Analysis of Fokker-Planck Approach for Foreign Exchange Market Statistics Study

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Stochastic methods are powerful means for foreign exchange markets analysis. Typically the differences in exchange rates with fixed time delays are investigated. These data can be regarded as a stochastic process. It was shown in [1] that empiric data of U.S. dollar – German mark exchange rates upon different delay times can be regarded as a stochastic Marcovian process. Further it was shown in the same paper that Kramers-Moyal coefficients can be estimated from the empirical data and finally, an explicit Fokker-Planck equation, which models the empirical probability distributions, for reverse logarithmic time can be obtained:

$$\frac{d}{d\tau} p(\Delta x, \tau) = \left[-\frac{\partial}{\partial \Delta x} D^{(1)}(\Delta x, \tau) + \frac{\partial^2}{\partial \Delta x^2} D^{(2)}(\Delta x, \tau) \right] p(\Delta x, \tau) \qquad (1)$$

Here $\tau = \ln(40\,960\,\mathrm{s} / \Delta \mathrm{t})$ where $\Delta t = t_2 - t_1$ is time difference and $\Delta x = x(t_2) - x(t_1)$ is the difference in exchange rate. Coefficients $D(\Delta x, \tau)$ are Kramers-Moyal coefficients.

In paper [1] numerical investigation of the obtained Fokker-Planck equation (1) was performed, but numerically-analytical study of the equation properties seems to be quite important. Equation (1) can be transformed into conservative form, which allows easy stationary solution obtaining. This solution has a singularity, and is equal to $\delta(\Delta x - \alpha)$, where $\delta(x)$ - Dirack function, α is determined from the Kramers-Moyal coefficients shape. This is easily explicable, as in case of reverse time, stationary solution corresponds to the distribution in the past, when no uncertainty existed (as the coefficients of the equation are obtained using empirical data for the past).

Analytical solution allows verification of numerical results, obtained in [1]. The authors state that according to the numerical results the distribution is not Gaussian, as it has heavy tails. It can be shown analytically that the distribution in any vicinity of the infinity is not Gaussian, but is described with a polynomial distribution function.

Finally the study of the equation (1) shows that problem for normal time is ill-posed as from mathematical point of view it is equivalent to thermal conductivity problem with reverse time.

We have verified our analytical investigations numerically and obtained good agreement.

References:

1. R. Friedrich, J. Peinke and Ch. Renner "How to Quantify Deterministic and Random Influences on the Statistics of the Foreign Exchange Market" J. Physical Review Letters volume 84, number 22. pp. 5224-5227