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Properties of on-line social systems

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Abstract. We study properties of five different social systems: (i) internet society of friends consisting of over 10^6 people, (ii) social network consisting of 3×10^4 individuals, who interact in a large virtual world of Massive Multiplayer Online Role Playing Games (MMORPGs), (iii) over 10^6 users of music community website, (iv) over 5×10^6 users of gamers community server and (v) over 0.25×10^6 users of books admirer website. Individuals included in large social network form an Internet community and organize themselves in groups of different sizes. The destiny of those systems, as well as the method of creating of new connections, are different, however we found that the properties of these networks are very similar. We have found that the network components size distribution follow the power-law scaling form. In all five systems we have found interesting scaling laws concerning human dynamics. Our research has shown how long people are interested in a single task, how much time they devote to it and how fast they are making friends. It is surprising that the time evolution of an individual connectivity is very similar in each system.

PACS. 89.75.Fb Structures and organization in complex systems – 89.65.Ef Social organizations; anthropology – 89.75.Da Systems obeying scaling laws

1 Introduction

In recent years investigations of complex networks have attracted the physics community's great interest. It has been discovered that the structure of various biological, technical, economical, and social systems has the form of complex networks [1]. The short length of the average shortest-path distance, the high value of the clustering coefficient and the scale-free distribution of connectivity are some of the of those networks' common properties [1,2]. Social networks, which are important examples of complex networks, share those properties.

The advent of modern database technology has greatly advanced the statistical study of networks. Because the available data sets are vast, it is possible to use techniques of statistical physics [2,3]. Studying the statistical properties of social, e.g. friendship, networks remains a challenge. It is possible to assess the form of distribution of connectivity with a survey, like in the case of the web of human sexual contacts [4]. However it is much more difficult to lern about other important properties of networks, because there is no data on their entirety. A survey often provides data on a small sample only. Progress in information technology has made it possible to investigate the structure of the social networks of interpersonal interactions maintained over the Internet, e.g. e-mail networks [5], web-based social networks of artificial communities [6] and blog networks [7].

The aim of this work is to introduce a data set describing the properties and evolution of five different social systems. The first one is a large social network of an Internet community (Grono), which consists of more than 10^6 individuals. The Grono (cluster) project was started in Poland on the website `www.grono.net`. During its existence, it has grown into a well-known social phenomenon among Internet users in Poland. Membership is strictly invitation only; existing members can invite an unlimited number of friends to the network via email, who, if they want to do so, join the network by an initial link connecting to the person who invited them.

The second system under investigation is a large social network of an on-line game [8], which consists of almost 3×10^4 individuals. In recent years on-line games have become increasingly popular and have attracted an increasing number of players, who interact in the large virtual world of Massive Multiplayer Online Role Playing Games (MMORPGs). MMORPG is a network game in which players enter a virtual world as characters they have invented-gaining virtual life. This virtual world takes the form of a game server connected to the Internet, on which accounts are registered for users who log in through special

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Table 1. Average properties of the whole network and the giant component (GC) for five systems: Grono, Allseron, LastFm, XFire, Shelfari. The total time (in years) over which each data set is collected is denoted by T. Values in parentheses indicate the uncertainty in the final digit.

Network	N	$\langle k \rangle$	$\langle l \rangle$	C	k_0	γ	τ	ϕ	$\langle n_G \rangle$	T
Grono	1 002 182	46.3	–	0.2	0	0.83(3)	4.50(5)	138	3.6	3
GC	994 381	46.3	4.4	(0.2)						
Allseron	28 011	1.4	–	0.02	6.2(9)	2.9(1)	4.99(2)	8.3	–	2
GC	6065	6.4	4.8	0.1						
LastFM	1 192 118	4.2	–	0.23	10(1)	4.0(1)	3.61(2)	12	1.9	5
GC	816 093	5.9	6.6	0.23						
XFire	5 241 578	1.2	–	0.17	0	1.92(1)	3.80(1)	18	–	3
GC	957 579	4.9	6.9	0.17						
Shelfari	253 967	2.7	–	0.06	1.5(3)	2.90(7)	2.1(2)	10.8	0.2	2
GC	250 621	2.7	10	0.06						

game client programs. Thousands of people can play on one server – they become a virtual society – so they share the common culture, area, identity and interactions in the network of interpersonal relationships. Individuals exploring this virtual world can collect funds, trade, organize in groups of different sizes, which can make alliances or wage war.

The third system under investigation is LastFM – the music community server, and more exactly the part of it known as Audioscrobbler project. Similar like in other presented systems, there is one server connected to the internet, on which accounts are registered for users. There is about 1×10^6 users of this system. The important part of the system is its special plugin installed into the music player application (e.g. winamp), which send information about every song played by user to the Audioscrobbler server. On that basis, people with similar music taste and songs they often listen to are presented and recommended to users who can see this information on theirs profile web site via web browser. This way people with similar music taste can meet each other and have the possibility to make friends (mutual consent is required) and gather into groups (music genre, performers).

The fourth system under investigation is XFire. It is gamers community program, having 5×10^6 users, similar to every Internet Chat systems, marked out its integration with almost all popular computer games. People which are already playing game do not need quit the game to chat with other people doing something else in this moment because XFire gives the possibility to talk inside games via special chat window. People who like to play computer games are using this application to keep in contact with other players even when they do not play any game in that moment or play two different games. For this purpose they add other people into their friend list and have possibility to see which game his/her friend plays, how much overall time they played and can always chat with this person when on-line. XFire allows to see friends of your friends so people have greater chance to make new acquaintanceship. XFire gives to each user their own web space with his statistics (i.e. overall time played, friend lists).

The fifth system described in this paper it is books admirer system Shelfari. Shelfari is web site server similar to LastFm. There is over 0.25×10^6 users of the system. Users can create account received in this way their own profiled WWW site where they can create virtual shelf with books which they enjoyed. System recommend users another ones with similar tastes and books which they read, show opinions other people about various books, give a possibility to create or gather into groups.

In the five systems all users can add, by mutual consent, and remove other people to their databases of friends. In this way undirected friendship network is formed. The networks under consideration consisted of a collection of individuals (network nodes) connected among one another by friendly relationships (network links). On-line time, a network of interpersonal relations (friendship network) and its expansion are the observables which show the activity of the virtual society and provide an opportunity to study human dynamics [9]. To study the structure of the networks and of human activity, we analyzed data containing the list of all friends, groups, creation date, and last login date of each individual. The total time over which each data set is collected is three years for Grono, two years for Allseron, five years for LastFM, three years for XFire, and two years for Shelfari.

The paper is organized as follows. In Section 2 we describe the basic network measures for all five networks under investigations. Next, in Section 3 we investigate the human dynamics. Section 4 has the conclusions.

2 Network structure

Basic network measures of networks and Giant Components (GC) are presented in Table 1. In all cases the value of the clustering coefficient C is two orders of magnitude larger than that of a random graph. The average path length $\langle l \rangle$ in GC is very small and comparable to that in a random graph. A high value of the clustering coefficient C and a short average path length $\langle l \rangle$ are characteristic features of social networks [2,10,11]; they are typical for small-world networks [12]. The degree distribution of networks is plotted in Figure 1. The graph shows power-law regime for all networks (the parameters of the power-law distributions were computed with maximum likelihood estimation [13] in other cases least-squares fitting

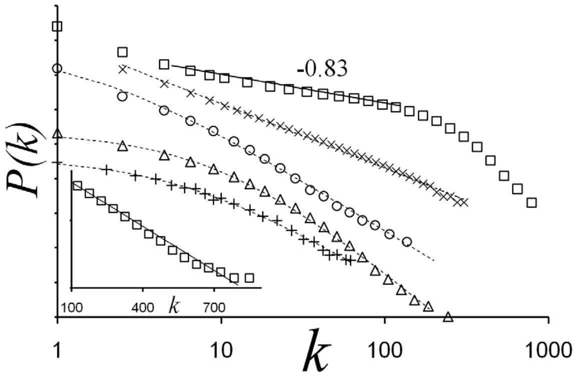

Fig. 1. Degree distribution for five systems: Grono (squares), Allseron (daggers), LastFm (triangles), XFire (crosses) and Shelfari (circles) in a double logarithmic scale. Results can be fit to power-law $P(k) \sim (k + k_0)^{-\gamma}$ (dashed line). The values of the exponents are presented in Table 1. The datasets are vertically offset.

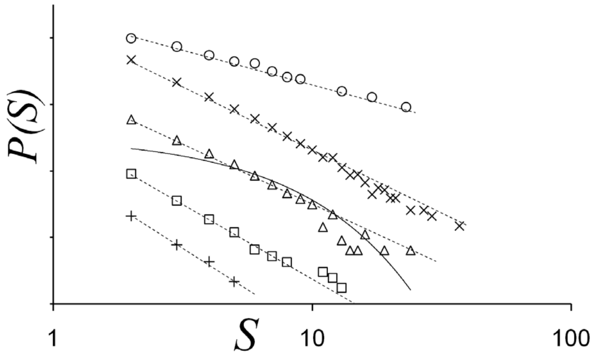

Fig. 2. Components size distribution denoted following: Grono (squares), Allseron (daggers), LastFm (triangles), XFire (crosses) and Shelfari (circles) in a double logarithmic scale. Results can be fit to power-law $P(S) \sim S^{-\tau}$ (dashed line; the comparison with exponential decay-solid line). The values of the exponents are presented in Table 1. The datasets are vertically offset.

were used). Such a power-law is common in many types of networks [1], also in social networks [4–6,14]. The connectivity distribution in each case can be approximated with the power-law $P(k) \sim (k + k_0)^{-\gamma}$ (the values of the parameters are presented in Tab. 1).

However, in the case of Grono network, two differences with other systems can be found. At first, it can be seen changes in degree distribution. For large k ($k > 100$) the degree distribution starts to take an exponential form $P(k) \sim e^{-0.01k}$ (see inset in Fig. 1a). The form of degree distribution indicates the existence of two underlying networks that are simultaneously present in the social network under investigation: the network of links that represent preexisting social contacts and the network of links represent social contacts maintained only via Internet. Similar form of degree distribution with two scaling regimes was found in another on-line social network [6].

The second difference results from a fact that Grono is the only web-service which main purpose is to facilitate interpersonal interaction between friends so the average connectivity is much larger and the average path length is smaller than in other networks, see Table 1. In other systems this is an additional function.

It is surprising that in five different systems the form of distribution of sizes of network components is very similar (Fig. 2). In all cases the results can be approximated with the power-law $P(S) \sim S^{-\tau}$. The values of the exponents are presented in Table 1. In three largest systems the value of the exponent is close to 4. The deviation from this value in systems Allseron and Shelfari can be connected with fact that their size is much smaller, and the number of network components is in consequence much lower. In this two systems the number of individuals with $k > 0$ and who are not included into GC is lowest. In second case there could be additionally reason. It is the only system when gathering into group (cooperation with large number of people) is necessary for individual development so almost all older characters are inside GC. In all systems the distributions have similar power-law form and values of exponents are comparable. It is surprising because there are two different mechanisms of adding users into system used. In Grono system, new user can be invited by already existing user in the system, then connection is created automatically (addition of the node is linked with network link creation) and this user is added into existing component. Occurrence of separate components is a result of the removing friends from the list. In the other networks new users can individually create accounts. In this way they create new components with one node size and they can gather into the other one afterwards. Similar value of τ exponent in each case suggests that such a form of $P(S)$ distribution can emerge as a result of different mechanisms of keeping interpersonal relationships.

For a random network of arbitrary degree distribution, the condition for the existence of a spanning cluster is $\phi = \frac{\langle k^2 \rangle}{2\langle k \rangle} > 1$ [15]. In the networks under investigations the values of the parameter ϕ are much greater than one (see Tab. 1), thus those networks are in percolative phase. The relation between parameter γ and the exponent τ for scale-free networks which are close to the percolation threshold is presented in reference [16]. It has been shown that the exponent τ bear a strong γ-dependence $\tau = \frac{2\gamma-3}{\gamma-2}$ for $2 < \gamma < 4$ (note that in the case of networks LastFM and XFire the value of the exponent τ seems to be independent on the form of the degree distribution, see Tab. 1). For $\gamma > 4$ usual mean field result $\tau = \frac{5}{2}$ is observed. Related results for growing networks of the Albert-Barabási model are presented in reference [17]. In networks under investigation the value of the parameter τ is high, however the fit to power-law relation is much better than to exponential decay (see solid line in Fig. 2). In order to determining whether a power-law is consistent with the empirical data is by computing the p-value for the power-law distribution [13]. In all systems under investigation the p-value is above threshold value (i.e. 0.05), hence the power-law model can not be rejected.

Table 2. The values of the exponents describing human dynamics in on-line communities under investigations and average life-span for all $\langle T_{LA} \rangle$ and $\langle T_{LI} \rangle$ inactive users (time units are in days). The percent of inactive users is denoted by n_I.

Network	n_I	α	β	λ	$\langle T_{LA} \rangle$	$\langle T_{LI} \rangle$
Grono	35%	0.60(2)	0.91(2)	0.71(2)	337	200
Allseron	77%	0.99(4)	0.64(2)	–	69	110
LastFm	40%	0.68(9)	0.5(1)	0.48(2)	207	180
XFire	88%	–	0.69(2)	–	57	38
Shelfari	20%	0.96(3)	0.24(2)	0.49(4)	32	15

However, in the case of Allseron very large value of the exponent τ is observed. It should be stressed that in Allseron, there are two main factors which are related to game mechanisms, leading to fast decrease of $P(S)$ distribution: (i) gamers are forced to cooperate, otherwise they are not able to develop further, (ii) the world in Allseron is very small (only few cities), thus there is very small chance that one cluster will not connect with another by even one person. The factors cause, that beside GC, there are only few small clusters (with small S) consisting of mainly new players. Because those mechanisms are not present in other systems, we suggest that high value of τ in Allseron could indicate, that in this kind of systems, the $P(S)$ distribution have a exponential form ($P(S) \sim \exp^{-S}$), but due to the short range of data (Allseron is the smallest among studied systems), it is hard to unambiguously certify, which form the $P(S)$ distribution has: power-law or exponential.

Large value of exponent τ suggest that the networks are far above percolation threshold, e.g. in the case of Grono the network disintegrates into smaller, disconnected parts when the fraction of bonds $p > 0.97$ is removed.

The average connectivity of the nearest neighbors k_{NN} of a node increases with k increasing. Hence, the networks under investigation are assortative mixed by degree; such a correlation is observed in many social networks [18]. In social networks it is entirely possible, and is often assumed in sociological literature, that similar people attract one another. On the other hand individuals organize in groups of different sizes in order to chat together. Since it is highly probable that members of a group are connected to other members, the positive correlations between degrees may at least in part reflect the fact that the members of a large (small) group are connected to the other members of the same large (small) group.

The relation $k_{NN}(k)$ can be approximated with the power-law. However, it was recently shown that such correlations can be explained by modularity and a heavy-tailed degree distribution [19].

The behavior of the clustering coefficient C is an interesting problem. In all systems under investigation significant correlation between the local clustering coefficient $C(k)$ and the node degree k are observed. The variations in the clustering coefficient C with the vertex degrees are reflecting the existence of degree correlations [20]. The degree distribution is responsible for some of the observed clustering-degree correlations, but cannot fully account for them.

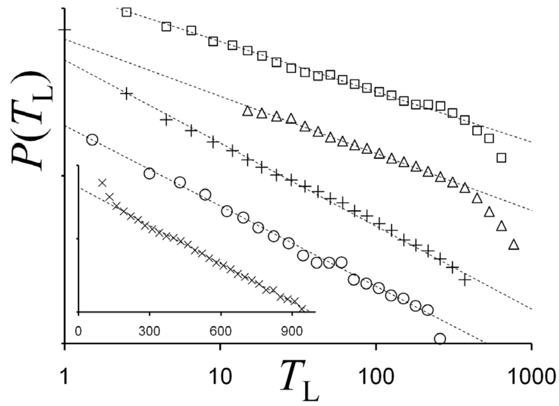

Fig. 3. The distribution of probability P_L that activity of an inactive individual in the community last T_L days: Grono (squares), Allseron (daggers), 3 LastFm (triangles), XFire (crosses) and Shelfari (circles) in a double logarithmic scale. Results can be fit to power-law $P_L(T_L) \sim T_L^{-\alpha}$ (dashed line). The values of the exponents are presented in Table 2. In the case of XFire the results have an exponential form $P_L(T_L) \sim e^{-0.004 T_L}$. The datasets are vertically offset.

3 Human dynamics

On-line communities offer a great opportunity to investigate human dynamics, because much information on individuals is registered in databases. To analyze how long people are interested in a single task, we studied creation date and last login date registered in the database. Individuals can lose interest in using the website after some time. We consider an individual as active when last login time is lesser than one month, in opposite case we consider an individual as inactive (the number of inactive individuals is presented in Tab. 2). In other words we assume that an individual is inactive if he or she does not login for 30 days before the data on this individual were collected.

The lifespan of an individual T_L is defined as the number of days from the time the individual was added (invited to the network) to the date of last logging. The distribution of the probability P_L that an inactive individual has the lifespan T_L is shown in Figure 3. This distribution can be approximated with the power-law $P_L(T_L) \sim T_L^{-\alpha}$ (the values of the exponents and the average time $\langle T_L \rangle$ are presented in Tab. 2). Thus, the probability that a human will devote the time t to a single activity has a fat-tailed distribution. However, in the case of XFire the distribution has exponential form $P_L(T_L) \sim e^{-0.004T_L}$ (see inset in Fig. 3). We suggest that this discrepancy from other results is caused by the fact that in all systems except XFire the friendship is made between people who are connected by common activity (cooperation) and discussions (by groups or forums) while the XFire offers only instant messaging system. It does not support creating close relationship like in case of meeting people who have common purposes, interests or meet each other in a real world. Additionally XFire system requires users to start additional application before they launch their target game and moreover their overall time is logged on XFire web site which can concern users about privacy issues. The entire reasons make that people are sooner daunted to use this program.

It should be noted that we have considered an individual as inactive when it does not login for more than one month (e.g. 60 or 90 days). For larger time period we do not observe significant changes in scaling results. On the other hand for low time period (e.g. seven days) the $P_L(T_L)$ relation has irregular, non-monotonic shape.

The value of exponent α may be connected with the type of activity. We suggest that, in the case of more demanding (time consuming) activity the users are more willing to stop this activity, and in consequence greater α is observed. That is because they cannot afford for waste to much time doing one thing and secondly they are boring faster. Presented results show that people are longer interested in systems which they are not able to devote much time. Then the MMORPG is much more time consuming (in average players spent few hours per day in virtual world) and demanding (the game is connected with stress because e.g. the avatar of the player can be killed) than simple sending messages to the friends and chatting (Grono). Similarly more time consuming is books reading and writing referees on them (Shelfari), than simple listening to the music (LastFM).

The time development of the connectivity of an (active or inactive) individual, i.e. the relation between lifespan of an individual and its connectivity is shown in Figure 4. The relation $k(T_L)$ can be approximated with the power-law relation $k(T_L) \sim T_L^{\beta}$. The values of the exponents are presented in Table 2. In the five systems the friendship network is undirected, i.e. new connection between individuals is added by mutual consent. Therefore, the number of connection of an inactive individual can not increase in time.

It is surprising that in all systems the $k(T_L)$ has similar (power-law) form, because the purpose of making friends in each case is different, e.g. in Allseron it is facility in hunting, in LastFM it is desire to knowing new songs, in XFire it is desire to keep in contact with people who start to pick up another activity. This result indicates that different processes of making new acquaintances can give the same scaling.

Although the form of relation between lifespan of an individual and its connectivity $k(T_L)$ is independent from system's destiny (and purpose of making friends), the value of exponent β is varying, and is greater if social interactions are connected with that how does system function. The increase in the number of connections is fastest in the case of Grono, because basic purpose of this web-service is to facilitate social interactions between people. Cooperation with other players is important part of each MMORPG and help of the friends is needed in order to score success in virtual world, therefore in the case of Allseron the number of connections increases fast also. In web-services systems contacts with other users are less helpful, therefore k increases slower.

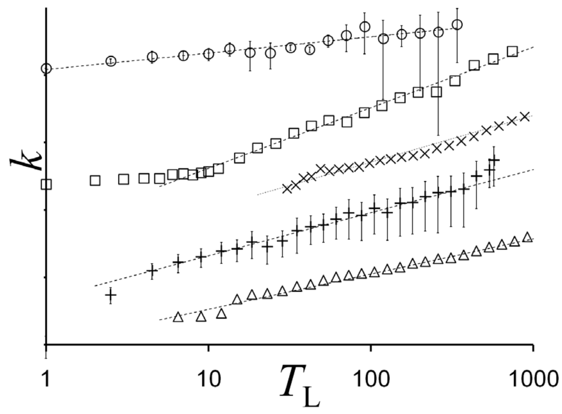

Fig. 4. Time development of the connectivity of an individual: Grono (squares), Allseron (daggers), LastFm (triangles), XFire (crosses) and Shelfari (circles) in a double logarithmic scale. Results can be fit to power-law $k(T_L) \sim T_L^{\beta}$ (dashed line). The values of the exponents are presented in Table 2. All individuals (active and inactive) were taken into account. The datasets are vertically offset. In the case of largest networks (Grono, LastFM, XFire) the size of errors bars is smaller than the size of marks.

The websites Grono, LastFM, Shelfari contain additional services such as discussion forums; hence users can organize themselves into groups of different sizes. Each individual can join to unlimited number of groups n_G. Average number of groups $\langle n_G \rangle$ in each system are shown in Table 1. Because an individual can be a member of many groups it is interesting to plot the relation between the number of groups in which the individual is involved n_G and life-span T_L of the individual. The results calculated for all individuals (active and inactive) are shown in Figure 5. The relation $n_G(T_L)$ can be approximated with the power-law relation $n_G(T_L) \sim T_L^{\lambda}$. The values of the exponents are presented in Table 2. Members of a group chat together on a single topic, however in the case of LastFM and Shelfari the subject of discussions is usually restricted to music and books, respectively, hence it is fairly limited number of ideas on new groups. In the case of Grono, there are no restrictions so the increase in the number of groups of an individual is fastest and $\langle n_G \rangle$ is largest.

It should be noted that in all cases the values of exponents $\beta < 1$ and $\lambda < 1$. It indicates that in all systems (despite of e.g. differences in mechanisms of making new acquaintances) users become less active over the time. They become less interested in making new friends and join to

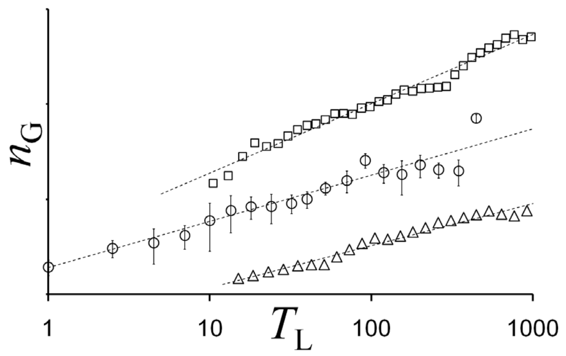

Fig. 5. Time development of the number of groups of an individual for the systems: Grono (squares), LastFm (triangles), Shelfari (circles) in a double logarithmic scale. Results can be fit to power-law $n_G(T_L) \sim T_L^{\lambda}$ (dashed line). The values of the exponents are presented in Table 2. All individuals (active and inactive) were taken into account. The datasets are vertically offset. In the case of largest networks (Grono and LastFM) the size of errors bars is smaller than the size of marks.

new groups (the greater T_L, the lower increase in k and n_G in one time unit).

Our results indicate that such a scaling law can be a typical relation and the value of the exponent depends on type of human activity (e.g. entertainment, interpersonal interactions). Similar relations concerning human dynamics have also been observed elsewhere [23] and can be a consequence of a decision-based queuing process. A model of such a process was recently proposed by Barabási [9]. It indicates that scale-free distributions are common in human dynamics.

4 Conclusions and future work

We have shown that a friendship networks maintained in Internet communities have similar properties (e.g. large clustering, a low value of the average path length, assortative mixing by degree) to other social networks.

We have compared form and exponent value of power-laws in all five presented systems and we noticed similar dynamic of human behavior in each cases. It is surprising because of different purposes and mechanisms of those systems. However, it is natural to see small differences between results which do not disturb the observability of universal rules.

We have noticed the biggest differences in $P(k)$ distribution when we compared Grono results with other systems. The value of the exponent is smaller than in other systems and for $k > 100$ the degree distribution has exponential form. It turns out to be result of the different purposes of the systems. It is only system where maintaining a friendship is main user activity and people create accounts for a sole purpose to meet each other. The second big difference occurs in the value of the exponent of $P(S)$ distribution. It is very similar for large networks ($N > 10^6$), however in the case of smaller networks large discrepancy in the results is observed. And lastly, the third big difference is observable in distribution of lifespan time in XFire system which can be a result of the weak social interaction between users and differences in interests. Despite all the differences, the power-law turns out to be good to describe friendship networks structure. We have found that the network components size distribution follow the power-law scaling form also. In the case of largest networks the value of the exponent $\tau \approx 4$, irrespectively of the form of the degree distribution, which is an unexpected result [16].

Next, we concentrated on the problem whether and how the power-law can describe the human dynamics. On the basis of creation date and last login of each individual recorded on the server, we have presented results concerning human dynamics. The time development of the connectivity and number of groups of an individual was calculated. The power-law form of distributions $P_L(T_L)$, $k(T_L)$, $n_G(T_L)$ and other authors' results [9] indicate that such a scaling law is common in human dynamics and should be taken into account in models of the evolution of social networks [24] and of human activity. It seems that different mechanism of evolution of on-line social systems can give similar scaling. The differences in values of exponents are related with the microscopic differences in these social systems, especially the destiny of the system and type of user's activity (like book reading or listening to radio). We suggest that presented in our work results concerning human dynamics and life-span of individuals can have a significant influence on the dynamic phenomena in the complex networks [25] (like rumor propagation or opinion formation [26]), e.g. the influence of the heavy-tailed distribution of the intercontact times between susceptible and infected individuals on spreading process of computer viruses was presented in [27].

In future work it will be considered to develop a mathematical model describing human dynamics in on-line communities on microscopic level. Next, of further interest would be a more careful exploration of the different dynamic phenomena in social networks taking into account the network evolution [14] and the lifespan T_L of an individual.

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