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Ising-based model of opinion formation in a complex network of interpersonal interactions

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Abstract

In our work the process of opinion formation in the human population, treated as a scale-free network, is modeled and investigated numerically. The individuals (nodes of the network) are characterized by their authorities, which influence the interpersonal interactions in the population. Hierarchical, two-level structures of interpersonal interactions and spatial localization of individuals are taken into account. The effect of the mass media, modeled as an external stimulation acting on the social network, on the process of opinion formation is investigated. It was found that in the time evolution of opinions of individuals critical phenomena occur. The first one is observed in the critical temperature of the system T_C and is connected with the situation in the community, which may be described by such quantifiers as the economic status of people, unemployment or crime wave. Another critical phenomenon is connected with the influence of mass media on the population. As results from our computations, under certain circumstances the mass media can provoke critical rebuilding of opinions in the population.

Keywords: Complex networks; Opinion formation; Ising model; Small world; Scale-free networks; Freezing by heating

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1. Introduction

The structure and the dynamics of complex networks have been extensively investigated in recent years [1–8]. It was found that many real-world networks, like the Internet [1], e-mail networks [2] and the web of human sexual contacts [3], have similar properties. They are called scale-free networks, because the probability that the number of k links connected to a node equals $P(k) \sim k^{-\gamma}$ [4]. Many authors have used this type of complex network to model a network of social contacts [8–12]. In particular, complex networks with a hierarchical structure corresponding to the real structure of human communities have been studied [7,13–15]. The most important properties of social networks are a small average shortest path between nodes (individuals) and a high value of the clustering coefficient [5,6], e.g. the probability that a friend of my friend is my friend in the community is high. These properties are typical for the structure of a social network and they have strong influence on the dynamical phenomena in the population e.g. the process of opinion formation.

For modeling of the process of opinion formation, in the case of two possible states (i.e., the positive or negative answer to a certain question), the Ising-based models are used by many authors [16–23]. In the human population the influence of one individual on others depends on their social status-authority. The individual who changes their opinion very often is not trustworthy. Hence, the authority may be connected to the probability of a change in opinion. The opinion of an individual is formed by their interpersonal interactions with other individuals which depends on the structure of the social network. Thus, the structure of a social network has an essential influence on the phenomenon of opinion formation.

In our work we investigate the process of opinion formation in the human population, treated as a scale-free network with nodes (individuals) of different degrees, taking into account their spatial localization and two-level hierarchical structure of interpersonal interactions (a similar model of network was used in Refs. [24,25]).

2. The model

In our model the population and its structure are described as follows: the population consists of $N = L \times L$ individuals S_{ij} with two permitted states meaning the opinion in a certain question: spin up ($S_{ij} = +1$) or spin down ($S_{ij} = -1$). Indices i,j in the two-dimensional lattice show the localization of an individual. To describe the social structure of a human population we take into account the location of each individual and the network of their interpersonal interactions. These interactions, i.e., connections and random contacts, have a hierarchical structure. The connections of each individual with k neighbors is the first level of interpersonal interactions (see Fig. 1a). All connections are symmetrical and have the same value. We have assumed that the network of the social connections is scale-free, i.e., the

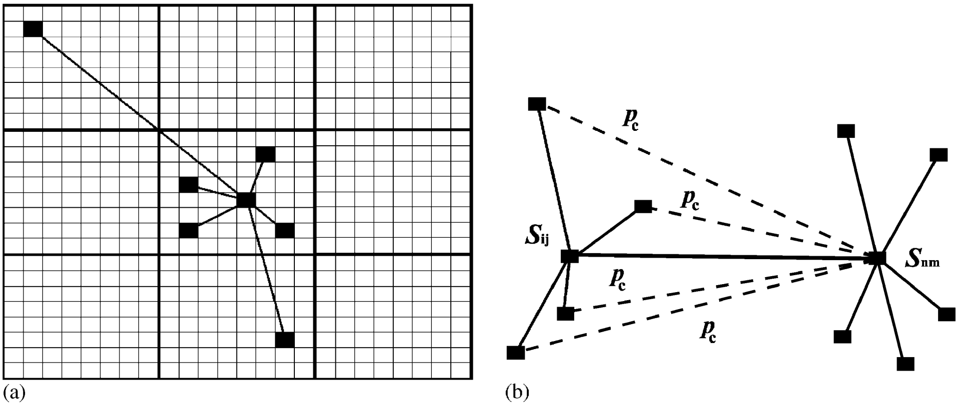

Fig. 1. An example of the network with $L = 24$ and $L_G = 8$ (nine local groups) from the point of view of the $S_{13,13}$ individual, which was connected with $k_{13,13} = 6$ neighbors and four of these connections are located in the local group (a). When the connection between two individuals S_{ij} and S_{nm} is created (solid line), the individual S_{nm} is connected with the neighbors of the individual S_{ij}. Each new connection (dashed line) is created with the probability p_C. Next, the connections between the neighbors S_{nm} and S_{ij} are created (this is not shown in the figure) (b).

distribution of connectivity of individuals has the form $P(k) \sim k^{-\gamma}$ ($\gamma = 3$ was used in most of the computation), with k generated from the range $(k_{\min}, k_{\max})$. Initially, all individuals are not connected. Next, connections between individuals are created with the probability $P(l)$, depending on the distance l between individuals S_{ij} and S_{nm}, where $n = i \pm l_1$; $m = j \pm l_2$ (l_1, l_2 are two independent random variables and a sign is generated with the probability 0.5):

$$P(l) \sim \frac{1}{1 + \exp[(l - a)/b]} + 0.001 \frac{L - l}{L} \ . \tag{1}$$

The second term in the Eq. (1) causes $P(l)$ to reach zero slowly enough. The whole population is divided into local groups of $N_G = L_G \times L_G$ individuals, where the size of these groups is connected by the parameters $a = L_G$ and $b = L_G/4$ of distribution (1). Thus, most connections are created between individuals located in the same local group. The structure of the network from the point of view of a certain individual is depicted in Fig. 1a. Having created the connection between S_{ij} and S_{nm}, the connections between the individual S_{nm} and each neighbors of the individual S_{ij} are created with probabilities p_C (Fig. 1b). Similarly, new connections between S_{ij} and neighbors of the individual S_{nm} are created, also with probabilities p_C. However, each pair of individuals can be connected only once, and a new connection is added to the ijth individual only when its actual number of connections is smaller than the value k_{ij} (where $i,j = 1, 2, \ldots, L$) generated with the distribution $P(k) \sim k^{-\gamma}$. In this way a desirable distribution of connectivity is obtained. It should be noted that in this procedure the value p_C influences the clustering coefficient C of the network [4,26]

$$C = \left\langle \frac{2E_{ij}}{k_{ij}(k_{ij}-1)} \right\rangle, \tag{2}$$

where E_{ij} is the number of connections between neighbors of the ijth individual and $\langle ... \rangle$ means averaging over all individuals.

The influence of parameters L_G, p_C, γ on the statistical properties of a network, with fixed values $k_{min} = 10$ and $k_{max} = 100$, is shown in Fig. 2. The distribution of connectivity $P(k)$ depends only on parameter γ (Fig. 2a), because in the process of creating a network connections are added to a node until its current number of links is smaller than value k_{ij} and k_{ij} is drawn from distribution $\sim k^{-\gamma}$. It is interesting to calculate the relation between the clustering coefficient of a node and its number of connections $C(k)$. As it is shown in Fig. 2b, the greater the p_C, the smaller the influence of parameter L_G on the $C(k)$ relation, and significant changes are observable only for very small values of L_G. On the one hand, for small values of p_C the clustering coefficient of a node increases with its connectivity (see the inset in Fig. 2b). On the other, for large enough p_C power-law $C(k) \sim k^{-\alpha}$, with $\alpha \approx 0.87$ for $p_C = 0.5$ and $\alpha \approx 0.96$ for $p_C = 1$, is visible. This indicates that for certain values of p_C there is a phase transition to the network described by a power-law. Such a power-law is observed in some real networks (e.g. actor network with $\alpha \approx 1$) [13,27] and reveals the hierarchical structure of a network generated by our model [13]. This

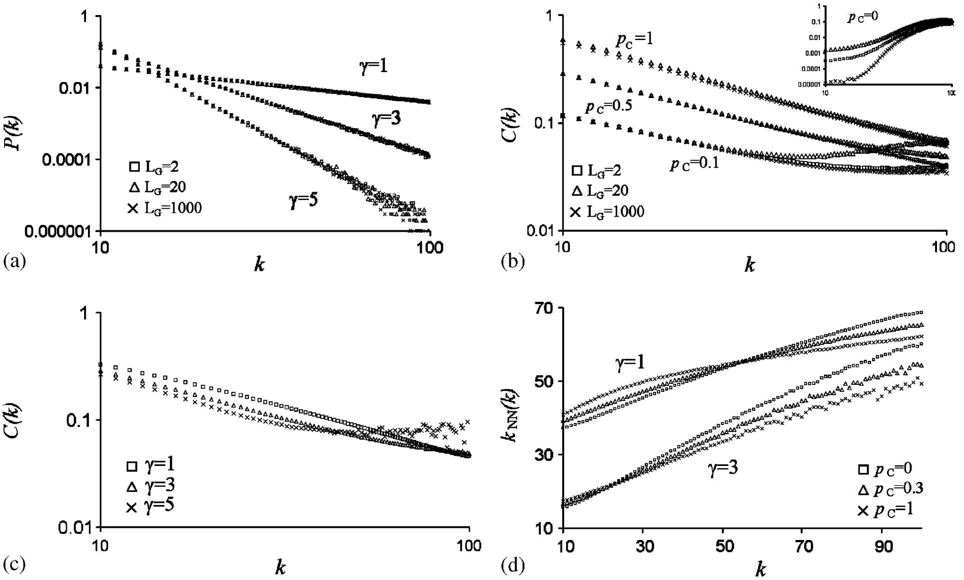

Fig. 2. Properties of a network: (a) distribution of connectivity; (b),(c) relation between the clustering coefficient of a node and its number of connections k; (d) average connectivity k_{NN} of nearest neighbors of a node of a function of their number of connections k. Values of other parameters are $L = 1000$; $k_{min} = 10$; $k_{max} = 100$; $p_C = 0.5$; and $L_G = 20$.

is so because in the procedure of network creation individuals with small values of k quickly obtain all connections. Next, individuals with larger k cannot create connections with individuals with smaller k—the individuals with high connectivity (hubs) have to create connections between themselves, creating in this way connections between groups (highly interconnected for large enough p_C) of individuals with small k. Thus, small groups are organized into increasingly larger groups in a hierarchical manner. Fig. 2c illustrates that exponent α is slightly influenced by parameter γ. Note that, for $\gamma = 5$ there are only a few individuals with very large connectivity. The average connectivity of nearest neighbors k_{NN} of a node with k connections is shown in Fig. 2d. It can be seen that the greater the k, the greater the k_{NN} (the value of k_{NN} does not depend on the value of L_G). Hence, a network generated by our model is assortative mixed by degree and such a correlation is observed in many real social networks [28].

Let us describe the effect of two levels of social structure on the phenomenon of opinion formation. Each individual is influenced by the local field h_{ij}, which depends on interactions with k_{ij} neighbors, effect of the local group and external stimulation I_E:

$$h_{ij}(t+1) = \frac{1}{k_{ij}}\left(\sum_{n=1}^{k_{ij}} A_{l(n)m(n)} S_{l(n)m(n)}(t) + \frac{1}{N_G}\sum_{n=1}^{N_G} A_{l(n)m(n)} S_{l(n)m(n)}(t)\right) + I_E\,. \qquad (3)$$

As we can see the most effective way of formation of an opinion of an individual are its interpersonal interactions with k_{ij} neighbors (the first term in Eq. (3)), while the influence of their local group is less effective (the second term in Eq. (3)). The parameter $A_{ij} \in (0, 1)$ is the authority of the ijth individual and its value is generated with a Gaussian distribution (with zero mean value and the variance σ). The relation between the fraction of individuals with $A_{ij} > 0.5$ and σ is shown in Fig. 3. This parameter describes the influence of an individual on other individuals. With increasing A_{ij} the importance of the opinion of the ijth individual in the community increases. The probability p_{ij} that an individual changes their state depends on the

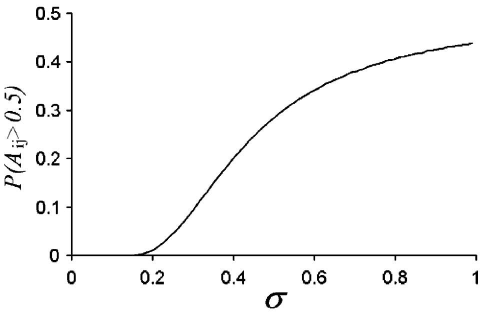

Fig. 3. The probability that an individual has authority $A_{ij} > 0.5$ vs. value of the parameter σ.

value of authority and local field:

$$p_{ij} = \begin{cases} (1 - A_{ij})(1 - \exp[h_{ij}S_{ij}/T]); & h_{ij}S_{ij} \leqslant 0 , \\ (1 - A_{ij})\exp[-h_{ij}S_{ij}/T]; & h_{ij}S_{ij} > 0 , \end{cases} \tag{4}$$

where T is the temperature—statistical parameter of the population. The probability that the ijth individual does not change their state increases with the value of A_{ij}. The greater the h_{ij}, the greater the probability that an individual has the state conforming with the local field. On the other hand, with an increase in temperature T there is an increase in the probability that the individual will have a state opposite to the local field h_{ij}. From Eq. (4) it is obtained that in $T = 0$ the probability of changing of the state is:

$$p_{ij} = \begin{cases} (1 - A_{ij}); & h_{ij}S_{ij} < 0 , \\ 0; & h_{ij}S_{ij} \geqslant 0 . \end{cases} \tag{5}$$

Hence, when the state of an individual agrees with the local field ($h_{ij}S_{ij} > 0$) the probability that the individual changes their state equals zero.

3. Results

Computations were performed for the initial conditions corresponding to a paramagnetic phase ($\langle S \rangle \equiv N^{-1}\sum_{i,j}^{L} S_{ij} = 0$) using synchronous dynamics.

In our model it is possible to investigate the influence of the clustering coefficient C on the value $\langle S \rangle$. For a small value of C, in most cases, the system settles into a ferromagnetic state ($\langle S \rangle = 1$). However, with increasing value of clustering coefficient $\langle S \rangle$ decreases (Fig. 4). For large C the population is divided into clusters (domains) of highly connected individuals with the same state. The individuals from different domains weakly interact among themselves. Therefore, the states of

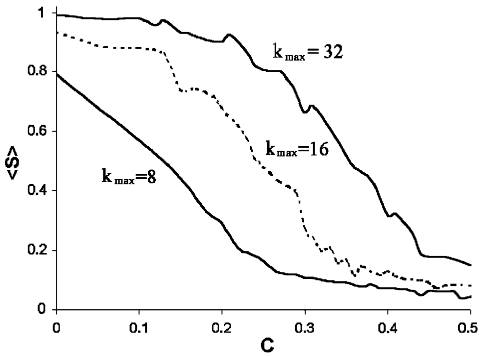

Fig. 4. Relation between $\langle S \rangle$ and clustering coefficient C for different values of k_{max} and $T = 0$. Values of other parameters are $L = 100$; $L_G = 20$; $k_{min} = 8$; $\sigma = 0.3$; and $I_E = 0$. Results are averaged over 100 independent simulations. Number of time steps of each simulation is 1000.

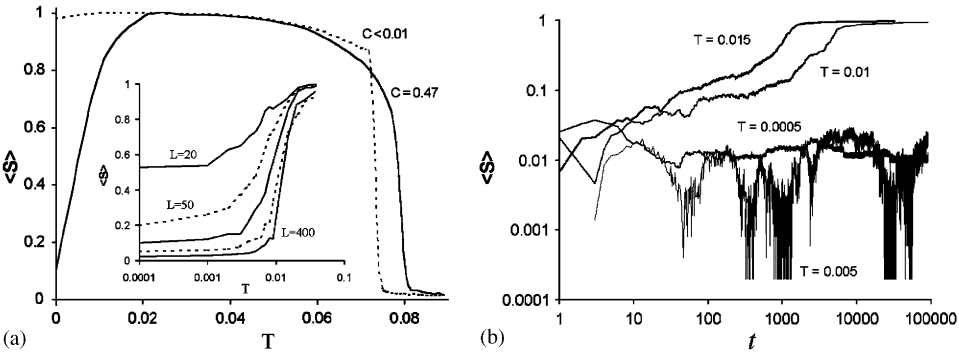

Fig. 5. Relation between $\langle S \rangle$ and temperature T for different values of clustering coefficient $C = 0.47$ (solid line) and $C < 0.01$ (dashed line). For the case of $C = 0.47$ and small temperatures the system remains in its initial (paramagnetic) state. However, for large enough T a ferromagnetic state occurs in which $\langle S \rangle$ reaches 1 (*freezing by heating*). For some critical temperature $T = T_C$ (depending on the value of the clustering coefficient) a phase transition occurs and a paramagnetic phase appears. In the inset the influence of the size of the network for $C \approx 0.5$ is shown ($L = 20; 50; 100; 200; 400$ from the top to the bottom respectively) (a). Changes of $\langle S \rangle$ over time t for different values of temperature T (b). Values of other parameters are $L = 100$; $L_G = 20$; $k_{min} = 8$; $k_{max} = 24$; $\sigma = 0.3$; $I_E = 0$. Results are averaged over 100 independent simulations. Number of time steps of each simulation is 2000.

domains are different and the system does not settle into ferromagnetic state. Significant importance the maximal number of connections k_{max}. With an increase in k_{max}, there is an increase in the probability that the system reaches a ferromagnetic state, even for a large value of C. Individuals with a large number of connections play the role of mediators between different domains and their presence leads to conforming of opinions of individuals belonging to different domains. For a number of connections large enough the ferromagnetic state occurs for all values of C.

It is interesting to investigate the influence of temperature T on the system for different values of C (Fig. 5a). For small temperatures and large values of a clustering coefficient the system remains in its initial, i.e., paramagnetic, state. However, an increase in T results in a change of the states of domains, in which individuals have minority opinion. There exists an interesting phenomenon called *freezing by heating* in which the system reaches a ferromagnetic state for large enough temperature. For further increase in the value of temperature the state of the system does not change until T reaches its critical value T_C, when a phase transition occurs. In $T = T_C$ the probability that an individual has a state opposite to the local field is large and the system is in a paramagnetic state.

With an increase in the value of the clustering coefficient C there is an increase in T_C and the changes of $\langle S \rangle$ as a function of T are less abrupt (see Fig. 5a). For large C the change of the state of one individual may lead to a change of the state of the whole domain (especially when this individual has great authority and large k). Therefore, the decreases of $\langle S \rangle$ is better observable for T smaller than T_C. On the other hand, the interactions between domains are weak—the change of the state of one domain slightly influences other domains. The majority of individuals have the same state. For small C the change of state of a small number of individuals may lead to a change of the state of a larger number of individuals exceeding the number of the individuals belonging to one domain. Hence, the critical temperature T_C increases with C. The inset in Fig. 5a illustrates the influence of the size of the network on the *freezing by heating* phenomenon for $C \approx 0.5$. The larger the network, the greater the temperature of transition to a ferromagnetic state. This is a result of a size effect and finite time of simulation (see the relation between $\langle S \rangle$ and time for different values of temperature T in Fig. 5b.). For a larger number of iterations the discrepancies between curves for different L are smaller and the changes in $\langle S \rangle$ are more abrupt.

In ferromagnets exceeding of the critical temperature T_C causes the disordering factor connected with the thermal fluctuations to dominate over the ordering factor, which are exchange interactions (of the quantum mechanical origin) between spins. Similarly in the case of the opinion formation in the community an increase of temperature T above T_C causes disorder trends (which will be described further in the text) to dominate over the influence of other individuals and media on the community. The phenomenon of phase transition in social systems was found by other authors [18,24,25,29–34].

With an increase in the average value of connectivity, there is an increase in the average influence of an individual on the network. Thus change of the state of a small number of individuals can lead to a large change of $\langle S \rangle$. Thus, the greater the value of parameter γ, the greater the temperature of phase transition T_C (Fig. 6a). This is because the number of connections in the network decreases. Note that a similar property occurs in ferromagnets—those with a greater value of exchange constant have a greater temperature of phase transition.

The greater the authority A_{ij}, the smaller the probability that the ijth individual does not obey the local field h_{ij}. Therefore in a population where the number of individuals with large authority is greater (greater σ) the temperature of phase transition T_C is larger (Fig. 6b).

Fig. 6c illustrates the influence of the size of the network, given by L, on the critical value of temperature. It can be seen that with an increase in L there is a decrease the value of T_C and the changes of $\langle S \rangle$ as a function of temperature are more abrupt. This result is quite different from that presented in Ref. [35], where T_C increases logarithmically with the size of the network, as a result of the numerical calculation of the Ising model in a Barabási–Albert scale-free network. Analytical calculations show that in scale-free networks with $\gamma > 5$ (i.e., in infinite networks $\langle k^4 \rangle$ takes on a finite value), the phase transition is similar to that in the Ising model on high-dimensional regular lattices [36]. These results are not fully comparable with the results presented here, because one of the parameters of our model is maximal connectivity k_{max}; this value does not depend on the size of the network (in a BA network k_{max} increases with N). Therefore, $\langle k^4 \rangle$ always takes on a finite value, irrespective γ, even for infinite networks. For initial condition $\langle S \rangle = 0$, the result that T_C decreases with N seems to be plausible in the case of opinion formation: in

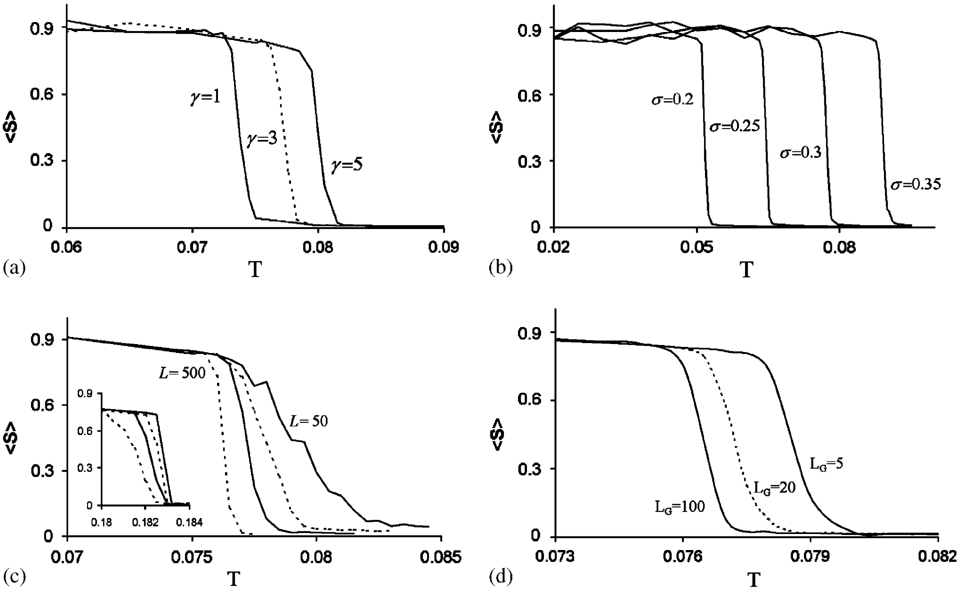

Fig. 6. The relation between $\langle S\rangle$ and value of the temperature for different values of γ and $k_{\max} = 48$ (a). The influences of the parameters σ (for $k_{\max} = 48$), size of the network given by L ($L = 50; 100; 200; 500$ from the right to the left, respectively) and the size of a local group L_G are shown in Figs. b,c,d respectively. The inset in (c) is described in the text. Values of other parameters are $L = 200$; $L_G = 20$; $k_{\min} = 8$; $k_{\max} = 24$; $p_C = 0$; $\sigma = 0.3$; $I_E = 0$. Results are averaged over 100 independent simulations. The number of time steps of each simulation is 1000.

greater communities it is more difficult to obtain a consensus. However, for initial condition $\langle S\rangle = 1$ and $A_{ij} = const$, from the inset in Fig. 6c, where the relation between $\langle S\rangle$ and T for $A_{ij} = 0.5$ and different values of L (50, 100, 200, 400 from the left to the right, respectively) are shown, we obtain results similar to those in Ref. [35,36].

With an increase in the spatial range of interactions (of local and long-range character—as results from Eq. (1)) given by L_G, there is a decrease in the temperature of the phase transition T_C (Fig. 6d). This is so because thermal fluctuation of the state of one individual influences a greater number of individuals when the spatial range of its interactions is greater.

It is interesting to investigate the behavior of the system at temperatures much larger than T_C (Fig. 7a). At very high temperatures the value of local field h_{ij} slightly influences the probability of a change of the state of an individual—the most important factor is the sign of h_{ij}, which has a decisive meaning for the behavior of an individual. It appears that for a specific range of control parameters and $T \gg T_C$ the system orders i.e., the majority of individuals have the same state. However, in every other time step their state becomes opposite i.e., limit cycle of length two is observed. This refers to the majority of individuals and is better observable for large

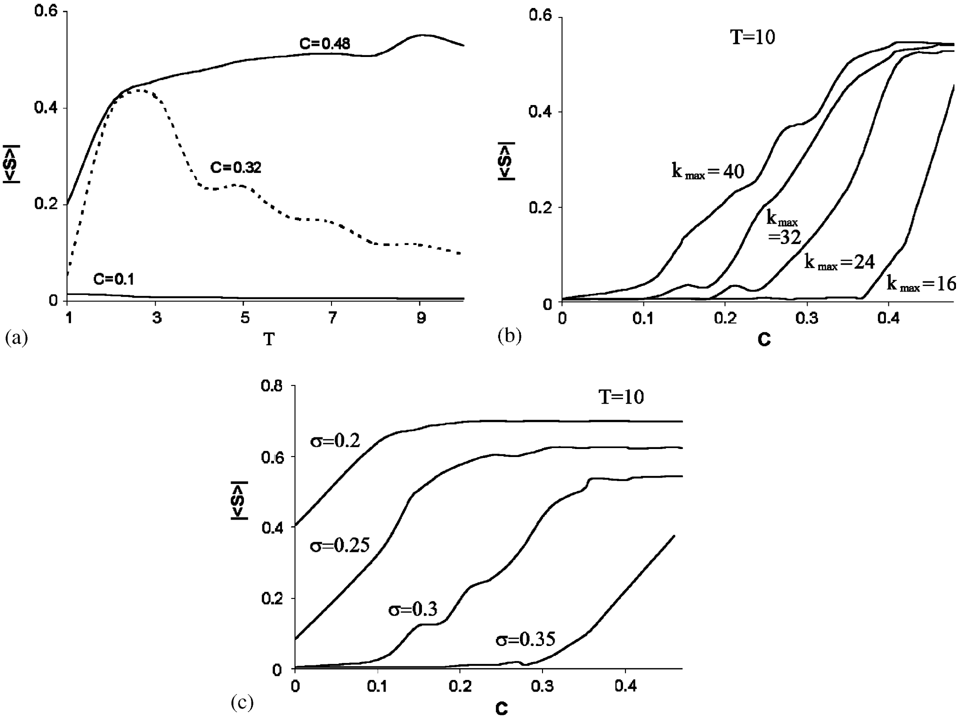

Fig. 7. The behavior of the system at high temperatures. The relation between $|\langle S\rangle|$ and T for different values of clustering coefficient C and $k_{\max} = 24$ is shown in (a). The relations between $|\langle S\rangle|$ and C for different values of $k_{\max}$ and for different values of σ (for $k_{\max} = 40$) are shown in (b) and (c), respectively. Values of other parameters are $L = 100$; $L_G = 20$; $k_{\min} = 8$; $k_{\max} = 24$; $\sigma = 0.3$; $I_E = 0$. Results are averaged over 100 independent simulations. The number of time steps of each simulation is 1000.

values of clustering coefficient C and small values of the parameter σ (see Figs. 7b and c).

The limit cycle of length two described above is similar to the attractor observed in the behavior of the electorate in young democracy, where in each subsequent elections the opposing party wins. With an increase in the value of the clustering coefficient there is an increase in the number of hermetic domains in which individuals oscillate in the same way. For small values of $k_{\max}$ different domains interact among themselves very weakly and their oscillations do not synchronize—the mean value of $|\langle S\rangle|$ is small (Fig. 7b). With an increase in $k_{\max}$ there is an increase in the number of individuals belonging to many different domains. This results in synchronization of oscillations in different domains and $|\langle S\rangle|$ increases with $k_{\max}$. Similarly in $T = 0$ the presence of individuals with high connectivity leads to system orders (see Fig. 4). With a decrease of the value of the parameter σ there is an increase in the probability that an individual will have a state opposite to the local field (Fig. 7c). As a consequence the number of individuals oscillating with the period two and the probability of synchronization of oscillations of different individuals are greater.

4. Influence of external stimulation

The presence of external stimulation can simulate the influence of mass media on the phenomenon of opinion formation in the community. In this section we investigate the possibility of change in the states of individuals due to external stimulation I_E, i.e., we assume that initially all individuals have states opposite to external stimulation.

Fig. 8 illustrates the relation between the number of individuals with the states conforming with external stimulation (denotes by $\langle S+\rangle$) as a function of I_E. It can be noticed that for I_E exceeding the critical value I_{EC} all individuals have state $S_{ij} = +1$. This means that a certain intensity of the influence of mass media can change the opinion of the community to the state constrained by the media. The value of I_{EC} increases approximately linearly with σ. The greater the average authority in the community (this may be connected e.g. to better education) the more difficult it is to manipulate the community.

With an increase in L_G and k_{max} there is a small increase in the critical value I_{EC}. Hence, it is easier to manipulate the community in which the structure of interpersonal interactions is less complex (i.e., when the number of connections and average path length are smaller and interactions have a more local character). Fig. 9 illustrates the influence of temperature T. At small values of temperature the opinion of most individuals is still opposite to the state constrained by the mass media ($\langle S+\rangle \approx 0$). For T large enough the states of all individuals conform with external stimulation and $\langle S+\rangle \approx 1$. As can be seen from Fig. 9, with an increase in T, there is a decrease in the critical value I_{EC}. This can be interpreted as follows: it is easier to manipulate a community at higher temperatures (e.g. when there exists high

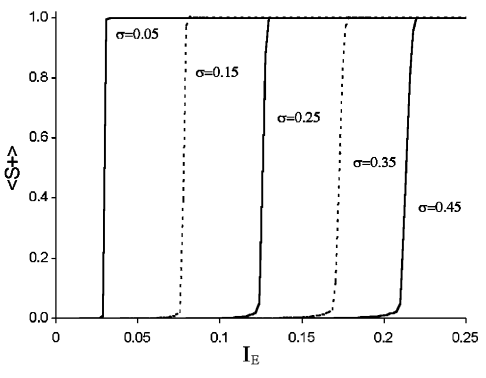

Fig. 8. The relation between $\langle S+\rangle$ and I_E for different values of σ. Values of other parameters are $L = 100$; $L_G = 20$; $k_{min} = 8$; $k_{max} = 24$; $p_C = 0$; $T = 0$. Results are averaged over 100 independent simulations. The number of time steps of each simulation is 1000.

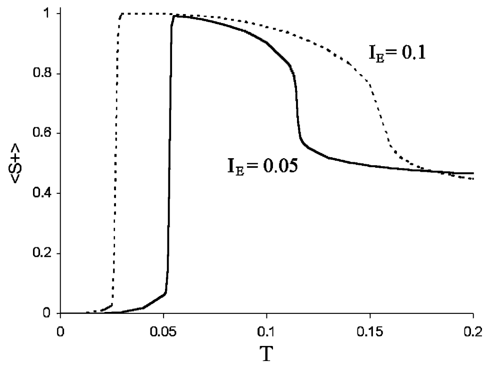

Fig. 9. The relation between $\langle S+\rangle$ and temperature T for $I_E] = 0.05$ (solid line) and $I_E = 0.1$ (dashed line). Note that $I_{EC}(T \approx 0.025) = 0.1$ and $I_{EC}(T \approx 0.05) = 0.05$ which means that with an increase of T, there is a decrease of the critical value I_{EC}. Values of other parameters are $L = 100$; $L_G = 20$; $k_{min} = 8$; $k_{max} = 24$; $\sigma = 0.3$; $p_C = 0$. Results are averaged over 100 independent simulations. The number of time steps of each simulation is 1000.

unemployment, crime wave, bad economic situations and people are dissatisfied). In these conditions small populist parties can take control of the community (like in Europe before the Second World War). With further increase in temperature a phase transition is observed (cf. Fig. 6), the temperature of phase transition T_C increases with I_E and the changes in $\langle S+\rangle$ are less abrupt for greater I_E. It can be seen that at very high temperatures the system reaches a paramagnetic state, because if $\langle S+\rangle = 0.5$ then 50% of individuals have an opinion conforming with the mass media and 50% of individuals have an opposite opinion ($\langle S\rangle = 0$).

As can be seen from Fig. 9 two phenomena of a type of phase transition are observed in the population. The first one is connected to exceeding of I_{EC} by the intensity of influence of mass media. The second one is connected to exceeding of T_C by the statistical temperature of the population.

5. Conclusions

In this paper we have studied the process of opinion formation in the human population, described as a scale-free network. We have taken into account hierarchical, two-level structures of interpersonal interactions, as well as a spatial localization of individuals. The network of interpersonal interaction used in our model also has other properties of real social networks, like a small average shortest-path, a large clustering, a power-law relation between clustering of a node and its connectivity, and assortative mixing.

It was found that at zero statistical temperature ($T = 0$), for a large value of clustering coefficient C, the system remains in its initial, paramagnetic state i.e., there is no dominant opinion in the community. However, with an increase in T a transition to a ferromagnetic state is observed—most of the population accepts the same opinion. This phenomenon is called *freezing by heating*.

When the temperature exceeds a certain critical value T_C there is an abrupt disappearance of dominant opinion in the community. This is a phenomenon of a type of phase transition observed in many other physical systems.

An other interesting behavior of the system is observed at very high temperatures, when there occurs an ordering of the opinion of most individuals and the opinion switches between two opposite states. This means that the system reaches a limit cycle of length two.

We have investigated the effect of the mass media on the process of opinion formation, modeled as an external stimulation acting on the social network. From our computations in the time evolution of opinions of individuals another critical phenomenon occurs. It depends on the statistical temperature of the system and are connected to the situation in the community, which may be described by such quantifiers as the economic status of the people, unemployment or crime wave. This means that the influence of the mass media, in certain circumstances, can provoke critical rebuilding of opinions in the population. It was found that it is easier to manipulate the community in which the structure of interpersonal interactions is less complex (i.e., when the number of connections and the average path length are smaller and interactions have a more local character).

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