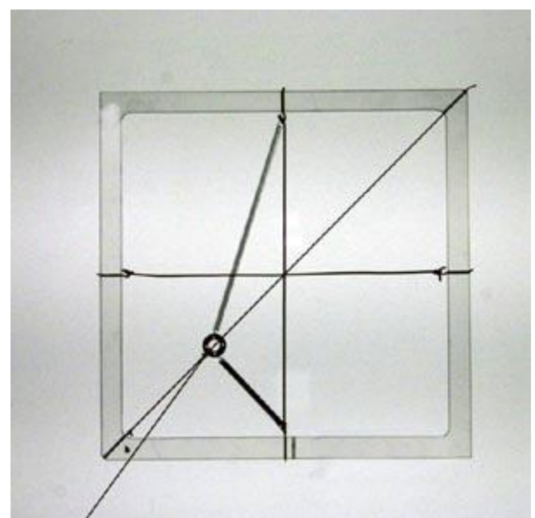




Horizontal force producing horizontal displacement (parallel response) ($\vartheta = \varphi = 90º$)

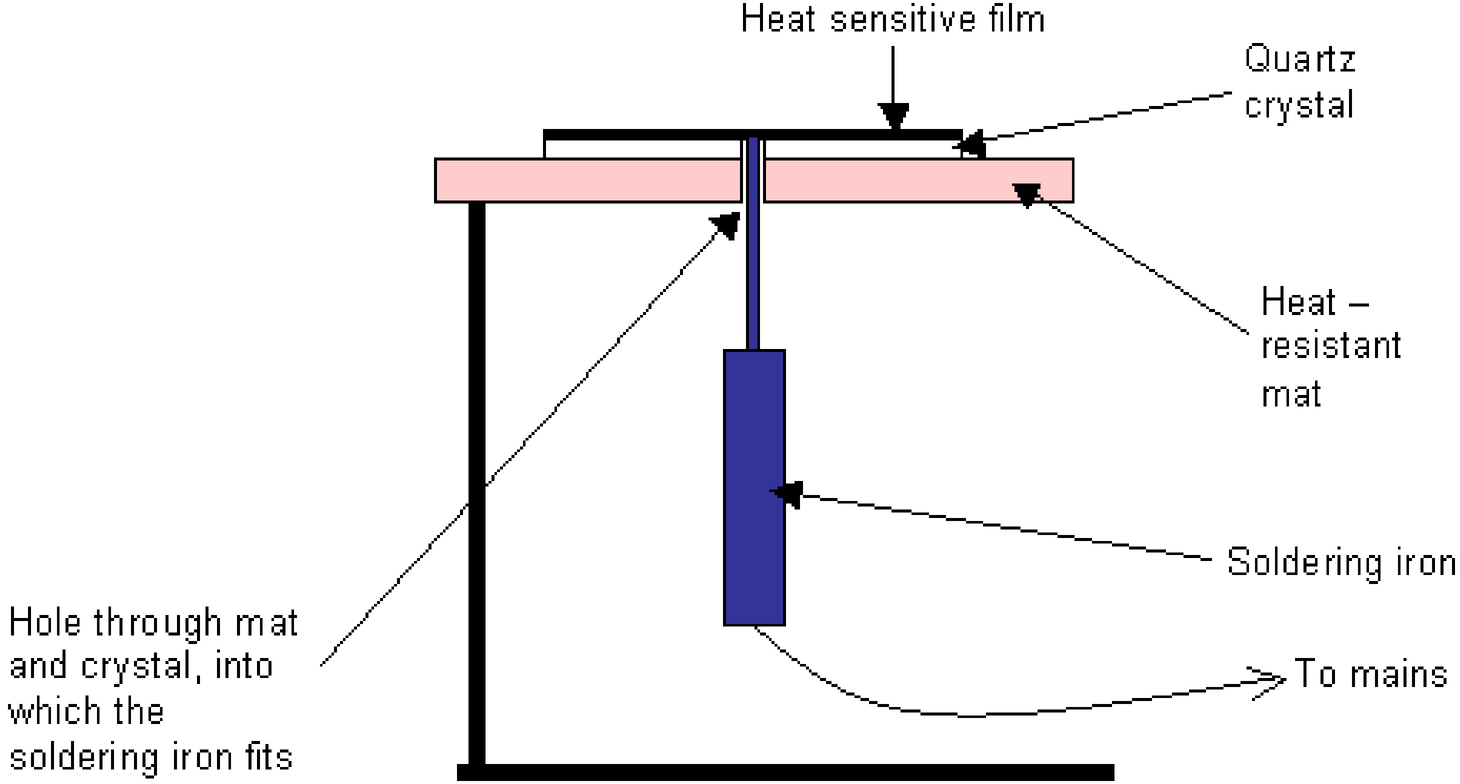
Vertical force producing vertical displacement (parallel response) ($\vartheta = \varphi = 0º$)





A 45º displacement from non-45º force (non-parallel anisotropic response) (ϑ = approx 35º, φ = 45º)

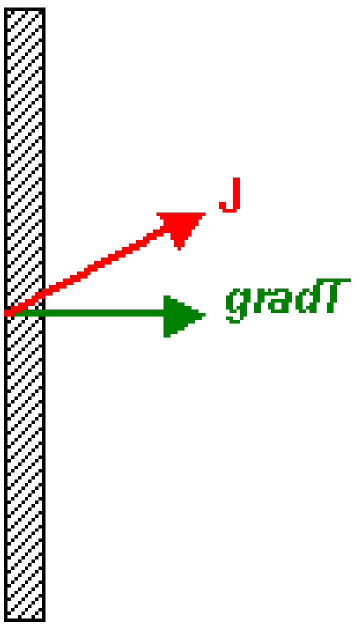
Anisotropic thermal conductivity - Quartz

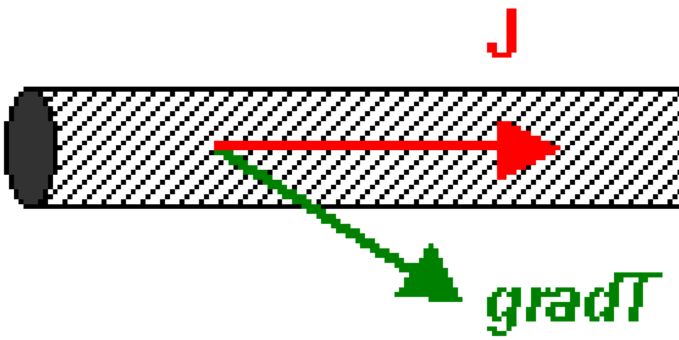


Quartz

Video of a section of quartz cut perpendicular to the c axis being heated from a point at its centre

Video of a section of quartz cut parallel to the c axis being heated from a point at its centre

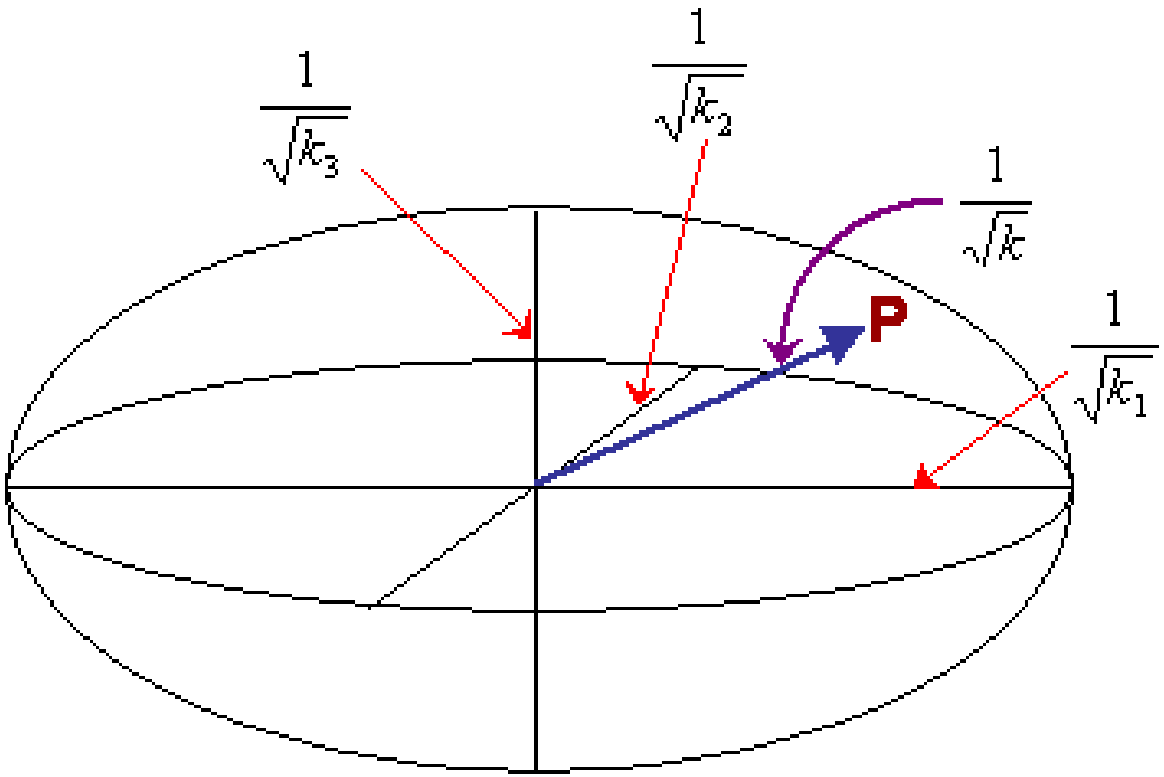




Rod:

Thin plate:

The anisotropy ellipsoid



The anisotropy ellipsoid

the components of the temperature gradient parallel to the principal axes will be

$$gradT_x = (gradT)l gradT_y = (gradT)m gradT_z = (gradT)n$$

The components of the heat flux are:

$$j_x = k_1(gradT)l j_y = k_2(gradT)m j_z = k_3(gradT)n$$

where k_1, k_2 and k_3 are the values of thermal conductivity along the principal axes, *x*, *y* and *z*, and are called the principal values.
Hence, resolving back along the direction of the temperature gradient, the heat flux is:

$$j_{||} = j_x l + j_y m + j_z n = (k_1 l^2 + k_2 m^2 + k_3 n^2) gradT$$

Thus the value of the thermal conductivity, k_{lmn}, defined by $k_{lmn} = \frac{j_{||}}{gradT}$

is related to the principal values and the directional cosines by:

$$k = k_1 . l^2 + k_2 . m^2 + k_3 . n^2$$

$$l = {}^{x}/_{r} \quad m = {}^{y}/_{r} \quad n = {}^{z}/_{r}$$

The anisotropy ellipsoid

Substituting in our equation for *k* gives:

$$k = \frac{k_1 x^2}{r^2} + \frac{k_2 y^2}{r^2} + \frac{k_3 z^2}{r^2} = \frac{1}{r^2}(k_1 x^2 + k_2 y^2 + k_3 z^2)$$

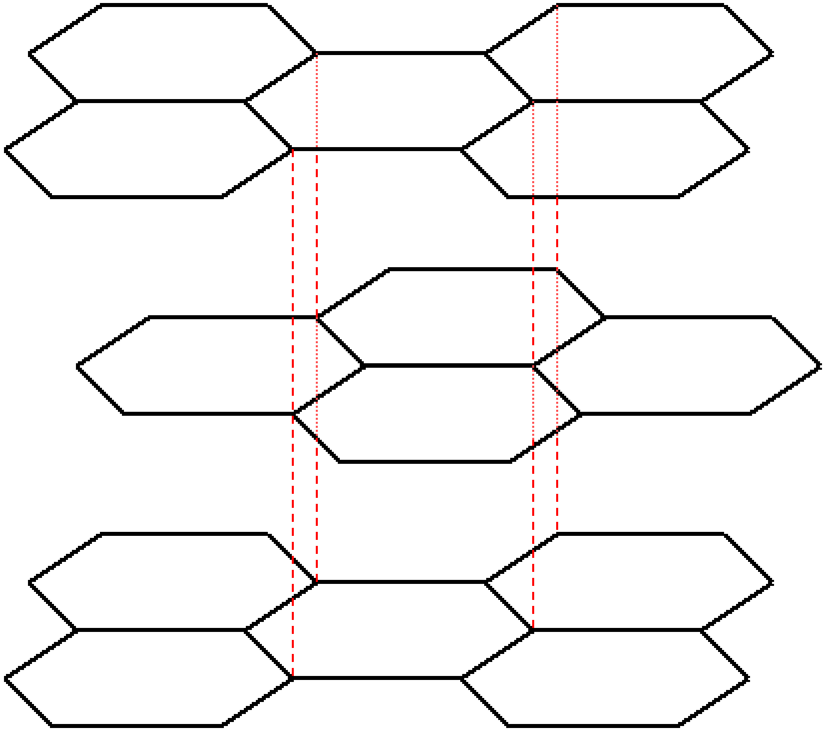
Setting $r = {}^{1}\!/\!\sqrt{k}$ then

$$k_1 x^2 + k_2 y^2 + k_3 z^2 = 1$$

If all the principal values are positive (as they must be for thermal conductivity), then this equation describes the surface of an ellipsoid.

Anisotropic electrical conductivity

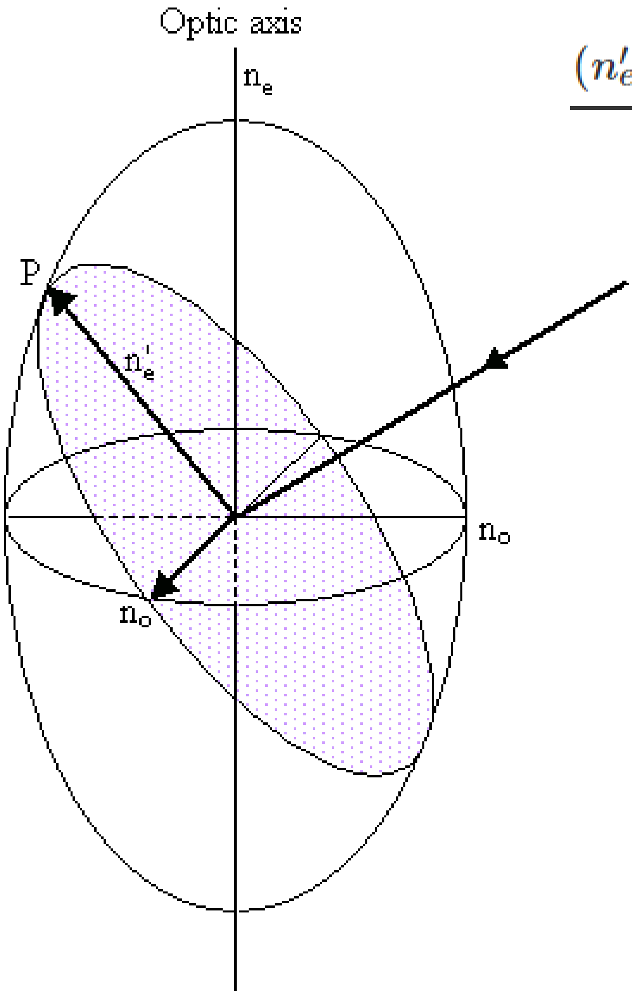
$$\boldsymbol{J} = \boldsymbol{\sigma E}$$



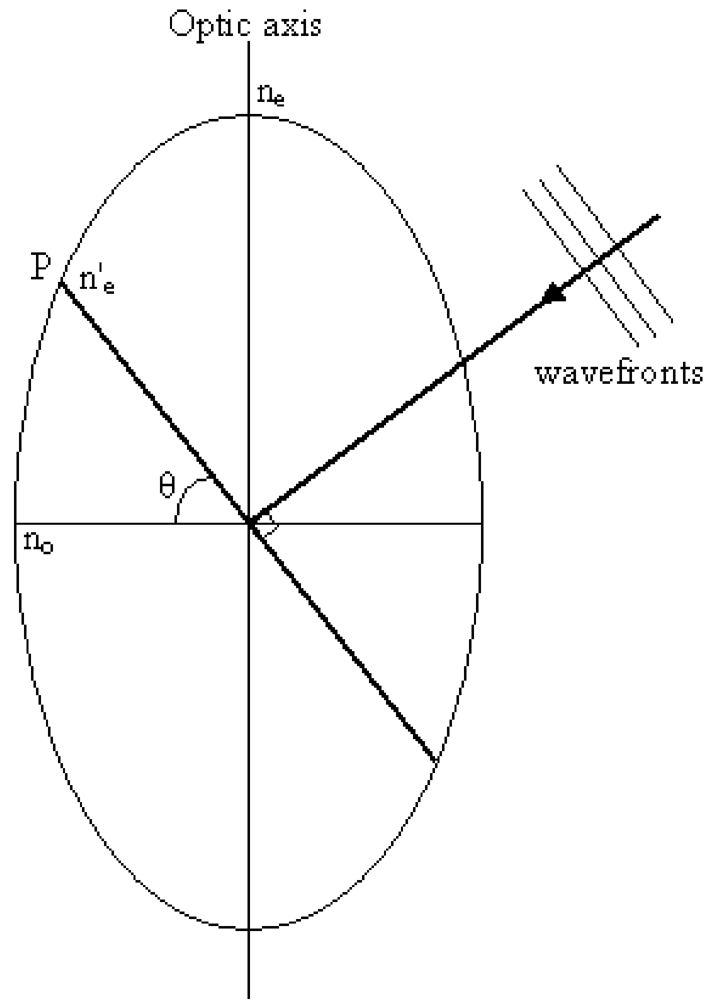
Graphite:

- easy conduction along the planes,
- 3 orders of magnitude smaller concuctivity in perpendicular direction

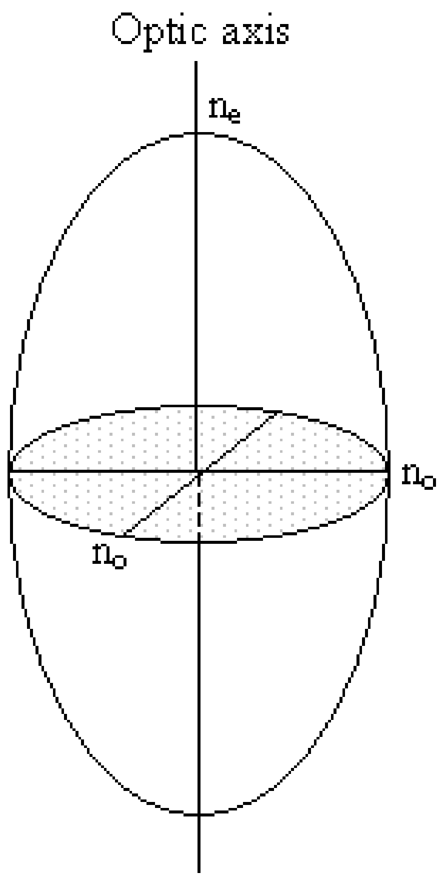
Optical anisotropy



$$\frac{(n_e' \cos\theta)^2}{n_o^2} + \frac{(n_e' \sin\theta)^2}{n_e^2} = 1$$



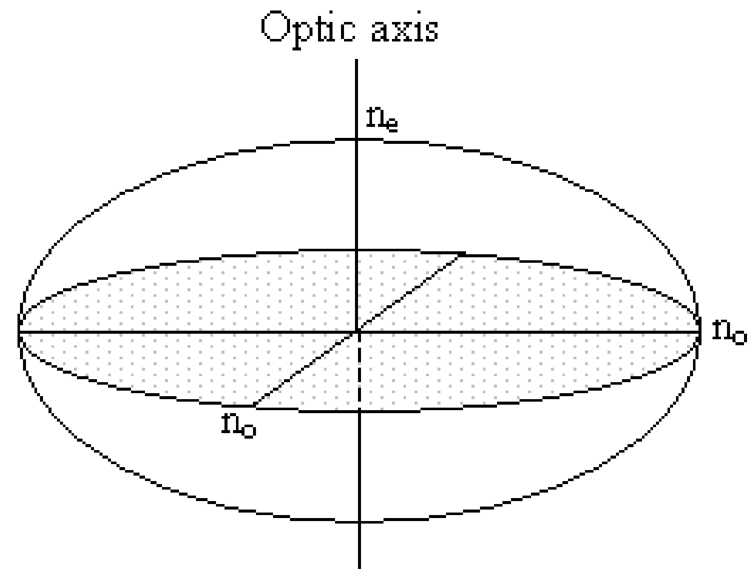
Optical anisotropy



"Positive"

$n_o < n_e$

$n_e - n_o > 0$



"Negative"

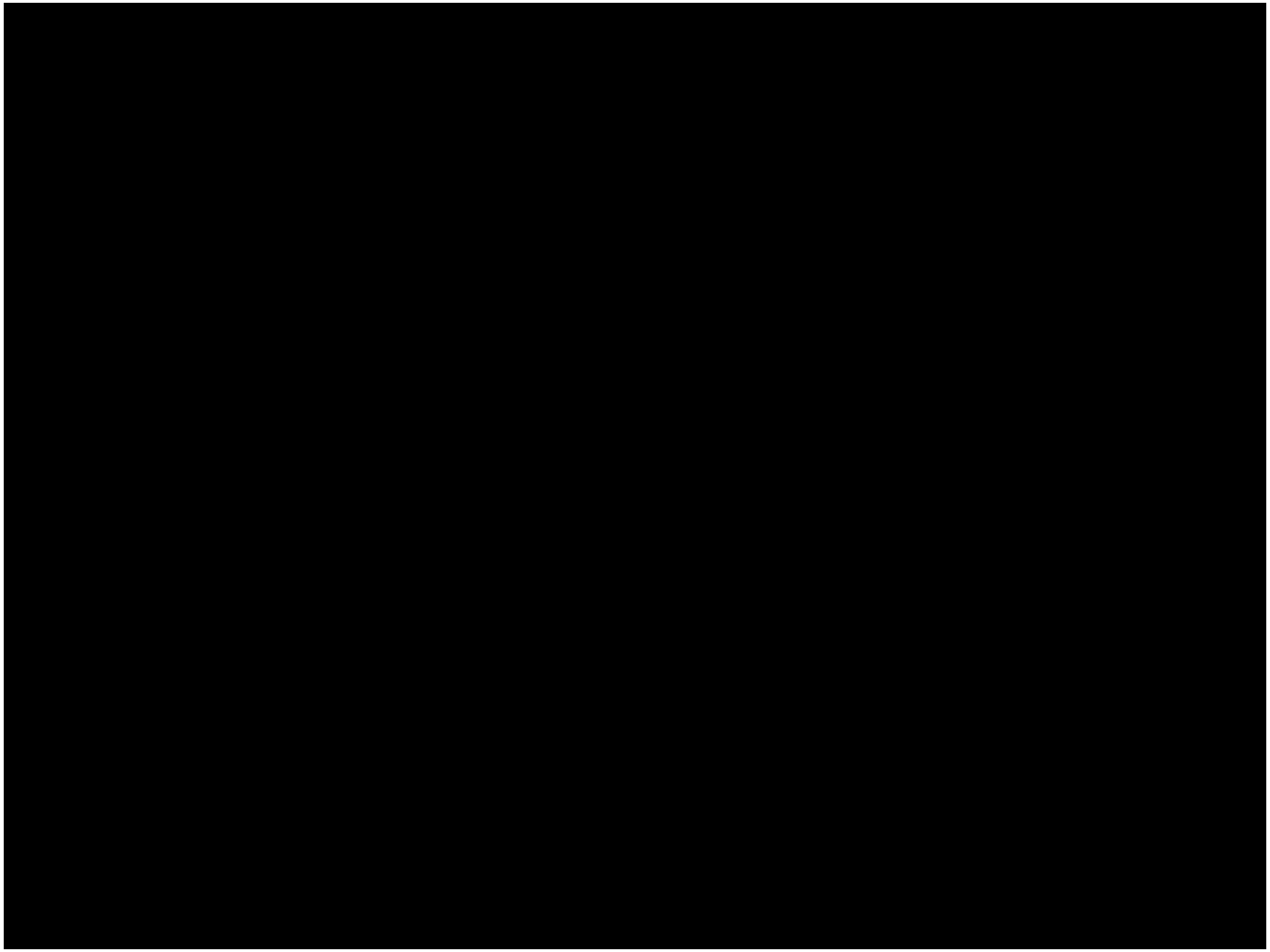
$n_o > n_e$

$n_e - n_o < 0$

Calcite rhomb

- large birefringence,
- we can observe two images,
- both images are due to ordinary and extraordinary wave that oscilates in two orthogonal directions

Calcite rhomb



Summary

- Some material properties are the same in all directions,
- Some materials can have different properties in different directions,
- Can be used in polarized light microscopy,
- Piezoelectricity,
- Electro-optic effect,
- Piezo-optical effect,
- Electrostriction,