

Programme

- 9⁰⁰ – 9¹⁵ *Registration*
- 9¹⁵ – 9³⁰ Welcome Janusz Hołyst
- 9³⁰ – 10⁰⁰ Biased random walks on complex networks: the role of local navigation rules.
A. Fronczak and P. Fronczak
- 10⁰⁰ – 10³⁰ Dynamics of non-conservative Voters.
R. Lambiotte, M. Ausloos and J.Holyst
- 10³⁰ – 11⁰⁰ *Coffee Break*
- 11⁰⁰ – 11³⁰ Bosonic correlations in weighted networks.
D. Garlaschelli and M.I. Loffredo
- 11³⁰ – 12⁰⁰ Majority dynamics on coupled networks – DuoNet simulator.
K. Suchecki, A. Scharnhorst and J.A. Hołyst
- 12⁰⁰ – 12³⁰ Collective Mind Reading via Blogs.
M. Thelwall
- 12³⁰ – 13³⁰ *Lunch Break*
- 13³⁰ – 14⁰⁰ Externally Triggered Opinion Avalanches on Networks.
M. Ausloos and F. Petroni
- 14⁰⁰ – 14³⁰ Approaching the thermodynamic limit: cutoffs in scale-free networks.
B. Waclaw
- 14³⁰ – 15⁰⁰ Felix Reed-Tsochas (to be announced)
- 15⁰⁰ – 15³⁰ Ising model on hierarchical scale-free networks.
S. Komosa and J.A. Hołyst
- 15³⁰ – 16⁰⁰ *Coffee Break*
- 16⁰⁰ – 16³⁰ Discussion and *Closing*.

Biased random walks on complex networks: the role of local navigation rules

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Theme: Traffic in complex networks , Random walk, Phase transition

Introduction

The problem of wandering at random in a network (or lattice) finds applications in virtually all sciences [1, 2]. With only minor adjustments random walks may represent thermal motion of electrons in a metal, or migration of holes in a semiconductor. The continuum limit of the random walk model is known as diffusion. It may describe Brownian motion of a particle immersed in a fluid, as well as heat propagation, bacterial motion, and even fluctuations in the stock market. Recently, the concept of random walks has been also applied to explore traffic in complex networks. The spectrum of network related problems include, among many others, ordinary traffic in a city, distribution of goods and wealth in economies, biochemical and gene expression pathways, and finally search (or routing) strategies in the Internet and other communication networks [3, 4, 5, 6, 7, 8, 9].

Biased random walk and navigation rules in routing strategies

The biased random walk model defined on scale-free networks is particularly interesting since it has been considered as a mechanism of transport and search in real networks, including the Internet. For a long time one has believed that the most optimum transport-related processes are based on shortest paths between two nodes under consideration. At the moment, one has however understood that such a routing strategy would require a global knowledge on network topology, which is often not available. Moreover, one can simply imagine that routing strategies based on shortest paths may create inconvenient queue congestions in scale-free networks, given that the majority of the shortest paths pass through hub nodes in such structures. It has been also realized that a possible alternative is to consider local navigation rules instead of global knowledge. As a consequence, a number of adequate models have been proposed (see e.g. [5, 8]). In general, the models mimic traffic in complex networks by introducing packets (particles) generation rate, as well as assigning a randomly selected source and a random destination to each packet. In these models, a common observation is that the traffic exhibits continuous phase transition from free flow to the congested phase as a function of the packet generation rate. In the free flow state, the numbers of created and delivered particles are balanced, while in the jammed state, the number of packets accumulated in the network increases with time.

Our results

During this talk, devoted to biased random walks on complex networks, I will explore the role of different local navigation rules on the mean first-passage (or transit) time between any pair of nodes [10]. I will show that the random walk model, although very simple, correctly describes properties of the proposed traffic models in the free flow state. I will provide a microscopic explanation of the phase transition from free flow to the jammed phase, and quantitative estimation of the critical value of the packet generation rate in classical random graphs and scale-free networks [8].

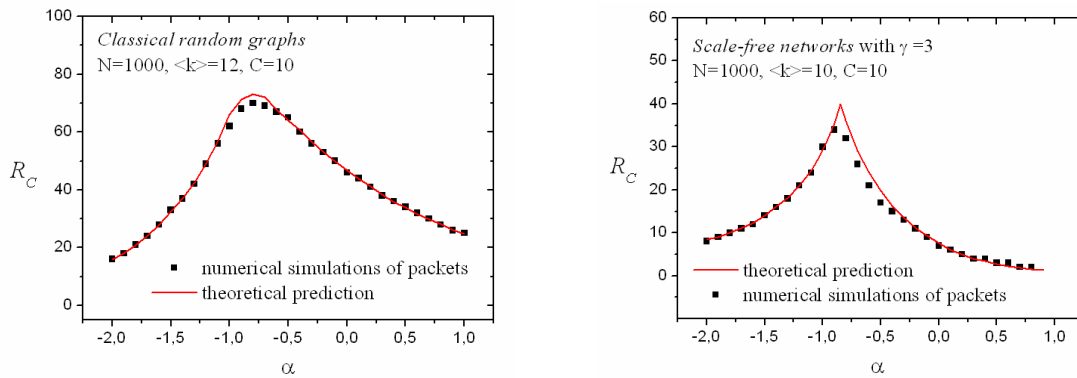


Fig.1. The critical value of the packet generation rate R_c versus routing strategy parameter α (please compare fig. 2 in [8]).

References

- [1] R.M. Mazo, *Brownian Motion: Fluctuations, Dynamics, and Applications*, Oxford Univ. Press (2002).
- [2] D. ben-Avraham and S. Havlin, *Diffusion and Reactions in Fractals and disordered Systems*, Cambridge Univ. Press (2000).
- [3] L.A. Adamic et al., *Phys. Rev. E* 64, 046135 (2001).
- [4] B. Tadić and G.J. Rodgers, *Adv. Complex Systems* 5, 445 (2002).
- [5] B. Tadić and S. Thurner, *Physica A* 332, 566 (2004).
- [6] B.J. Kim et al., *Phys. Rev. E* 65, 027103 (2002).
- [7] M. Rosvall, P. Minnhagen, and K. Sneppen, *Phys. Rev. E* 71, 066111 (2005).
- [8] W.-X. Wang et al., *Phys. Rev. E* 73, 026111 (2006).
- [9] R. Germano and A.P.S. de Moura, *Phys. Rev. E* 74, 036117 (2006).
- [10] S. Redner, *A guide to first-passage processes*, Cambridge Univ. Press (2001).

Dynamics of non-conservative Voters

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Theme:

We study a family of opinion formation models in one dimension where the propensity for a voter to align with its local environment depends non-linearly on the fraction of disagreeing neighbors [1,2]. Depending on this non-linearity in the voting rule, the population may exhibit a bias toward zero magnetization or toward consensus and the average magnetization is generally not conserved. We use a decoupling approximation to truncate the equation hierarchy for multi-point spin correlations and thereby derive the probability to reach a final state of +1 consensus as a function of the initial magnetization. The case when voters are influenced by more distant voters is also considered by focusing in detail on the Sznajd model. Finally, the role played by the underlying topology is considered by focusing on heterogeneous networks [3] and networks with communities [4].

References

- [1] R. Lambiotte and S. Redner (2008): Dynamics of Non-Conservative Voters. *Europhys. Lett.* 82, 18007
- [2] R. Lambiotte and S. Redner (2007): Dynamics of Vacillating Voters. *J. Stat. Mech.*, L10001
- [3] R. Lambiotte (2007): How does degree heterogeneity affect an order-disorder transition? *Europhys. Lett.* 78, 68002
- [4] R. Lambiotte, M. Ausloos and J. Holyst (2007): Majority Model on a network with communities. *Phys. Rev. E*, 030101(R)

Bosonic correlations in weighted networks

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Theme: Correlations and weighted networks.

Introduction

All the empirical results in identifying complex patterns in real networks rely on underlying assumptions on the correct null hypotheses. As our understanding of the correct null models improves, our picture of network structure evolves (Caldarelli, 2007). Correlations are detected by comparing an observed property with the behavior expected in the uncorrelated or null case. Therefore, the choice of the correct null model is of fundamental importance.

In unweighted networks, many properties which were first interpreted as nontrivial were later shown to be merely due to a lower-level structure of the graph (Maslov *et al.*, 2004; Park and Newman, 2003). For instance, even under the null hypothesis of statistically independent links, unavoidable structural correlations were shown to arise in unweighted networks. In particular, it was shown that even maximally random networks with specified degree sequence are unavoidably biased towards disassortativity (the average nearest-neighbors degree k^n decreases with the degree k) and hierarchy (the clustering coefficient c decreases with k) (Maslov *et al.*, 2004; Park and Newman, 2003). This is due to an ‘effective repulsion’ between vertices with large degree, which can be connected by at most one link, and this is in some sense ‘less’ than expected by randomly matching them. Thus the local properties alone determine higher-order structural correlations, and purely uncorrelated unweighted networks do not exist.

Results

Since structural correlations are due to the ‘Fermionic’ constraint that at most one link exists between two vertices (Park and Newman, 2003) in unweighted networks, they are unexpected for weighted graphs, where large weights (equivalent to multiple edges) are allowed. However, here we show that significant structural constraints, stronger than previously believed (Barrat *et al.*, 2004; Serrano *et al.*, 2006) are present in weighted networks as well, as a result of their ‘Bosonic’ or more generally ‘mixed’ nature.

We show that all the null models that have been proposed for weighted networks can be fully characterized at a microscopic level. This allows us to obtain analytically the correct null hypotheses for topological and weighted properties. Remarkably, we find that these Bosonic correlations correspond to a type of weighted structural constraints biasing many properties in a direction opposite to what happens in the unweighted case.

Discussion

Our results show that the correct null behaviour of weighted quantities is different from what previously believed (Barrat *et al.*, 2004; Serrano *et al.*, 2006). This clearly indicates that a systematic redefinition of unbiased measures is necessary for weighted networks. We propose a suitable class of weighted quantities that correctly filter out structural correlations.

References

- [1] G. Caldarelli (2007): Scale-free Networks. Complex Webs in Nature and Technology. Oxford University Press, UK.
- [2] S. Maslov, K. Sneppen, and A. Zaliznyak (2004): Detection of topological patterns in complex networks: correlation profile of the Internet. *Physica A* 333, 529.
- [3] J. Park, and M. E. J. Newman (2003): Origin of degree correlations in the Internet and other networks. *Phys. Rev. E* 68, 026112.
- [4] A. Barrat, M. Barthélemy, R. Pastor-Satorras, and A. Vespignani (2004): The architecture of complex weighted networks. *PNAS* 101, 374.
- [5] M. A. Serrano, M. Boguna, and R. Pastor-Satorras (2006): Correlations in weighted networks. *Phys. Rev. E* 74, 055101R.

Majority dynamics on coupled networks – DuoNet simulator

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Themes: Network Theory; Applications to Communications and Management; Social Processes Modeling

Statistical physics approaches to study structure and dynamics of networks have produced a lot of interesting insights of what has been labeled as “complex networks” (Albert, Barabasi 2002; Newman 2003; Dorogovtsev, Mendes book 2003). Knowledge about complex networks has been and is still travelling from physics also into areas of social sciences (Scharnhorst 2003) as for example information and communication sciences (Boerner et al. 2007), or economic theory (Pyka, Scharnhorst book 2008).

In this paper we present a model implementing majority rule on coupled networks. Its mathematical structure is based on the Ising model used in physics to explain how spontaneous magnetization emerges. Meanwhile in the context of sociophysics and complex networks, this dynamics has been applied to opinion changes. In particular, it models the spreading out of new characteristics in a modular network where closer connected areas can be interpreted as sign for groups and communities. In this model, the majority rule means that agents aim for the same state as a majority of their neighbors and will change their state accordingly. In reality, several situations can occur where this rule might be in place. Examples are when deciding on

political choices under the influence of your social network; when deciding to replace a CD-ROM drive with DVD-ROM drive if most of your friends already have them (and then you can't borrow DVD from them if you don't upgrade); or deciding to follow a certain scientific idea if your collaborators are already working on it. Recently, a specific situation for such network dynamics has been explored (Suchecki, Hołyst 2006). Starting from an initial state of coexistence, this means two network modules (communities) where nodes have opposite states (opinions, ideas), a consensus can be reached when the coupling strength between networks is increased or when the overall temperature is increased. By temperature we mean something like the rate of fluctuations in the system. More concretely, the probability with which agents disregard the influence of majority and act in an unrelated (random) way. It was found, that a smaller but densely connected community can “take over” a larger one by imposing their initial state on the latter. This is a counterintuitive result to the usual “critical mass” argumentation for the survival of something new. We show that “size” is not the only decisive parameter.

The model seems to be appropriate to shed light, generally spoken, on conditions how something “new”, an innovation in a very broad system-theoretical sense, can survive in a network environment using specific network structures. In particular it sheds light how innovation can survive even in a lock-in situation. The higher degree of connectivity can then be interpreted as “network externality”. This way, beside niches or different growth regimes an additional survival pre-condition for innovation is proposed. (Bruckner et al. 1996, Sonntag et al 2003)

In this paper we present a simulation tool which allows non-experts to play with features of coupled networks. Aiming for a wider audience than physicists an interface is needed which uses labels and legends understandable for outsiders. The process of building the simulation entails a lot of work of what has been called “articulation and translation” in science and technologies studies (Galison, 1996; Beaulieu et al. 2008). Such kind of work is core to interdisciplinary projects as the CREEN project, but is often not addressed explicitly. Our presentation is partly also a story about how to build a simulation tool which is useful also in not graph-educated fields of social sciences and humanities.

References

- [1] R. Albert, A.-L. Barabasi (2002): Statistical mechanics of complex networks. *Reviews of Modern Physics* 74, 47-97
- [2] A. Beaulieu, M. Ratto, A. Scharnhorst (2008): Learning in a landscape, forthcoming
- [3] K. Börner, S. Sanyal, A. Vespignani (2007): Network Science. In Cronin, Blaise (eds.), *Annual Review of Information Science & Technology* 41, 537-607, chapter 12, Medford, NJ: Information Today, Inc./American Society for Information Science and Technology.
- [4] E. Bruckner, W. Ebeling, M.A. Jiménez-Montaño, A. Scharnhorst (1996): Nonlinear Effects of Substitution - an Evolutionary Approach. *Journal of Evolutionary Economics* 6, pp 1-30
- [5] S.N. Dorogovtsev, J.F.F. Mendes (2003): Evolution of networks. From Biological Nets to the Internet and WWW. Oxford University Press
- [6] P. Galison (1996): Computer Simulations and the Trading Zone. In: P. Galison, D.J. Stump (eds.) (1996): *The Disunity of Science: Boundaries, Contexts, and Power*, Stanford University Press, pp. 118-157
- [7] I. Hartmann-Sonntag, A. Scharnhorst, W. Ebeling (2004): Modelling Self-Organization and Innovation Processes in Networks. <http://arxiv.org/abs/cond-mat/0406425>
- [8] M.E.J. Newman (2003): The structure and function of complex networks. *Society for Industrial and Applied Mathematics Review* 45 (2), 167-256
- [9] A. Pyka, A. Scharnhorst (eds.) (2008): Innovation networks. Springer, Berlin, forthcoming

- [10] A. Scharnhorst (2003): Complex Networks and the Web: Insights from Nonlinear Physics. *Journal of Computer-Mediated Communication* 8(4)
- [11] K. Suchecki, J.A. Hołyst (2006): Ising model on two connected Barabasi-Albert networks. *Phys. Rev. E* 74, 011122

Collective Mind Reading via Blogs

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Theme: Blogs, data mining, social software, sentiment.

Although the best-known blogs tend to be carefully constructed, tightly themed productions that are similar to newspapers or magazines in content, the majority of Blogs and similar social publishing software environments contain the casual thoughts of millions of ordinary people. These diary-like Blogs are easily accessed by humans and computers, making huge quantities of thoughts potentially available for analysis on a daily basis. The increasing online informal publication has created an unprecedented opportunity to judge the mood of the nation on specific topics more quickly than ever before. This talk will discuss the software issues and practicalities relevant to using blogs for collective “mind reading”.

[1] Kanayama, H., & Nasukawa, T. (2006). Fully automatic lexicon expanding for domain-oriented sentiment analysis. In *EMNLP: Empirical Methods in Natural Language Processing* (pp. 355-363). Stroudsburg, PA: ACL.

[2] Pang, B., & Lee, L. (2004). Sentimental education: Sentiment analysis using subjectivity summarization based on minimum cuts. In *Proceedings of the ACL 2004* (pp. 271-278). New York: ACL Press.

[3] Thelwall, M. (2007). Blog searching: The first general-purpose source of retrospective public opinion in the social sciences? *Online Information Review*, 31(3), 277-289.
<http://www.scit.wlv.ac.uk/~cm1993/mycv.html>

Externally Triggered Opinion Avalanches on Networks

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Theme: Network Theory

Introduction

From a statistical physics point of view, opinion formation is similar to epidemic or forest fire spreading, landslide, crystal growth, fracture, percolation and pertains to the abundant literature on phase transition studies. The modeling of such phenomena, for reproducing stylized features, is highly important in order to connect statistical physics with the socio-economic world. Some conceptual difficulty has been resolved due to the consideration of concepts in

studies of cases implying elaborate networks in which the statistical characteristics, like the number of sites, neighbors, distances, weights and directivity of links are not trivial.

Therefore, based on a theoretical model for opinion spreading on a network, through avalanches, the effect of an external field is considered, using methods from non-equilibrium statistical mechanics. The original part contains the implementation that the avalanche is only triggered when a local variable (a so called awareness) reaches and goes above a threshold. The dynamical rules are presently constrained to be as simple as possible, in order to sort out the basic features, though more elaborated variants are proposed and discussed.

Results

Several results are obtained for an Erdos-Renyi network and interpreted through simple analytical laws, scale free or logistic map-like, i.e., (i) the sizes, durations, and number of avalanches, including the respective distributions, (ii) the number of times the external field is applied to one possible node before all nodes are found to be above the threshold, (iii) the number of nodes still below the threshold and the number of hot nodes (close to threshold) at each time step.

We have found evolution laws similar, though with different exponents, as those found in other problems, e.g. even in earthquake dynamics (Rundle *et al.*, 1997). As an example we will discuss the Fisher-Stauffer relation about the frequency of spinodal fluctuations of areas, i.e. clusters of excited nodes here, above a threshold. It is seen that only the field strength dependence remains for a scaling law.

Discussion

For the present perspective, let us recall that opinion avalanche problems are similar to epidemics with threshold, see (Pastor-Satorras and Vespignani, 2001; Boguna *et al.*, 2003) when there is some self-organized information propagation, restricted to realistic "social" constraints. Connection with landslide problems can also be made, as e.g. with Piegari *et al.* (2006) and with forest fires (Duarte, 1997; Perry, 1998) in particular in the case when a tree is ignited only if a neighbouring tree burns long enough. This fact that wet wood does not yet burn corresponds to our awareness below the threshold. The model is also related to a sand pile model in which the threshold height to be reached before toppling is NOT due to the number of neighbors, but to the network structure.

In all cases, the results indicate that the dynamics can be seen as geared by triggered or damped competition.

Acknowledgement

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References

- [1] J. B. Rundle, W. Klein, S. Gross and C. D. Ferguson (1997): Traveling density wave models for earthquakes and driven threshold systems. *Phys. Rev. E* 56, 293.
- [2] J.A.M.S. Duarte (1997): Fire spread in natural fuel: computational aspects, in *Ann. Rev. Comput. Phys.*, D. Stauffer (ed.), World Scientific Singapore, V, 1.
- [3] G. L.W. Perry (1998): Current approaches to modelling the spread of wildland fire: a review. *Progress in Physical Geography* 22, 222.

- [4] R. Pastor-Satorras and A. Vespignani (2001): Epidemic spreading in scale-free networks. *Phys. Rev. Lett.* 86, 3200.
- [5] M. Boguna, R. Pastor-Satorras, and A Vespignani (2003): Absence of epidemic threshold in scale-free networks with degree correlations. *Phys. Rev. Lett.* 90, 028701.
- [6] E. Piegari, V. Cataudella, R. Di Maio, L. Milano, and M. Nicodemi (2006): Finite driving rate and anisotropy effects in landslide modeling. *Phys. Rev. E* 73, 026123.

Format of the 2-page extended abstracts to be submitted to the International Conference of Network Science 2008, Norwich, UK

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Theme: Approaching the thermodynamic limit: cutoffs in scale-free networks

Abstract

Recent progress in understanding the structure and function of complex networks has been largely influenced by the application of statistical methods of modern physics. The statistical mechanics of networks, if restricted to structural properties, deals with two classes of problems. In the first, one considers networks being in a sort of equilibrium, where the concept of the statistical ensemble applies naturally. The second class deals with growing networks, which are usually treated by the rate-equation formalism. Both approaches, however, share one common thing: the majority of considered models is analytically solvable only in the thermodynamic limit. This causes several difficulties. First of all, a question arises about statistical equivalence of different ensembles of networks. Second, considering dynamical processes on networks, one can ask how they are influenced by approaching the thermodynamical limit in different ways.

The third question, which I want to face in this talk is how fast some network features converge towards their limiting values, for different models. Because the most striking structural property of many complex networks is the power-law degree distribution, I will focus on it. For any finite network, the power law cannot extend to infinity and must have a cutoff. Since the cutoff has direct impact on many processes taking place on networks, for example disease spreading or percolation, it is important to know how it scales with the network size N .

In my talk I will discuss cutoffs for several models belonging to both growing and equilibrated networks. I will show that one should speak about two different cutoffs; one related to the scaling of the maximal degree k_{\max} and the other one which describes how fast the degree distribution approaches its limiting shape. Both cutoffs have in general different scaling with N . I will demonstrate that the degree distribution for a finite network can be written as a product of the limiting power-law distribution and some function depending on properly rescaled degree. This function has some universal properties.

Among other things, I will show also that if the exponent γ in the power-law equals three, logarithmic corrections to the scaling appear for equilibrated networks, but not for growing ones. Thus the point $\gamma=3$ is the critical one – the convergence towards the thermodynamic limit is the slowest at this point.

Acknowledgment

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References

- [1] S. N. Dorogovtsev, J. F. F. Mendes, and A. N. Samukhin, *Principles of statistical mechanics of random networks*, Nucl. Phys. B 666 (2003), 396.
- [2] Z. Burda and A. Krzywicki, *Uncorrelated random networks*, Phys. Rev. E 67 (2003), 046118.
- [3] S. N. Dorogovtsev, J. F. F. Mendes, A. M. Povolotsky, and A. N. Samukhin, *Organization of complex networks without multiple connections*, Phys. Rev. Lett. 95 (2005),
- [4] B. Waclaw and I. M. Sokolov, *Finite size effects in Barabási-Albert growing networks*, Phys. Rev. E 75, 056114 (2007).
- [5] B. Waclaw, L. Bogacz and W. Janke, in preparation.

Ising model on hierarchical scale-free networks

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Theme: Hierarchical networks , Ising dynamics, phase transition

Introduction

We studied the behavior of an Ising spin model on different hierarchical scale-free networks using Monte Carlo simulations. We observed a phase transition from ferromagnetism to paramagnetism and a power-law behavior of critical temperature with network size and on the ratio $\langle k^2 \rangle / \langle k \rangle$. Two different order parameters were used: a standard average network spin and a weighted network spin.

Model

We considered three models of hierarchical scale-free networks: the first, which is the deterministic, Ravasz - Barabasi (RB) mode [1], the second, the P1 model [2], which is stochastic generalization of the deterministic one, and the next one, the PD model [2], which is a variation of the P1 model. Ising spins were placed at every node of such a network and ferromagnetic interactions between nearest neighbors were taken into account.

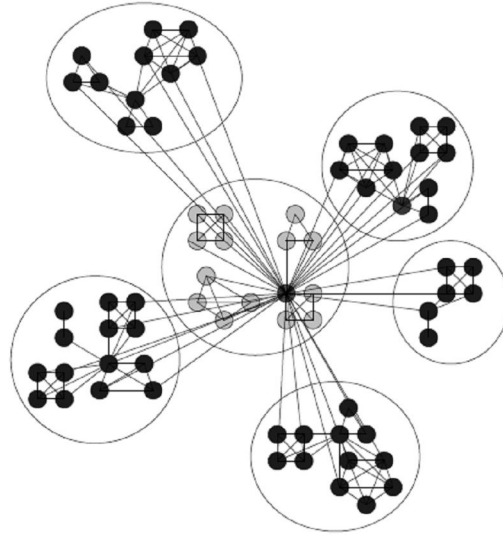


Fig. 1. Network of hierarchy $d=2$ in P1 model.

We conducted standard Monte Carlo simulation with Metropolis Algorithm (average number of time steps for relaxation was between 100 and 200 Monte Carlo time steps [MCS], depending on the size (hierarchy) of the network).

Results

We observed phase transitions in all considered networks. Below a critical temperature T_c there was a spontaneous magnetization that disappeared above T_c . Numerical simulations show that the value of the critical temperature is a power law function of a ratio between the mean values of the square of node degree and the expected node degree

$$T_c \sim \left(\frac{\langle k^2 \rangle}{\langle k \rangle} \right)^\alpha$$

where the exponent α is similar for all studied models $\alpha = 0.58$. There exists also a power-law behavior of critical temperature with network size but corresponding exponents are different for different models. We observed also that reversing and pinning a few central spins of such hierarchical networks can invert the magnetization of the whole system consisting of $N=20.000$ nodes.

Acknowledgement

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References

- [1] E. Ravasz, A.-L.Barabasi, Phys. Rev. E 67, 026112 (2003).
- [2] K. Suchecki, J.A. Hołyst, A.P.P. B 36(8), 2499-2511 (2005).
- [3] S. Komosa, J.A. Hołyst, to be published.

